Karlsruher Institut für Technologie

BELLE

Belle II

# Experimental overview: $|V_{cb}|$ & $|V_{ub}|$ at Belle/Belle II

Raynette van Tonder

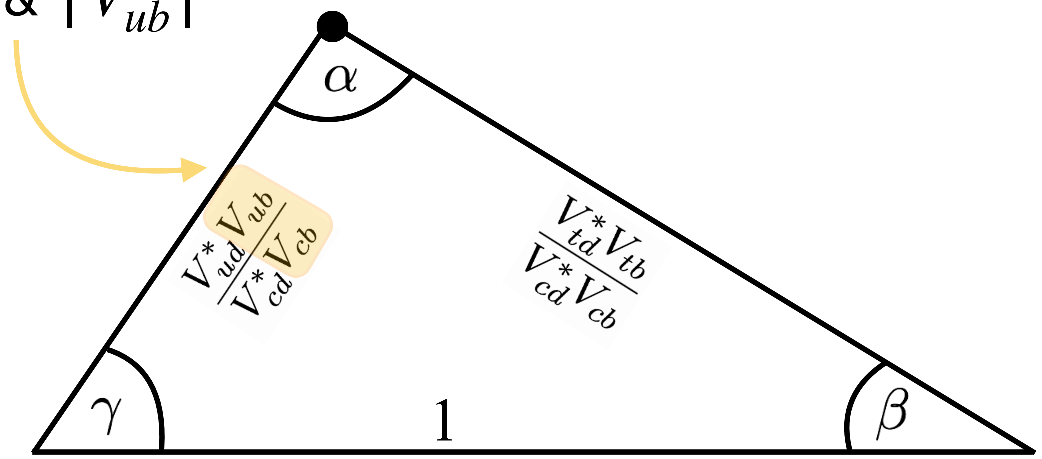
Lattice meets Continuum 2024

Walter Benjamin-Programm

DFG Deutsche Forschungsgemeinschaft

# Semileptonic decays

- SL $B$ decays ideal to extract CKM Matrix elements $|V_{cb}|$ & $|V_{ub}|$
- $|V_{qb}|$ limiting the constraining power of global fits.
- Important inputs to predictions of SM rates for ultra-rare decays.
- Significant tension between inclusive & exclusive determinations poses a longstanding puzzle.




**Exclusive $|V_{ub}|$**

$\bar{B} \to \pi \ell \bar{\nu}_\ell$

**Exclusive $|V_{cb}|$**

$\bar{B} \to D \ell \bar{\nu}_\ell, \bar{B} \to D^* \ell \bar{\nu}_\ell$

$$\mathcal{B} \propto |V_{qb}|^2 f^2$$ ← Form Factors

**Inclusive $|V_{ub}|$**

$\bar{B} \to X_u \ell \bar{\nu}_\ell$

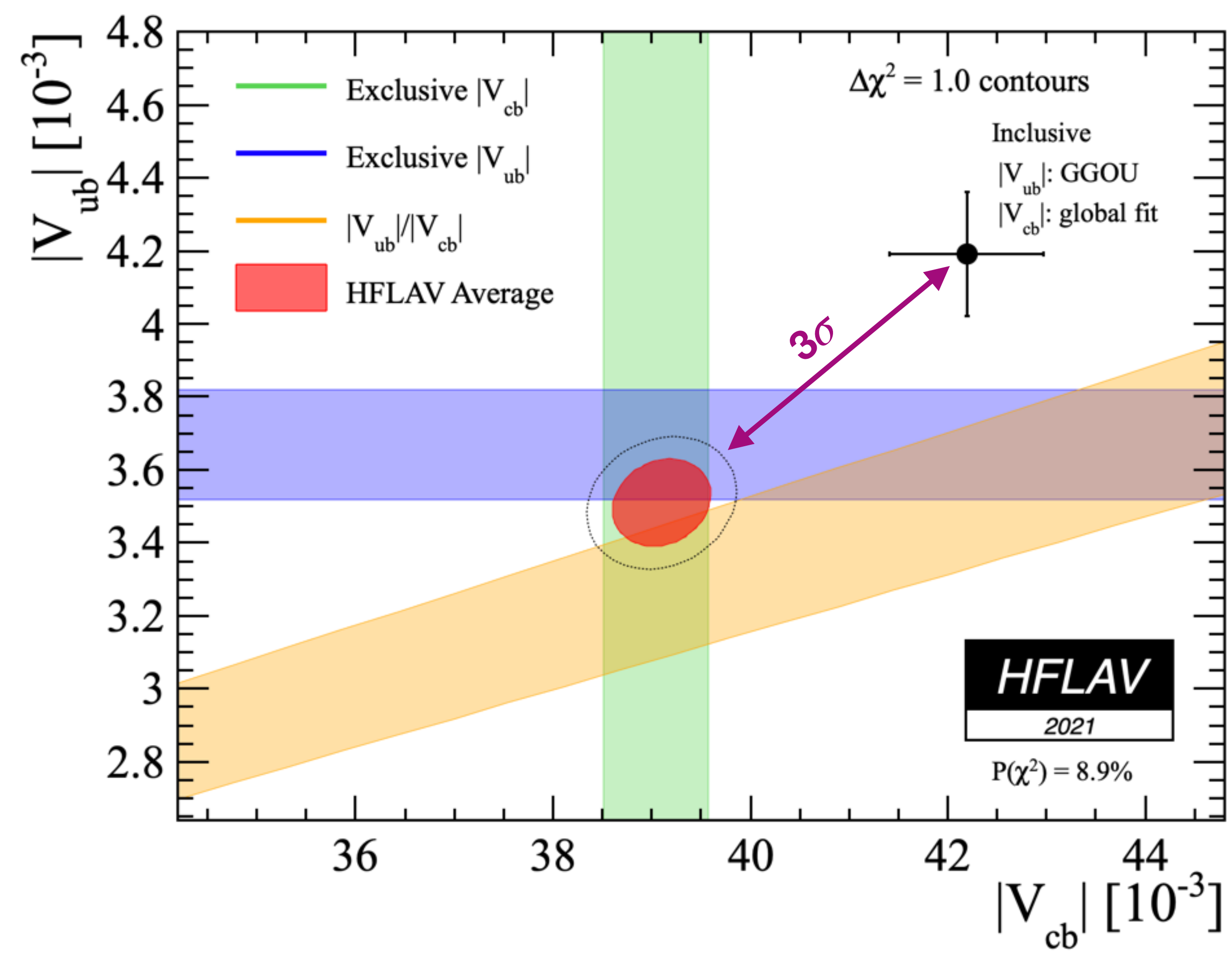
**Inclusive $|V_{cb}|$**

$\bar{B} \to X_c \ell \bar{\nu}_\ell$

Heavy Quark Expansion

$$\mathcal{B} = |V_{qb}|^2 \left[ \Gamma(b \to q \, \ell \, \bar{\nu}_\ell) + 1/m_{c,b} + \alpha_s + \ldots \right]$$

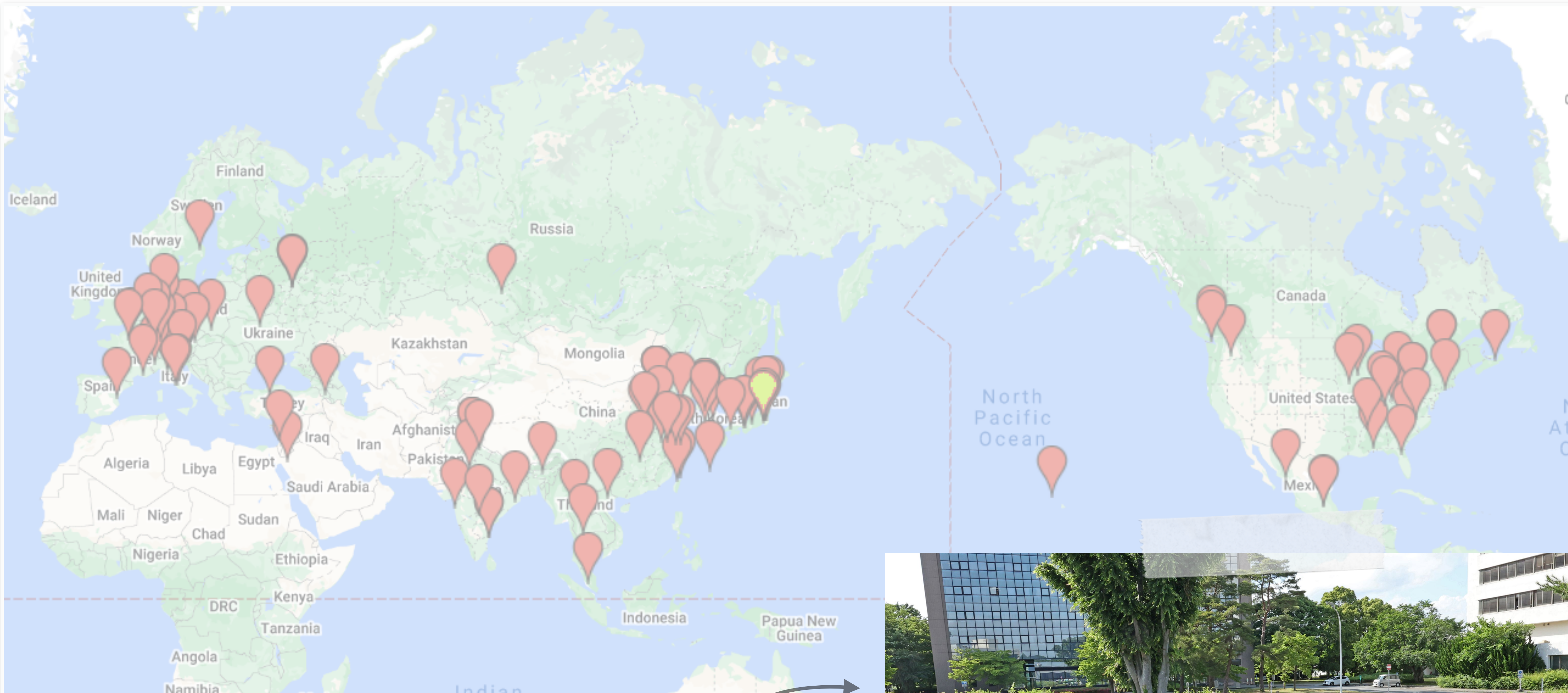
**A longstanding tension…**



# Meet the people!

Collaboration map

- The Belle II Collaboration comprises 1188 researchers from 125 institutes in 29 countries!




48th Belle II General Meeting
3-7 June 2024, KEK

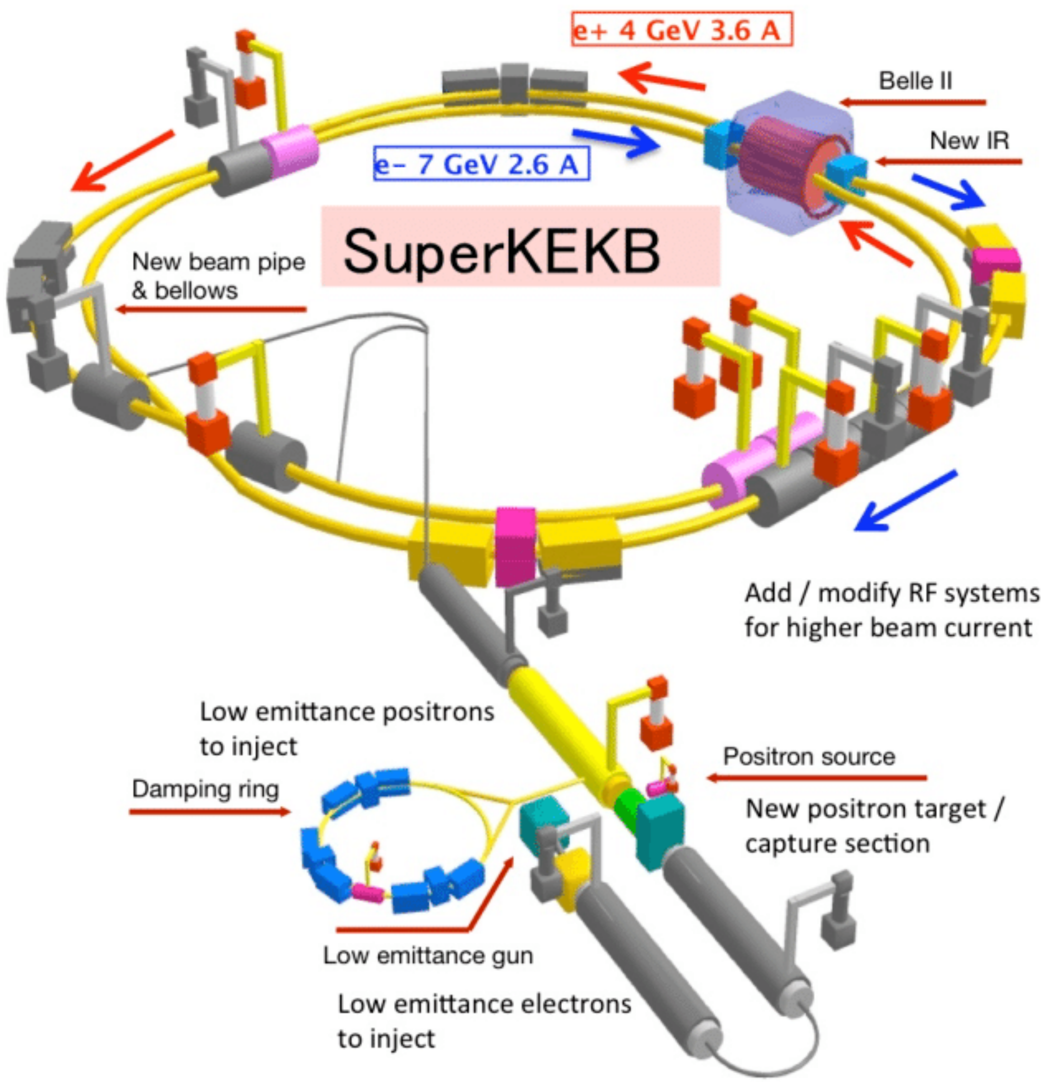
# SuperKEKB in a nutshell

$$\mathscr{L}_{\text{Belle}} = 2.11 \times 10^{34}\,\text{cm}^{-2}\,\text{s}^{-1}$$

x30!

Goal: Achieve instantaneous luminosity of $\mathscr{L}_{\text{Belle II}} = 6 \times 10^{35}\,\text{cm}^{-2}\,\text{s}^{-1}$

with record $4.7 \times 10^{34}\ \text{cm}^{-2}\text{s}^{-1}$ already achieved!



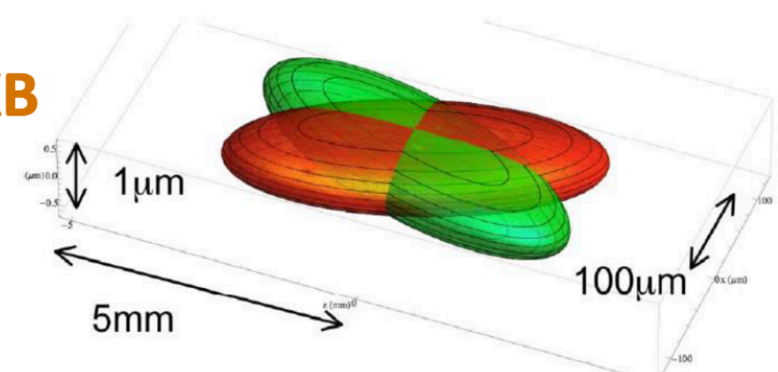

How to increase luminosity:

$$L = \frac{\gamma_\pm}{2 e r_e}\left(1 + \frac{\sigma_y^*}{\sigma_x^*}\right)\left(\frac{I_\pm \zeta_{\pm y}}{\beta_y^*}\right)\left(\frac{R_L}{R_y}\right)$$

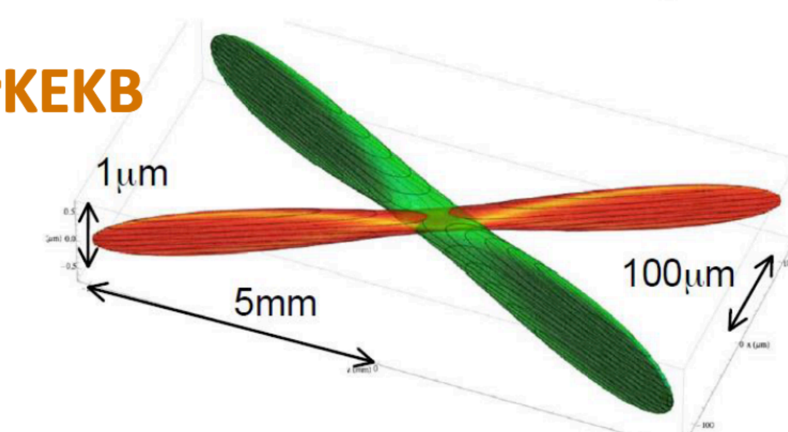
- Lorentz factor
- Beam current **x 1.5**
- Beam-beam parameter
- Beam size
- Vertical **β** function **x 1/20**
- Geometric factors

**Nano-beam scheme**: Squeeze vertical beam spot size down to ≈ 50 nm using superconducting focusing magnets.

KEKB

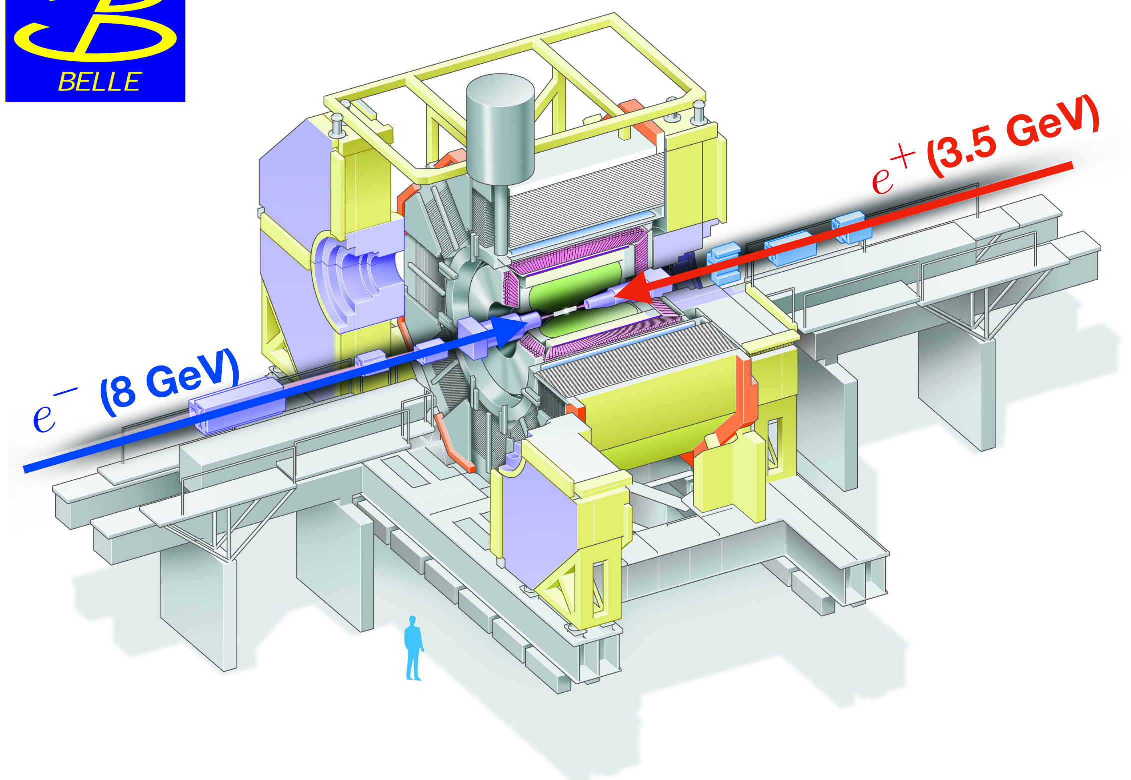



SuperKEKB

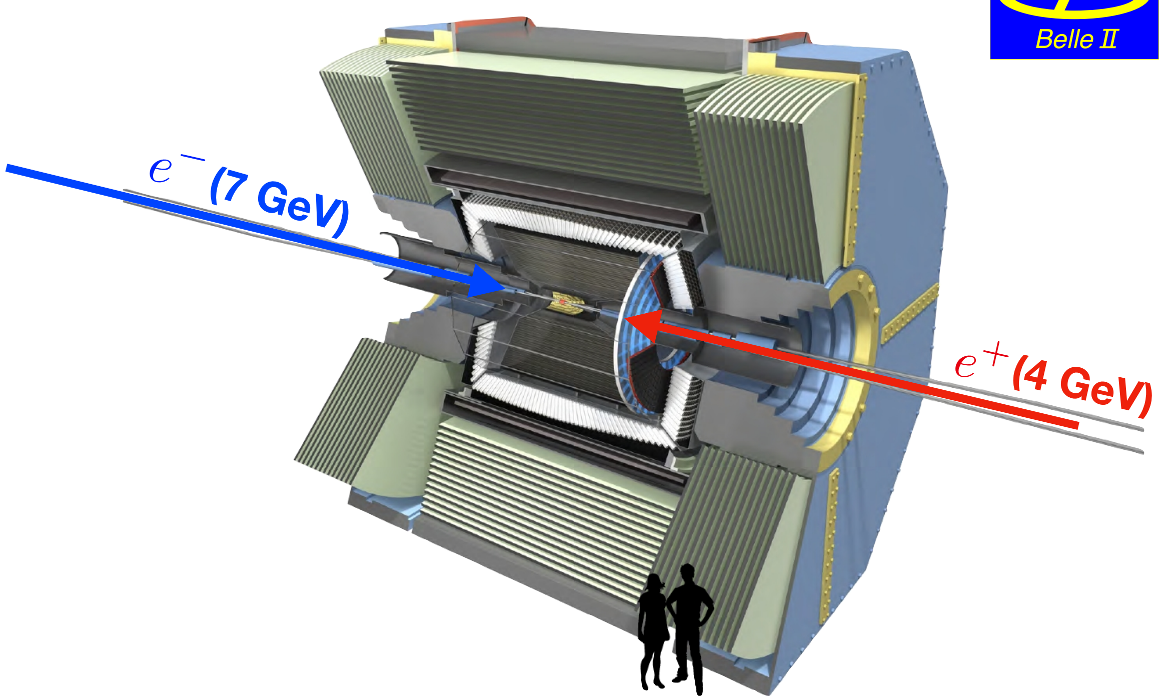



Mt. Tsukuba
Belle (II)
Detector
SuperKEKB
(HER + LER)
Damping ring
$(e^+)$
Linac
KEK Tsukuba
Campus
Japan
Belle II
Tsukuba
/ KEK
Tokio

# Belle & Belle II Detectors



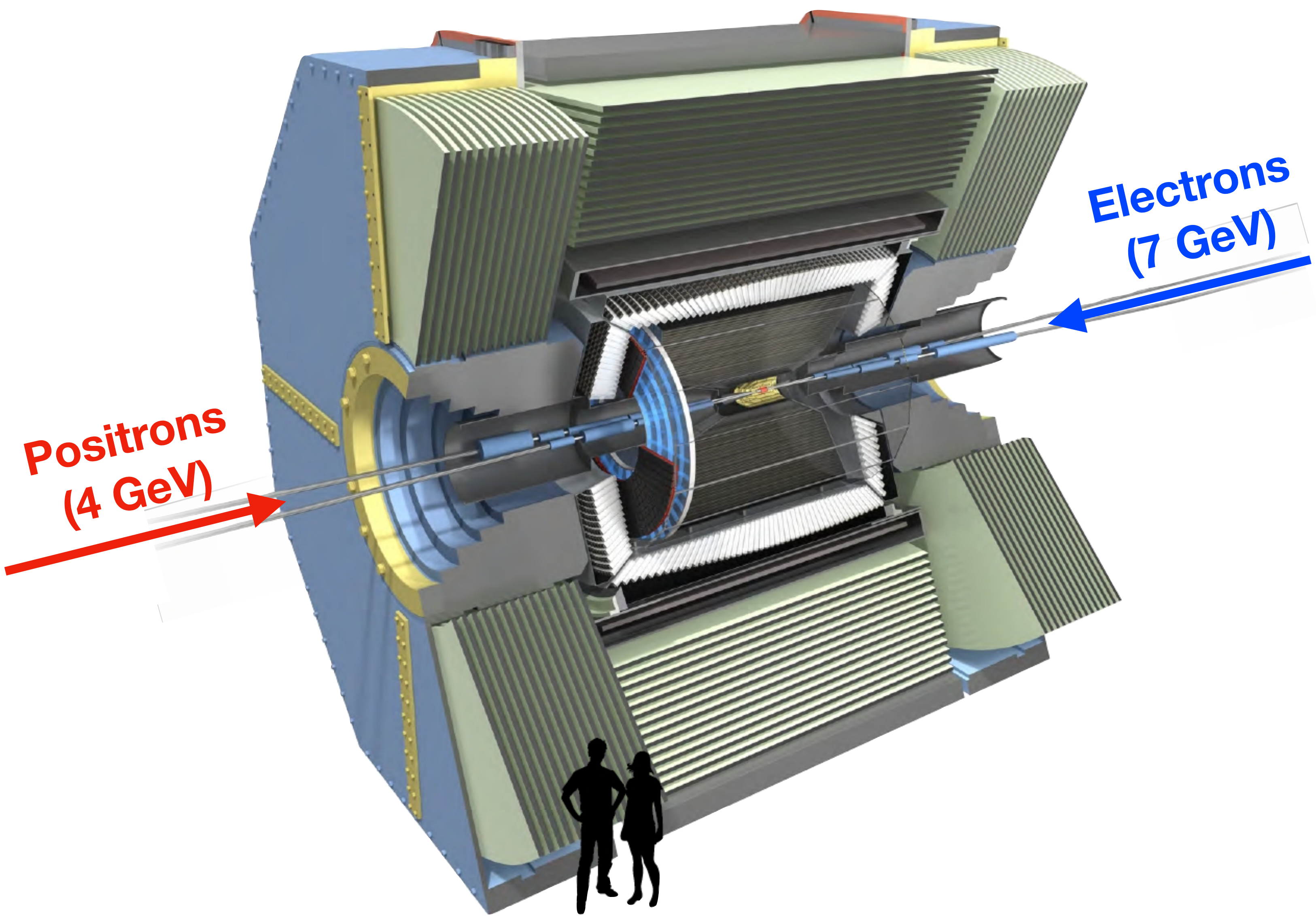

- Operated from 1999 to 2010.
- Asymmetric $e^+$ (3.5 GeV) - $e^-$ (8 GeV) collider.
- Collected total of 1 ab$^{-1}$ of data.
- Collected 711 fb$^{-1}$ at $\Upsilon(4S)$ resonance.




- Asymmetric $e^+$ (4 GeV) - $e^-$ (7 GeV) collider.
- Recorded 531 fb$^{-1}$ of data: equivalent to BaBar and 1/2 of Belle dataset.
- Run I data at $\Upsilon(4S)$ resonance: 365 fb$^{-1}$
- Run II started February 2024.
- **Aims to collect many-ab$^{-1}$ of data!**

# B-meson production at B-Factories

- An ideal laboratory to study rare decays or decays with missing energy



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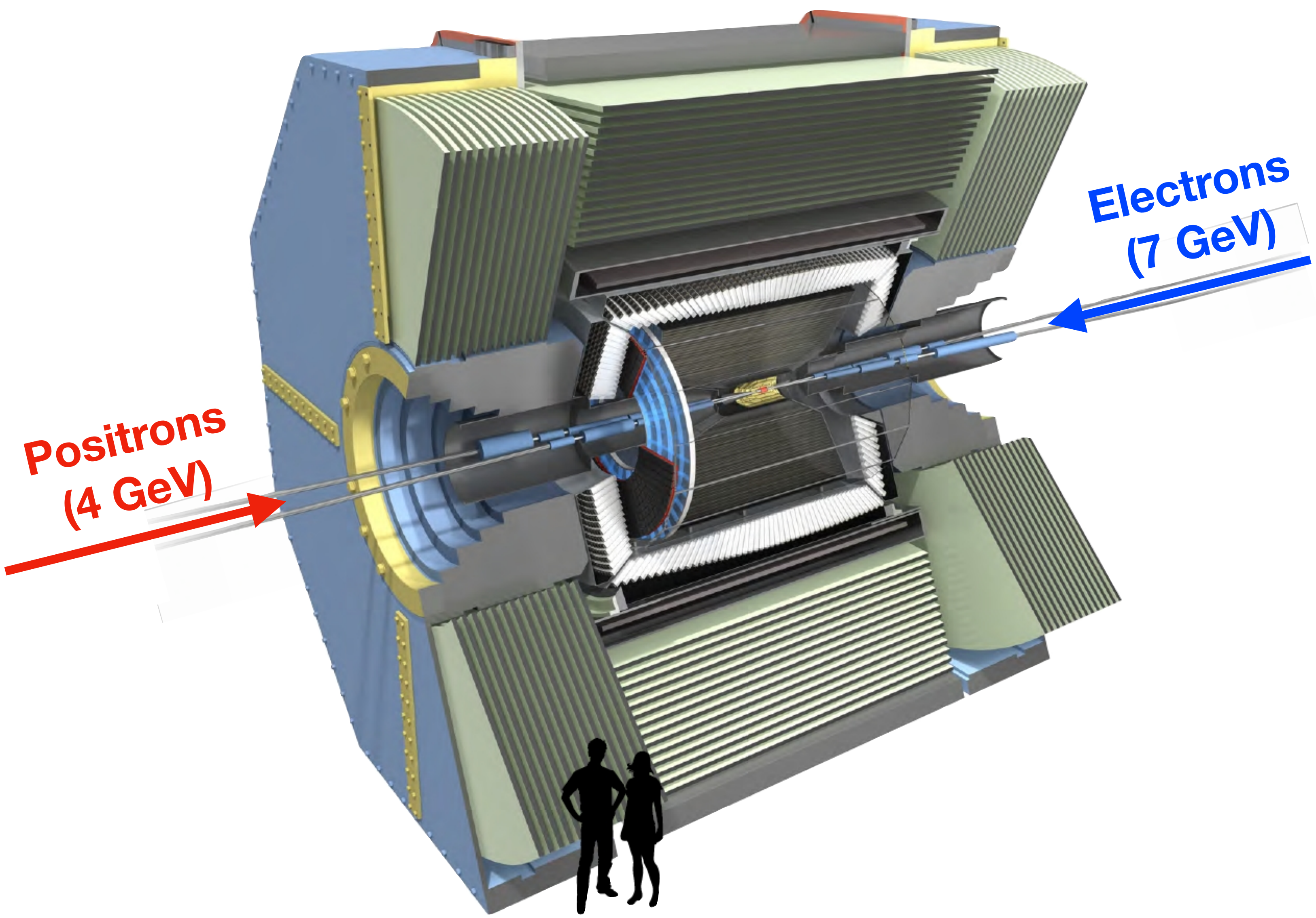


**Collide** electrons and positrons at a **centre of mass energy** of about twice the B meson mass:

$$\sqrt{s} = 10.58 \text{ GeV}$$

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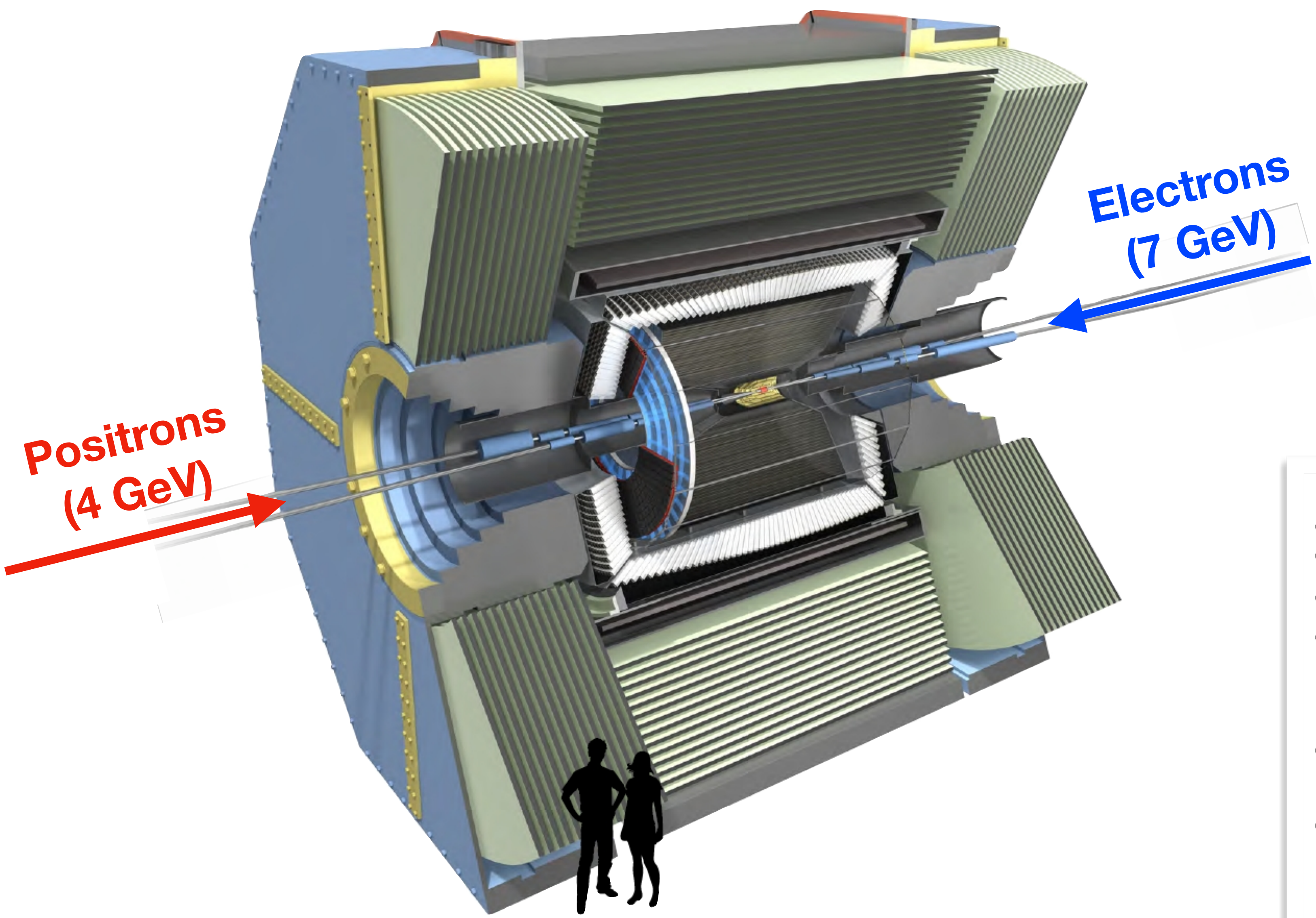


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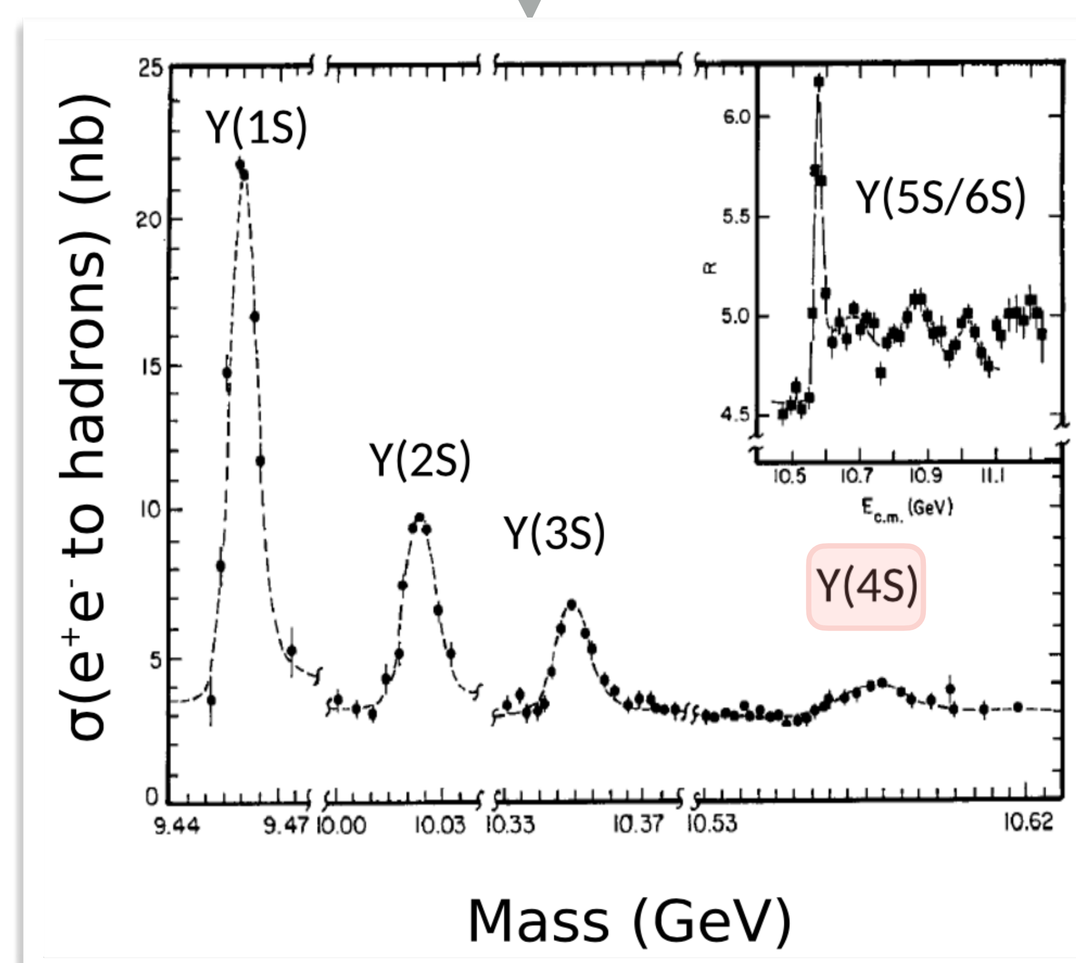

