Hadronic B Decays at Belle II

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on behalf of the Belle II collaboration

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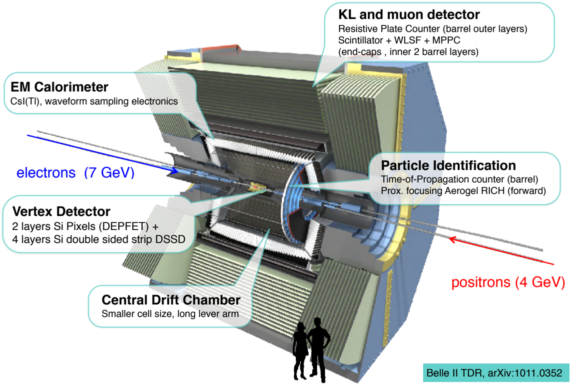
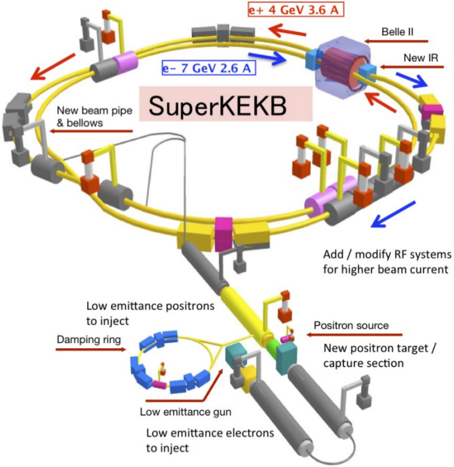
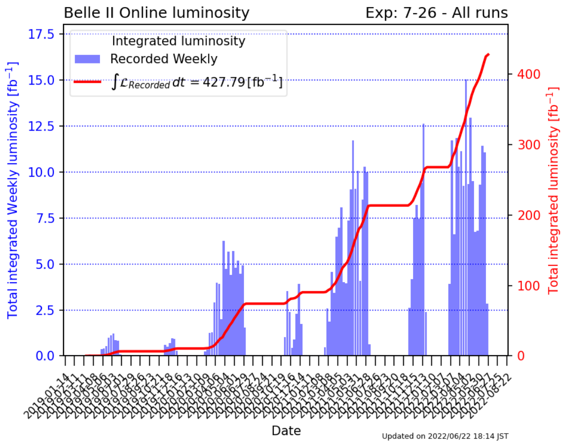
The Belle II experiment

- asymmetric collision of e^+e^-
- center-of-mass energy mostly at $\Upsilon(4S)$ resonance
- $\Upsilon(4S) \to B^+B^-$ (~51.5 %), $\Upsilon(4S) \to B^0\overline{B}^0$ (~48.5 %)





Status



- from 2019 to 2022 collected $424\ \mathrm{fb}^{-1}$ of data
  - $363\ \mathrm{fb}^{-1}$ at the $\Upsilon(4S)$ resonance
    $\Rightarrow 387\,\mathrm{M}\ B\overline{B}$ pairs
  - $61\ \mathrm{fb}^{-1}$ below and above the $\Upsilon(4S)$ resonance
- several analyses combine the Belle and Belle II data
  $\Rightarrow$ adding $711\ \mathrm{fb}^{-1}$ on resonance data
- hadronic B decays used to measure CP asymmetries and branching fraction (ratios)

CP violation in the SM quark sector

- mass eigenstates $\neq$ eigenstates of weak interaction
- superposition described via complex CKM matrix
- quark transition proportional to matrix elements
- unitarity of CKM matrix:

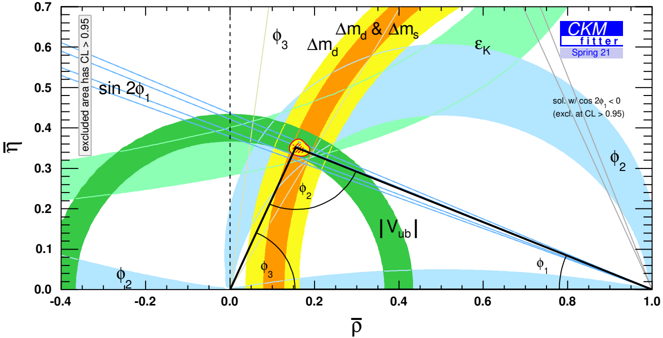
$$\begin{pmatrix} d' \\ s' \\ b' \end{pmatrix} = \boldsymbol{V}_{\text{CKM}} \begin{pmatrix} d \\ s \\ b \end{pmatrix} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \begin{pmatrix} d \\ s \\ b \end{pmatrix}$$

$$V_{ud}V_{ub}^* + V_{cd}V_{cb}^* + V_{td}V_{tb}^* = 0$$

$$\phi_1 = \beta \equiv \text{arg}\left(-\frac{V_{cd}V_{cb}^*}{V_{td}V_{tb}^*}\right)$$