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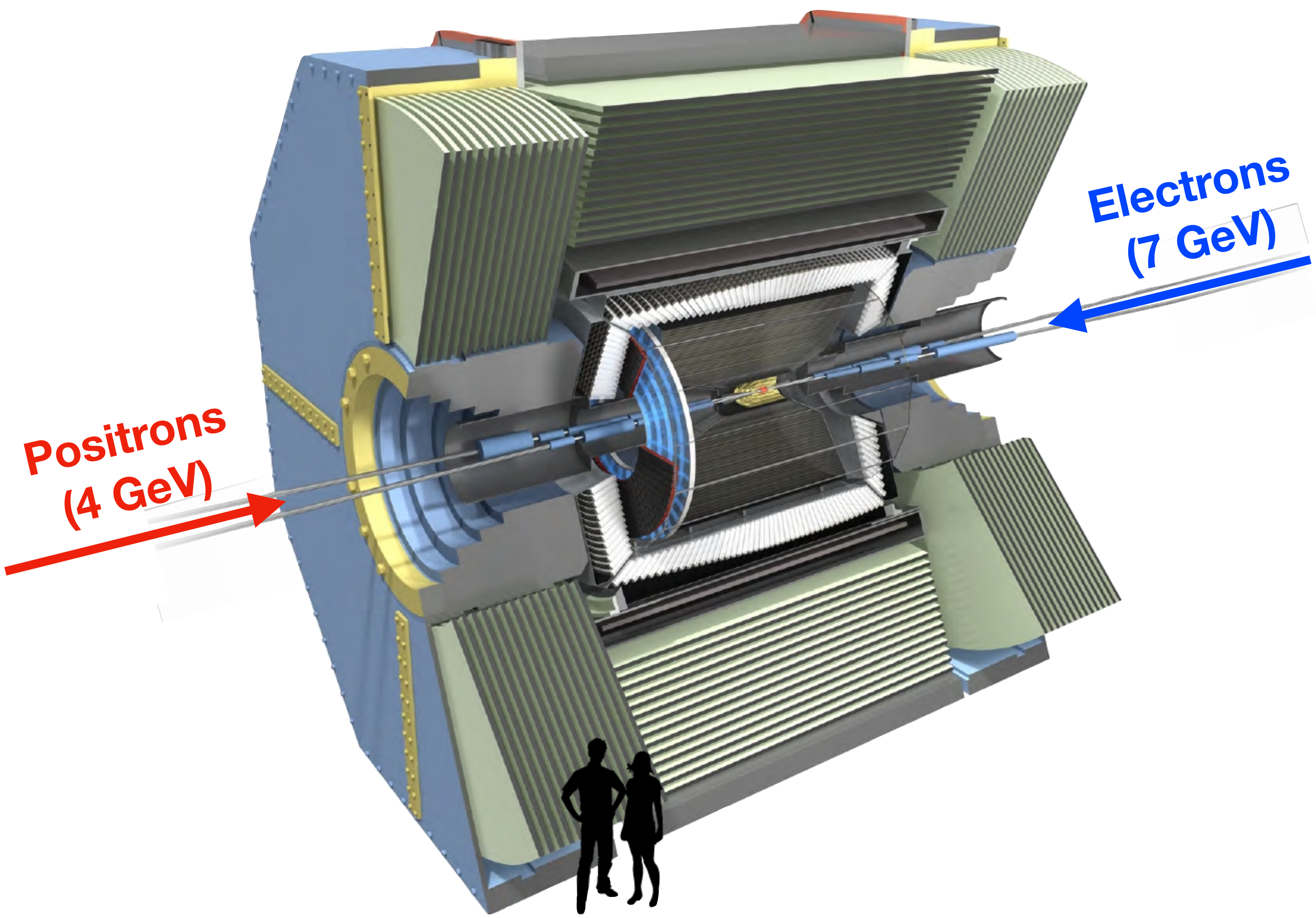


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$$\Upsilon(4S)$$

$$\langle b\bar{b} \rangle$$

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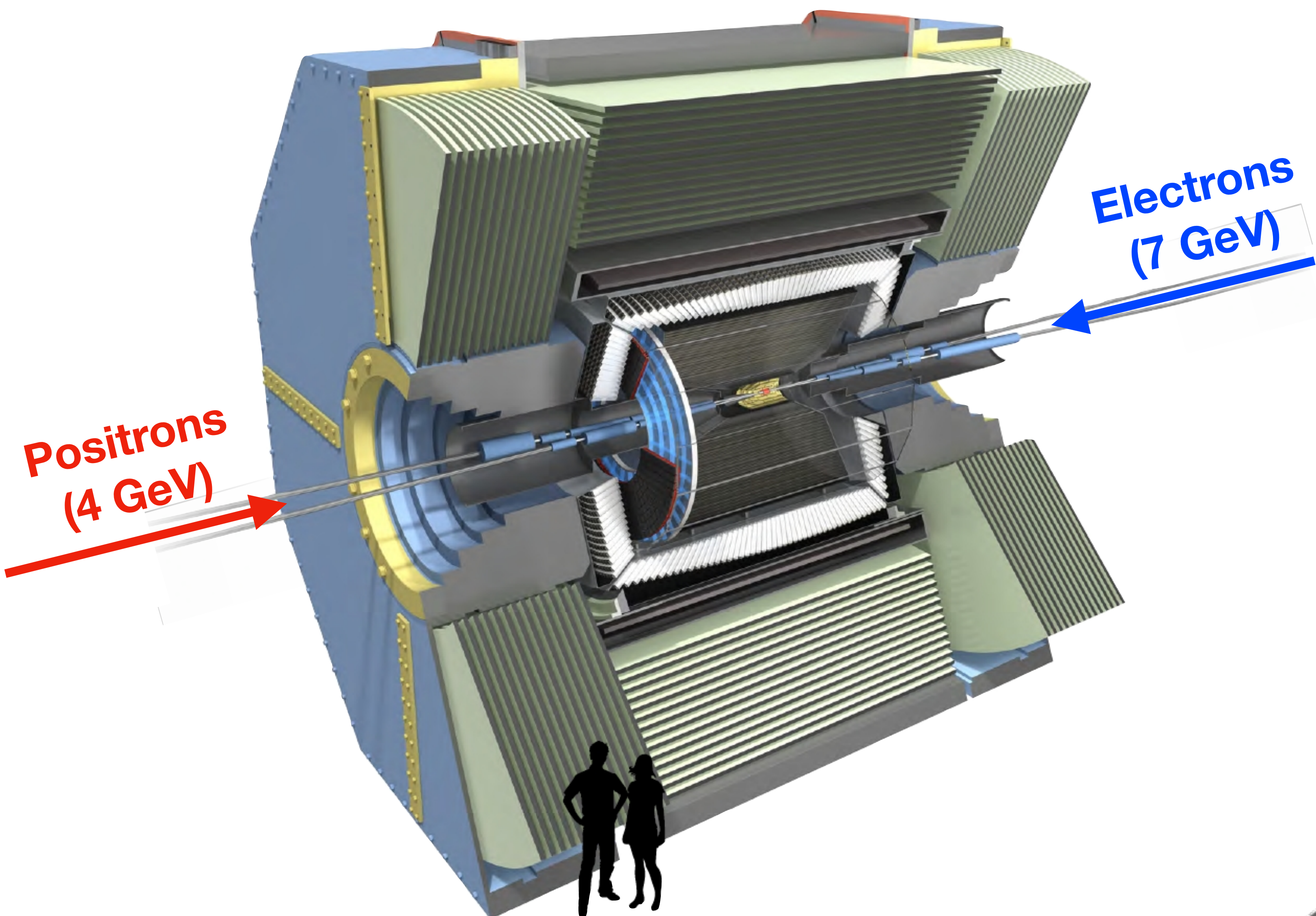


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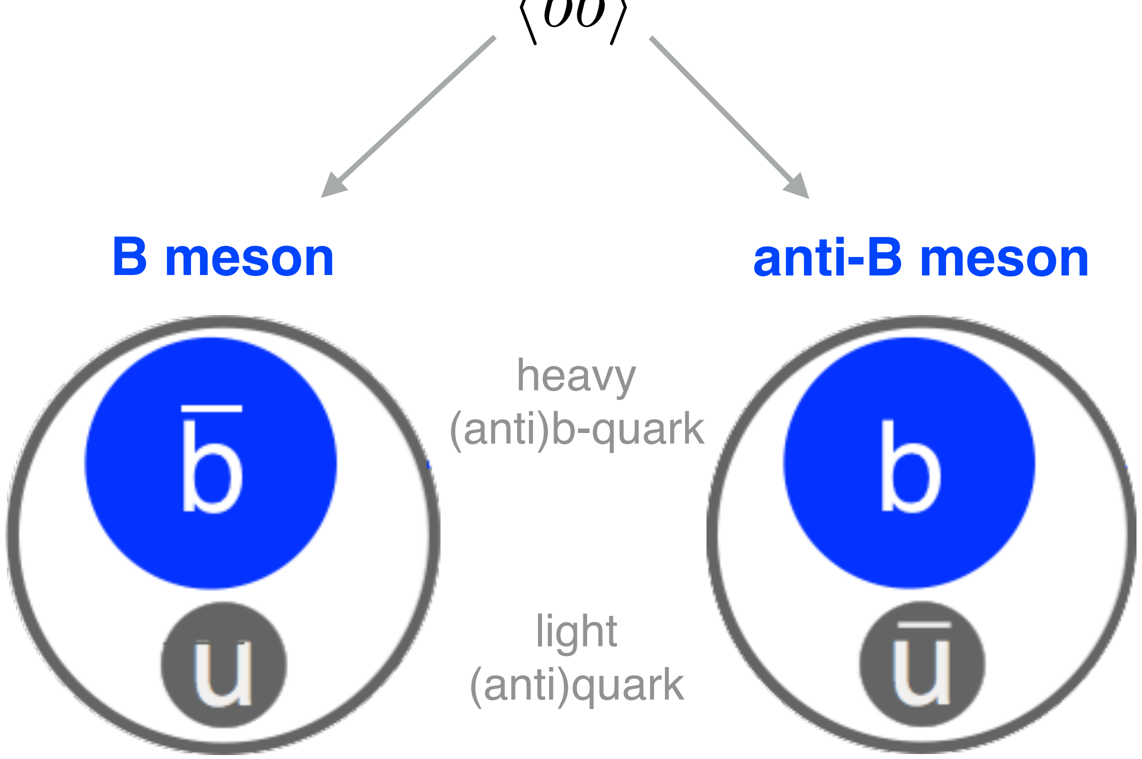
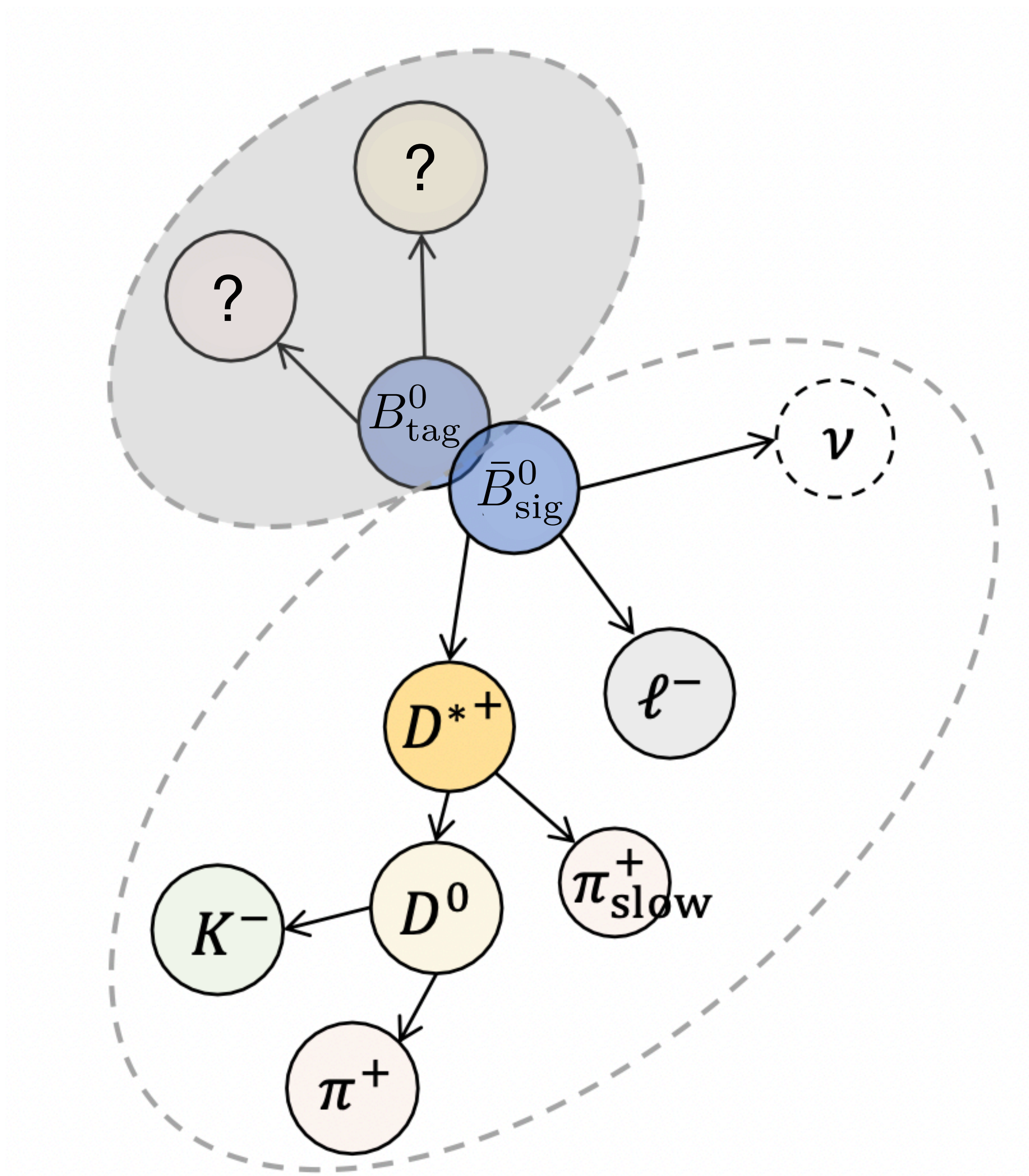



**Advantages**: Precisely known initial state, unique event topology & experimentally clean environment

# Tagging strategies at B-Factories

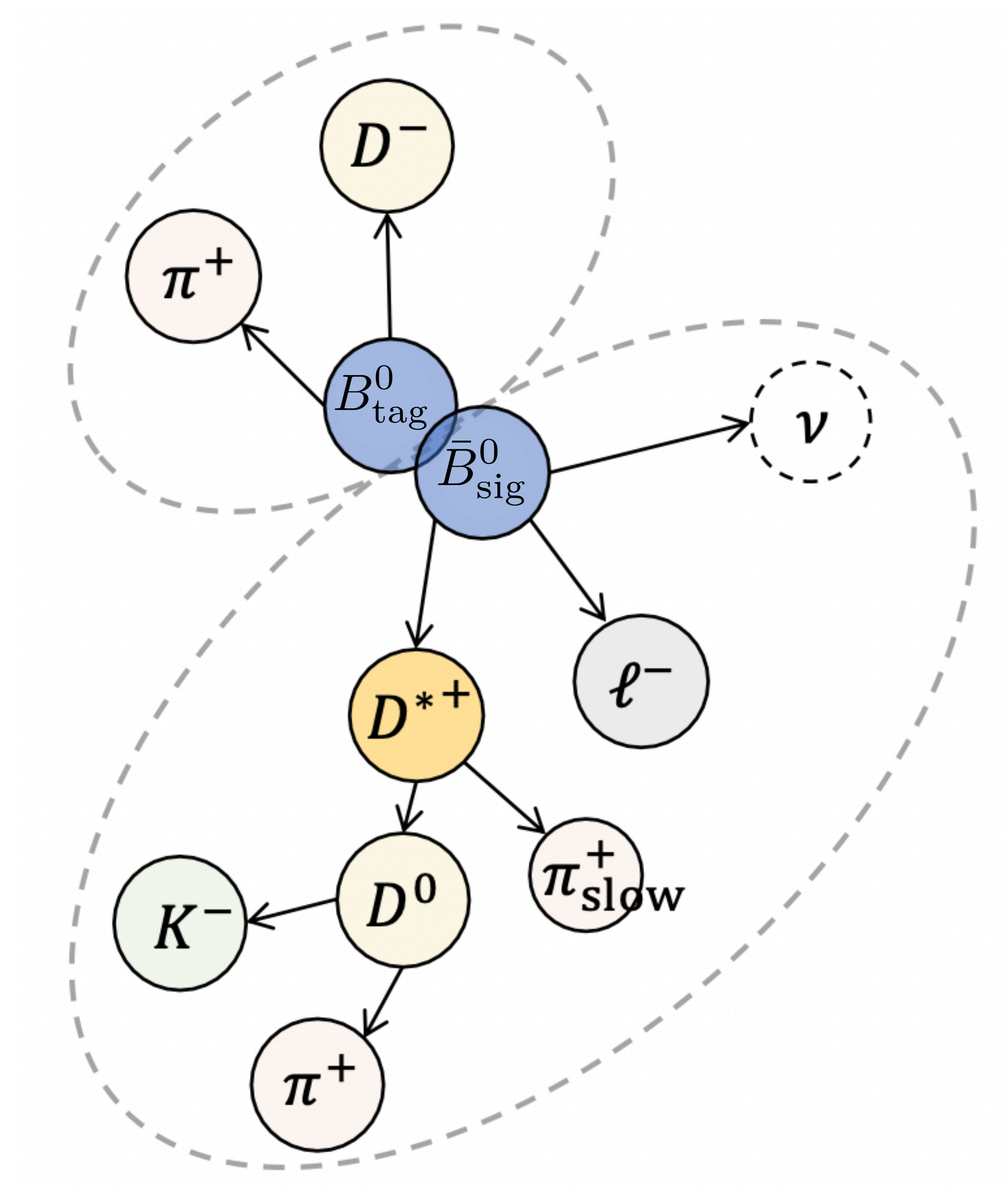
**Untagged**

Only reconstruct the signal B meson ($B_{sig}$).




**Tagged**

Reconstruct $B_{tag}$ with hadronic decay modes.




← Efficiency, backgrounds

Purity, available observables →

Image credit: K. Kojima

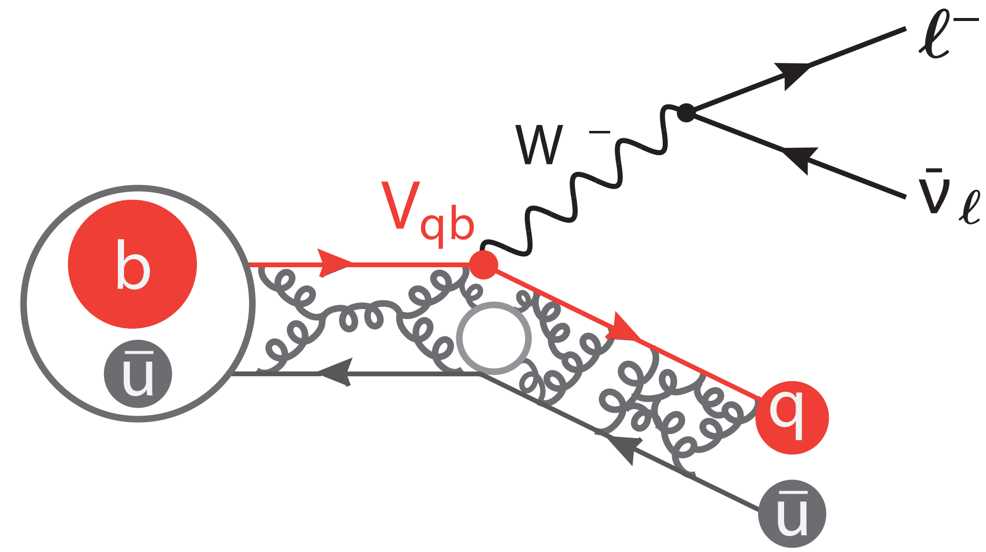
# Exclusive determinations

# The anatomy of an exclusive semileptonic decay



Semileptonic decays have the advantage of being **theoretically clean**, since leptonic and hadronic currents factorize:

$$d\Gamma \propto |\mathcal{A}|^2 = G_F^2 |V_{qb}|^2 \cdot |H^\mu L_\mu|^2$$



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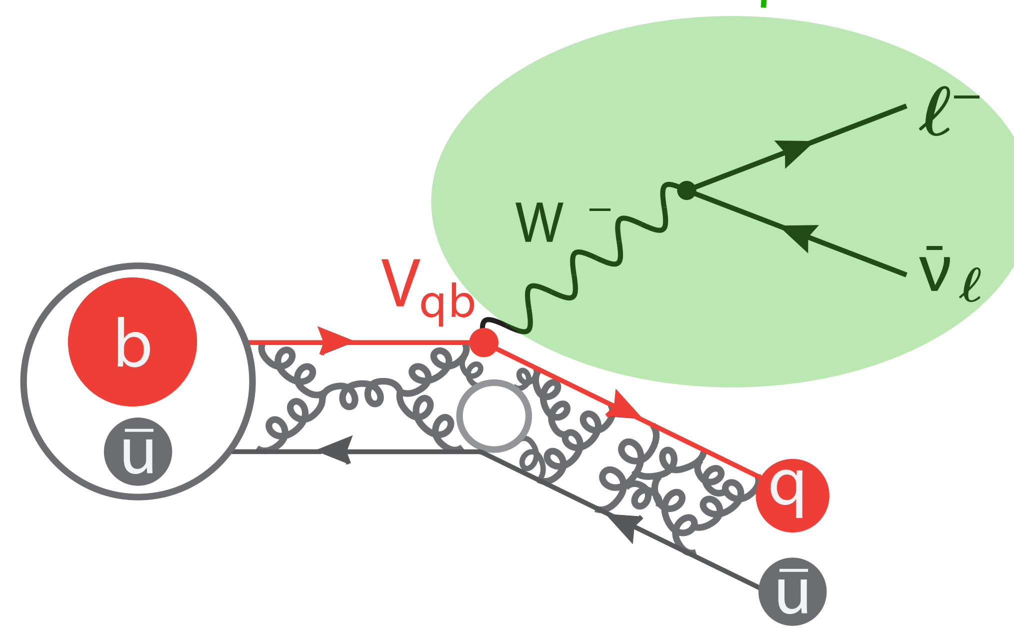
Leptonic matrix element

$$L_\mu = \langle P_\ell P_\nu | \bar{\ell} \gamma_\mu (1-\gamma^5) \nu_\ell | 0 \rangle$$



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- **Hadronic matrix elements** cannot be calculated analytically!
- Non-perturbative physics parametrized with **hadron transition form factors** as functions of $q^2 = (p_B - p_X)^2$.
- For $\ell = e, \mu$ the contribution from $f_-(q^2)$ **is negligible**, thus the decay rate only depends on the $f_+(q^2)$ form factor.
- Fit to available data to determine values.
- We need **input** from lattice QCD for at least the **normalization**.

See talks by Tobias, Takashi, Alejandro, Martin and many more!

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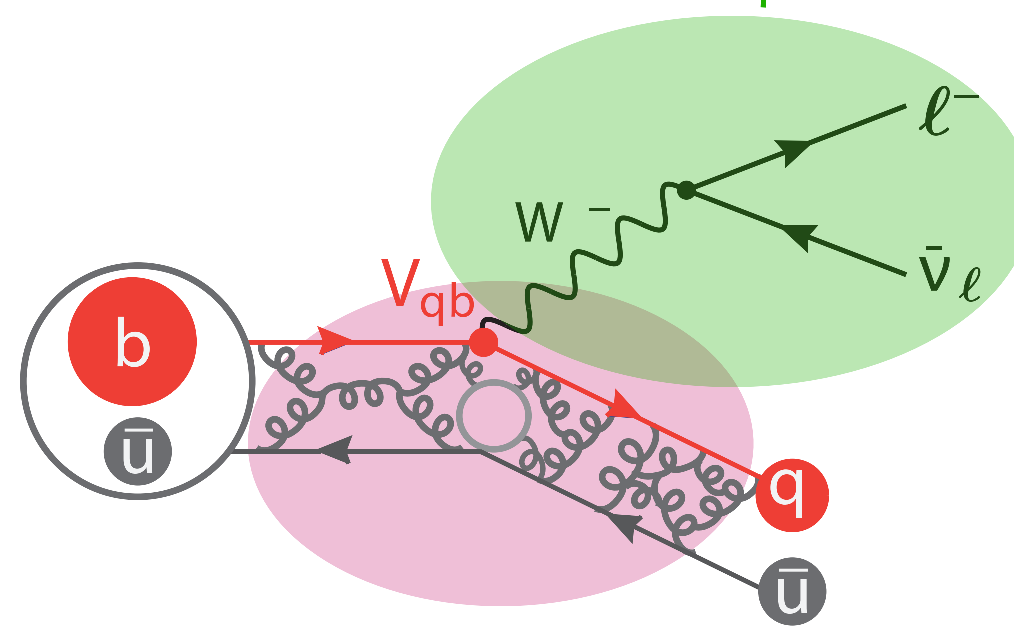



Hadronic matrix element

$$H^\mu = \langle D | \bar{c} \gamma^\mu b | B \rangle$$
$$= f_+ (P_B + P_D)^\mu + f_- (P_B - P_D)^\mu$$

Form Factors

$$= f_+(q^2)(P_B + P_D)^\mu + f_-(q^2) q^\mu$$

where $q^\mu \equiv (P_B - P_D)^\mu$

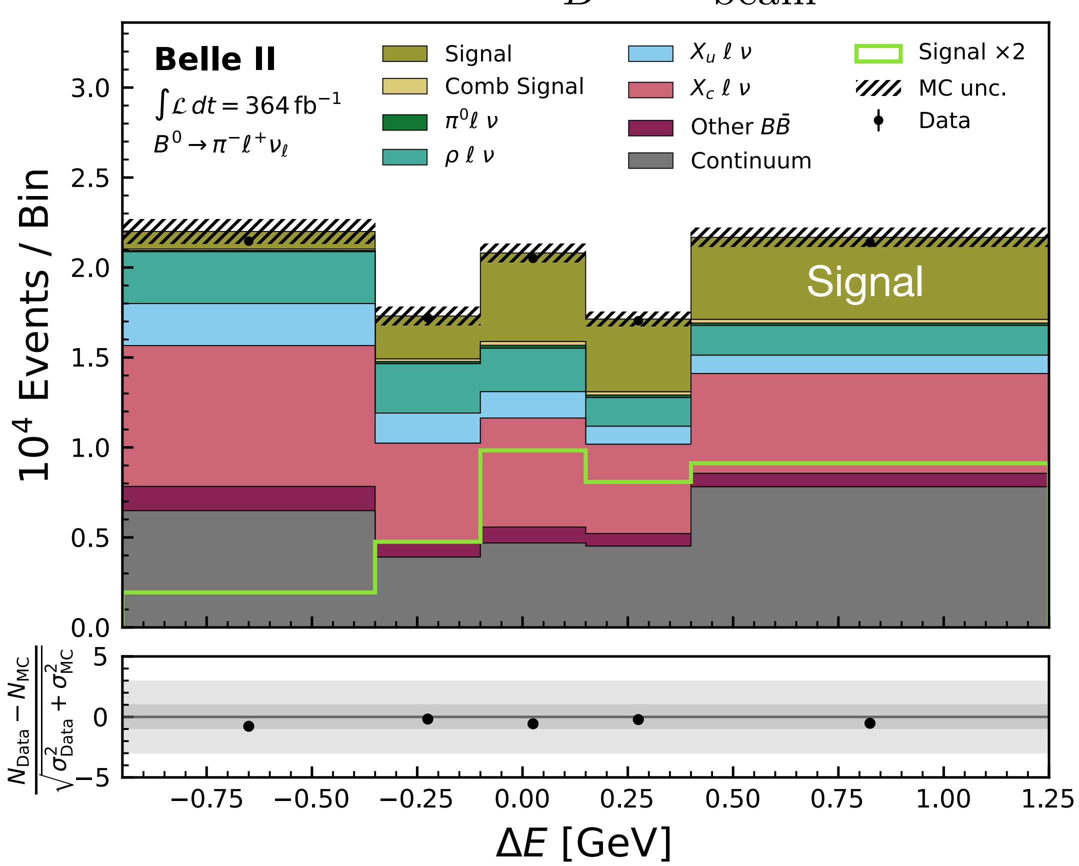
# $|V_{ub}|$ from $B \to \pi/\rho\ell\nu$

Belle II

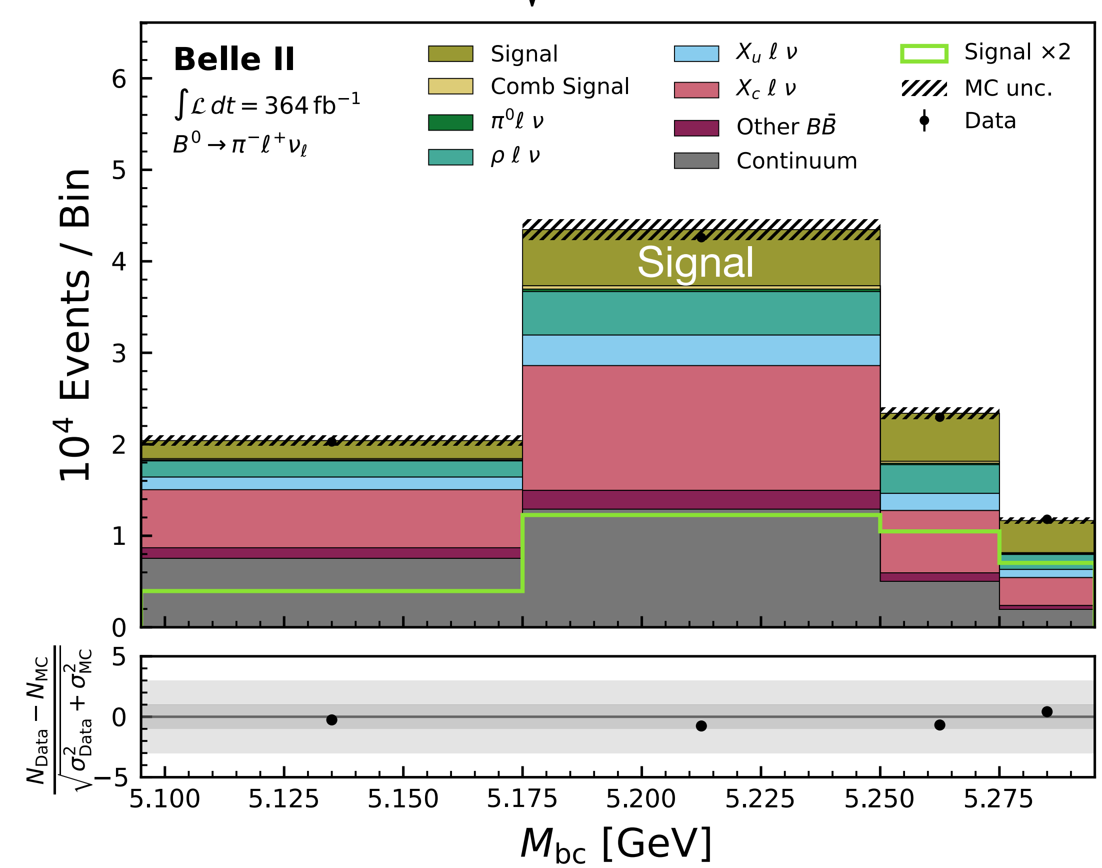
arXiv:2407.17403

- **Untagged** reconstruction of $B^0 \to \pi^- \ell^+ \nu_\ell$ and $B^+ \to \rho^0 \ell^+ \nu_\ell$ using full Belle II **Run I dataset**.
- **New idea:** simultaneously extract signal yields via binned 3D fits using beam-constrained mass $M_{bc}$ and energy difference $\Delta E$ in bins of $q^2 = (p_B - p_\pi)^2 = (p_\ell + p_\nu)^2$.
- **Main challenge:** large backgrounds from $e^+e^- \to q\bar{q}$ processes (a.k.a. continuum) and other semileptonic $B \to X_c \ell \nu$ decays.

$$\Delta E = E_B^* - E_{\text{beam}}^*$$




$$M_{bc} = \sqrt{E_{\text{beam}}^{*2} - |\vec{p}_B^{\,*}|^2}$$



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- **Main challenge:** large backgrounds from $e^+e^- \to q\bar{q}$ processes (a.k.a. continuum) and other semileptonic $B \to X_c \ell \nu$ decays.
- Take into account **cross-feed** signal yields and correlations between backgrounds.

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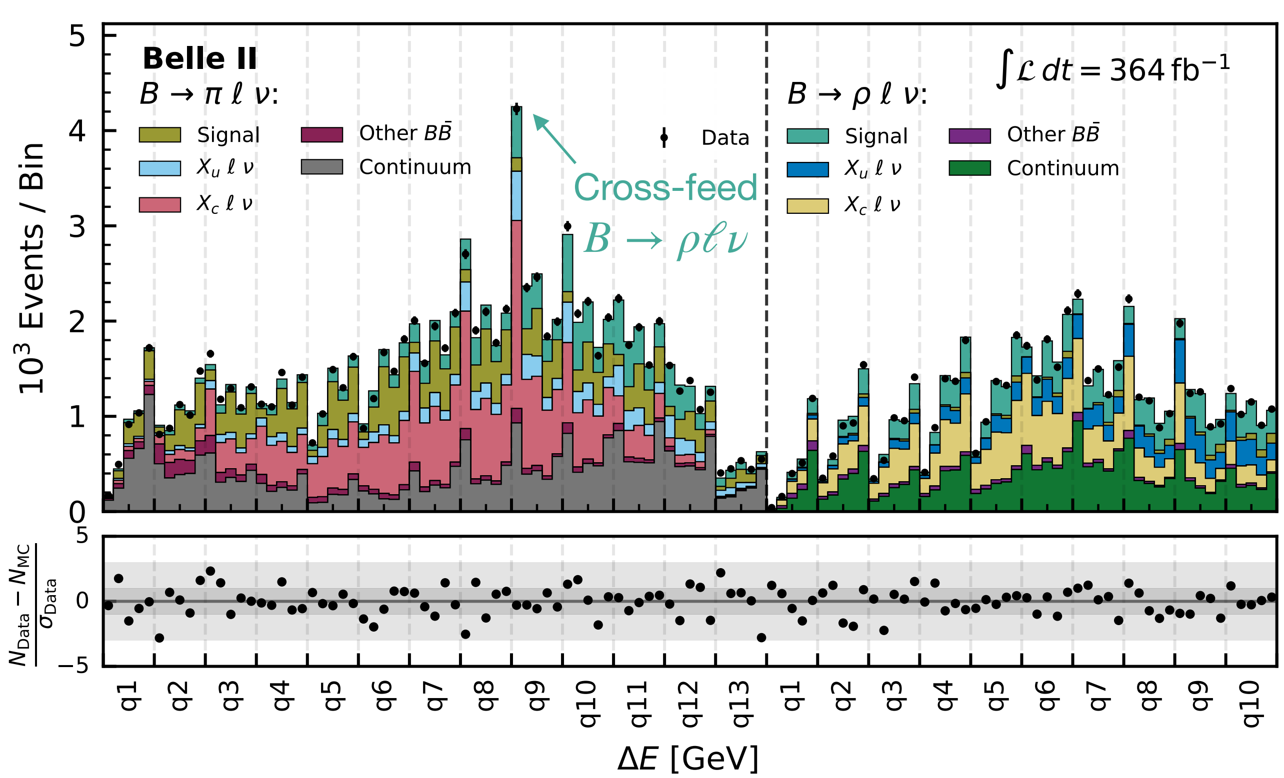


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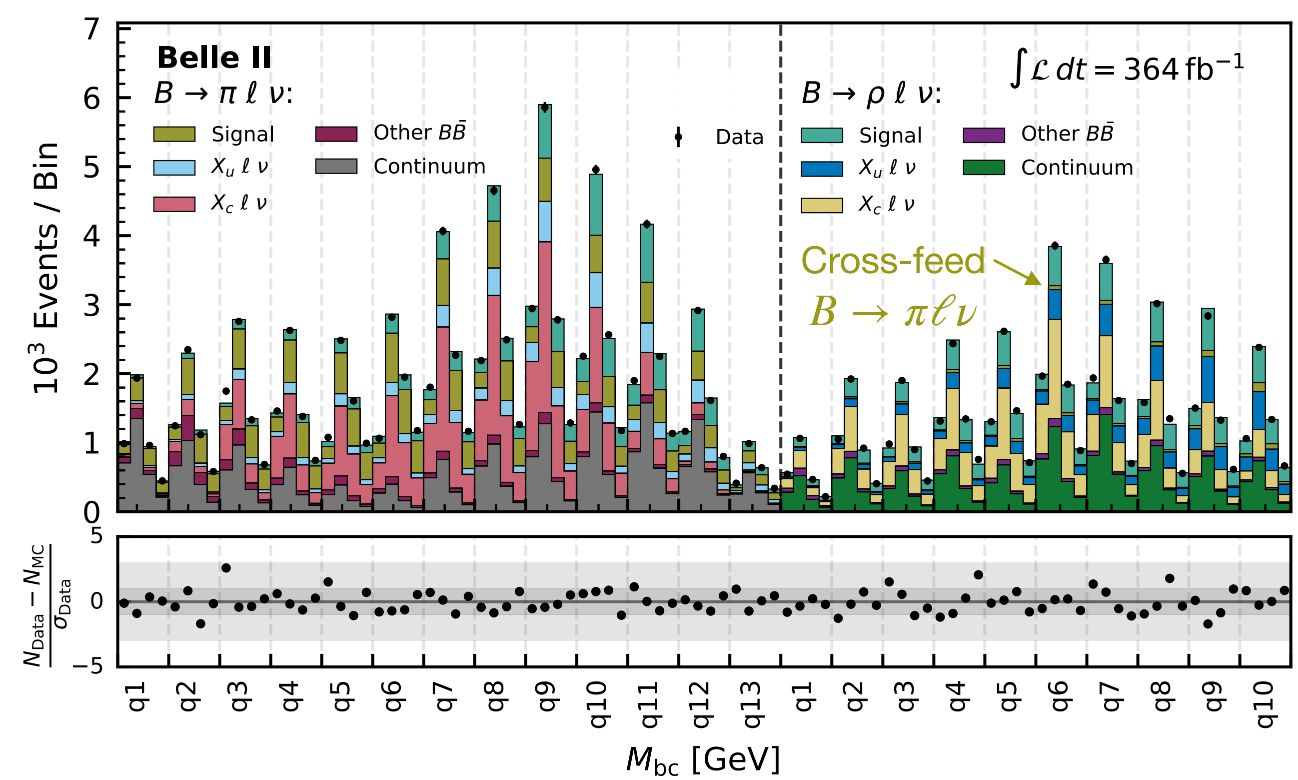

# $|V_{ub}|$ from $B \to \pi/\rho\ell\nu$

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- Convert to partial branching fractions $\Delta\mathscr{B}_i$ using **reconstruction efficiencies**.
- Total branching ratios **consistent with world averages:**

$$\mathcal{B}(B^0 \to \pi^-\ell^+\nu) = (1.516 \pm 0.042 \pm 0.059) \times 10^{-4}$$
$$\mathcal{B}(B^+ \to \rho^0\ell^+\nu) = (1.625 \pm 0.079 \pm 0.180) \times 10^{-4}$$

- Determine $|V_{ub}|$ by **fitting differential decay widths** using the relevant form factor expansions with constraints from LQCD/LCSR:

$B^0 \to \pi^-\ell^+\nu$

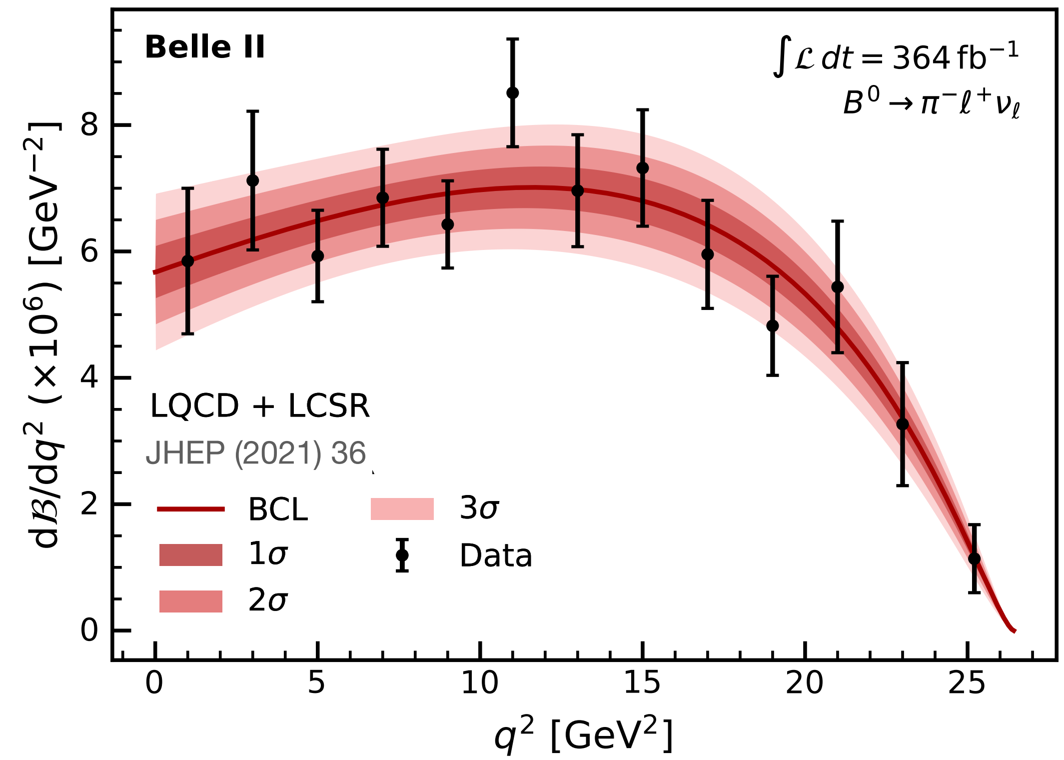
$$|V_{ub}|_{\mathrm{LQCD+LCSR}} = (3.73 \pm 0.07 \pm 0.07 \pm 0.16) \times 10^{-3}$$

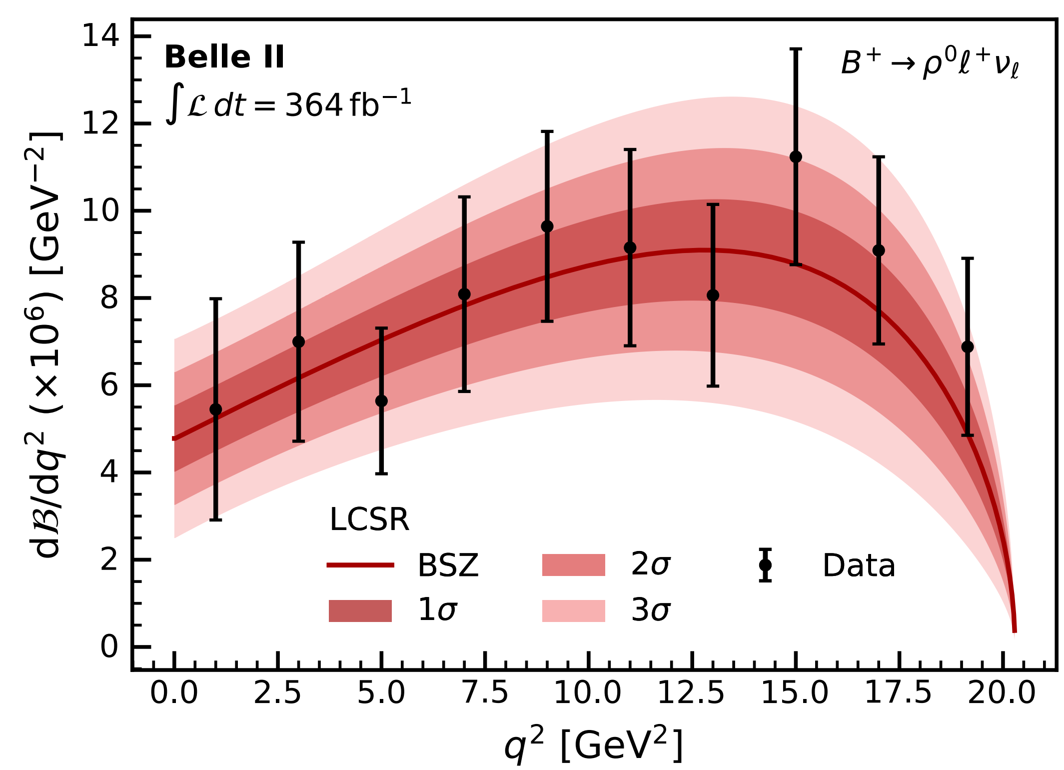
$B^+ \to \rho^0\ell^+\nu$

$$|V_{ub}|_{\mathrm{LCSR}} = (3.19 \pm 0.12 \pm 0.17 \pm 0.26) \times 10^{-3}$$

Largest systematic: Continuum & $B \to \pi\pi\ell\nu$ modelling

**In agreement with exclusive world average**




Bourrely-Caprini-Lellouch (BCL): PRD 82 (2009) 099902
Bharucha-Straub-Zwicky (BSZ) : J. High Energ. Phys. 08 (2016) 96

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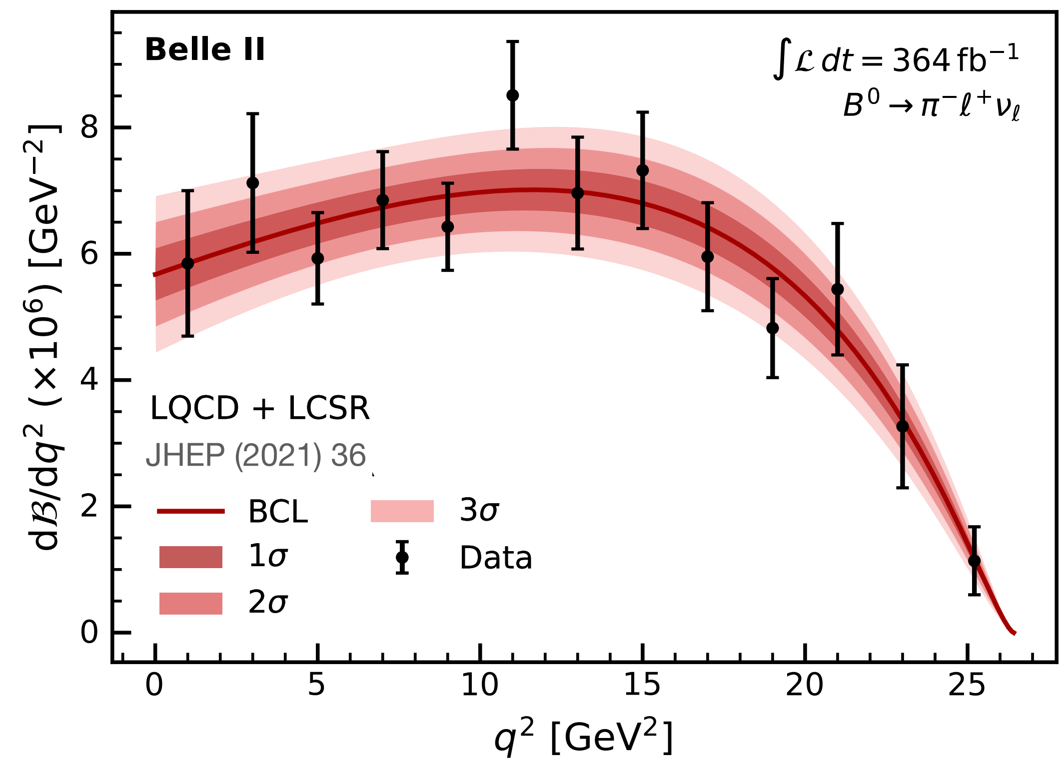


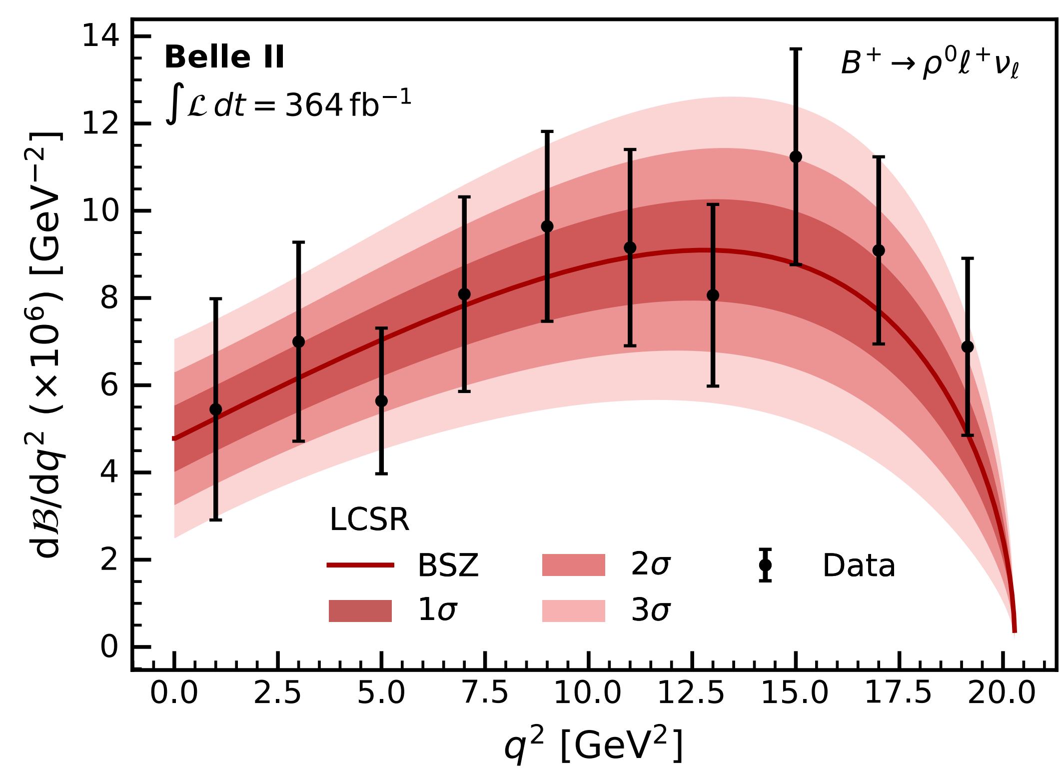


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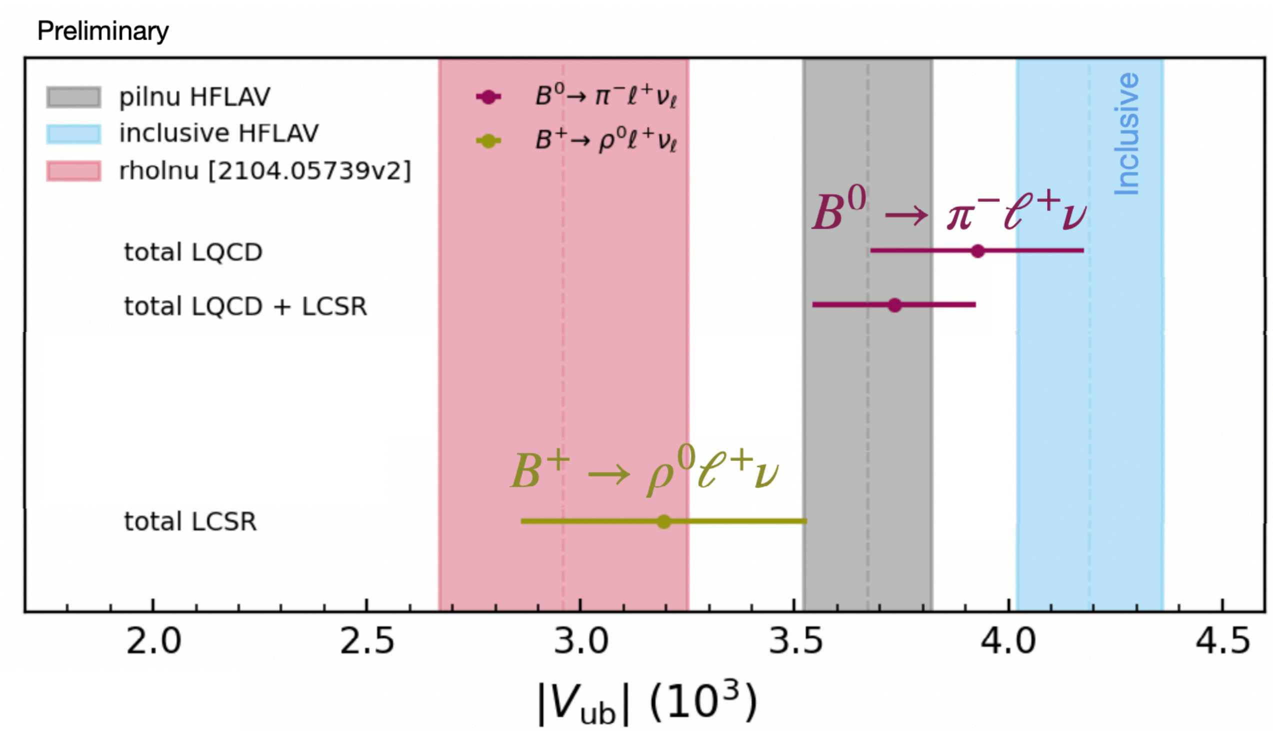


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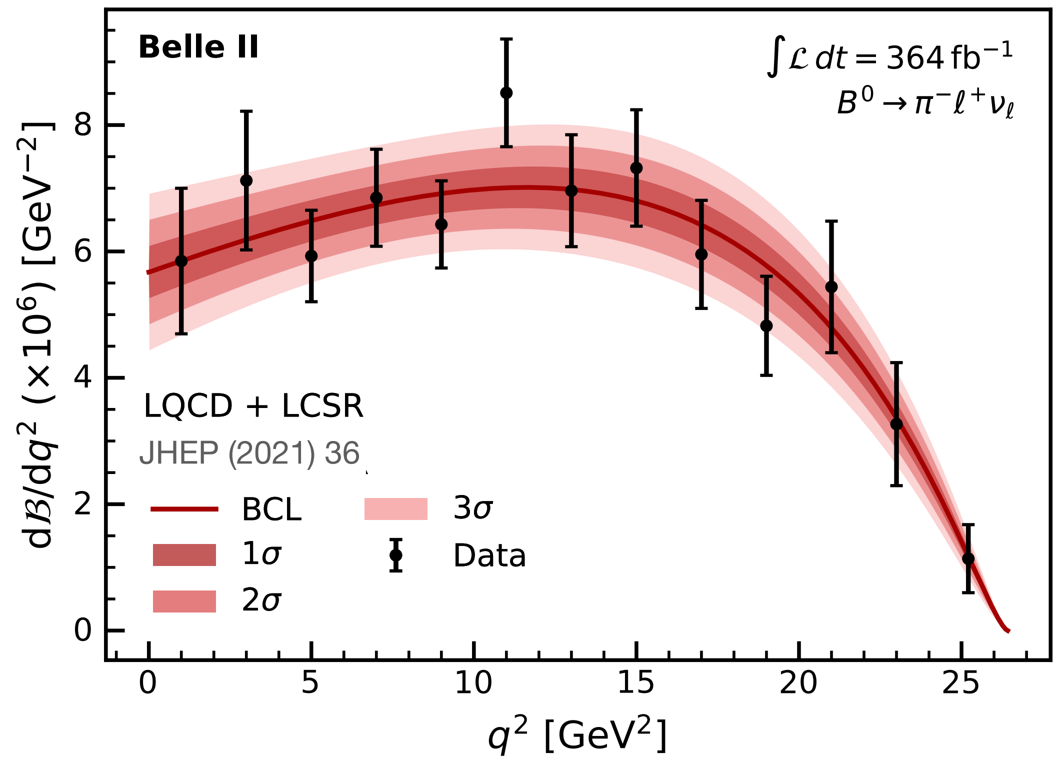


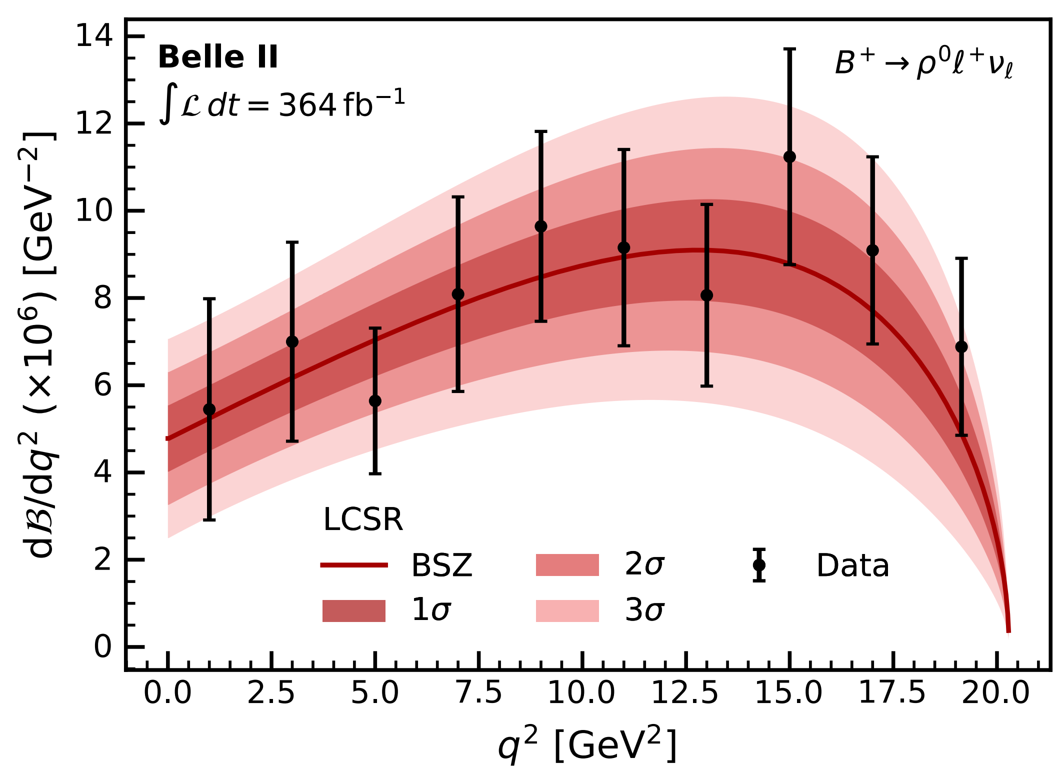


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# $|V_{cb}|$ from untagged $B^0 \to D^{*+}\ell^-\nu$

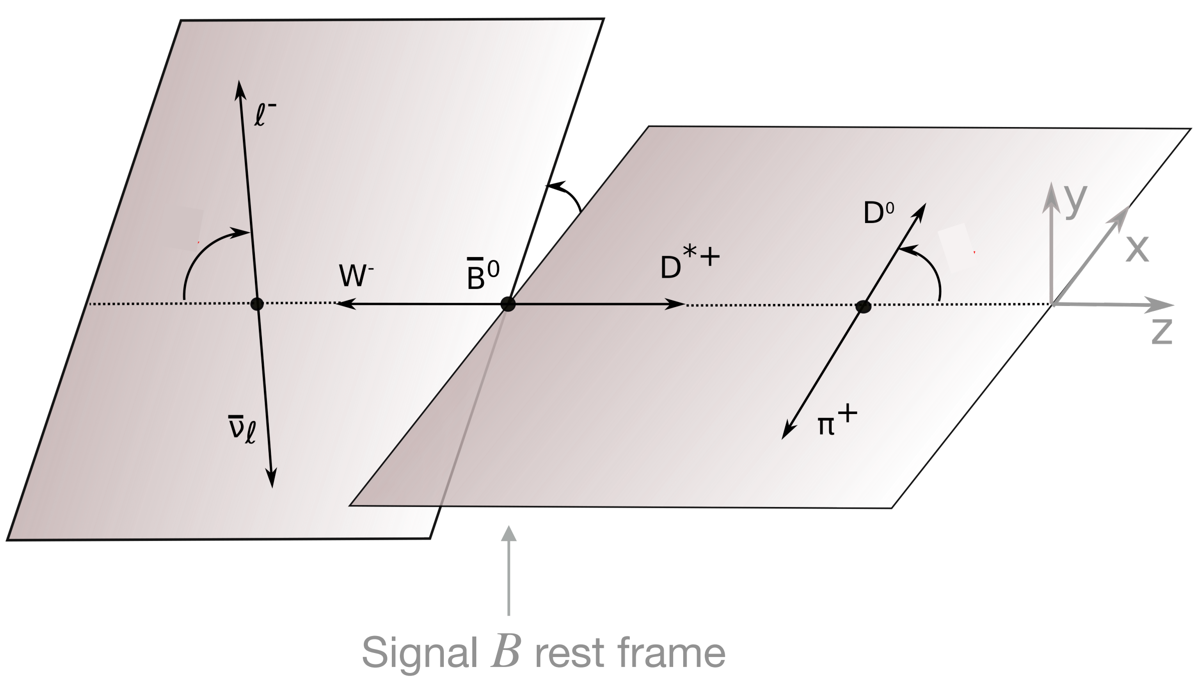
Belle II

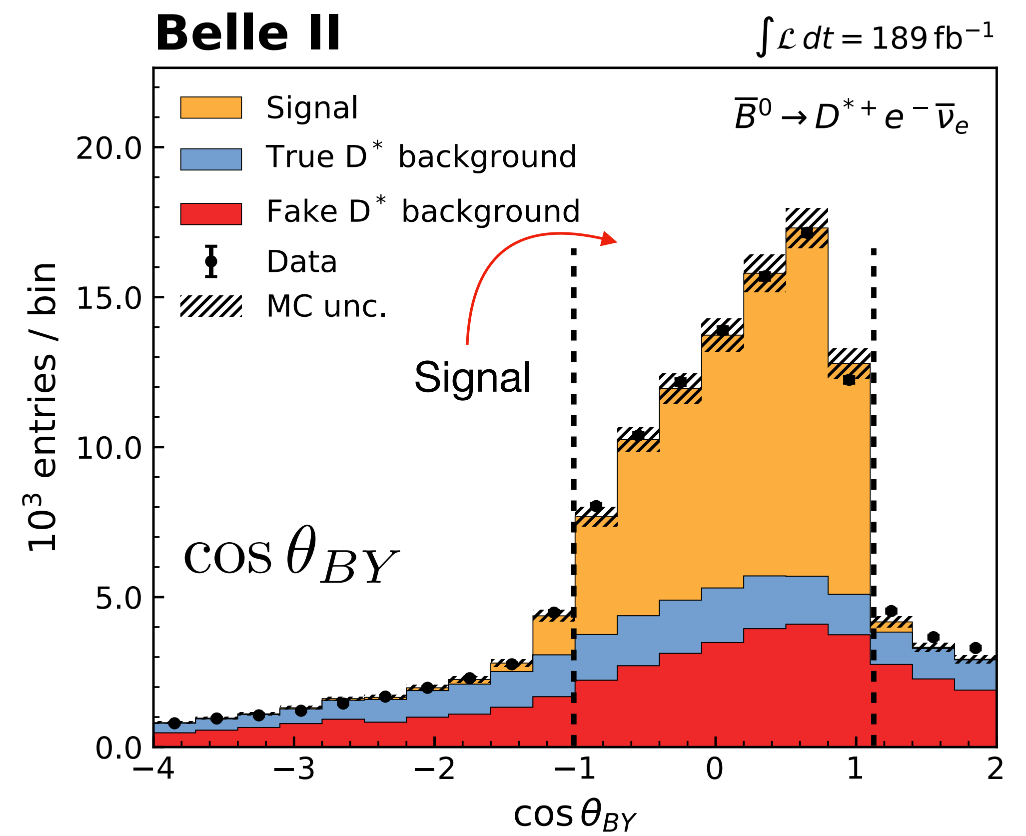
PRD 108, 092013 (2023)

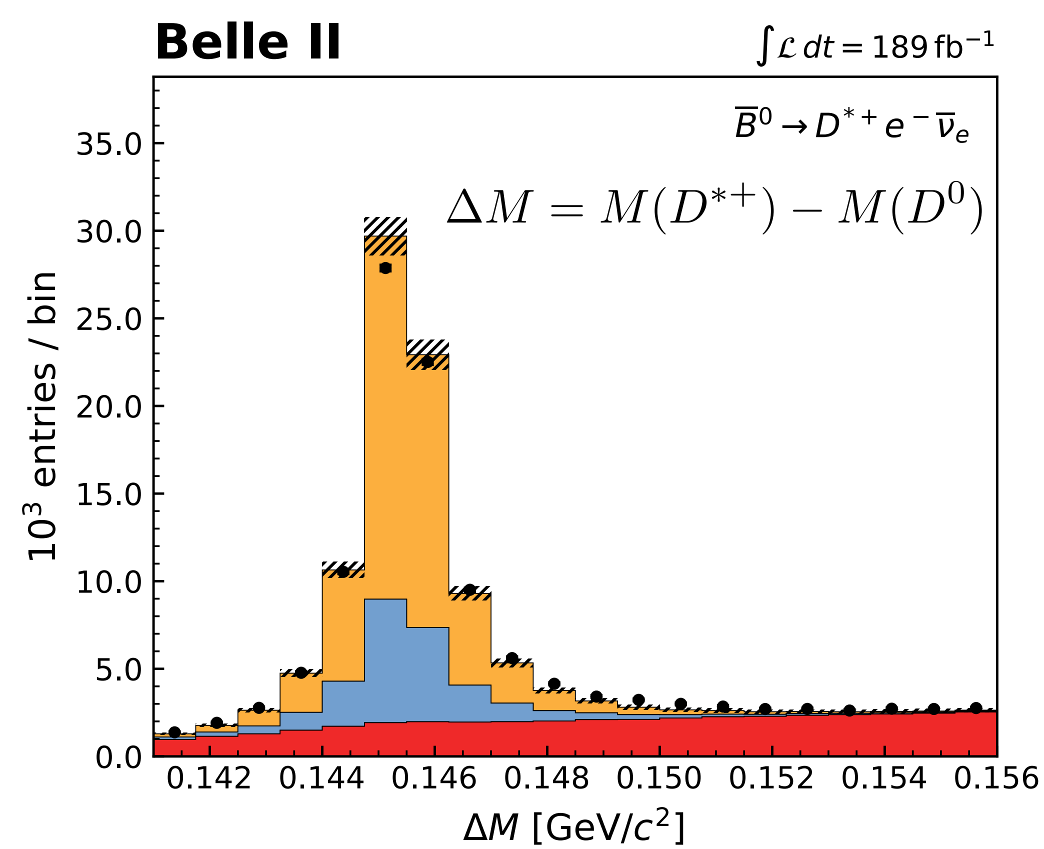
- **Reconstruct** $D^{*+} \to D^0(\to K^-\pi^+)\pi^+_{\text{slow}}$ and combine with appropriately **charged lepton** ($\ell = e$ or $\mu$).
- **Main challenge:** accurate background model, slow pion (p < 0.4 GeV) tracking and statistical correlations between bins.
- Reconstruct the angle between $B$ and $Y = D^*\ell$:

$$\cos\theta_{BY} = \frac{2E^*_B E^*_Y - m^2_B - m^2_Y}{2|p^*_B||p^*_Y|}$$

- **Extract signal yield** with 2D fit to $\cos\theta_{BY}$ and $\Delta M = M(D^{*+}) - M(D^0)$ in bins of…



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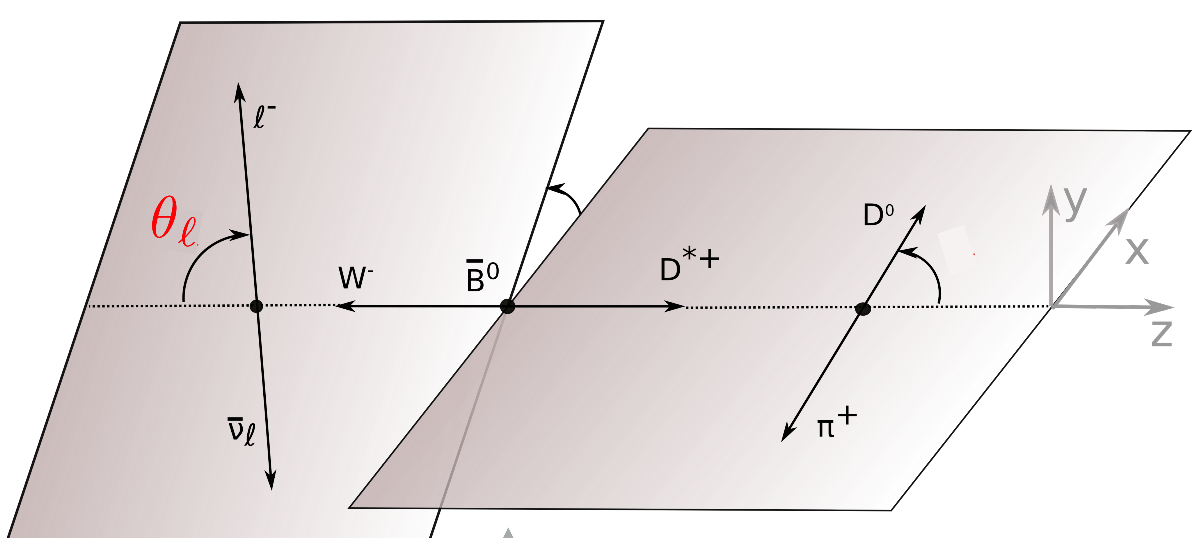
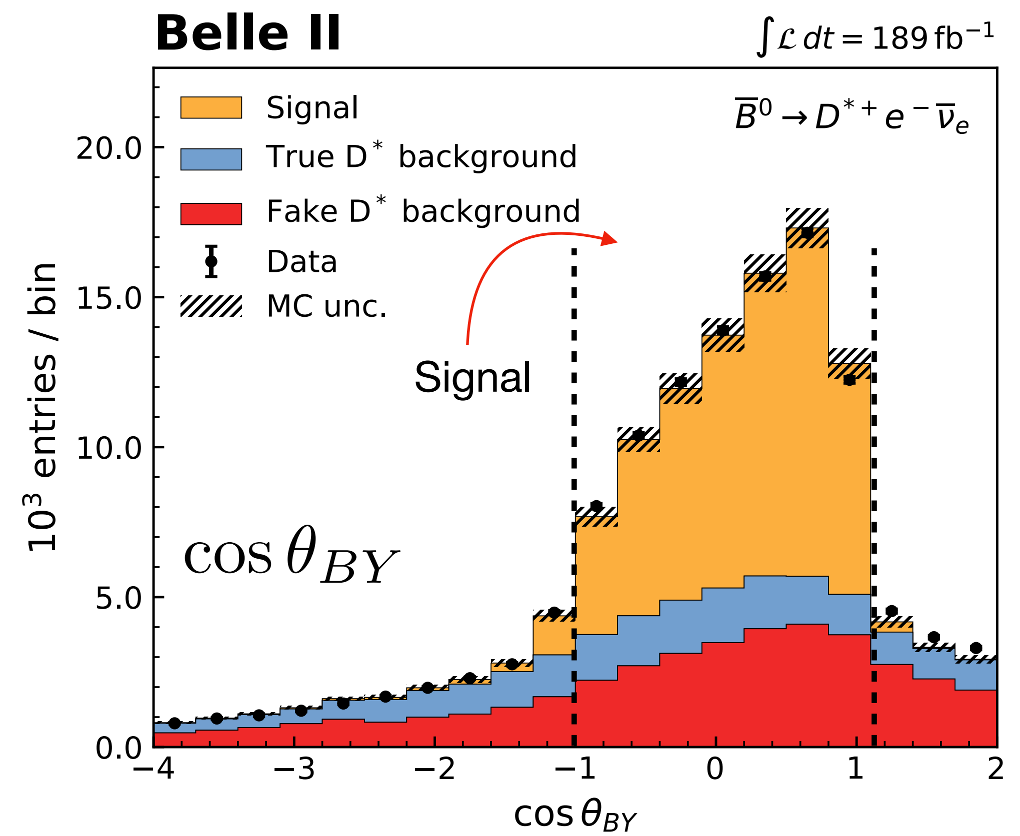
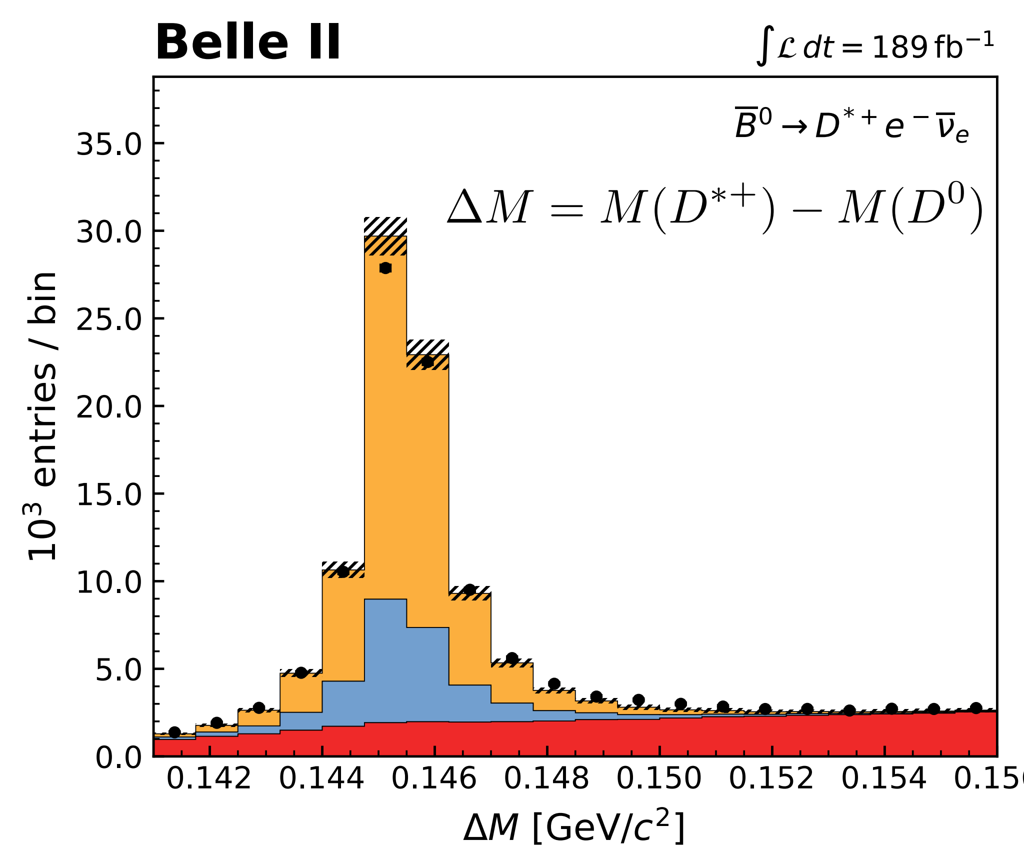