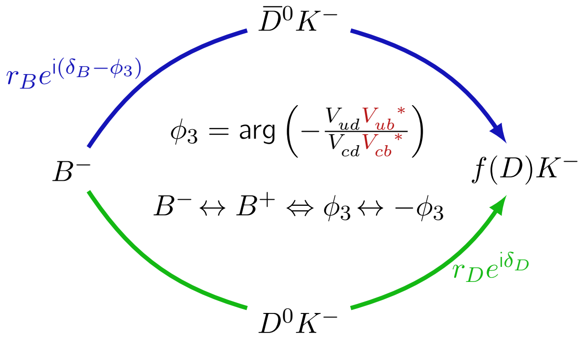
$$\phi_2 = \alpha \equiv \text{arg}\left(-\frac{V_{td}V_{tb}^*}{V_{ud}V_{ub}^*}\right)$$

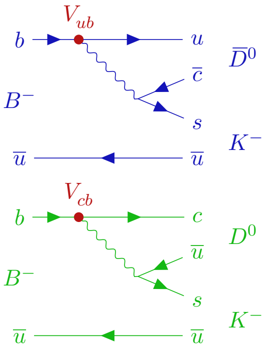
$$\phi_3 = \gamma \equiv \text{arg}\left(-\frac{V_{ud}V_{ub}^*}{V_{cd}V_{cb}^*}\right)$$



Measurements of the CKM angle ϕ_3

Concept of tree-level ϕ_3 measurements





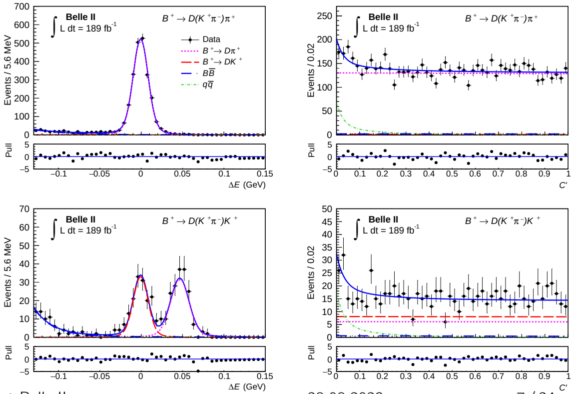
- interference between $b \to u$ and $b \to c$ transitions
- δ_B: strong phase difference, r_B: ratio between favored and suppressed amplitudes $\Rightarrow \sigma_{\phi_3} \propto \frac{1}{r_B}$
- r_D and δ_D may require external charm input
- GLW: D decay to CP eigenstates, like $D \to K^+K^-$ or $D \to \pi^+\pi^-$ PLB 265 (1991) 172, PLB 253 (1991) 483
- GLS: singly Cabibbo-suppressed D decays PRD 67 (2003) 071301
  - Belle + Belle II result (arXiv:2306.02940) in backup

GLW measurement of ϕ_3 using $B^\pm \to D_{CP\pm} K^\pm$ decays arXiv:2308.05048

- using full Belle data (711 fb^{-1}) and first half of Belle II data (189 fb^{-1})
- reconstructed D final states: $K^-\pi^+$ (flavor-specific), $K_S^0\pi^0$ (CP-odd), and K^+K^- (CP-even)
- measured CP asymmetries

$$\mathcal{A}_{CP\pm} \equiv \frac{\mathcal{B}(B^-\to D_{CP\pm}K^-) - \mathcal{B}(B^+\to D_{CP\pm}K^+)}{\mathcal{B}(B^-\to D_{CP\pm}K^-) + \mathcal{B}(B^+\to D_{CP\pm}K^+)} \qquad \mathcal{R}_{CP\pm} \equiv \frac{\mathcal{B}(B^-\to D_{CP\pm}K^-) + \mathcal{B}(B^+\to D_{CP\pm}K^+)}{(\mathcal{B}(B^-\to D_{\text{flav}}K^-) + \mathcal{B}(B^+\to \bar{D}_{\text{flav}}K^+))/2}$$

- K-π separation based on PID likelihood
- continuum suppression via BDT
- extraction of signal yields
  - 2D fit of ΔE and transformed BDT output C'
  - simultaneous fit in 12 subsets split by B charge, D final state, bachelor hadron
- found peaking $B \to K K^+ K^-$ background using D mass sideband
  - fix shape from MC and yield from data