Signal $B$ rest frame



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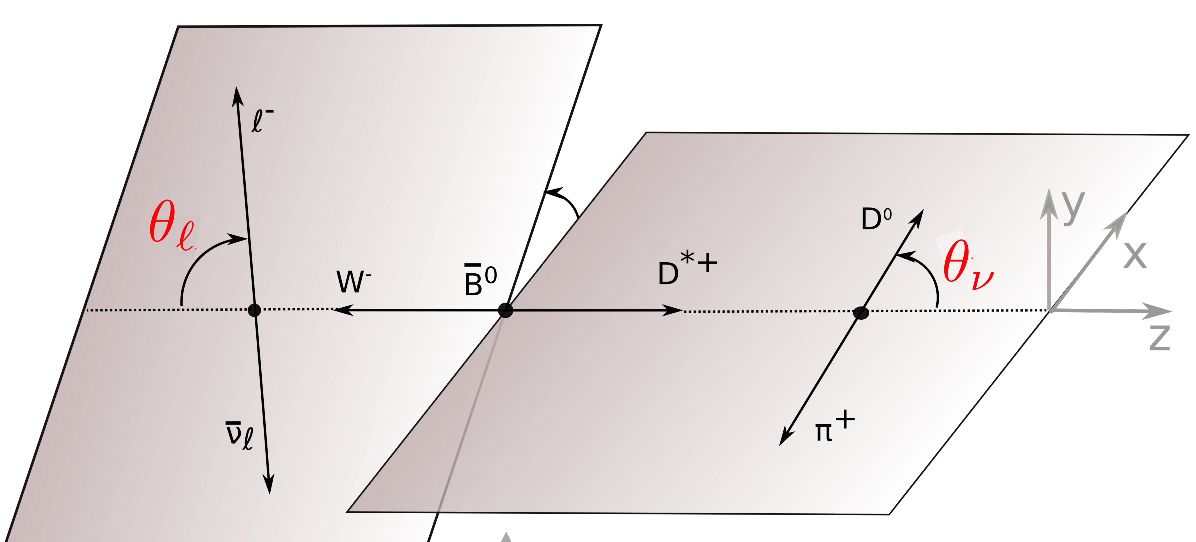
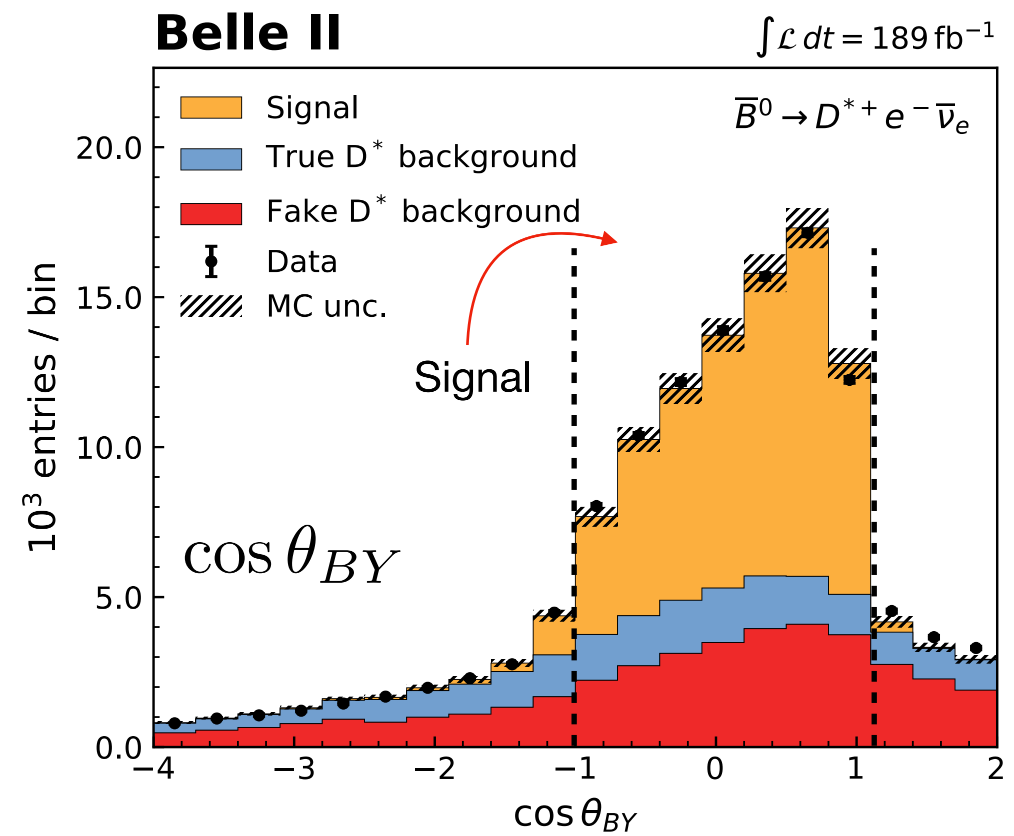
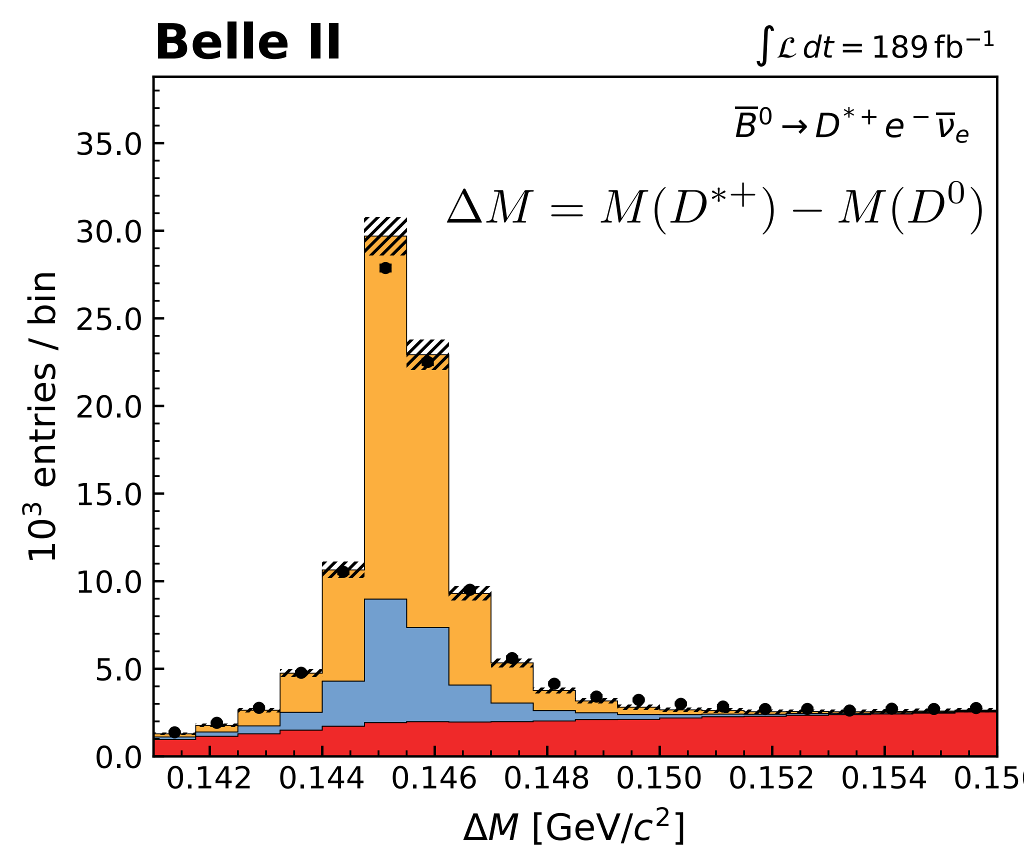
- **Extract signal yield** with 2D fit to $\cos\theta_{BY}$ and $\Delta M = M(D^{*+}) - M(D^0)$ in bins of $\cos\theta_\ell$, $\cos\theta_\nu$




Signal $B$ rest frame



# $|V_{cb}|$ from untagged $B^0 \to D^{*+}\ell^-\nu$



Belle II

PRD 108, 092013 (2023)

- **Reconstruct** $D^{*+} \to D^0(\to K^-\pi^+)\pi^+_{\text{slow}}$ and combine with appropriately **charged lepton** ($\ell = e$ or $\mu$).
- **Main challenge:** accurate background model, slow pion (p < 0.4 GeV) tracking and statistical correlations between bins.
- Reconstruct the angle between $B$ and $Y = D^*\ell$:

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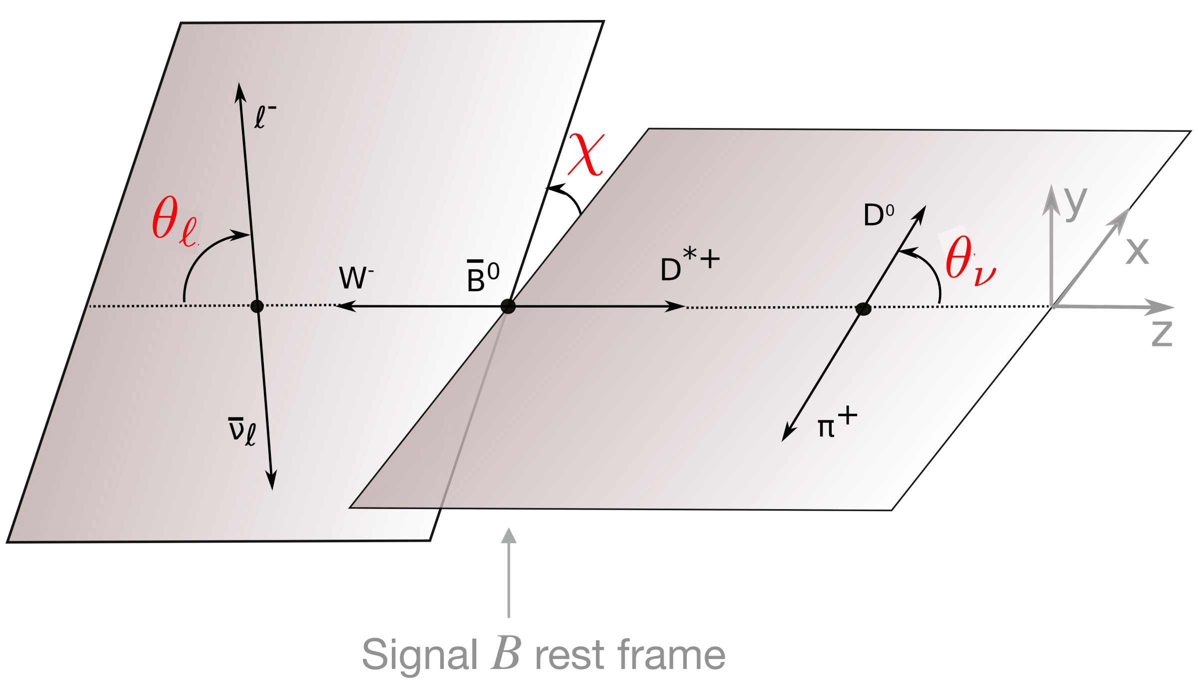


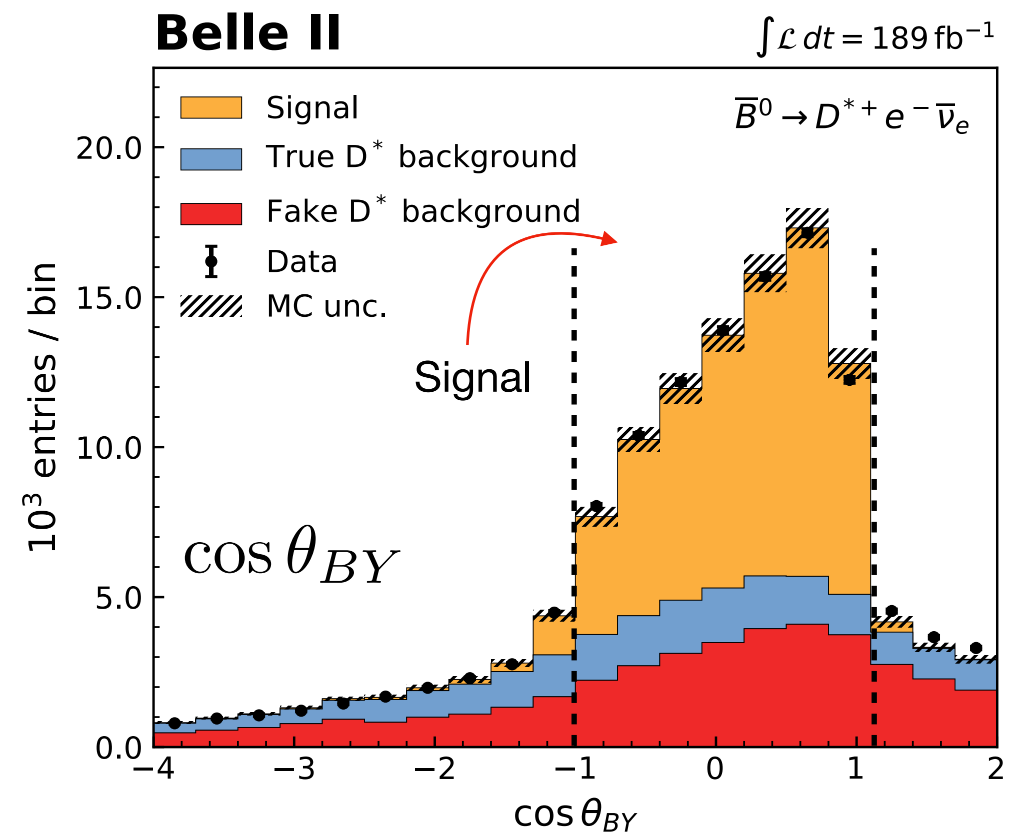


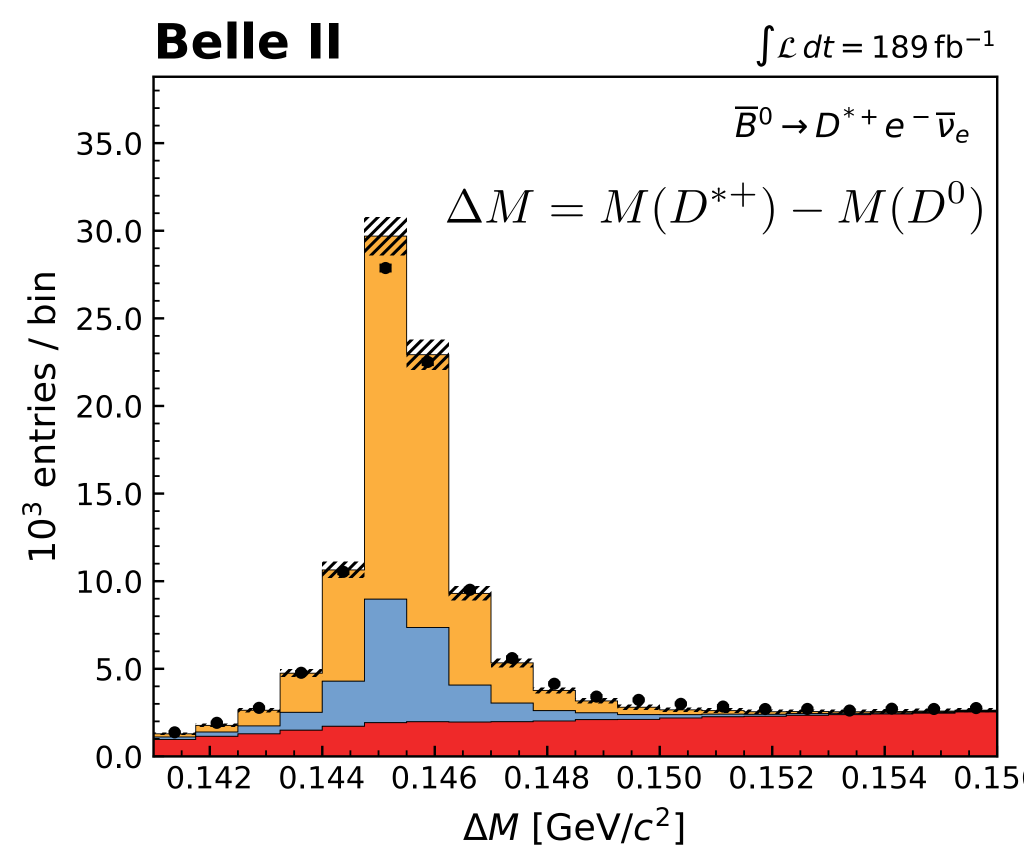

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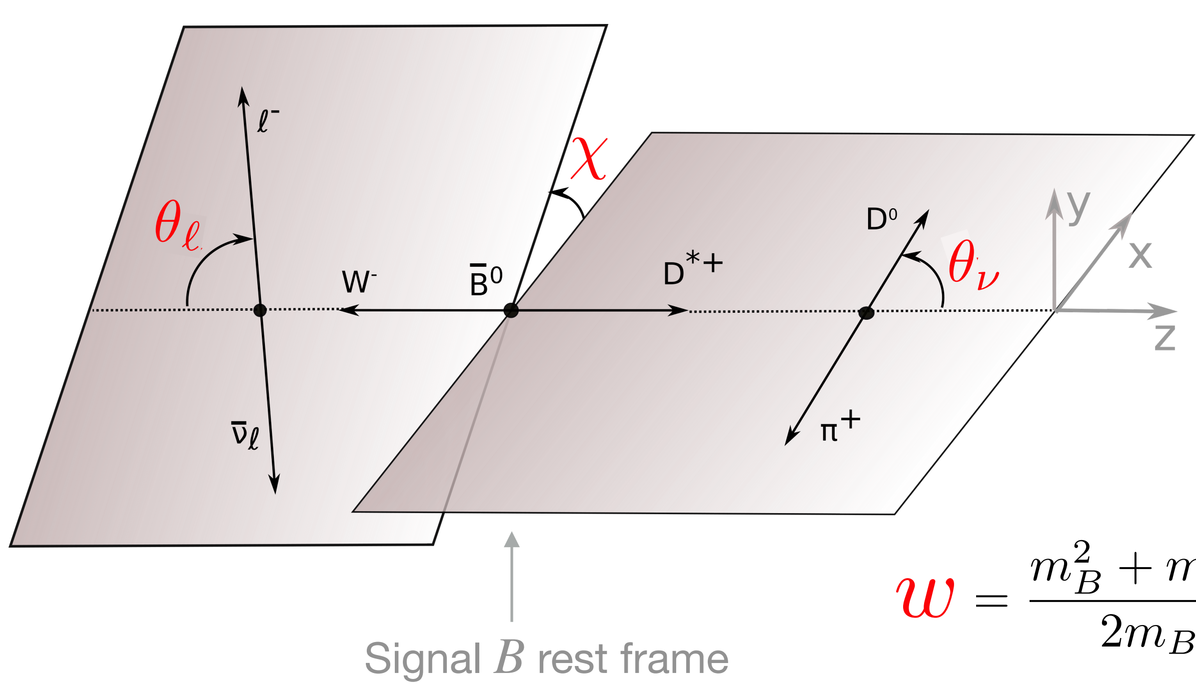
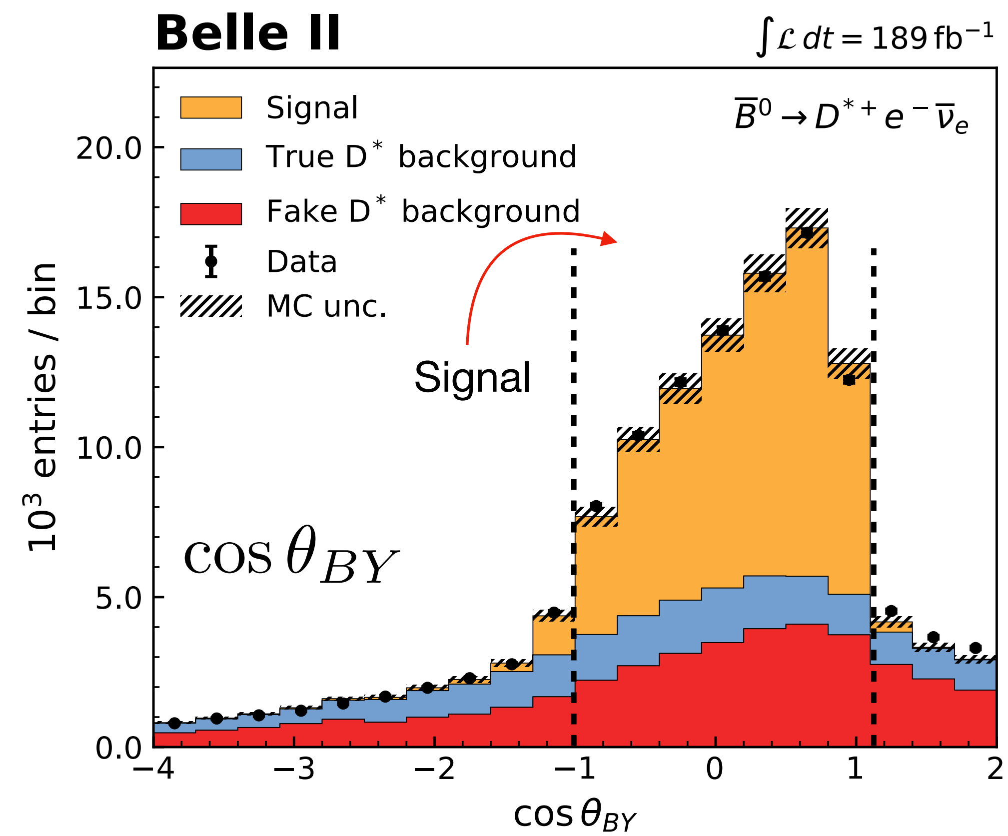
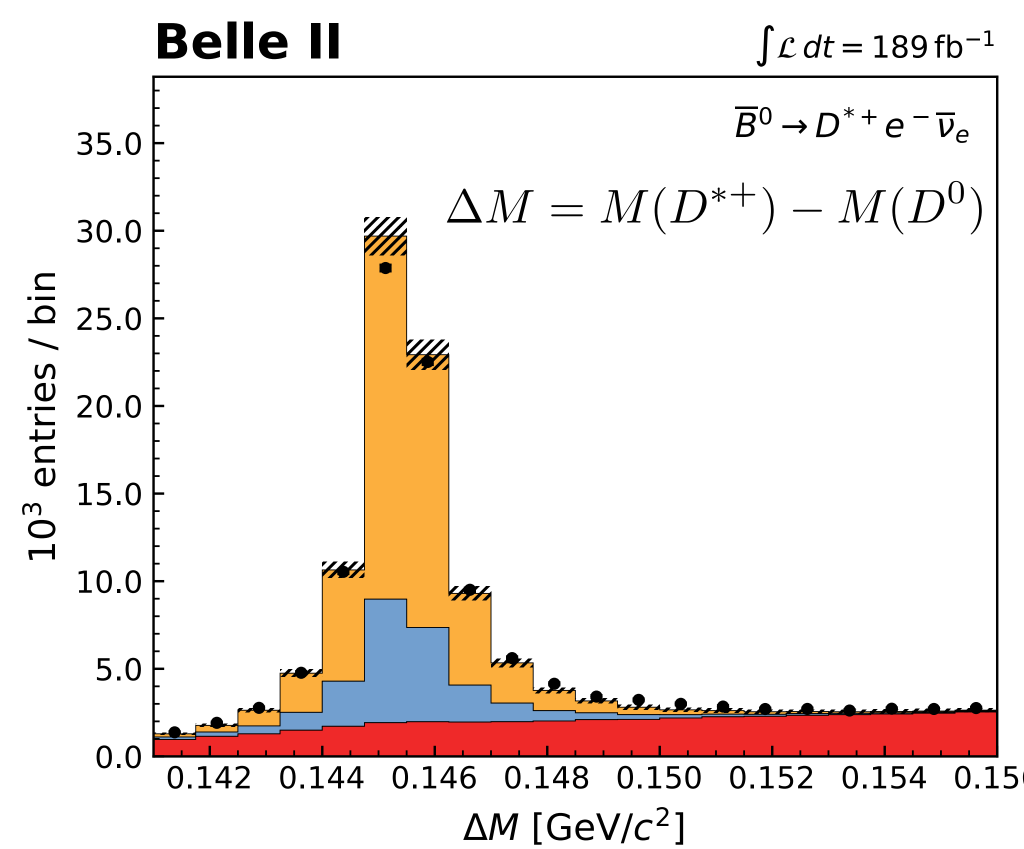



$$w = \frac{m_B^2 + m_{D^*}^2 - q^2}{2m_B m_{D^*}}$$



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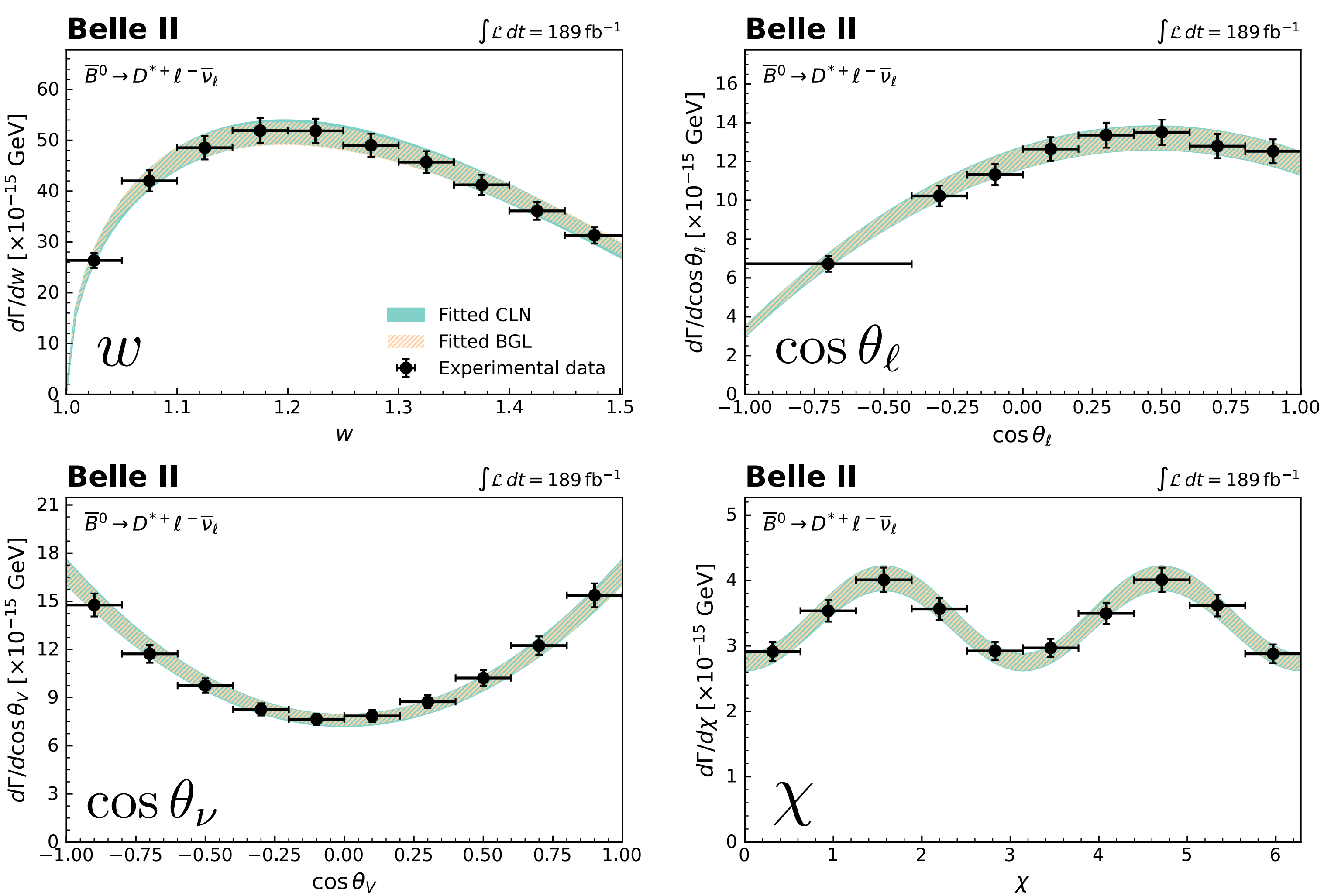
Belle II

PRD 108, 092013 (2023)

- Convert to partial branching fractions $\Delta\Gamma_i$ using **reconstruction efficiencies**.
- **Fit differential shapes** with different form factor expansions to obtain $|V_{cb}|$.
- Use FNAL/MILC lattice QCD data at zero recoil ($w = 1$) for **normalisation.** Phy. Rev. D 89 (2014) 114504




***e* mode:** $\mathcal{B}(\bar{B}^0 \to D^{*+}e^-\bar{\nu}_e) = (4.92 \pm 0.03 \pm 0.22)\%$

***μ* mode:** $\mathcal{B}(\bar{B}^0 \to D^{*+}\mu^-\bar{\nu}_\mu) = (4.93 \pm 0.03 \pm 0.23)\%$

**Ratio** $R(D^*_{e/\mu}) = 0.998 \pm 0.009 \pm 0.020$

$$\frac{d^4\Gamma}{dw\, d\cos\theta_\ell\, d\cos\theta_\nu\, d\chi} \propto |V_{cb}|^2 F^2(w, \cos\theta_\ell, \cos\theta_\nu, \chi)$$

$$|V_{cb}|_{\mathrm{BGL}} = (40.6 \pm 0.3 \pm 1.0 \pm 0.6) \times 10^{-3}$$

$$|V_{cb}|_{\mathrm{CLN}} = (40.1 \pm 0.3 \pm 1.0 \pm 0.6) \times 10^{-3}$$

Slow pion eff. plays leading role in syst.

Input from LQCD at zero-recoil F(1)

**Fom factor parameterizations:**

Caprini-Lellouch-Neubert (CLN) parameterization
Phy. Rev. D 56 (1997) 6895

Boyd-Grinstein-Lebed (BGL) parameterization
Nucl. Phy. B 530 (1998) 152

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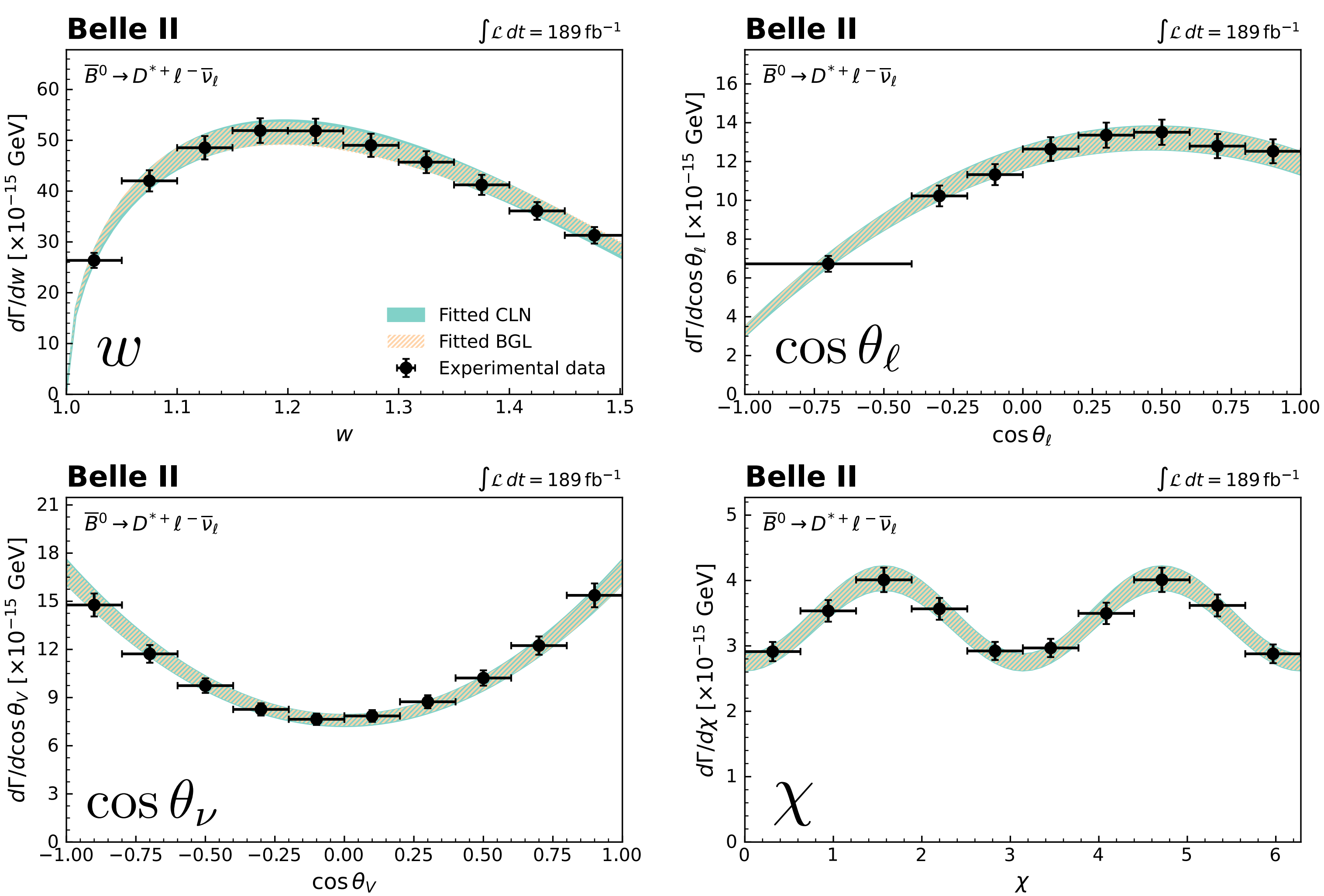


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WA values (HFLAV 21):

$|V_{cb}|_{\text{Excl.}}$ = (39.10 ± 0.50) x $10^{-3}$

$|V_{cb}|_{\text{Incl.}}$ = (42.19 ± 0.78) x $10^{-3}$

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PRD 108, 092013 (2023)

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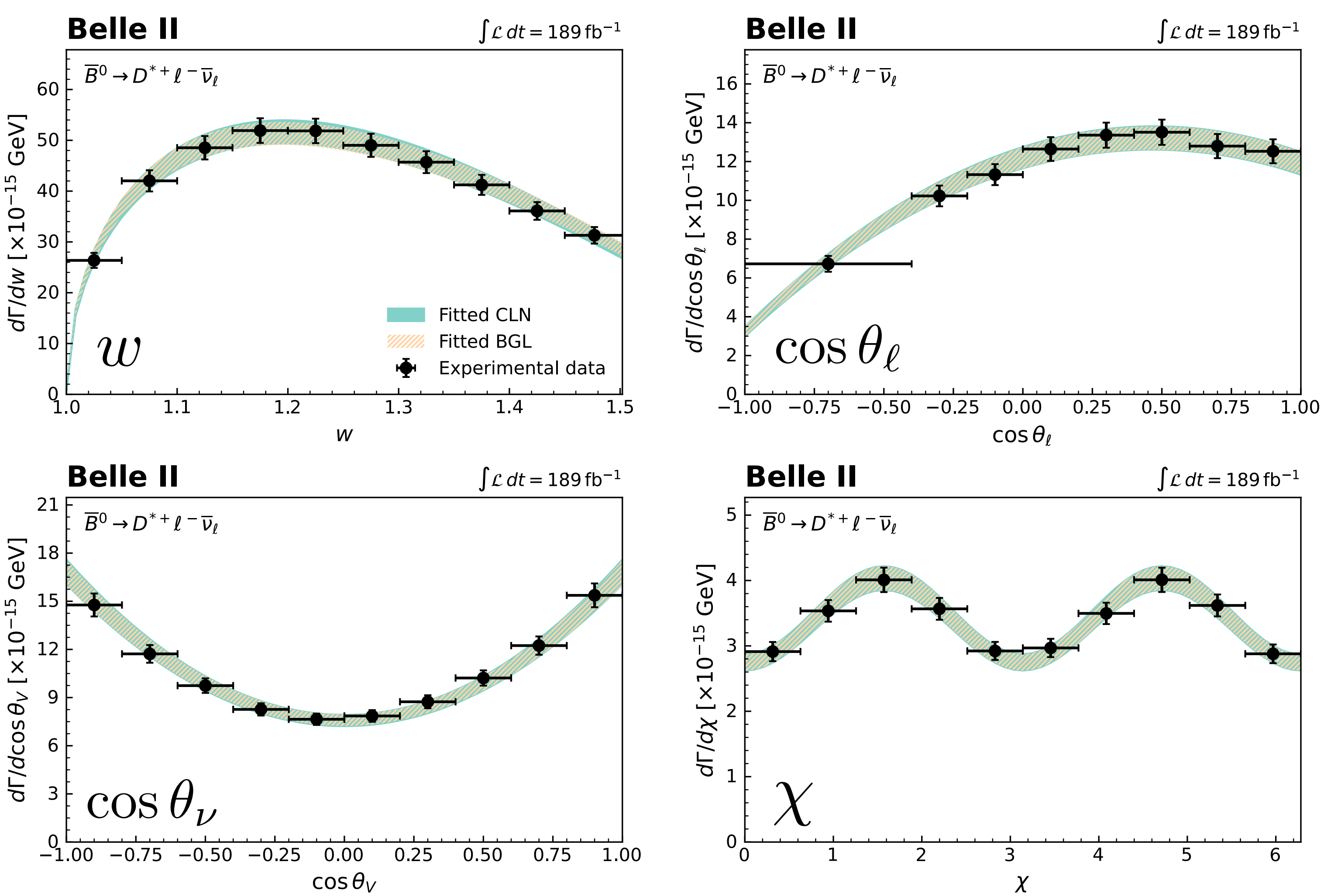


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**Shifts exclusive average closer to inclusive average**

**Fom factor parameterizations:**

Caprini-Lellouch-Neubert (CLN) parameterization
Phy. Rev. D 56 (1997) 6895

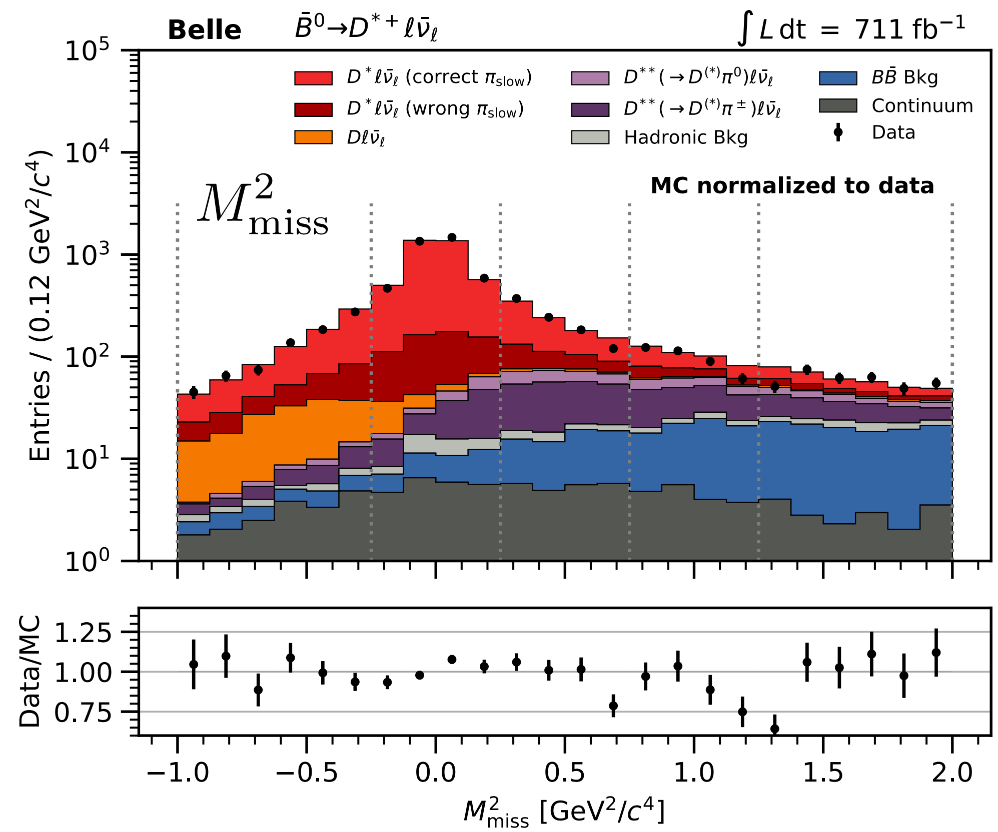
Boyd-Grinstein-Lebed (BGL) parameterization
Nucl. Phy. B 530 (1998) 152

# $|V_{cb}|$ from angular coefficients of $\bar{B} \to D^*\ell\nu$

BELLE

arXiv:2310.20286

- Angular coefficients capture **full differential information** allowing for SM tests —some $J_i = 0$ for SM.
- Hadronic **tagged analysis** using complete Belle dataset of 711 fb$^{-1}$ while implementing **improved** Belle II tagging algorithm.
- Reconstruct $B^+ \to D^{*0}\ell\nu$ and $B^0 \to D^{*+}\ell\nu$ with $D^{*+} \to D^0\pi^+, D^+\pi^0$.
- Determine **signal yields** by fitting the mass of undetected neutrinos in the event: $M^2_{miss} = (p_{e^+e^-} - p_{B_{tag}} - p_{D^*} - p_\ell)^2$.
- Measure 12 angular coefficients $J_i$ in four bins of $w$.