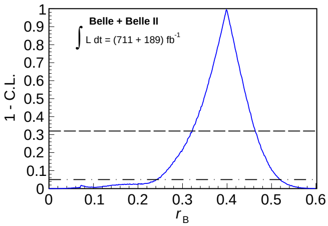
GLW measurement of ϕ_3 using $B^\pm \to D_{CP\pm} K^\pm$ decays arXiv:2308.05048

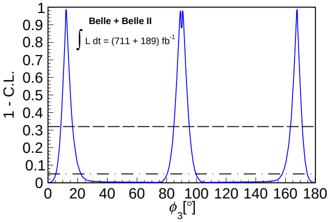
$$\mathcal{R}_{CP+} = 1.164 \pm 0.081 \pm 0.036$$
$$\mathcal{R}_{CP-} = 1.151 \pm 0.074 \pm 0.019$$
$$\mathcal{A}_{CP+} = (+12.5 \pm 5.8 \pm 1.4)\%$$
$$\mathcal{A}_{CP-} = (-16.7 \pm 5.7 \pm 0.6)\%$$

- significance for CP violation: $2.0\,\sigma$ $(2.8\,\sigma)$ in CP-even (CP-odd) mode
- $3.5\,\sigma$ evidence for $\mathcal{A}_{CP+} \neq \mathcal{A}_{CP-}$
- $\mathcal{R}_{CP+}$ $2.2\,\sigma$ larger than world average
- good agreement for $\mathcal{R}_{CP-}$ with world average

- convert CP asymmetries to angle ϕ_3 via

$$\mathcal{R}_{CP\pm} = 1 + r_B^2 \pm 2\,r_B \cos\delta_B \cos\phi_3 \qquad \mathcal{A}_{CP\pm} = \pm 2\,r_B \sin\delta_B \sin\phi_3 / \mathcal{R}_{CP\pm}$$



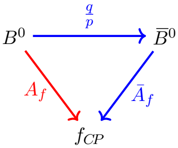


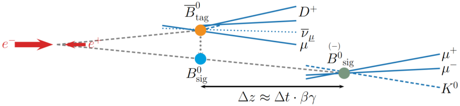
- large $\mathcal{R}_{CP+} \to$ large r_B
  $\Rightarrow$ low uncertainty on ϕ_3
- at 68.3% CL three intervals for ϕ_3
  - $[8.5^\circ, 16.5^\circ] \cup [84.5^\circ, 95.5^\circ] \cup [163.3^\circ, 171.5^\circ]$

Time-dependent CP violation measurements

Basics of time-dependent CP violation measurements

- CP violation in the interference between
  - direct decay
  - decay after mixing



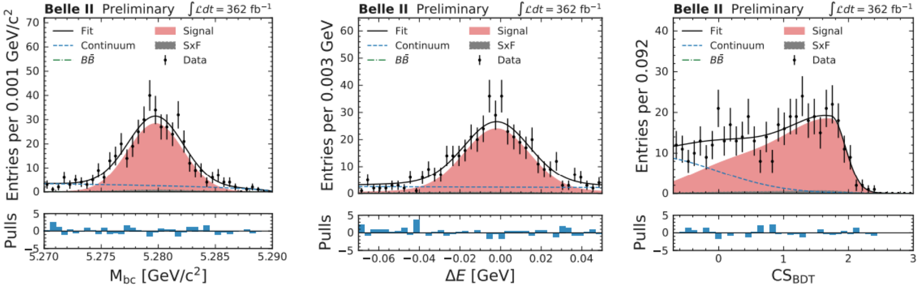


$$\mathcal{A}(\Delta t) \equiv \frac{\Gamma(\overline{B}^0(\Delta t) \to f_{CP}) - \Gamma(B^0(\Delta t) \to f_{CP})}{\Gamma(\overline{B}^0(\Delta t) \to f_{CP}) + \Gamma(B^0(\Delta t) \to f_{CP})} = S_f \sin(\Delta m\, \Delta t) - C_f \cos(\Delta m\, \Delta t)$$

- initial flavor $\to$ flavor tagging
  - introduces diluting mistag fraction ω
- proper-time difference $\to$ reconstruction of distance between B_{sig} and B_{tag} vertices
  - requires description of proper-time resolution, often via per-event uncertainties $\sigma_{\Delta t}$
- CP observables $\to$ parameters of interest
  - $S_{J/\psi\, K^0_S} = \sin 2\phi_1$ in SM if no direct CP violation
- mixing parameters $\to$ from external measurements

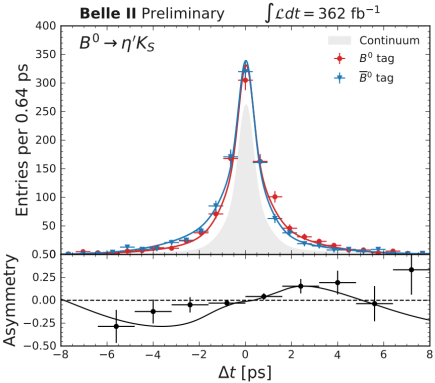
Measurement of CP asymmetries in $B^0 \to \eta' K^0_S$ decays

- $B^0 \to \eta' K^0_S$ is $b \to s q\bar{q}$ transition that proceeds via loop amplitudes
- comparison of CP observables with $\sin 2\phi_1$ probes beyond standard model contributions
- reconstruct $\eta' \to \eta(\to \gamma\gamma)\pi^+\pi^-$ and $\eta' \to \rho^0(\to \pi^+\pi^-)\gamma$
- perform B vertex fit and remove multiple candidates via best fit probability
- continuum suppression with BDT (validated on off-resonance data)
- four components in maximum-likelihood fit: signal, self-cross-feed, $q\bar{q}$ background, and $B\bar{B}$ background
- perform maximum likelihood fit to beam-constrained mass M_{bc}, energy difference ΔE, BDT output CS_{BDT}



Measurement of CP asymmetries in $B^0 \to \eta' K^0_S$ decays

- add proper-time difference as fourth fit observable
- flavor tag and proper-time uncertainty conditional observables
- resolution function determined with $B^0 \to D^{(*)-}\pi^+$ decays
- 358 ± 20 signal in $\eta' \to \eta(\to \gamma\gamma)\pi^+\pi^-$ channel at 79% purity
- 346 ± 21 signal in $\eta' \to \rho^0(\to \pi^+\pi^-)\gamma$ channel at 24% purity
- largest systematic uncertainties from
  - fixing shape parameters
  - Δt resolution model
  - motion of B-meson in $\Upsilon(4S)$ frame
- results of CP observables in agreement with current world averages
- sensitivity approaching that of Belle and BaBar



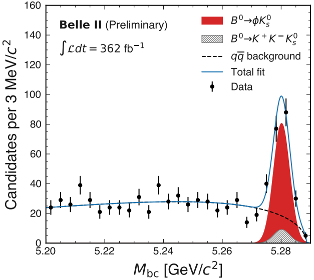
$$S_{\eta' K^0_S} = 0.67 \pm 0.10 \pm 0.04$$
$$C_{\eta' K^0_S} = 0.19 \pm 0.08 \pm 0.03$$

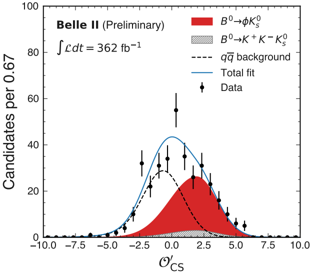
- correlation(S, C): 3.4%

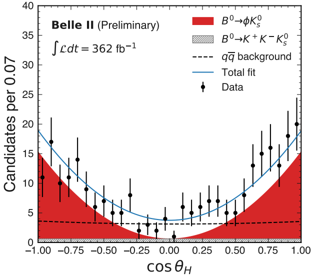
Measurement of CP asymmetries in $B^0 \to \phi K^0_S$ decays with Belle II

arXiv:2307.02802

- $B^0 \to \phi K^0_S$ proceeds via penguin diagram $\Rightarrow$ sensitive to physics beyond the SM
- reconstruct $\phi \to K^+K^-$ and $K^0_S \to \pi^+\pi^-$
- restrict sample to narrow region around ϕ mass $\Rightarrow$ quasi-two-body analysis
- continuum suppression via BDT
- unbinned extended maximum-likelihood fit to four dimensions:
  - beam-constrained mass M_{bc} and transformed BDT output $\mathcal{O}'_{CS}$ to separate signal from continuum
  - helicity angle $\cos\theta_H$ to distinguish non-resonant $B^0 \to K^+K^-K^0_S$
  - proper-time difference Δt to extract CP observables