Phys.Rev.D 90 (2014) 9, 094003

$$\bar{J}_i = \frac{1}{N_i}\sum_{j=1}^{8}\sum_{k,l=1}^{4} \eta^{\chi}_{i,j}\eta^{\theta_\ell}_{i,k}\eta^{\theta_V}_{i,l} \left|\chi^{(j)} \otimes \chi^{(k)} \otimes \chi^{(l)}\right|$$

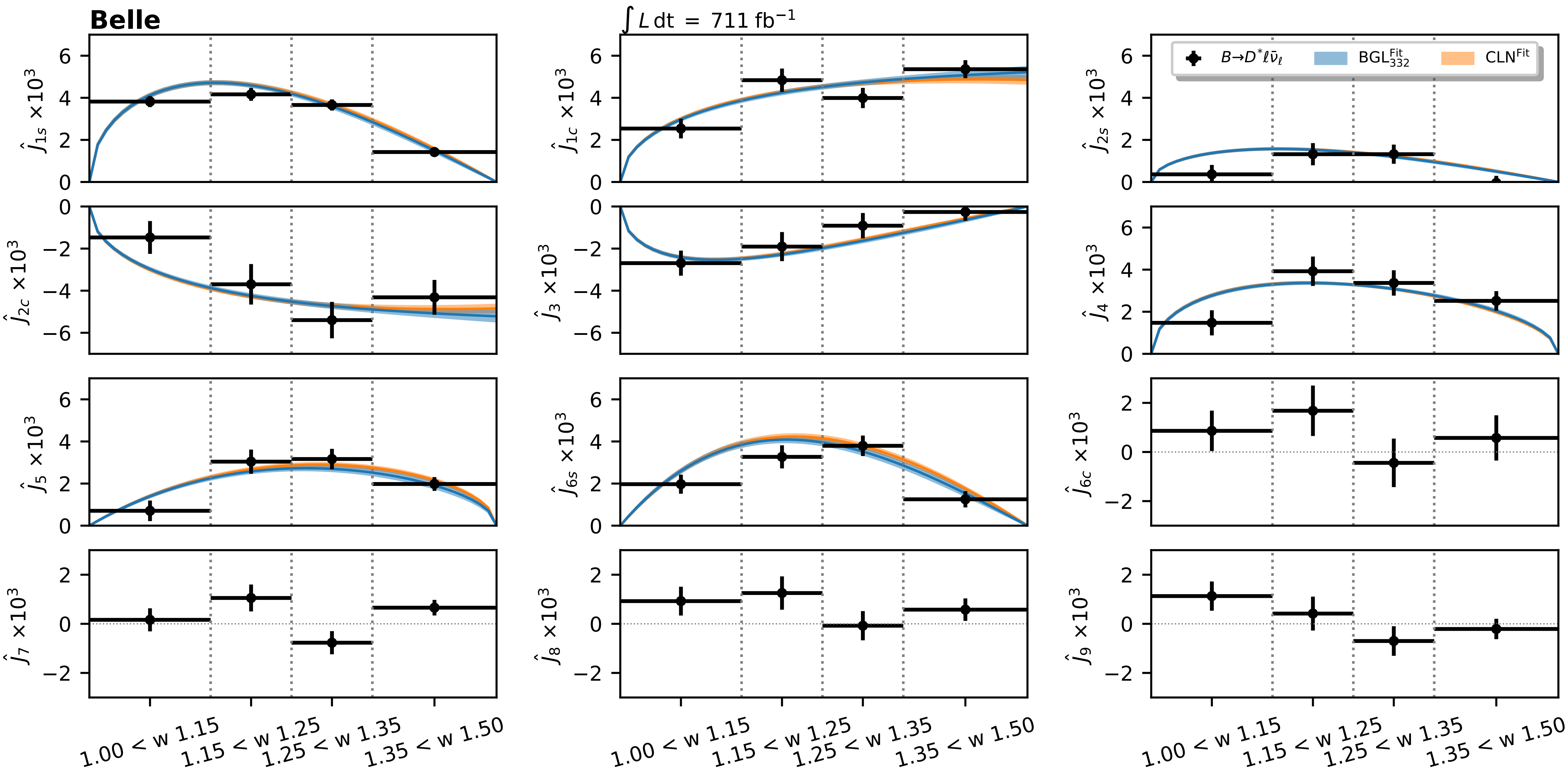
Normalization — Weights — Unfolded Yields

$$\frac{d\Gamma(\bar{B} \to D^*\ell\bar{\nu}_\ell)}{dw\, d\cos\theta_\ell\, d\cos\theta_V\, d\chi} = \frac{2G_F^2\eta_{EW}^2|V_{cb}|^2 m_B^4 m_{D^*}}{2\pi^4} \times \Big(J_{1s}\sin^2\theta_V + J_{1c}\cos^2\theta_V + (J_{2s}\sin^2\theta_V + J_{2c}\cos^2\theta_V)\cos 2\theta_\ell + J_3\sin^2\theta_V\sin^2\theta_\ell\cos 2\chi + J_4\sin 2\theta_V\sin 2\theta_\ell\cos\chi + J_5\sin 2\theta_V\sin\theta_\ell\cos\chi + (J_{6s}\sin^2\theta_V + J_{6c}\cos^2\theta_V)\cos\theta_\ell + J_7\sin 2\theta_V\sin\theta_\ell\sin\chi + J_8\sin 2\theta_V\sin 2\theta_\ell\sin\chi + J_9\sin^2\theta_V\sin^2\theta_\ell\sin 2\chi\Big).$$

# $|V_{cb}|$ from angular coefficients of $\bar{B} \to D^* \ell \nu$

BELLE

arXiv:2310.20286

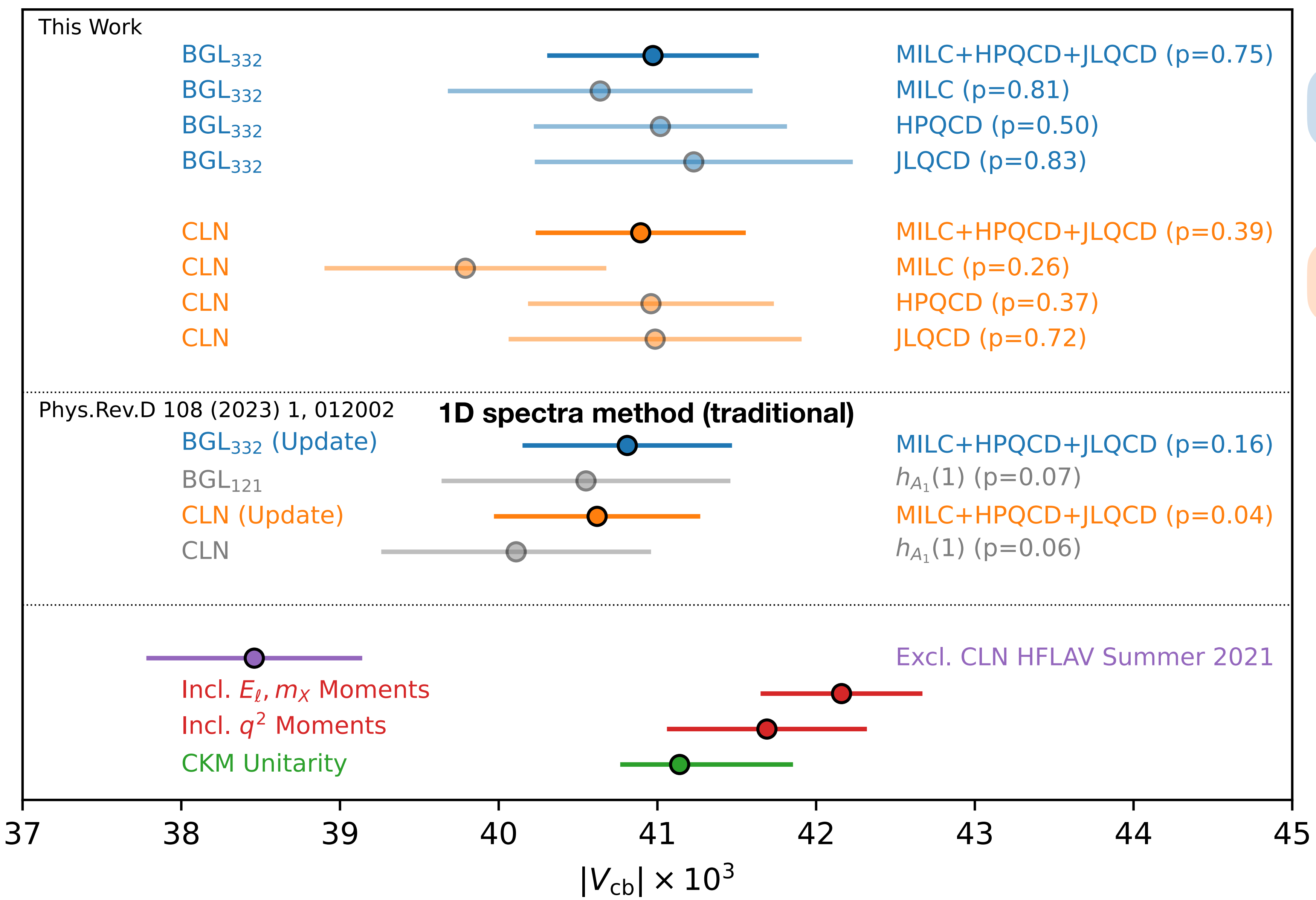


# $|V_{cb}|$ from angular coefficients of $\bar{B} \to D^* \ell \nu$

BELLE

arXiv:2310.20286

- Extract $|V_{cb}|$ with **external constraint on normalisation** (HFLAV 2021) & LQCD beyond zero-recoil.
- Results in agreement with **fits to 1D partial rates** from the same data set: Phys. Rev. D 108 (2023) 012002
- Also agrees with latest and most precise determinations of **inclusive** $|V_{cb}|$.



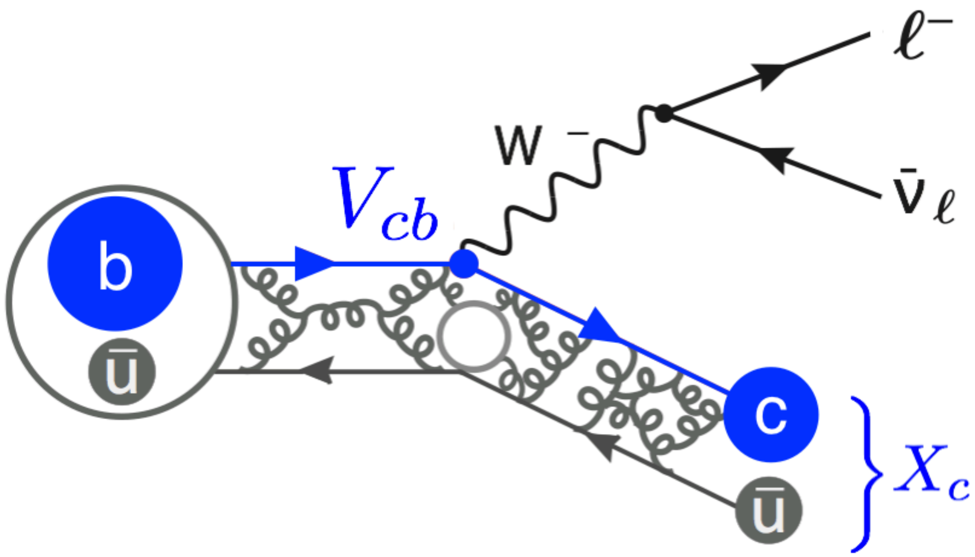

$$|V_{cb}|_{\rm BGL} = (41.0 \pm 0.3 \pm 0.4 \pm 0.5) \times 10^{-3}$$

$$|V_{cb}|_{\rm CLN} = (40.9 \pm 0.3 \pm 0.4 \pm 0.4) \times 10^{-3}$$

- BGL truncation based on nested hypothesis test.
- Systematic uncertainty dominated by limited sample size for deriving migration & efficiency corrections, branching fractions of D decay…

# Inclusive determinations

# Inclusive determinations in a nutshell




Total decay rate **determined** from Heavy Quark Expansion (HQE)

$$\mathcal{B} = |V_{qb}|^2 \left[ \Gamma(b \to q\,\ell\,\bar{\nu}_\ell) + 1/m_{c,b} + \alpha_s + \ldots \right]$$

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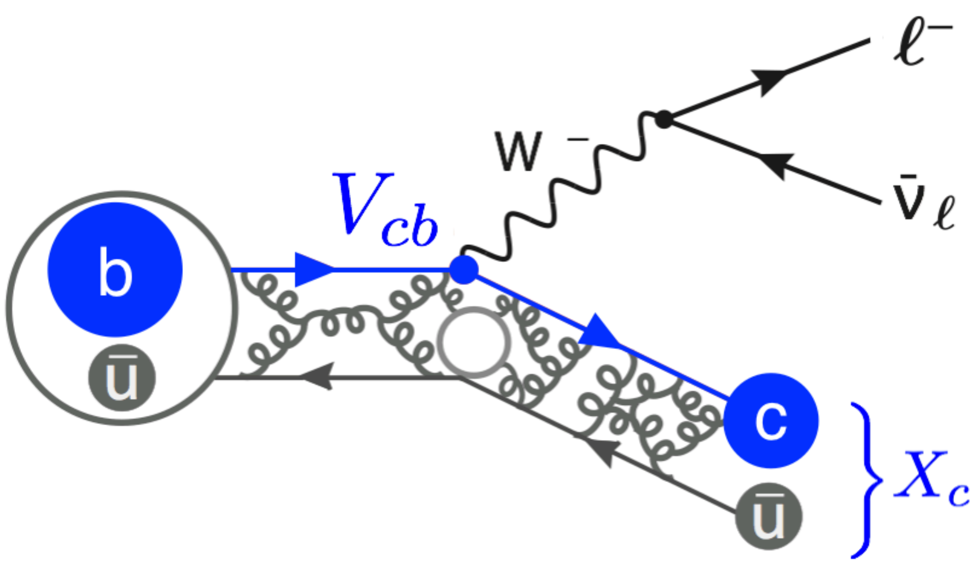


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How to determine $V_{cb}$ and the HQE matrix elements in 2 **easy** steps:

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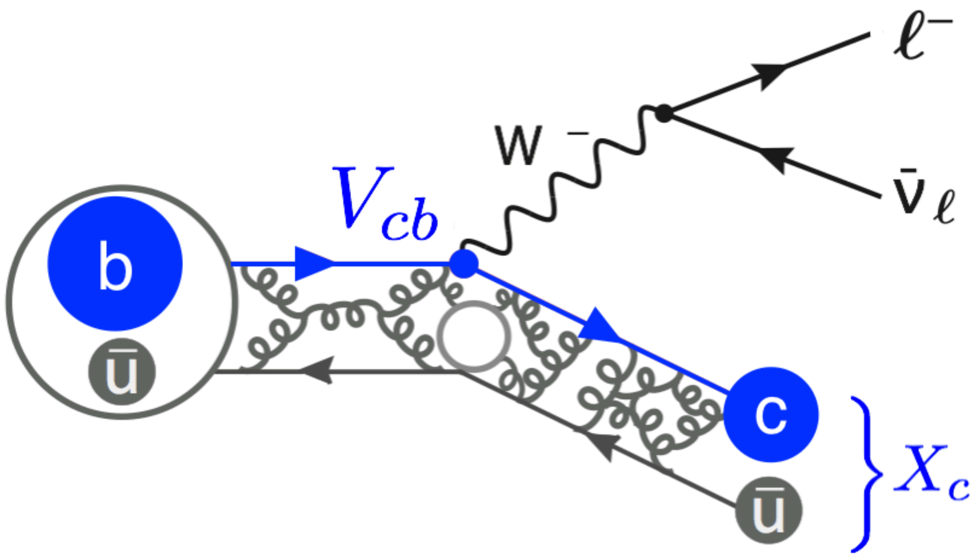


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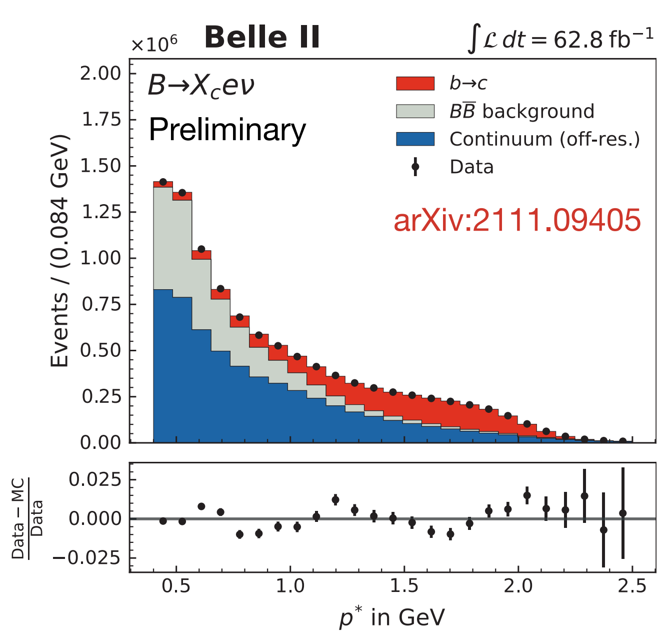
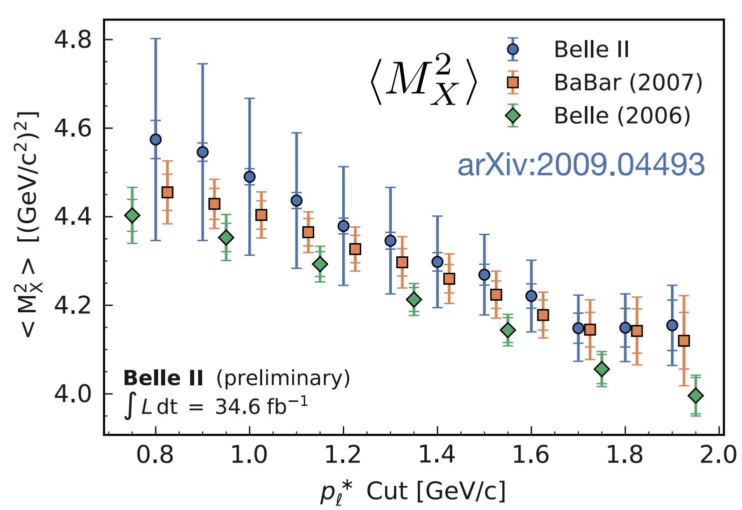
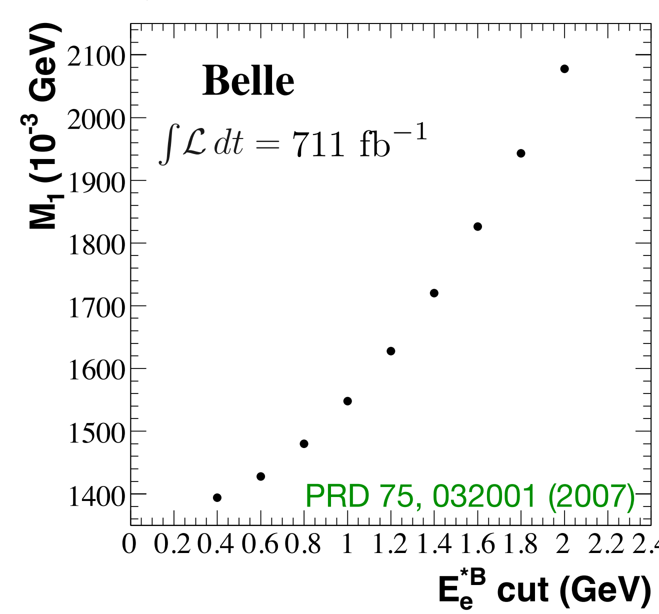
Step 1: Measure spectral moments.

$\Gamma(B \to X_c \ell \nu_\ell)$ $\langle M_X^n \rangle$ $\langle E_\ell^n \rangle$



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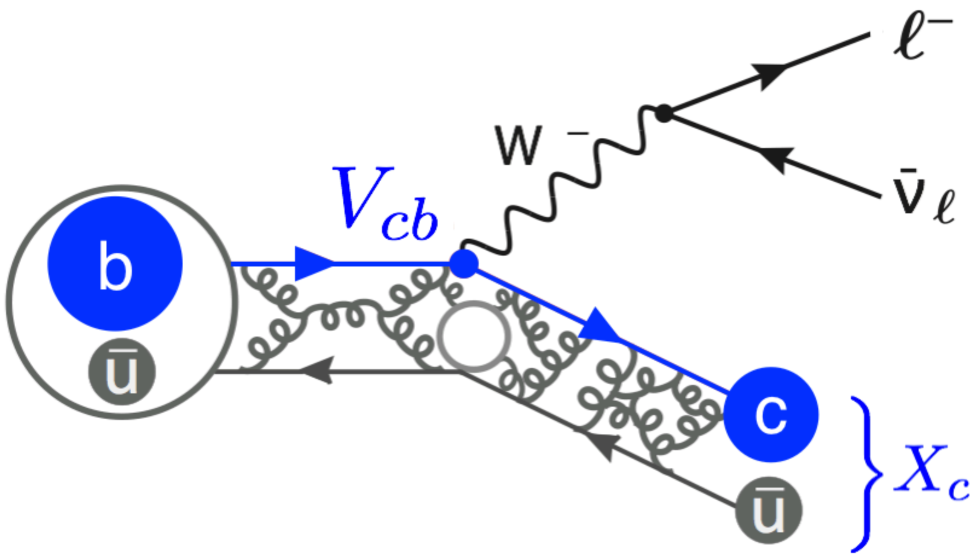


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↓

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$$d\Gamma = d\Gamma_0 + d\Gamma_{\mu_\pi} \frac{\mu_\pi^2}{m_b^2} + d\Gamma_{\mu_G} \frac{\mu_G^2}{m_b^2} + d\Gamma_{\rho_D} \frac{\rho_D^3}{m_b^3} + d\Gamma_{\rho_{LS}} \frac{\rho_{LS}^3}{m_b^3} + \mathcal{O}(1/m_b^4)$$

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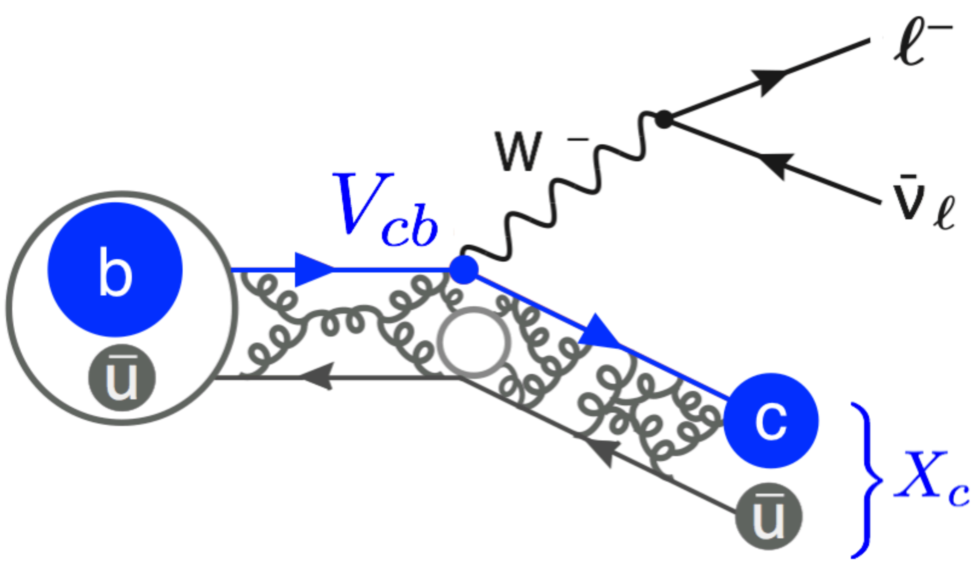


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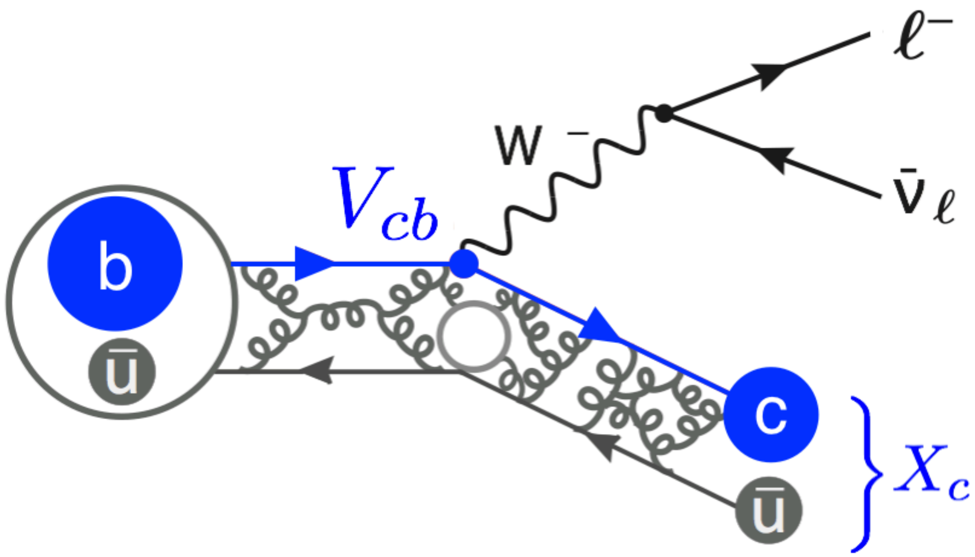
$d\Gamma$ are calculated perturbatively.

See Maria Laura's talk for comprehensive overview.

# Inclusive determinations in a nutshell

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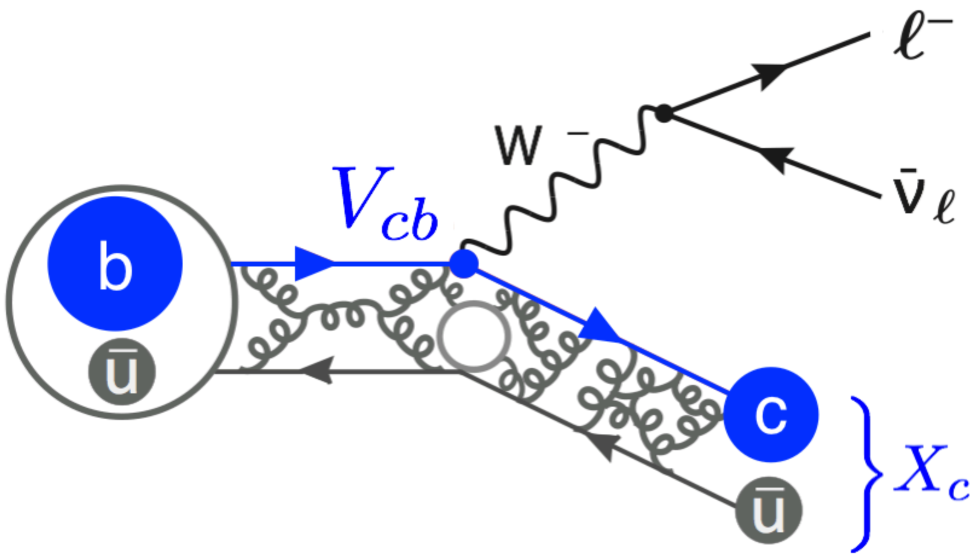
HQE elements encapsulate non-perturbative dynamics.

# Inclusive determinations in a nutshell

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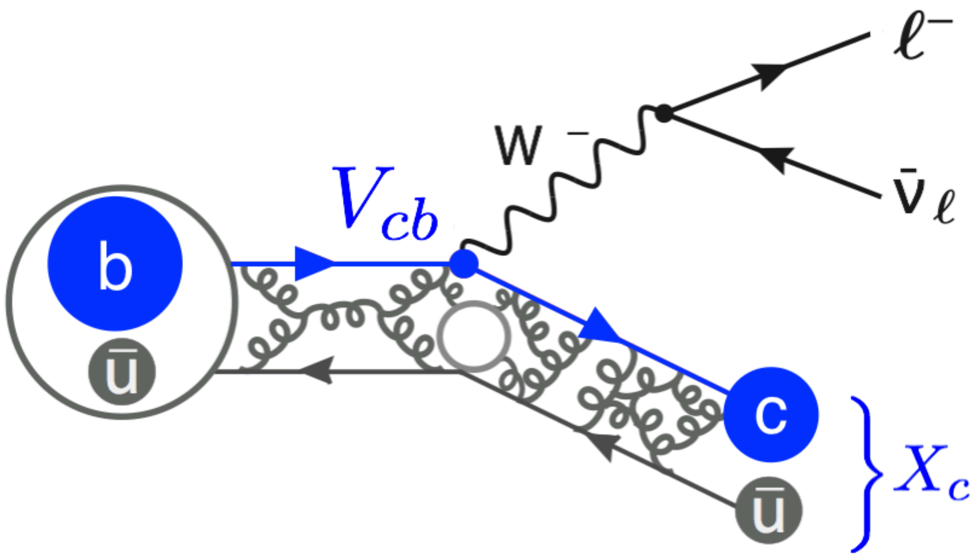
↓

$V_{cb}!$ ← Not factorial!

# Inclusive determinations in a nutshell

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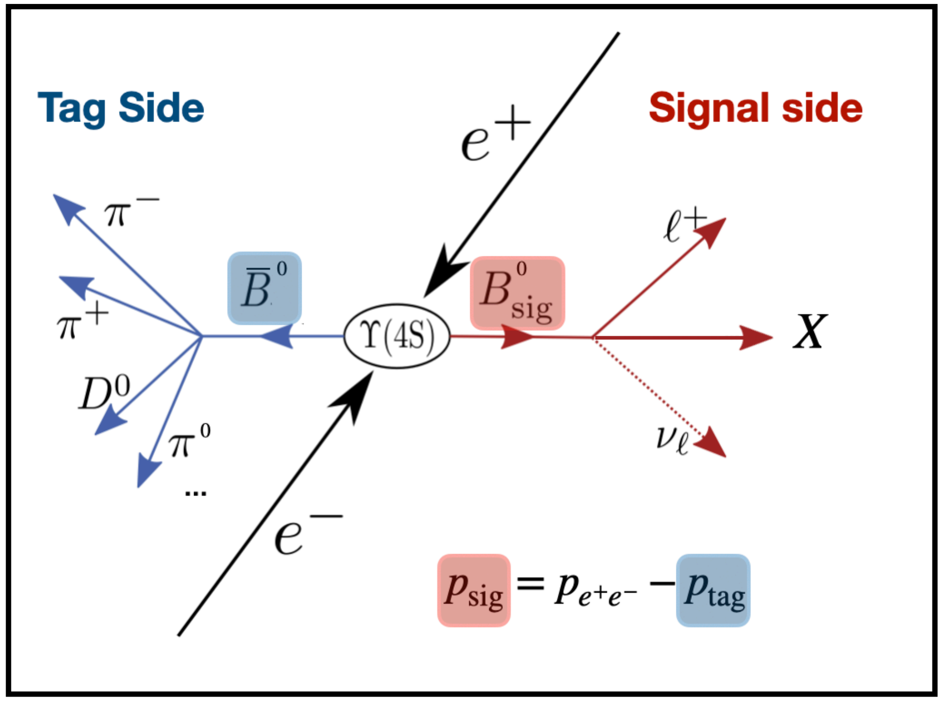
Parameters proliferate at higher orders. Can be reduced with reparametrization invariance.

↓

$V_{cb}$!

# Inclusive reconstruction

**Key-technique**: hadronic tagging



# Inclusive reconstruction

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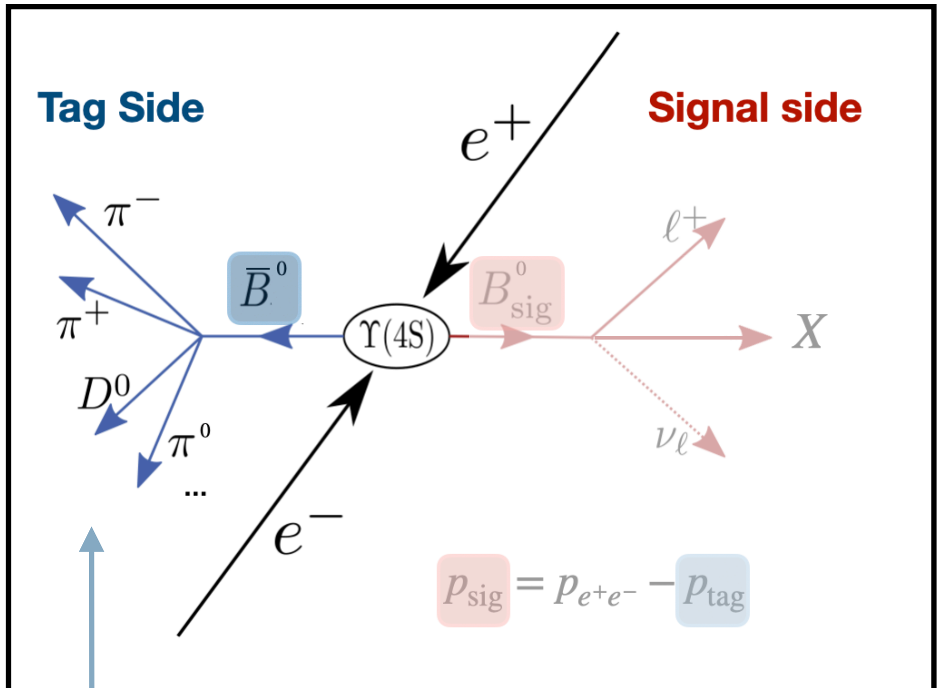
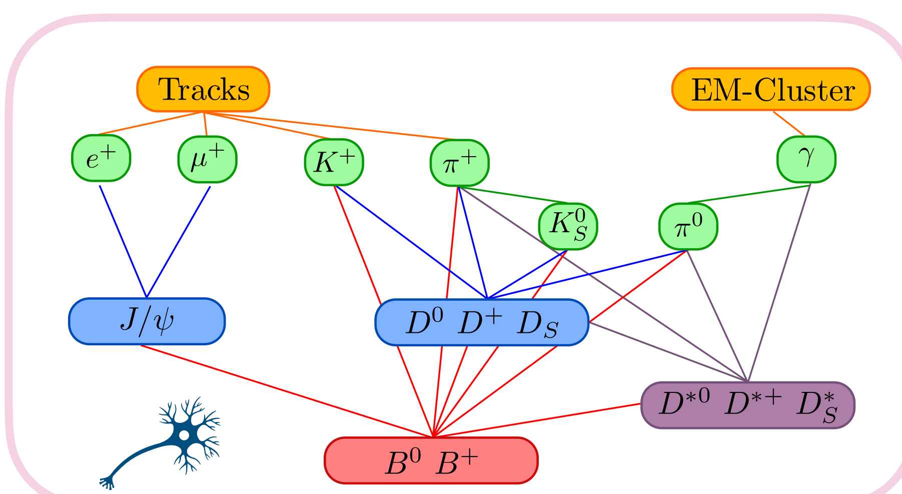



**Full Reconstruction** =
Belle tagging algorithm
(Efficiency: 0.28% / 0.18% for $B^\pm$ & $B^0/\bar{B}^0$)

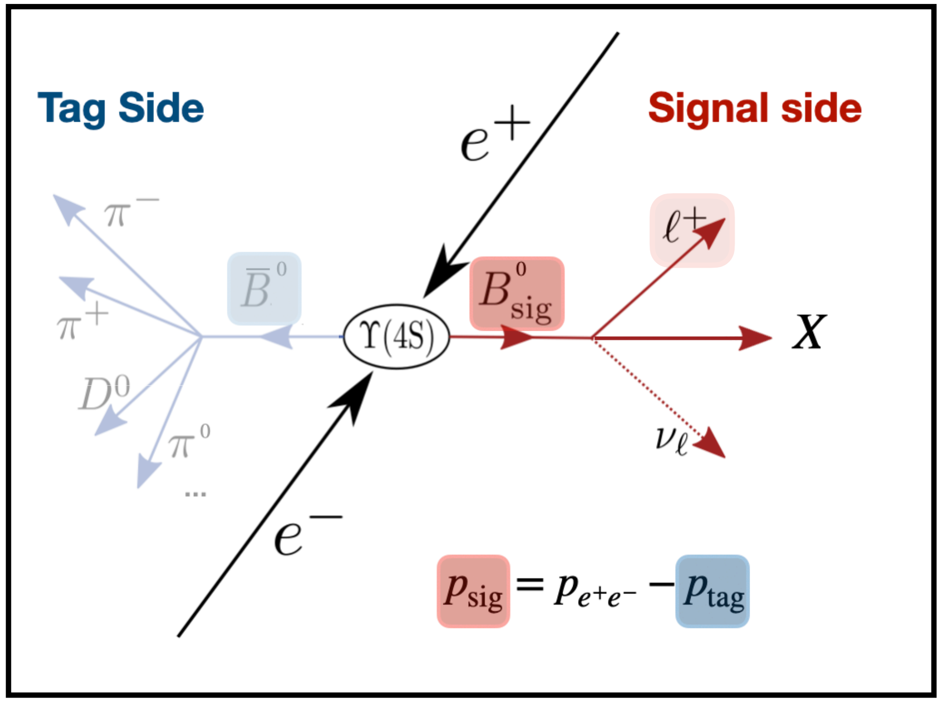
**Full Event Interpretation** =
Belle II tagging algorithm
(Efficiency: 0.6% /0.3% for $B^\pm$ & $B^0/\bar{B}^0$)

Candidates reconstructed using a **hierarchical** approach & **neural networks** in **hadronic modes**



# Inclusive reconstruction

**Key-technique**: hadronic tagging




**Full Reconstruction** =
Belle tagging algorithm
(Efficiency: 0.28% / 0.18% for $B^{\pm}$ & $B^0/\bar{B}^0$)

**Full Event Interpretation** =
Belle II tagging algorithm
(Efficiency: 0.6% /0.3% for $B^{\pm}$ & $B^0/\bar{B}^0$)

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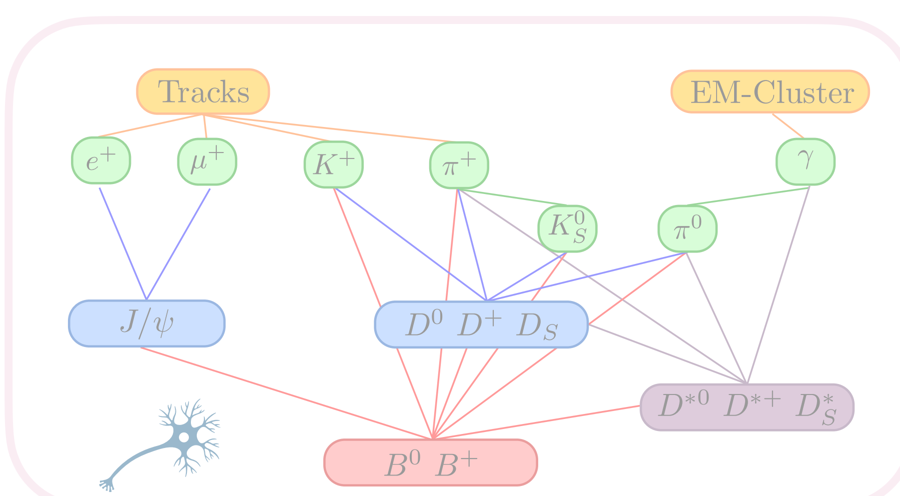

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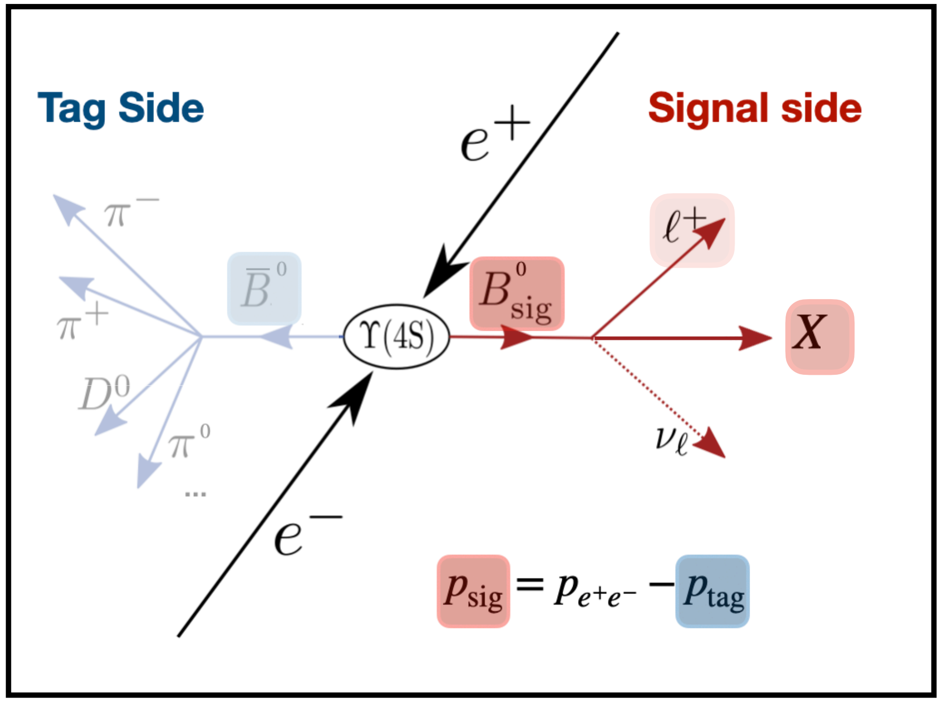


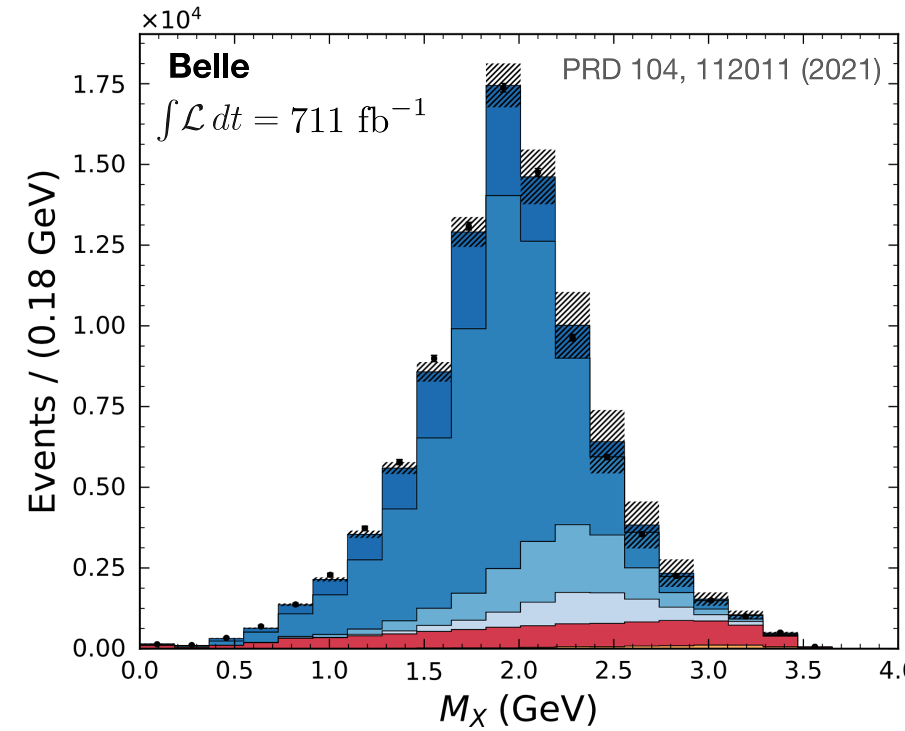

**Can identify X constituents**

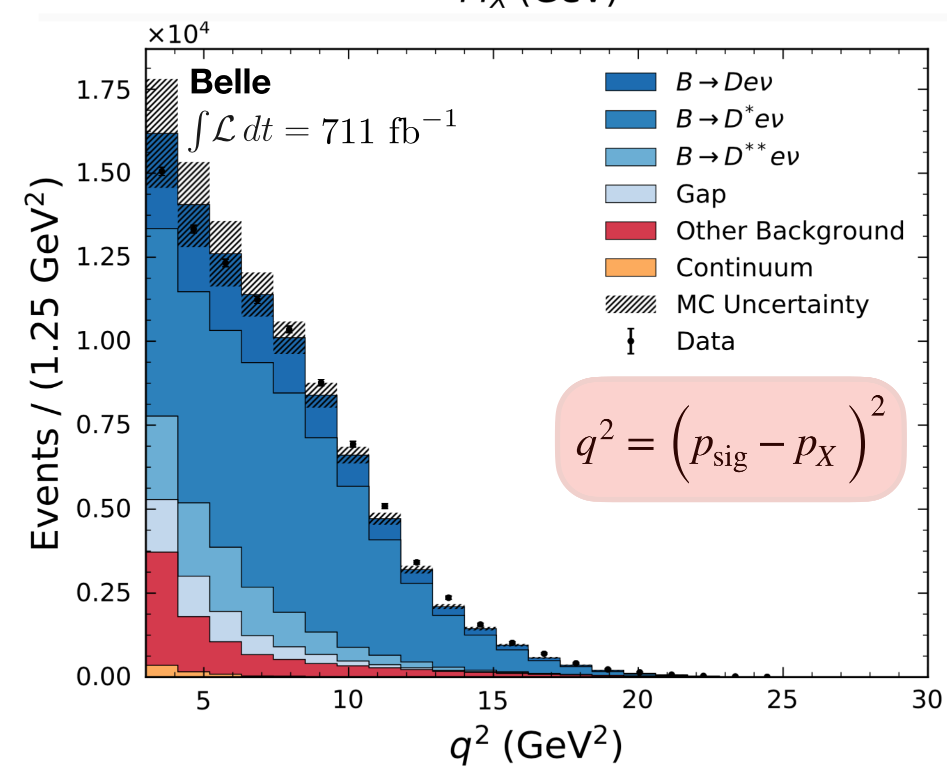
$$M_X = \sqrt{(p_X)^\mu (p_X)_\mu}$$

Charged Tracks

Neutral Clusters

$$p_X = \sum_i \left( \sqrt{m_\pi^2 + |\mathbf{p}_i|^2}, \mathbf{p}_i \right) + \sum_j (E_j, \mathbf{k}_j)$$

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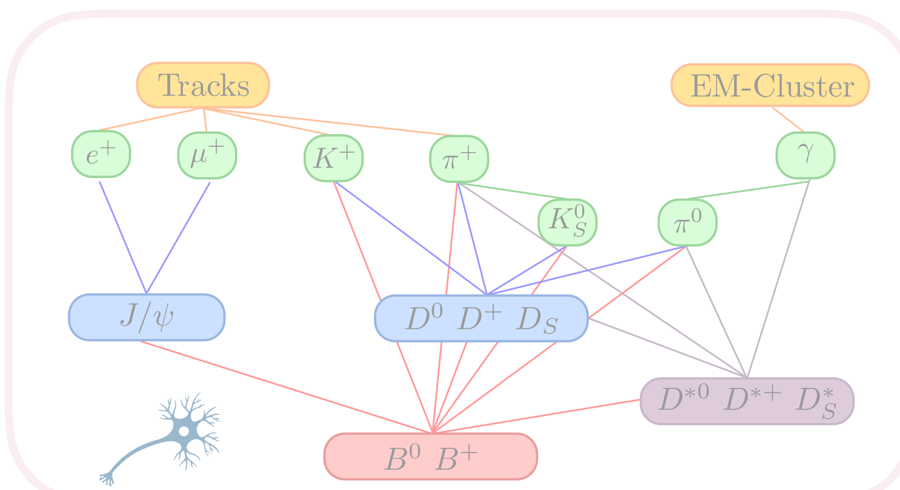
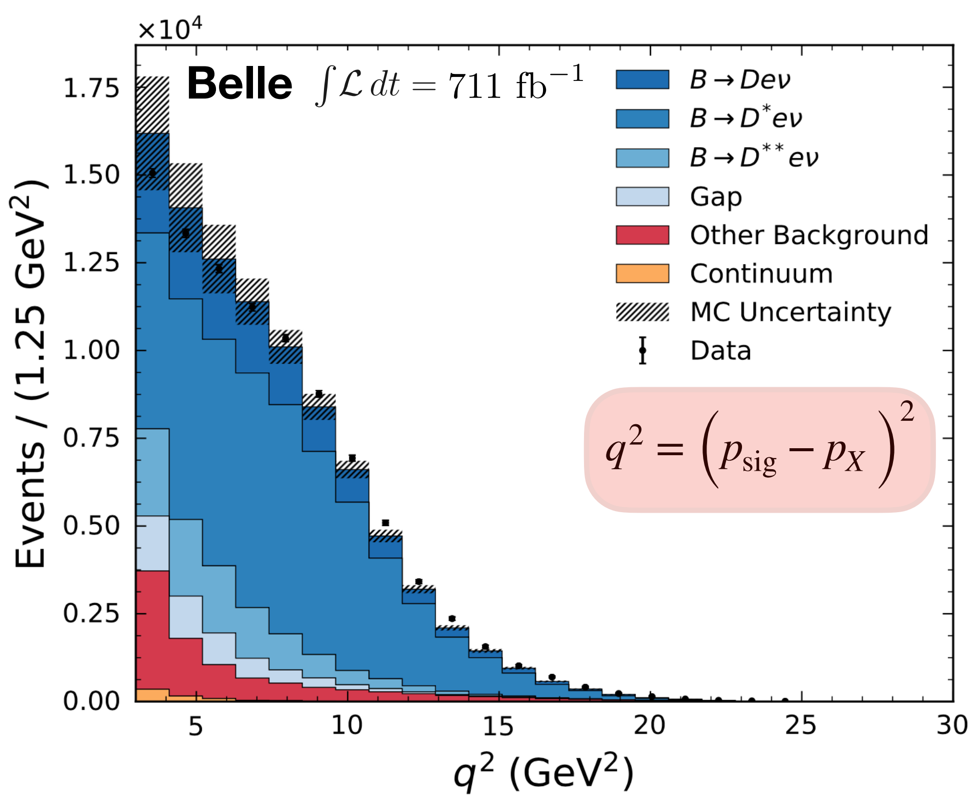


# Incl. $|V_{cb}|$ from $q^2$ moments

BELLE

PRD 104, 112011 (2021)

- **Novel theoretical approach** to determine incl. $|V_{cb}|$ with a reduced set of higher order HQE parameters at $\mathcal{O}(1/m_b^4)$ in a completely data-driven approach. JHEP 02 177 (2019)
- Requires the reconstruction of $q^2$ for $B \to X_c \ell \nu_\ell$ decays
  - Only possible through **hadronic tagging at B-factories!**
- **Main challenge**: non-resonant $X_c \ell \nu_\ell$ 'gap' modelling. (See Florian's talk for new insights.)




How to measure moments:

$$\langle q^{2n} \rangle = \frac{\sum w_i(q^2)(q^{2n}_{\mathrm{calib},i})}{\sum_i w_i(q^2)} \times \mathcal{C}_{\mathrm{cal}} \times \mathcal{C}_{\mathrm{acc}}$$

- Calibrated $q^2$ dist. accounting for data/MC differences
- Residual bias corr. factor
- Background subtraction weights
- Correct for resolution & selection effects

# Incl. $|V_{cb}|$ from $q^2$ moments

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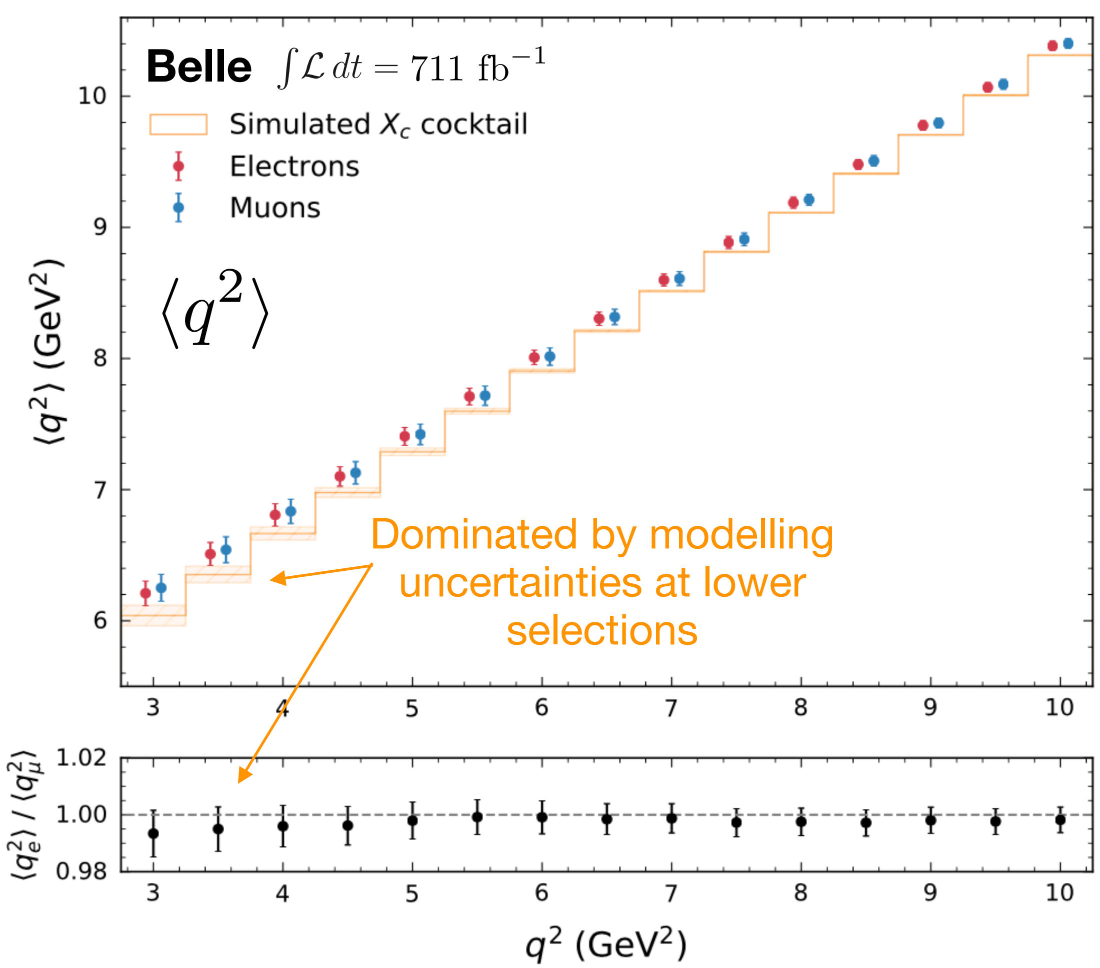


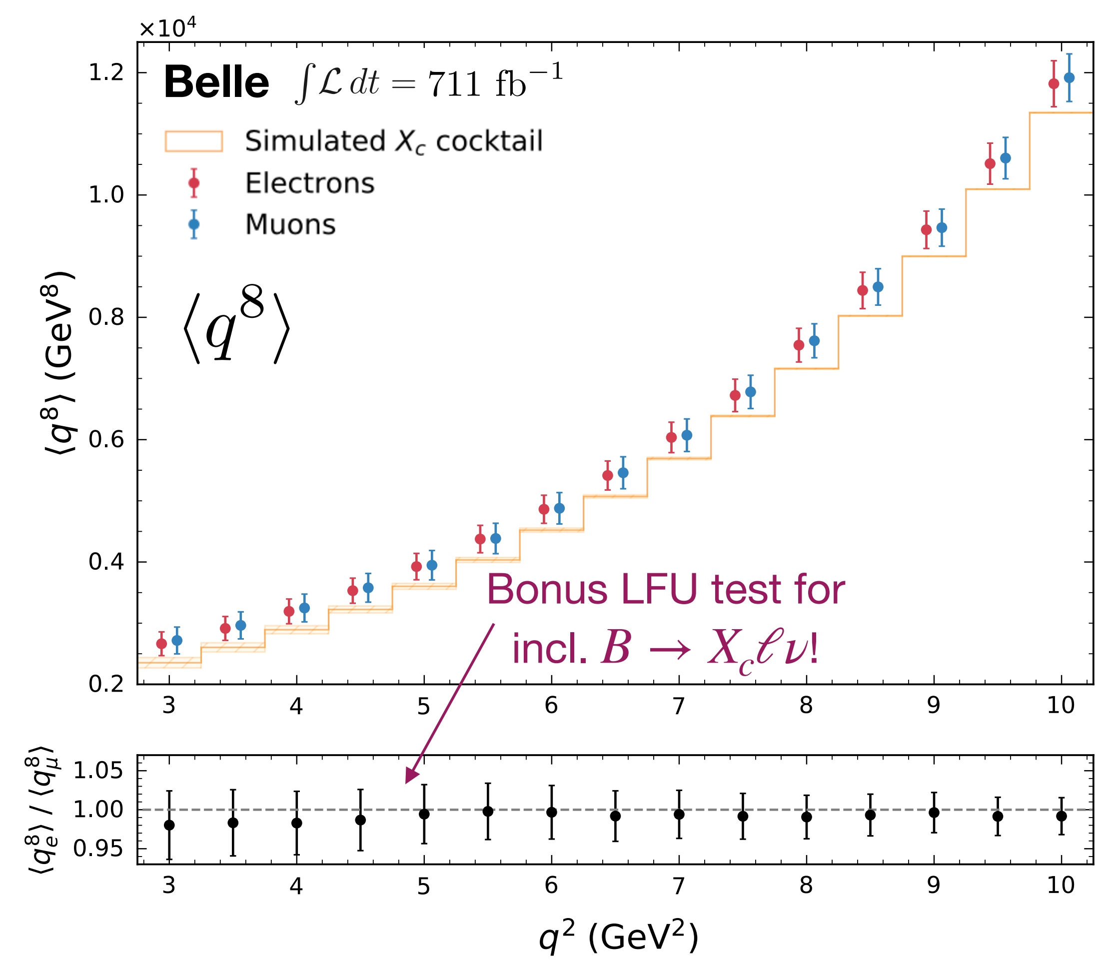

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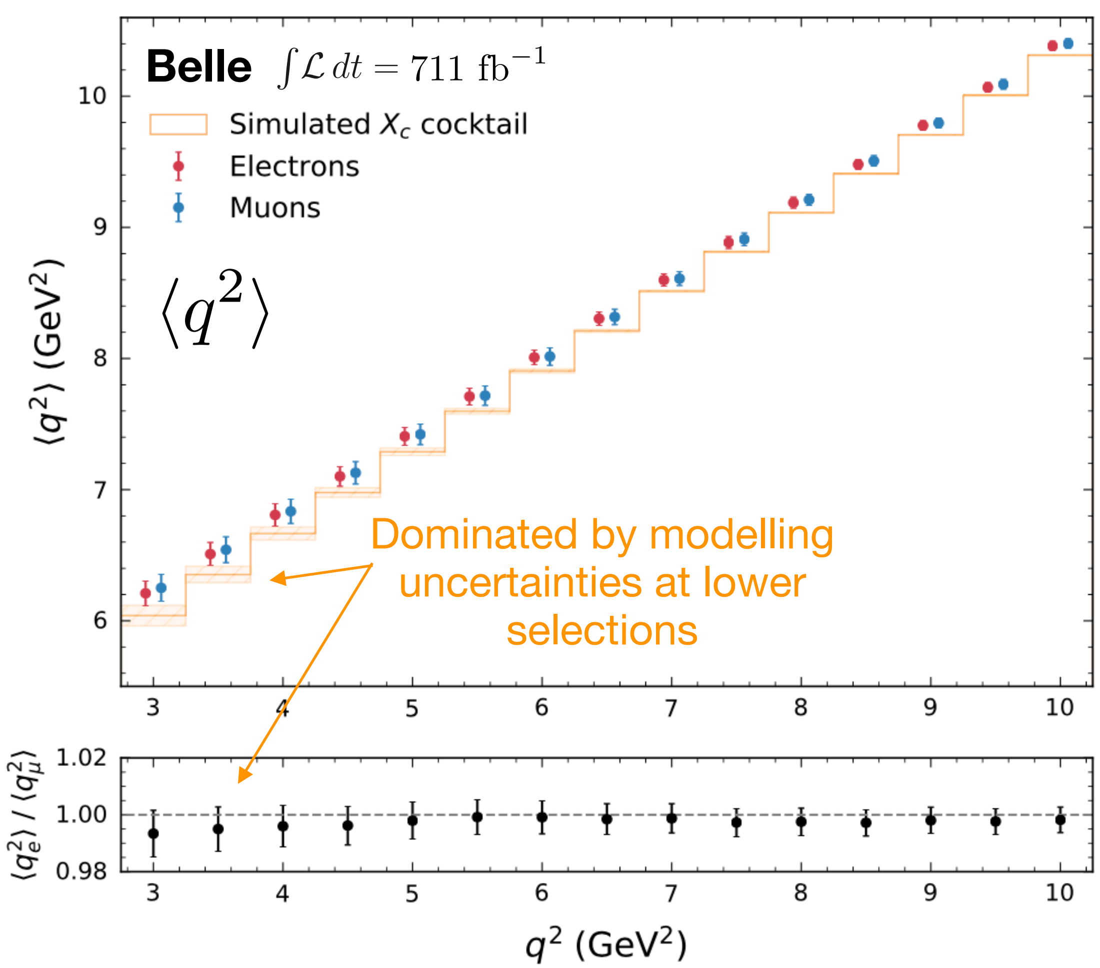


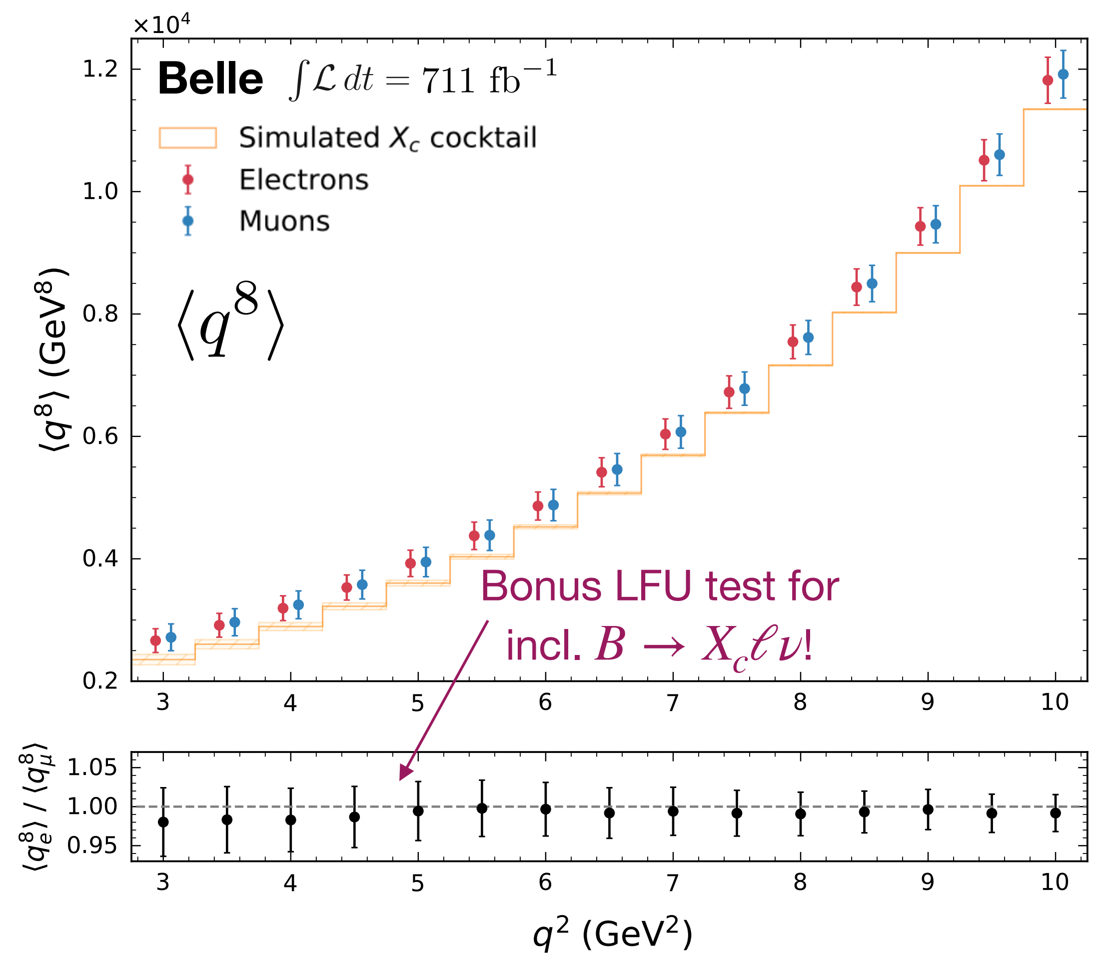


See Keri's talk for details!

- Combined Belle & Belle II fit:

$$|V_{cb}| = (41.69 \pm 0.63) \cdot 10^{-3}$$

Belle II moments: Phy. Rev. D 107 (2023) 7, 072002

# Ratio of $|V_{ub}|$ and $|V_{cb}|$ from inclusive decays

BELLE

arXiv:2311.00458

- Measuring $B \to X_u \ell \nu$ is challenging due to large background component from $B \to X_c \ell \nu$.
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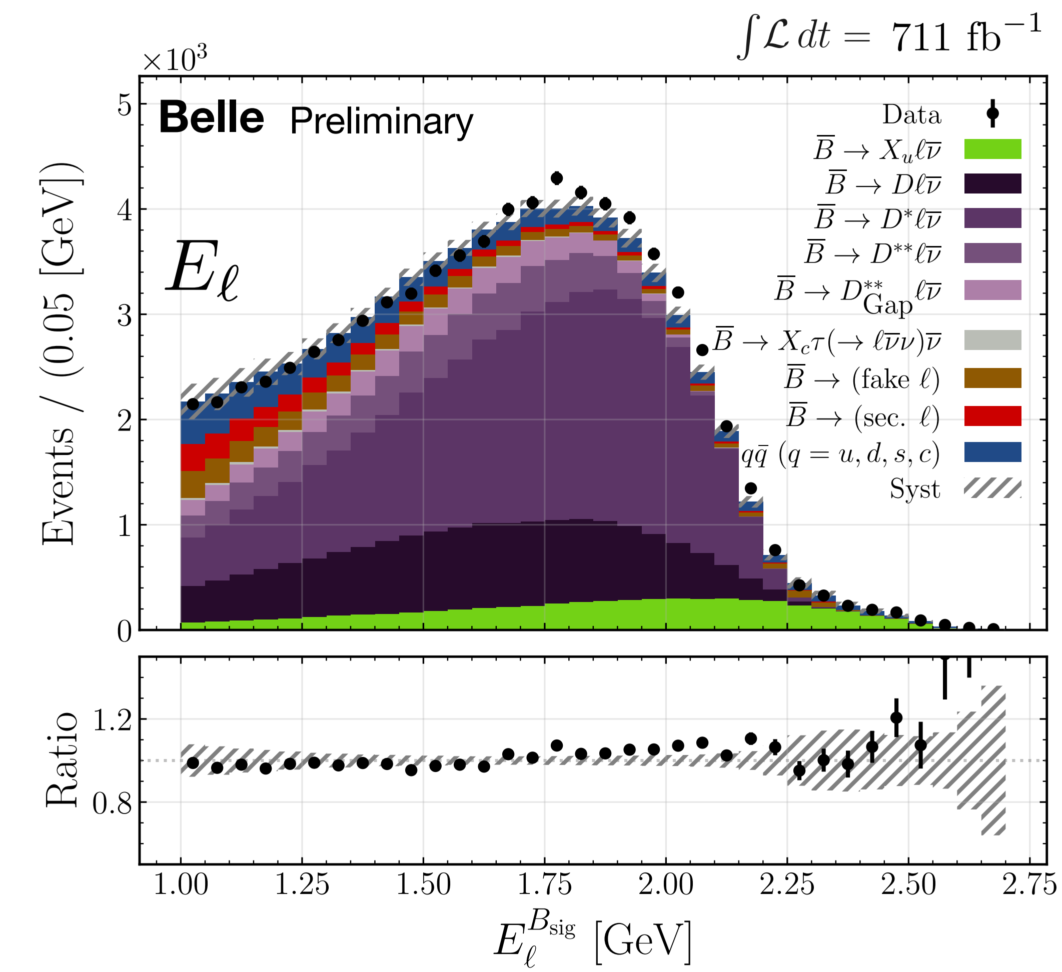


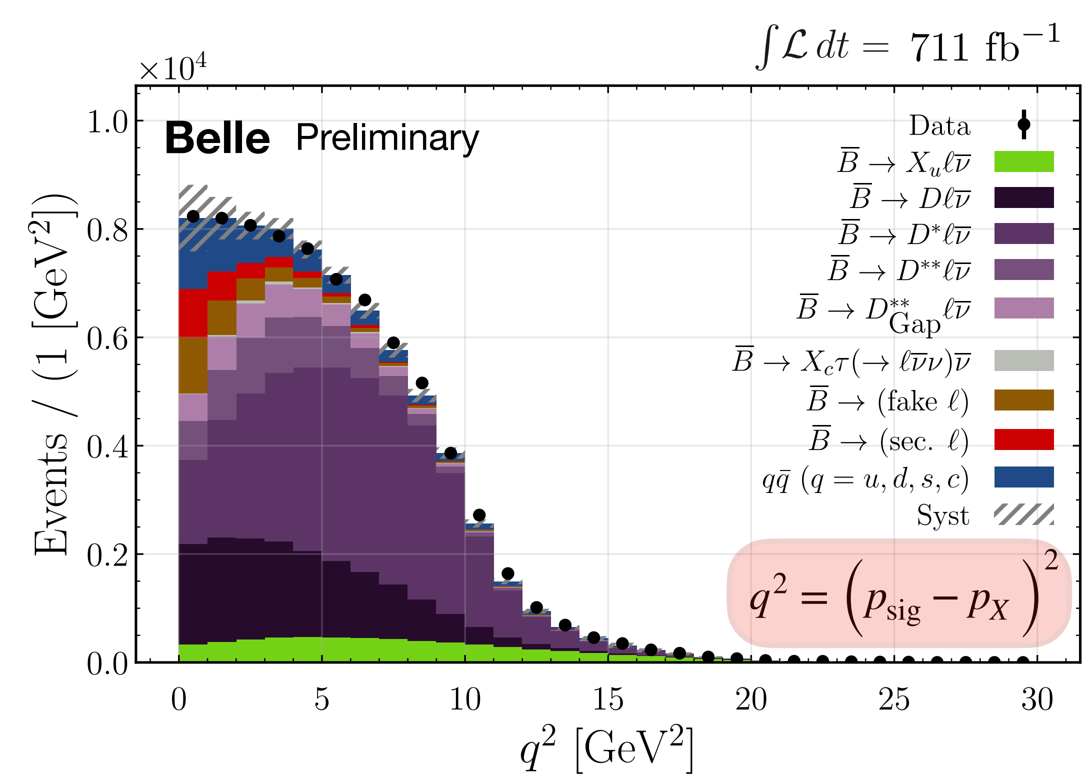

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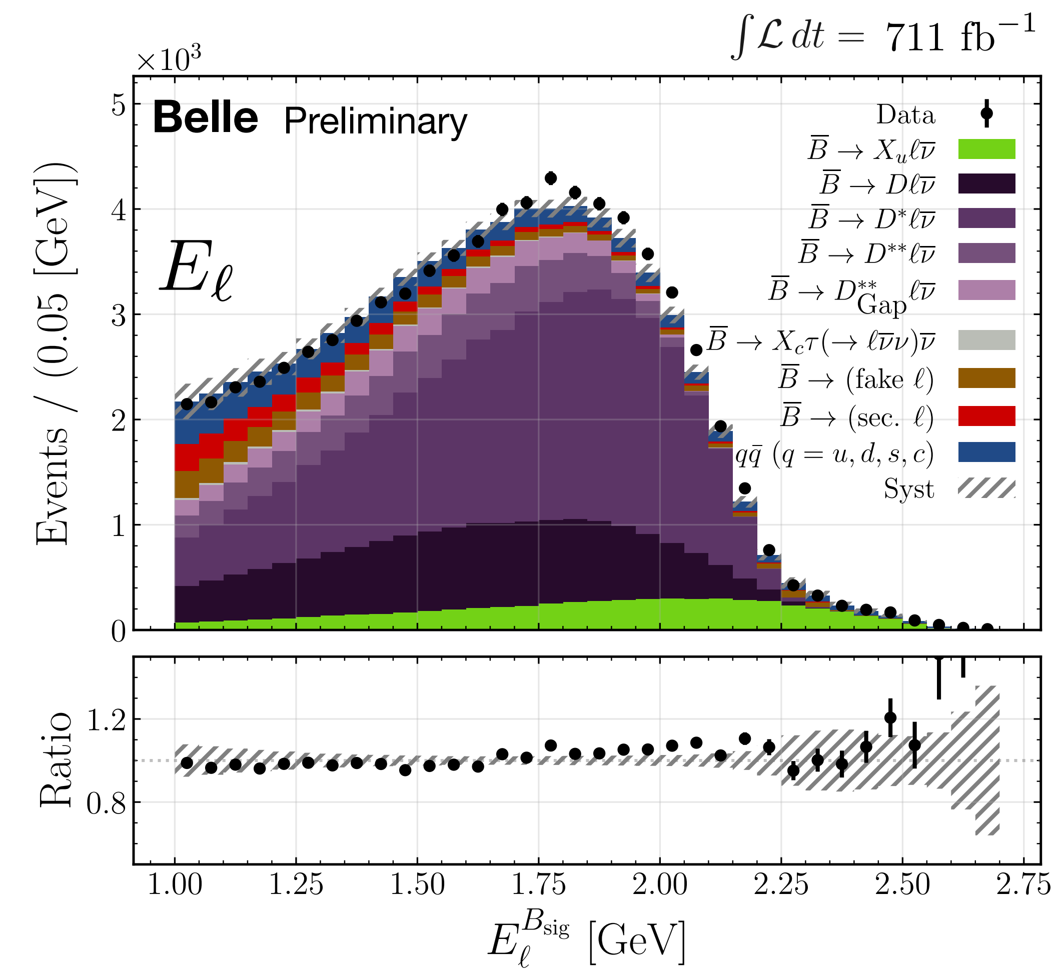


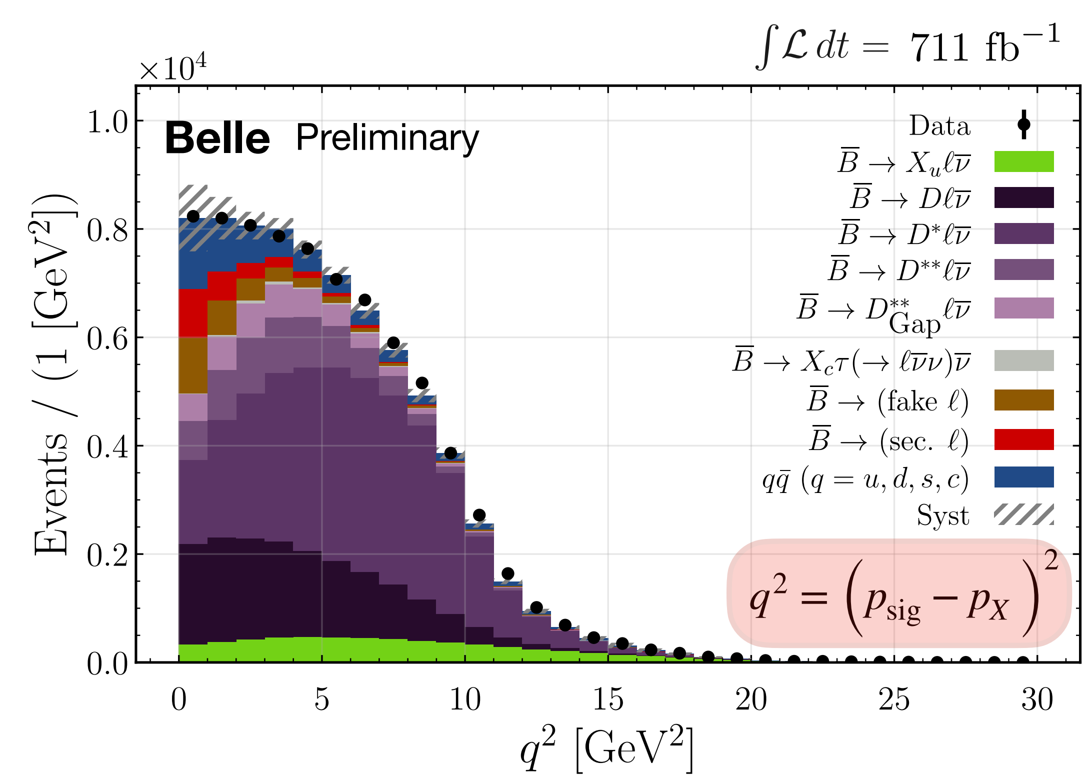

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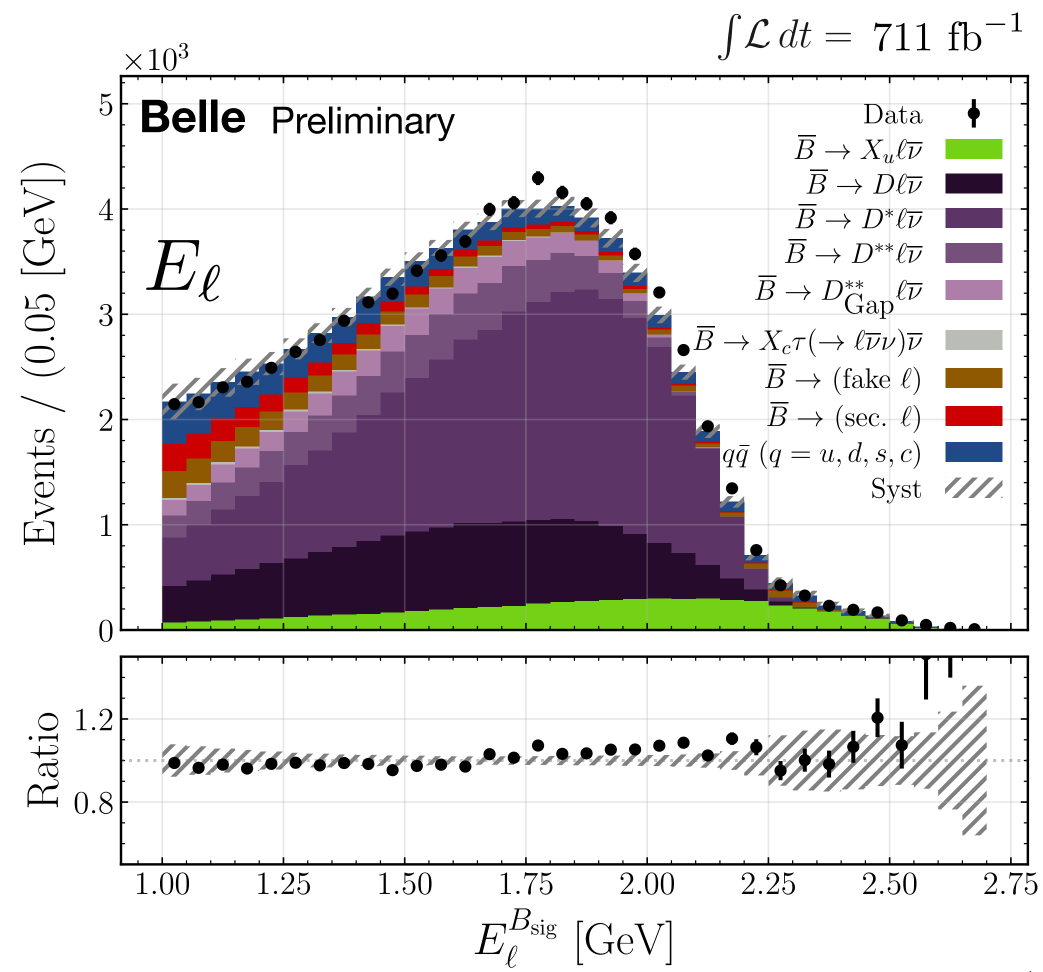


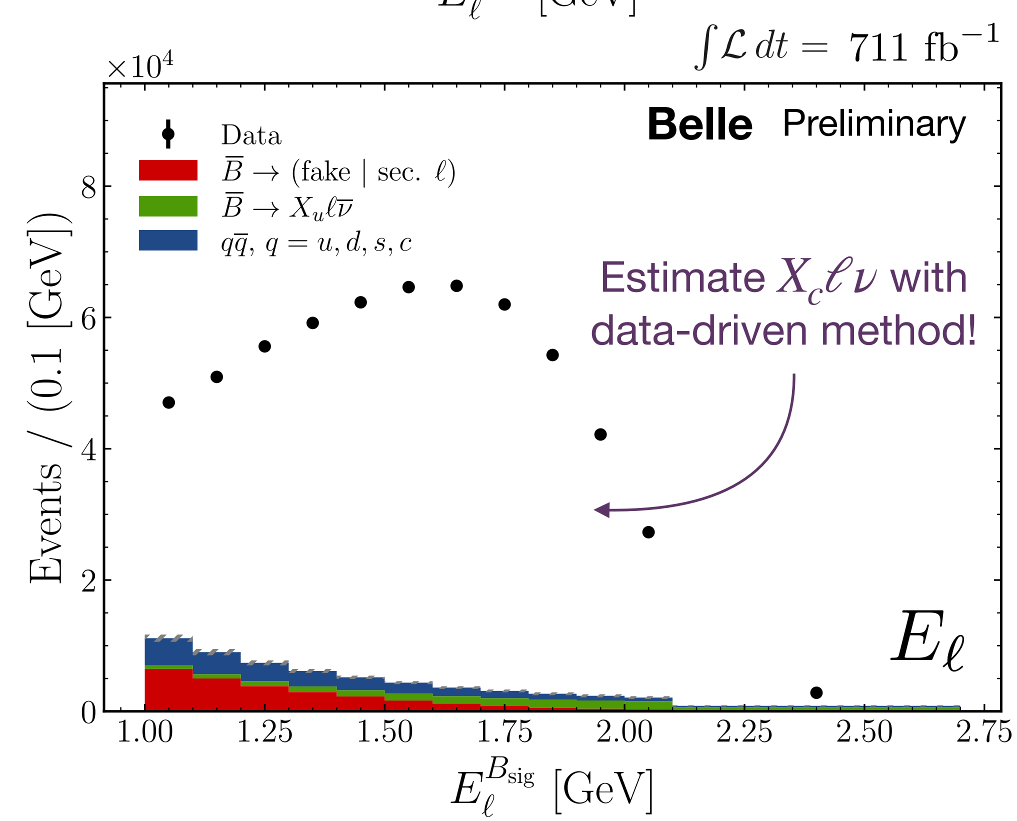

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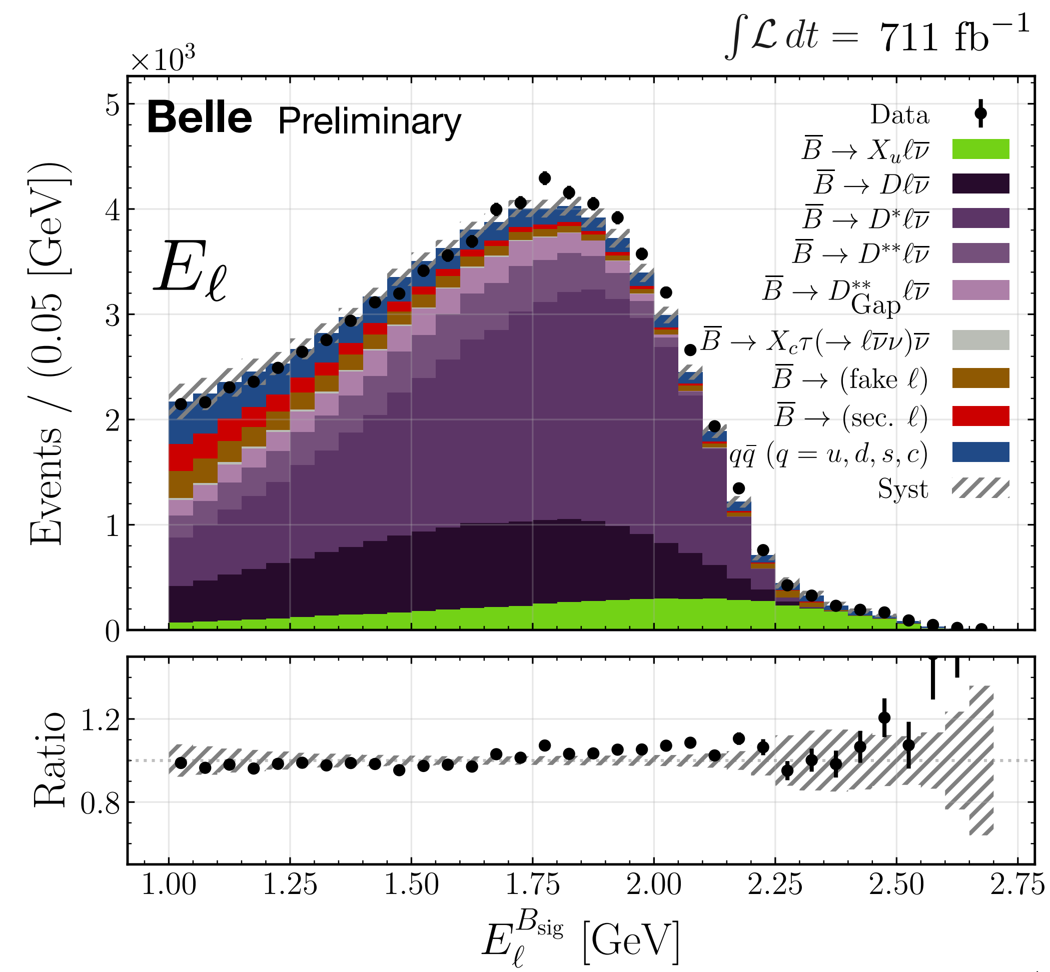


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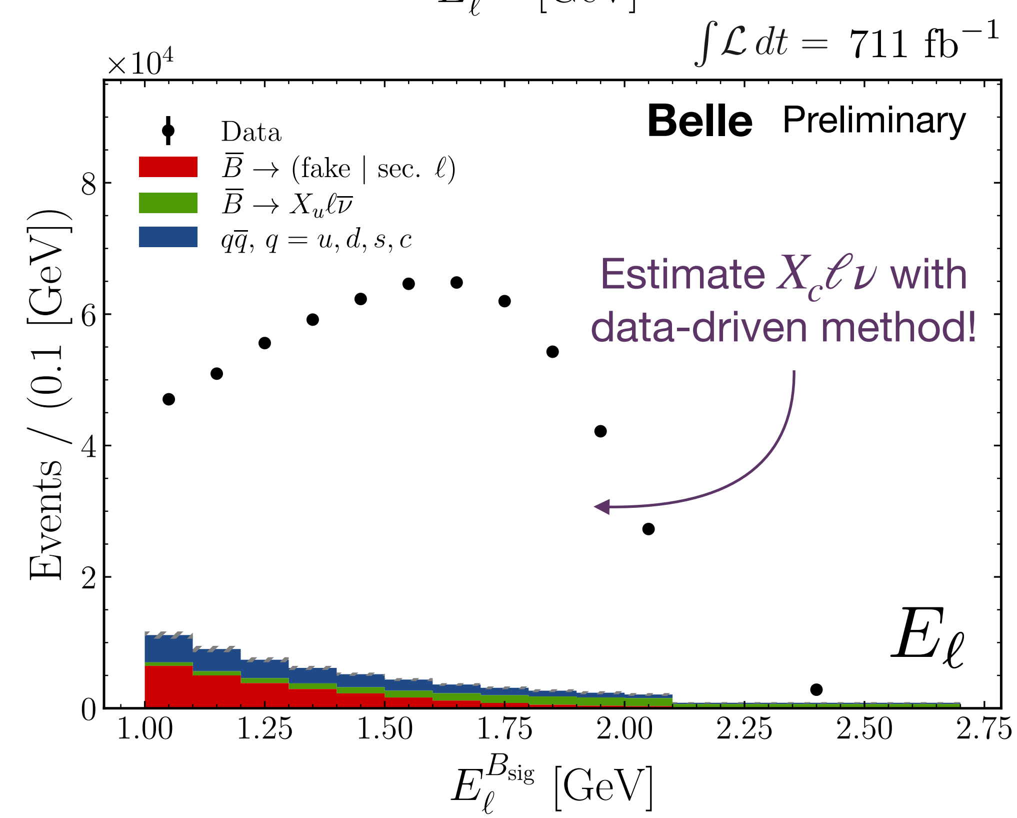



Measure partial BF for the region $E_\ell^B > 1$ GeV:

$$\frac{\Delta\mathcal{B}(\bar{B} \to X_u \ell \bar{\nu})}{\Delta\mathcal{B}(\bar{B} \to X_c \ell \bar{\nu})} = 1.96(1 \pm 8.4\%_{\mathrm{stat}} \pm 7.9\%_{\mathrm{syst}}) \times 10^{-2}$$

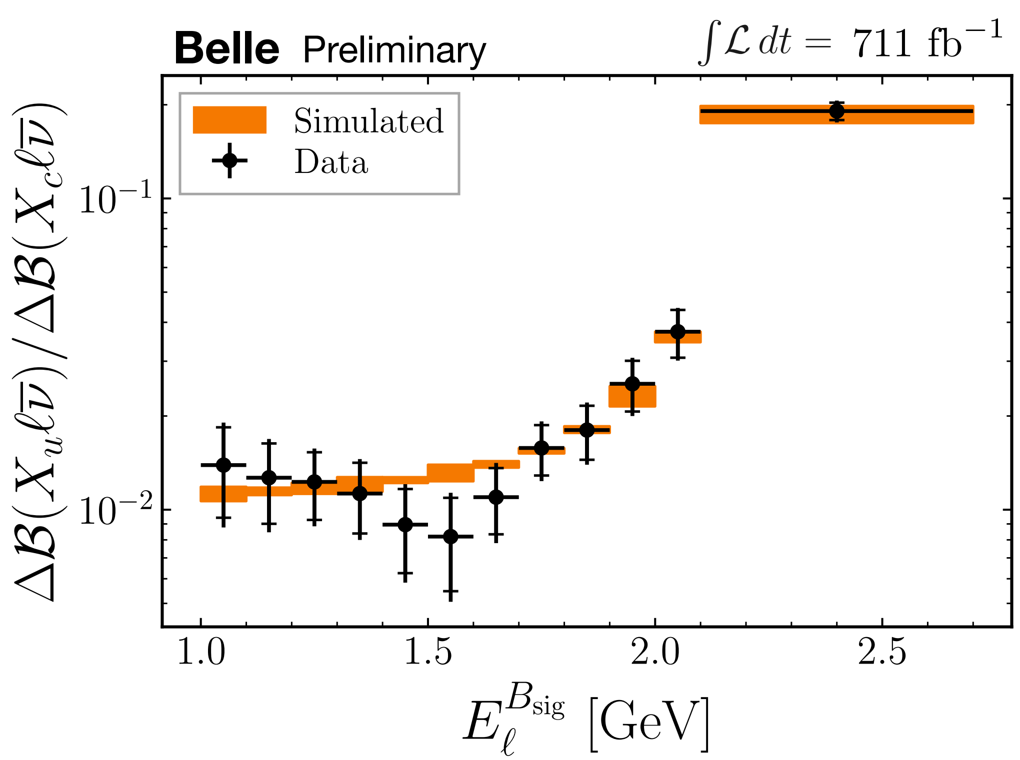
Leading systematics:
Modelling of $B \to X_u \ell \nu$ component &
composition of fake leptons and secondary decays

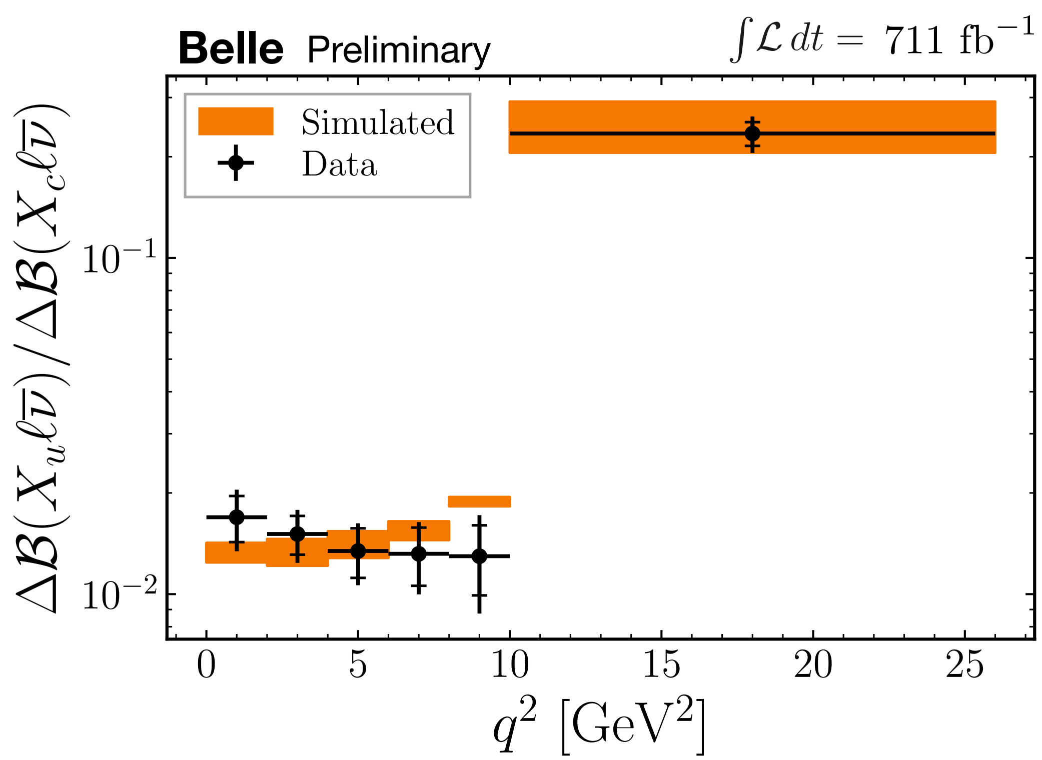
# Ratio of $|V_{ub}|$ and $|V_{cb}|$ from inclusive decays

BELLE

arXiv:2311.00458

- Unfold $B \to X_u \ell \nu$ & $B \to X_c \ell \nu$ yields via singular value decomposition (SVD). arXiv:hep-ph/9509307
- Take ratio and correct for efficiency to form differential ratios.



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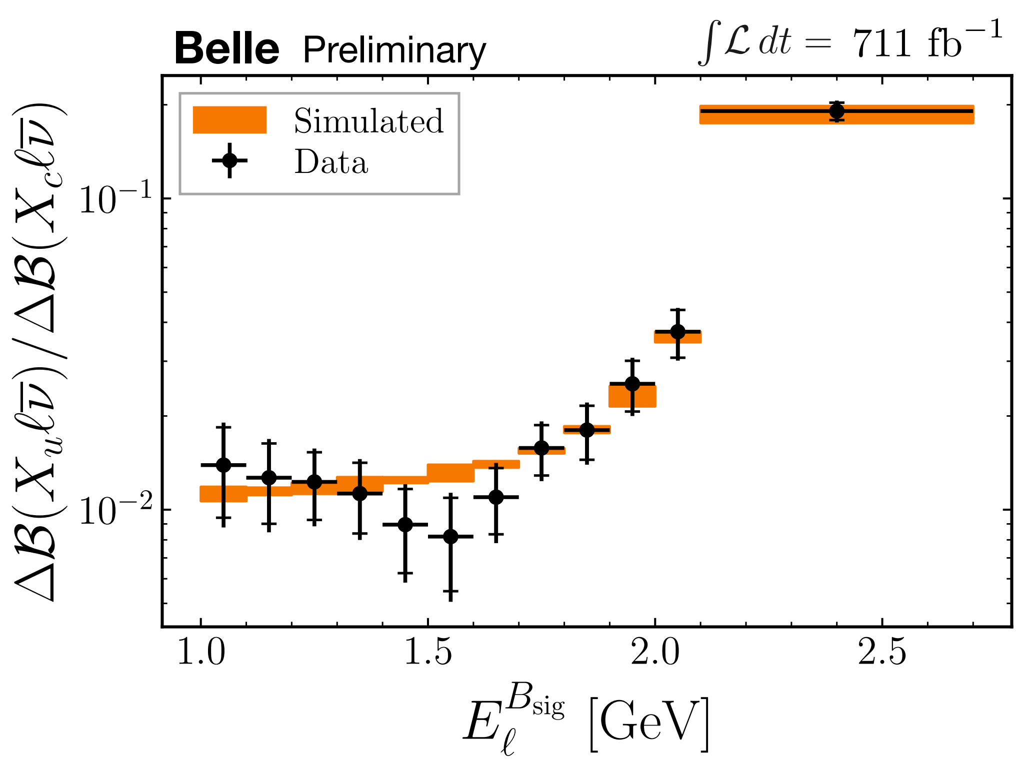


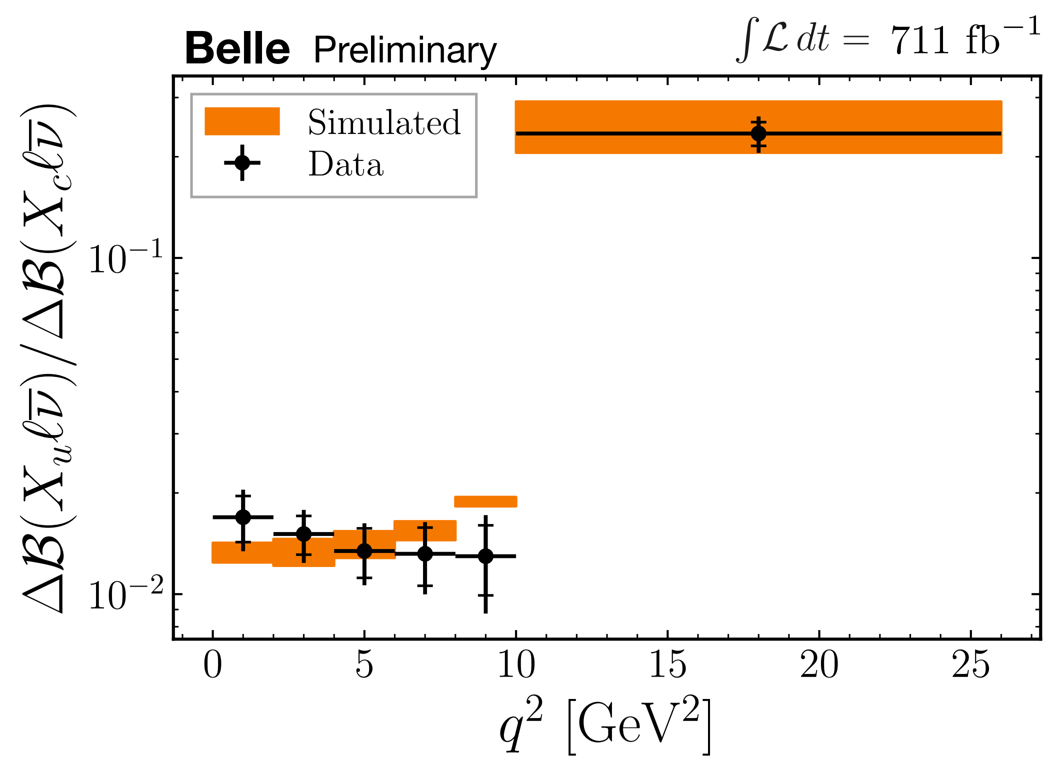


Final step: Extract $|V_{ub}|/|V_{cb}|$ with **partial BF**

$$\frac{|V_{ub}|}{|V_{cb}|} = \sqrt{\frac{\Delta\mathcal{B}(\bar{B} \to X_u \ell \bar{\nu})}{\Delta\mathcal{B}(\bar{B} \to X_c \ell \bar{\nu})} \frac{\Delta\Gamma(\bar{B} \to X_c \ell \bar{\nu})}{\Delta\Gamma(\bar{B} \to X_u \ell \bar{\nu})}}$$

**Theo. decay rates:**

| | |
|---|---|
| J. High Energ. Phys. 10 (2007) 058 | $\Delta\Gamma^{\text{GGOU}}(B \to X_u \ell \nu) = 58.5^{+2.7}_{-2.3}\ \text{ps}^{-1}$ |
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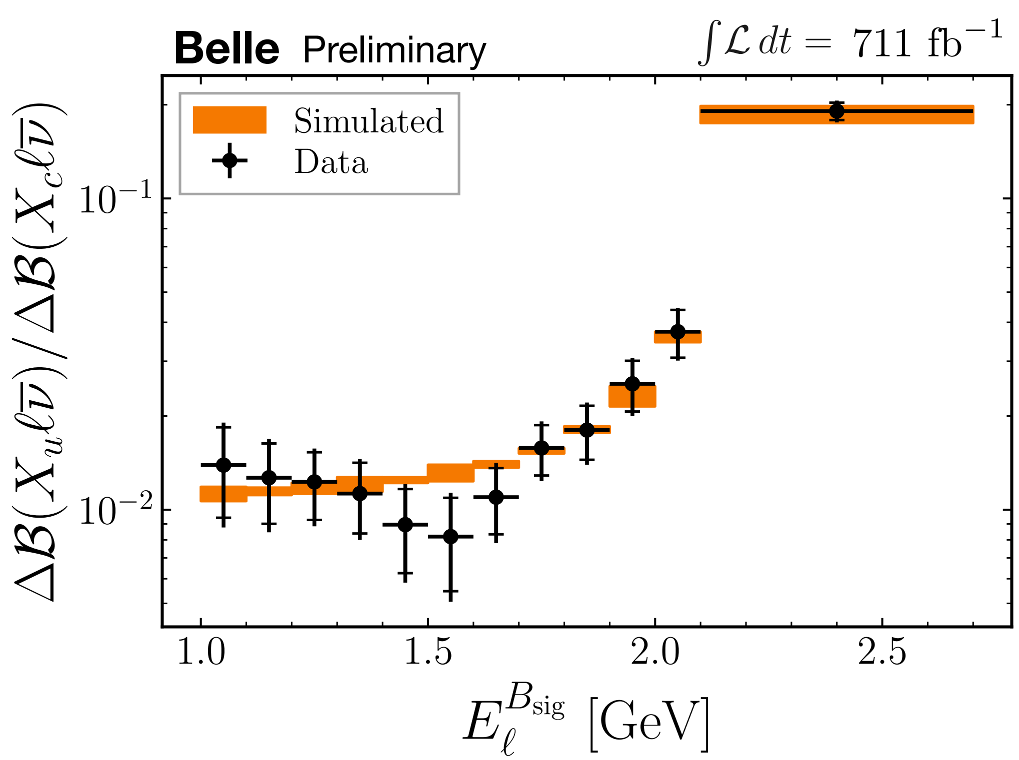


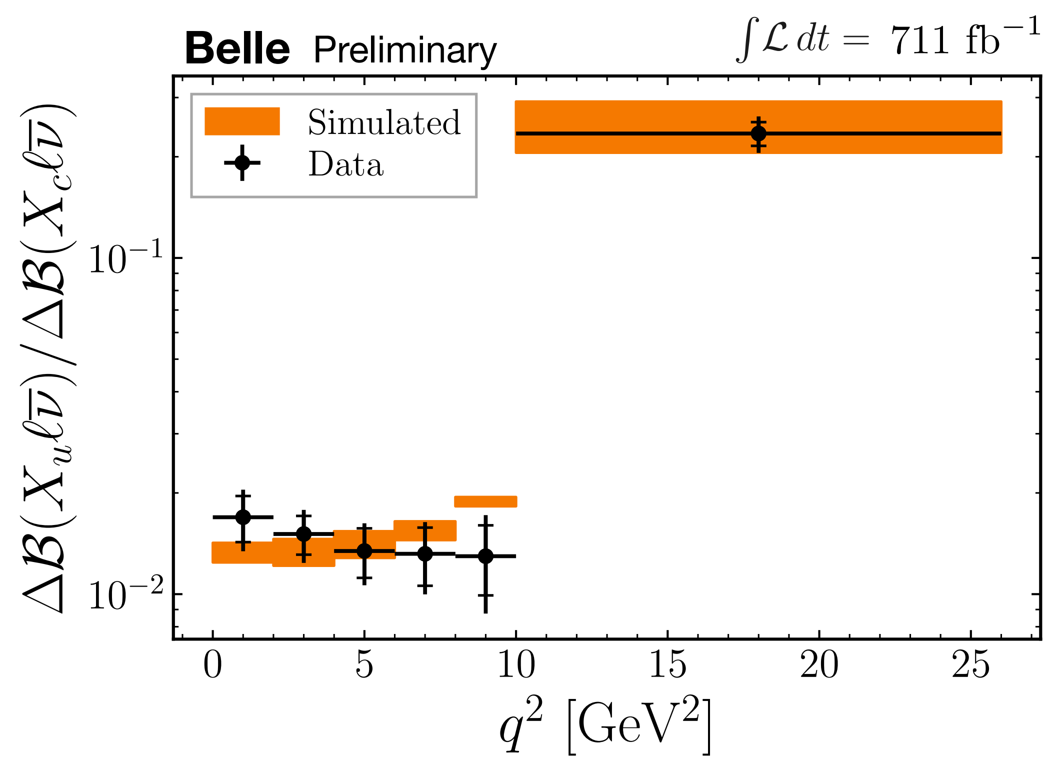


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$$\frac{|V_{ub}|}{|V_{cb}|}^{\mathrm{BLNP}} = 0.0972(1 \pm 4.2\%_{\mathrm{stat}} \pm 3.9\%_{\mathrm{syst}} \pm 5.2\%_{\Delta\Gamma(\bar{B} \to X_u \ell \bar{\nu})} \pm 2.0\%_{\Delta\Gamma(\bar{B} \to X_c \ell \bar{\nu})})$$

$$\frac{|V_{ub}|}{|V_{cb}|}^{\mathrm{GGOU}} = 0.0996(1 \pm 4.2\%_{\mathrm{stat}} \pm 3.9\%_{\mathrm{syst}} \pm 2.3\%_{\Delta\Gamma(\bar{B} \to X_u \ell \bar{\nu})} \pm 2.0\%_{\Delta\Gamma(\bar{B} \to X_c \ell \bar{\nu})})$$

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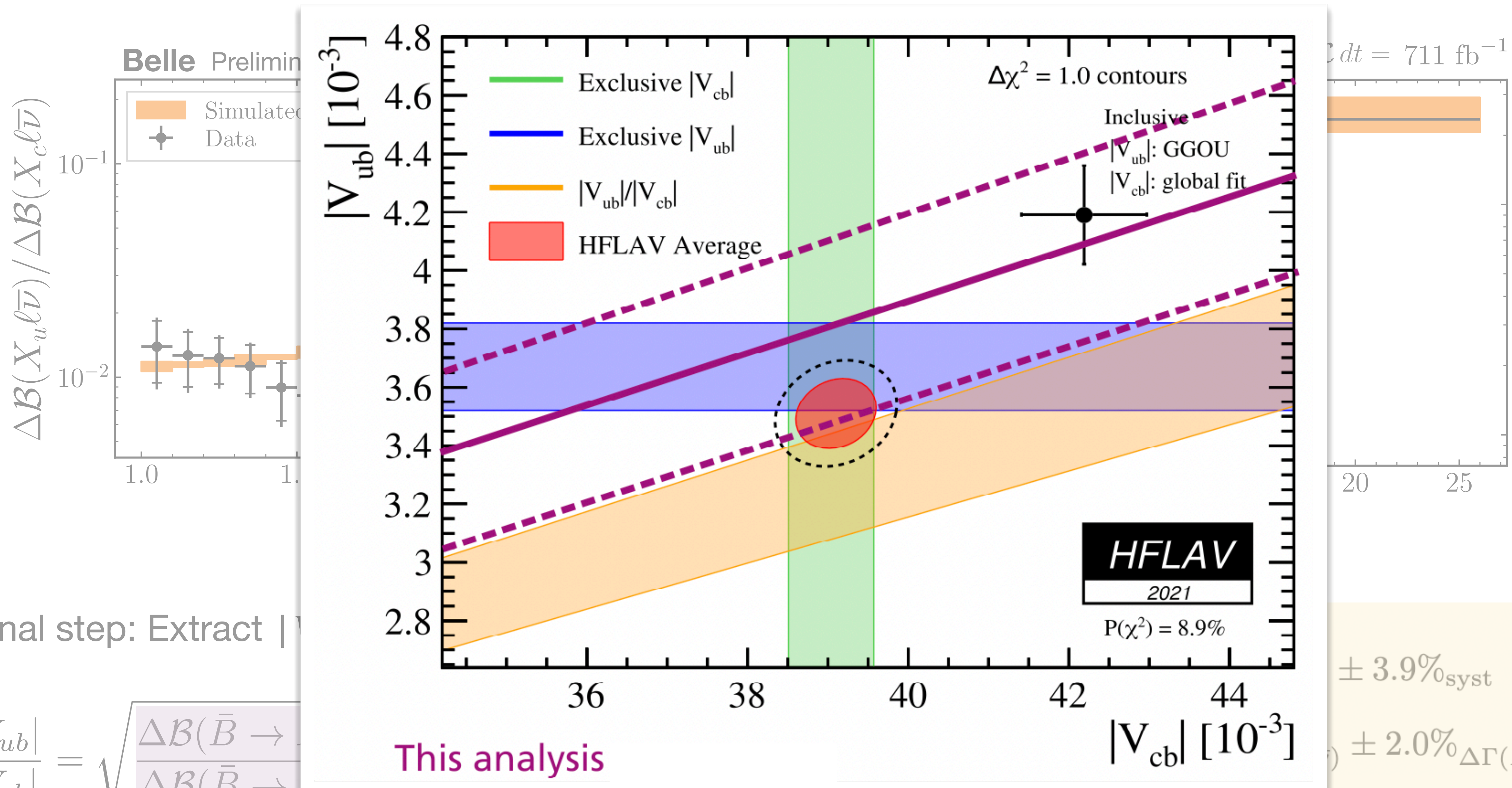


Final step: Extract |

$$\frac{|V_{ub}|}{|V_{cb}|} = \sqrt{\frac{\Delta\mathcal{B}(\bar{B} \to}{\Delta\mathcal{B}(\bar{B} \to}}$$

$\pm 3.9\%_{\mathrm{syst}}$

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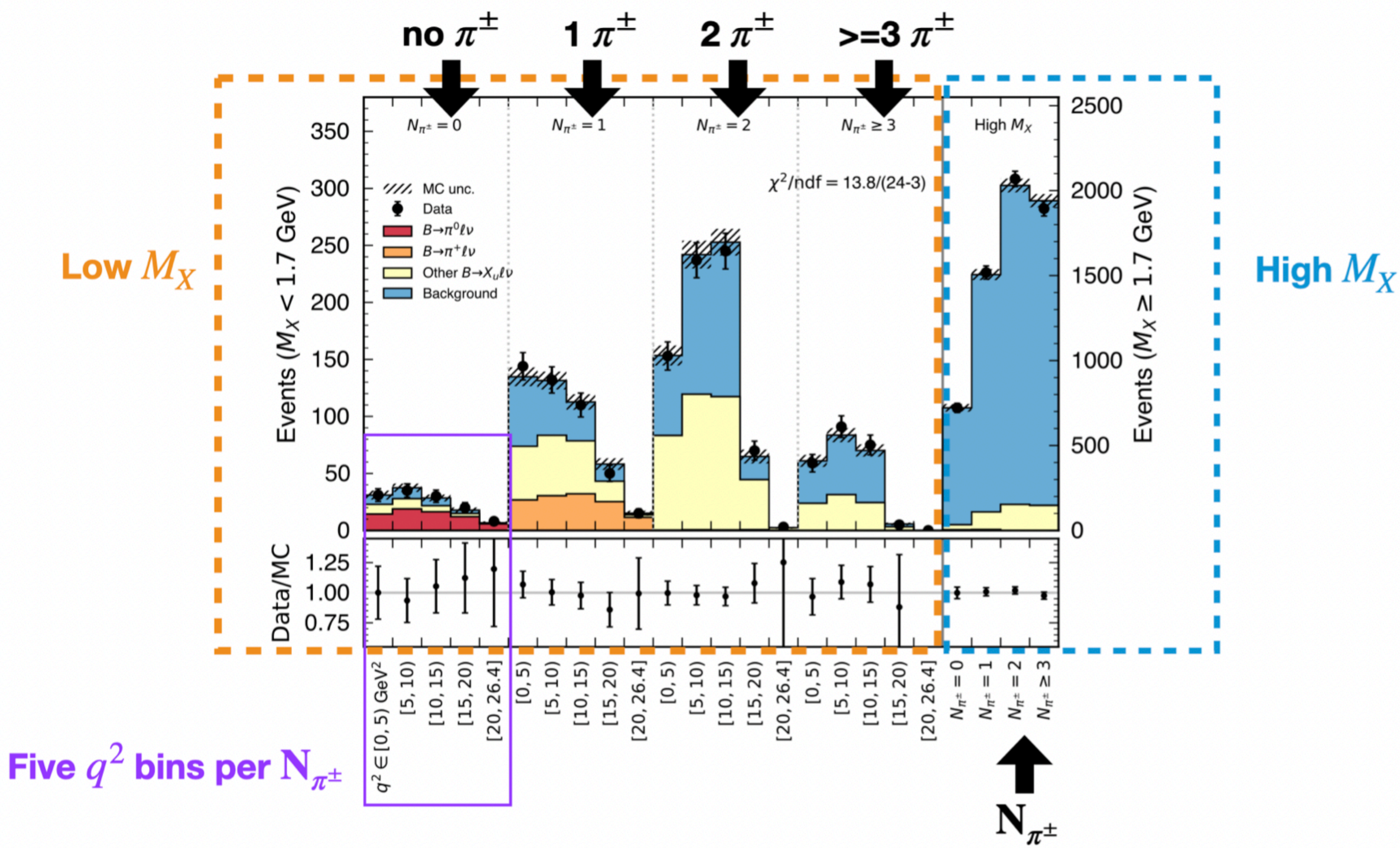
# Simultaneous determinations of incl. & excl. $|V_{ub}|$

BELLE

PRL 131, 211801 (2023)

- Inherits **similar analysis strategy** from former inclusive $|V_{ub}|$ Belle analysis. Phy. Rev. D 104 ( 2021) 012008
- **Simultaneously extract signal for** $B \to \pi \ell \nu$ and $B \to X_u \ell \nu$ in $q^2$ and charged pion multiplicity $N_{\pi^\pm}$.
- Normalizations and $B \to \pi \ell \nu$ form factors ($q^2$ shape) determined from fit.



# Simultaneous determinations of incl. & excl. $|V_{ub}|$

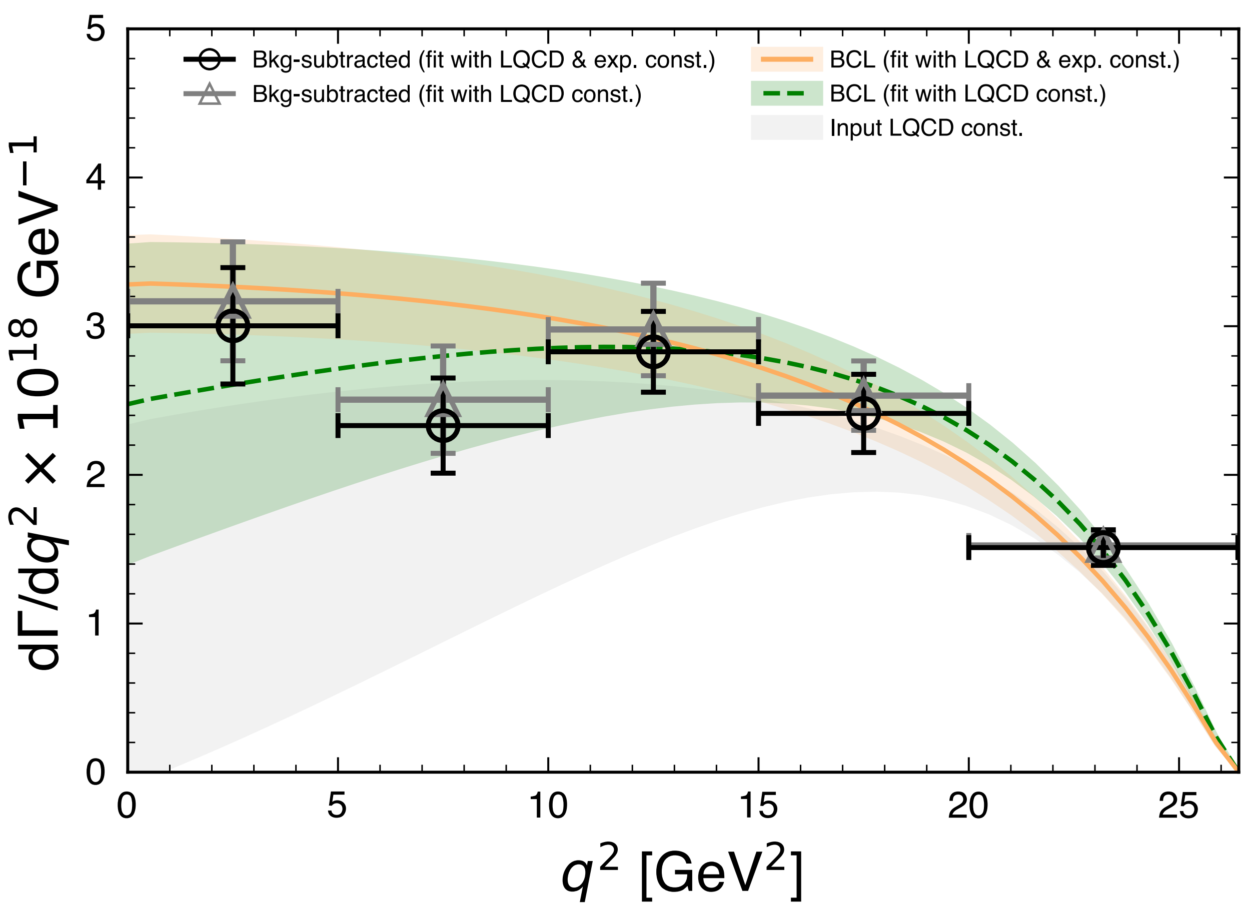
BELLE

PRL 131, 211801 (2023)

**Exclusive $|V_{ub}|$:**

- Fit BCL $B \to \pi\ell\nu$ form factors with two constraints:
  - LQCD only, Eur. Phys. J. C 82 (2022) 869
  - LQCD + experimental information.
- Combined or separate $B \to \pi^+\ell\nu$ & $B \to \pi^0\ell\nu$.

Largest systematic: tagging efficiency (±4%)




**Inclusive $|V_{ub}|$:**

- Use theoretical prediction of inclusive partial rate. J. High Energ. Phys. 10 (2007) 058

Largest systematic: $B \to X_u\ell\nu$ modelling (±10.9%)

$$|V_{ub}|_{\rm excl} = (3.78 \pm 0.23 \pm 0.16 \pm 0.14) \times 10^{-3}$$
$$|V_{ub}|_{\rm incl} = (3.88 \pm 0.20 \pm 0.31 \pm 0.09) \times 10^{-3}$$

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PRL 131, 211801 (2023)

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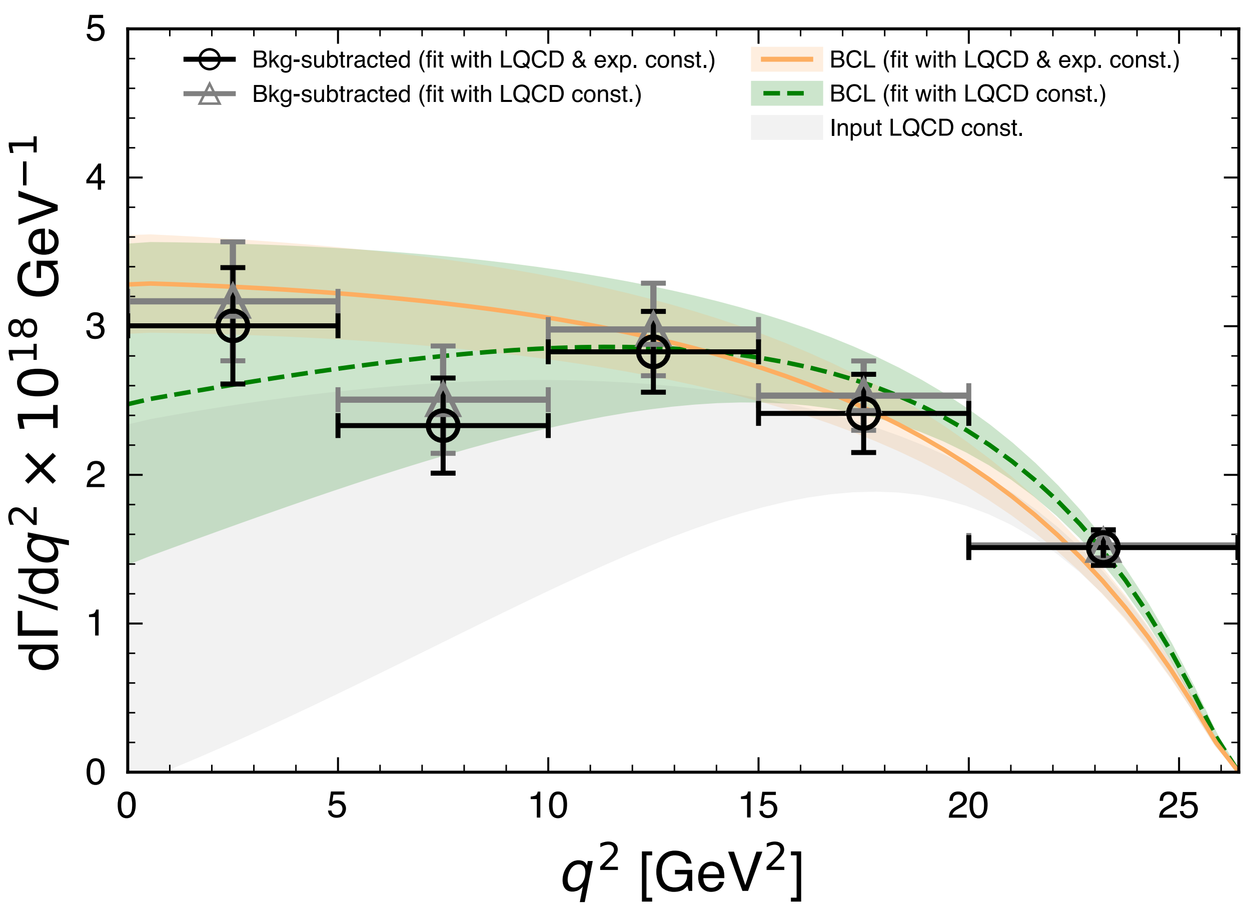


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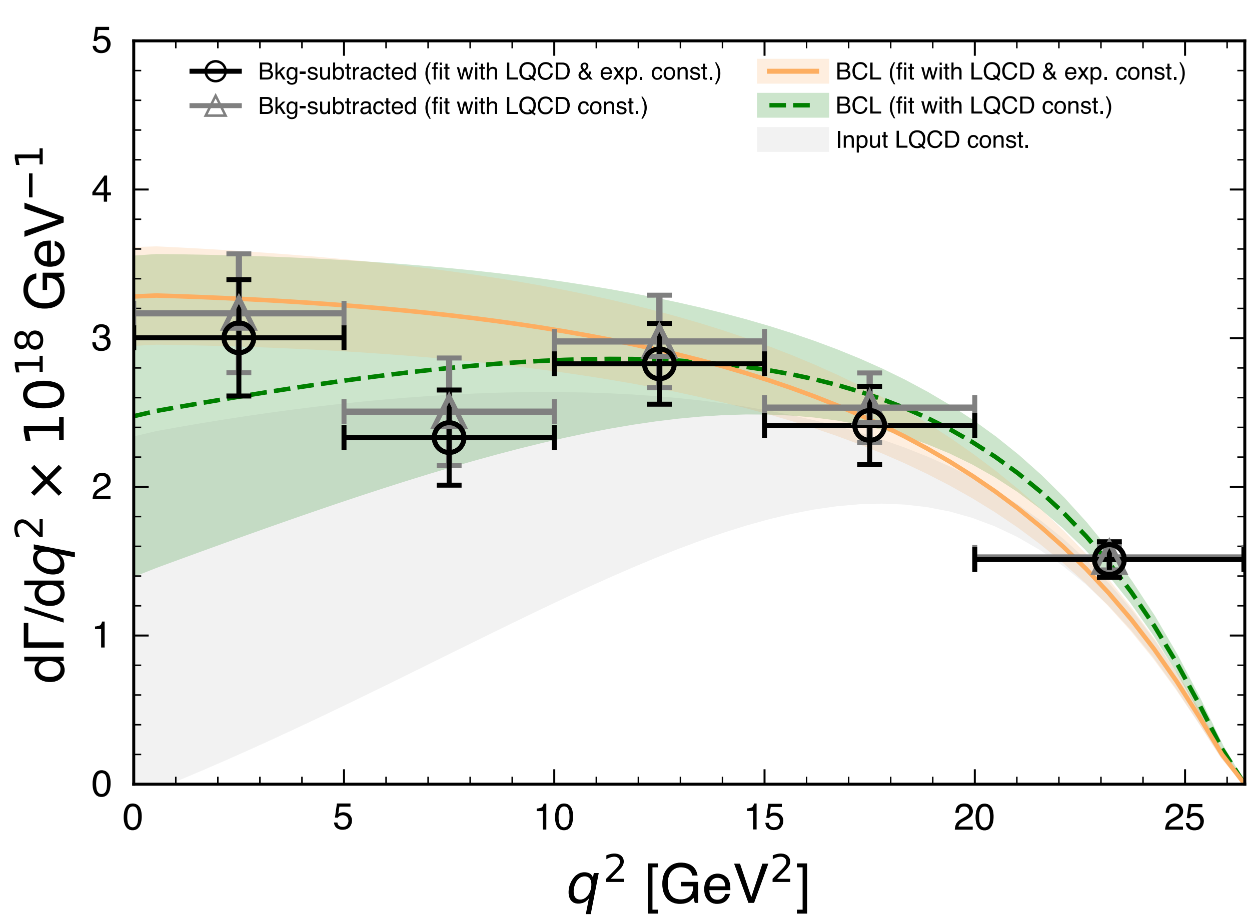
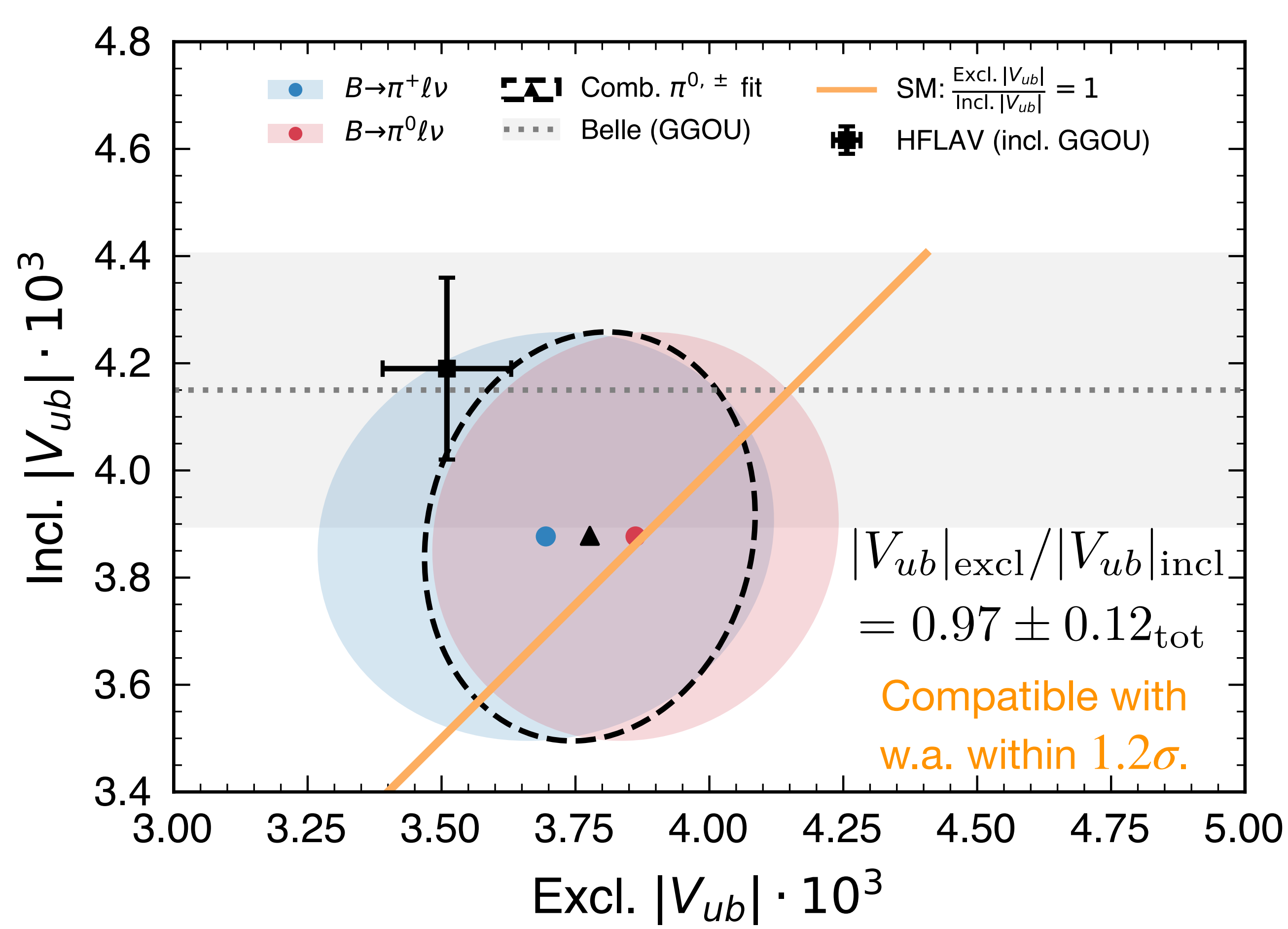
Weighted average of excl. & incl:

$$|V_{ub}|_{\rm avg} = (3.84 \pm 0.26) \times 10^{-3}$$

Consistent with CKMFitter (without $|V_{ub}|$) within $0.8\sigma$.



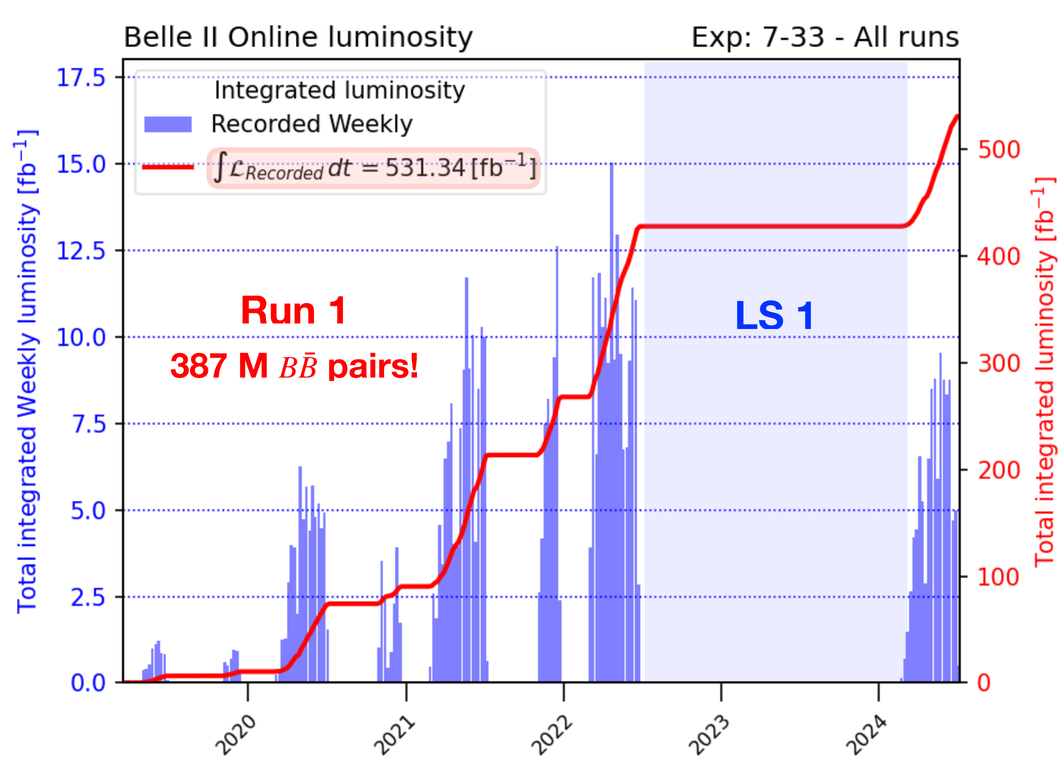



# Conclusion & Outlook

Belle (II) offers a unique and fertile environment for precision measurements of semileptonic $B$ decays.

Very active field, with **innovative strategies** of measuring $V_{ub}$, $V_{cb}$ and tests of lepton flavour universality.

- With the current collected data set, Belle II already produces **world-leading and unique** results!
- The well-understood Belle data set is still used to squeeze out **interesting measurements**.
- Collaboration between **theory and experiment** crucial to solve ongoing puzzles!

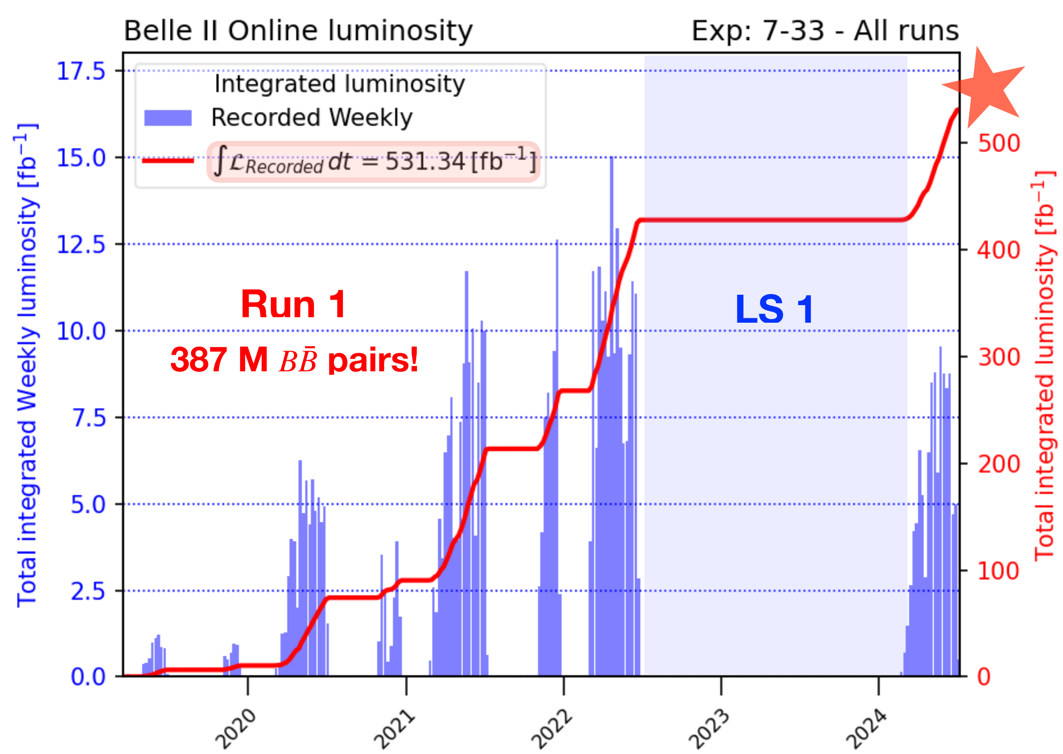



# Conclusion & Outlook

Belle (II) offers a unique and fertile environment for precision measurements of semileptonic $B$ decays.

Very active field, with **innovative strategies** of measuring $V_{ub}$, $V_{cb}$ and tests of lepton flavour universality.

- With the current collected data set, Belle II already produces **world-leading and unique** results!
- The well-understood Belle data set is still used to squeeze out **interesting measurements**.
- Collaboration between **theory and experiment** crucial to solve ongoing puzzles!




Exciting, new **results** are on the way!

# Summary of today's results

## Exclusive $|V_{ub}|$

Belle II

$B^0 \to \pi^- \ell^+ \nu$ $|V_{ub}|_{\mathrm{LQCD+LCSR}} = (3.73 \pm 0.07 \pm 0.07 \pm 0.16) \times 10^{-3}$

$B^+ \to \rho^0 \ell^+ \nu$ $|V_{ub}|_{\mathrm{LCSR}} = (3.19 \pm 0.12 \pm 0.17 \pm 0.26) \times 10^{-3}$

arXiv:2407.17403
(Submitted to PRD)

## Exclusive $|V_{cb}|$

Belle II

$B^0 \to D^{*+} \ell^- \nu$ $|V_{cb}|_{\mathrm{BGL}} = (40.6 \pm 0.3 \pm 1.0 \pm 0.6) \times 10^{-3}$

PRD 108, 092013 (2023)

BELLE

$\bar{B} \to D^* \ell \nu$ $|V_{cb}|_{\mathrm{BGL}} = (41.0 \pm 0.3 \pm 0.4 \pm 0.5) \times 10^{-3}$

arXiv:2310.20286
(Accepted by PRL)

## Inclusive $|V_{ub}|$ / $|V_{cb}|$

BELLE

$$\frac{|V_{ub}|}{|V_{cb}|}^{\mathrm{BLNP}} = 0.0972(1 \pm 4.2\%_{\mathrm{stat}} \pm 3.9\%_{\mathrm{syst}} \pm 5.2\%_{\Delta\Gamma(\bar{B}\to X_u \ell\bar{\nu})} \pm 2.0\%_{\Delta\Gamma(\bar{B}\to X_c \ell\bar{\nu})})$$

$$\frac{|V_{ub}|}{|V_{cb}|}^{\mathrm{GGOU}} = 0.0996(1 \pm 4.2\%_{\mathrm{stat}} \pm 3.9\%_{\mathrm{syst}} \pm 2.3\%_{\Delta\Gamma(\bar{B}\to X_u \ell\bar{\nu})} \pm 2.0\%_{\Delta\Gamma(\bar{B}\to X_c \ell\bar{\nu})})$$

arXiv:2311.00458
(Submitted to PRD)

## Incl./Excl. $|V_{ub}|$

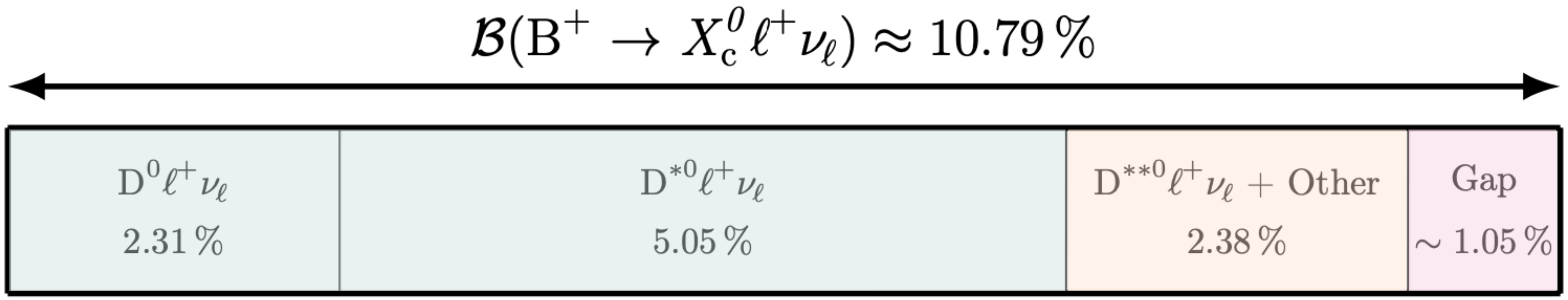
$|V_{ub}|_{\mathrm{avg}} = (3.84 \pm 0.26) \times 10^{-3}$

PRL 131, 211801 (2023)


Thank you for your attention!

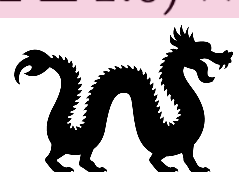
# $B \to X_c \ell \nu$ modelling

A **leading systematic** for many analyses (not just semileptonic):




…or is it even bigger?

| Decay | $\mathcal{B}(B^+)$ | $\mathcal{B}(B^0)$ |
|---|---|---|
| $B \to D\,\ell^+\,\nu_\ell$ | $(2.4 \pm 0.1) \times 10^{-2}$ | $(2.2 \pm 0.1) \times 10^{-2}$ |
| $B \to D^*\,\ell^+\,\nu_\ell$ | $(5.5 \pm 0.1) \times 10^{-2}$ | $(5.1 \pm 0.1) \times 10^{-2}$ |
| $B \to D_1\,\ell^+\,\nu_\ell$ | $(6.6 \pm 0.1) \times 10^{-3}$ | $(6.2 \pm 0.1) \times 10^{-3}$ |
| $B \to D_2^*\,\ell^+\,\nu_\ell$ | $(2.9 \pm 0.3) \times 10^{-3}$ | $(2.7 \pm 0.3) \times 10^{-3}$ |
| $B \to D_0^*\,\ell^+\,\nu_\ell$ | $(4.2 \pm 0.8) \times 10^{-3}$ | $(3.9 \pm 0.7) \times 10^{-3}$ |
| $B \to D_1'\,\ell^+\,\nu_\ell$ | $(4.2 \pm 0.9) \times 10^{-3}$ | $(3.9 \pm 0.8) \times 10^{-3}$ |
| $B \to D\pi\pi\,\ell^+\,\nu_\ell$ | $(0.6 \pm 0.9) \times 10^{-3}$ | $(0.6 \pm 0.9) \times 10^{-3}$ |
| $B \to D^*\pi\pi\,\ell^+\,\nu_\ell$ | $(2.2 \pm 1.0) \times 10^{-3}$ | $(2.0 \pm 1.0) \times 10^{-3}$ |
| $B \to X_c \ell \nu_\ell$ | $(10.8 \pm 0.4) \times 10^{-2}$ | $(10.1 \pm 0.4) \times 10^{-2}$ |



Fairly well known.

Broad states based on 3 measurements. (BaBar, Belle, DELPHI)

Some hints from BaBar & recent Belle result.

# A tale of two ‘gap’ models

**Model 1**:
Equidistribution of all final state particles in phase space

| Decay | $\mathcal{B}(B^+)$ | $\mathcal{B}(B^0)$ |
|---|---|---|
| $B \to D\,\ell^+\,\nu_\ell$ | $(2.4 \pm 0.1) \times 10^{-2}$ | $(2.2 \pm 0.1) \times 10^{-2}$ |
| $B \to D^*\,\ell^+\,\nu_\ell$ | $(5.5 \pm 0.1) \times 10^{-2}$ | $(5.1 \pm 0.1) \times 10^{-2}$ |
| $B \to D_1\,\ell^+\,\nu_\ell$ | $(6.6 \pm 0.1) \times 10^{-3}$ | $(6.2 \pm 0.1) \times 10^{-3}$ |
| $B \to D_2^*\,\ell^+\,\nu_\ell$ | $(2.9 \pm 0.3) \times 10^{-3}$ | $(2.7 \pm 0.3) \times 10^{-3}$ |
| $B \to D_0^*\,\ell^+\,\nu_\ell$ | $(4.2 \pm 0.8) \times 10^{-3}$ | $(3.9 \pm 0.7) \times 10^{-3}$ |
| $B \to D_1'\,\ell^+\,\nu_\ell$ | $(4.2 \pm 0.9) \times 10^{-3}$ | $(3.9 \pm 0.8) \times 10^{-3}$ |
| $B \to D\pi\pi\,\ell^+\,\nu_\ell$ | $(0.6 \pm 0.9) \times 10^{-3}$ | $(0.6 \pm 0.9) \times 10^{-3}$ |
| $B \to D^*\pi\pi\,\ell^+\,\nu_\ell$ | $(2.2 \pm 1.0) \times 10^{-3}$ | $(2.0 \pm 1.0) \times 10^{-3}$ |
| $B \to D\eta\,\ell^+\,\nu_\ell$ | $(4.0 \pm 4.0) \times 10^{-3}$ | $(4.0 \pm 4.0) \times 10^{-3}$ |
| $B \to D^*\eta\,\ell^+\,\nu_\ell$ | $(4.0 \pm 4.0) \times 10^{-3}$ | $(4.0 \pm 4.0) \times 10^{-3}$ |
| $B \to X_c \ell \nu_\ell$ | $(10.8 \pm 0.4) \times 10^{-2}$ | $(10.1 \pm 0.4) \times 10^{-2}$ |

**Model 2**:
Decay via intermediate broad $D^{**}$ state

| Decay | $\mathcal{B}(B^+)$ | $\mathcal{B}(B^0)$ |
|---|---|---|
| $B \to D_0^*\,\ell^+\,\nu_\ell$ ($\hookrightarrow D\pi\pi$) | $(0.03 \pm 0.03) \times 10^{-2}$ | $(0.03 \pm 0.03) \times 10^{-2}$ |
| $B \to D_1^*\,\ell^+\,\nu_\ell$ ($\hookrightarrow D\pi\pi$) | $(0.03 \pm 0.03) \times 10^{-2}$ | $(0.03 \pm 0.03) \times 10^{-2}$ |
| $B \to D_0^*\pi\pi\,\ell^+\,\nu_\ell$ ($\hookrightarrow D^*\pi\pi$) | $(0.108 \pm 0.051) \times 10^{-2}$ | $(0.101 \pm 0.048) \times 10^{-2}$ |
| $B \to D_1^*\pi\pi\,\ell^+\,\nu_\ell$ ($\hookrightarrow D^*\pi\pi$) | $(0.108 \pm 0.051) \times 10^{-2}$ | $(0.101 \pm 0.048) \times 10^{-2}$ |
| $B \to D_0^*\,\ell^+\,\nu_\ell$ ($\hookrightarrow D\eta$) | $(0.396 \pm 0.396) \times 10^{-2}$ | $(0.399 \pm 0.399) \times 10^{-2}$ |
| $B \to D_1^*\,\ell^+\,\nu_\ell$ ($\hookrightarrow D^*\eta$) | $(0.396 \pm 0.396) \times 10^{-2}$ | $(0.399 \pm 0.399) \times 10^{-2}$ |

(Assign 100% BR uncertainty in systematics covariance matrix)