Measurement of CP asymmetries in $B^0 \to \phi K^0_S$ decays with Belle II

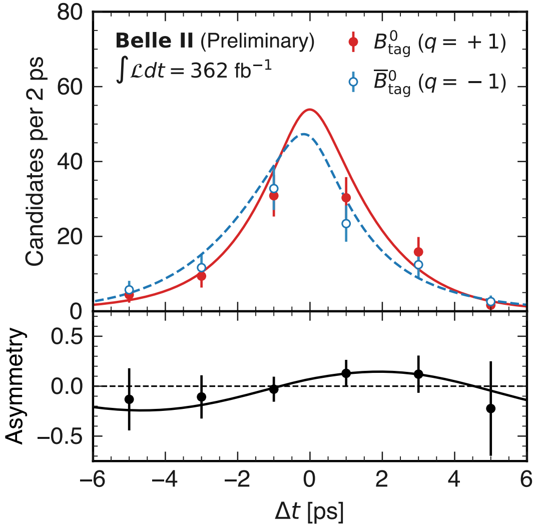
arXiv:2307.02802

- found 162 ± 17 signal candidates

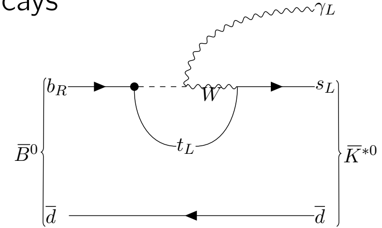
$$S_{\phi K^0_S} = 0.54 \pm 0.26^{+0.06}_{-0.08}$$
$$C_{\phi K^0_S} = -0.31 \pm 0.20 \pm 0.05$$

- largest systematic uncertainties arise from
  - fit bias
  - neglecting $B\overline{B}$ backgrounds

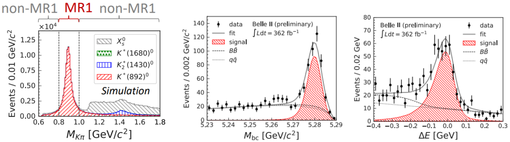
  and for C also
  - treatment of multiple candidates
  - non-resonant background
- results compatible with previous measurements
- no significant shift wrt $\sin 2\phi_1$ observed



Measurement of CP asymmetries in $B^0 \to K_S^0\pi^0\gamma$ decays



- $b \to s\gamma$ proceeds via one-loop diagrams at lowest order
  $\Rightarrow$ sensitive to physics beyond the SM
- weak interaction couples to left-handed fermions
  $\Rightarrow$ for $\overline{B}^0$ decay photon predominantly left-handed
- interference between B^0 and $\overline{B}^0$ highly suppressed
- select highest energetic photon and reject potential π^0 and η combinations with BDT
- define two mass regions, around K^{*0} and everywhere else
- separate signal from background with fit to M_{bc} and ΔE



- extract CP observables from simultaneous fit to Δt and time-integrated rate
- most precise results to date
- E and p scale as well as $B\overline{B}$ background asymmetries dominant systematics

$$S_{K^{*0}\gamma} = 0.00^{+0.27+0.03}_{-0.26-0.04} \qquad S_{K_S^0\pi^0\gamma} = 0.04^{+0.45}_{-0.44} \pm 0.10$$
$$C_{K^{*0}\gamma} = 0.10 \pm 0.13 \pm 0.03 \qquad C_{K_S^0\pi^0\gamma} = -0.06 \pm 0.25 \pm 0.07$$

Improvements of flavor tagging

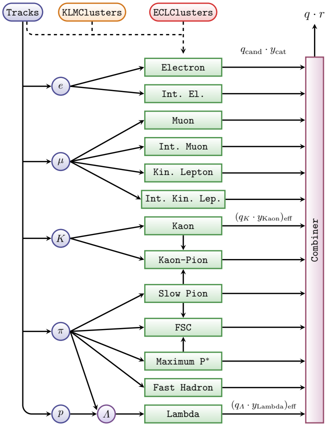
- so far used BDT-based two-stage approach of categories that are then combined

Categories	Targets for $\overline{B}^0$
Electron	e^-
Intermediate Electron	e^+
Muon	μ^-
Intermediate Muon	μ^+
Kinetic Lepton	ℓ^-
Intermediate Kinetic Lepton	ℓ^+
Kaon	K^-
Kaon-Pion	K^-, π^+
Slow Pion	π^+
Maximum p^*	ℓ^-, π^-
Fast-Slow-Correlated (FSC)	ℓ^-, π^+
Fast Hadron	π^-, K^-
Lambda	Λ

$\overline{B}^0 \to D^{*+}\ \bar{\nu}_\ell\ \ell^-$
$\hookrightarrow D^0\ \pi^+$
$\hookrightarrow X\ K^-$

$\overline{B}^0 \to D^+\ \pi^-\ (K^-)$
$\hookrightarrow \overline{K}^0\ \nu_\ell\ \ell^+$

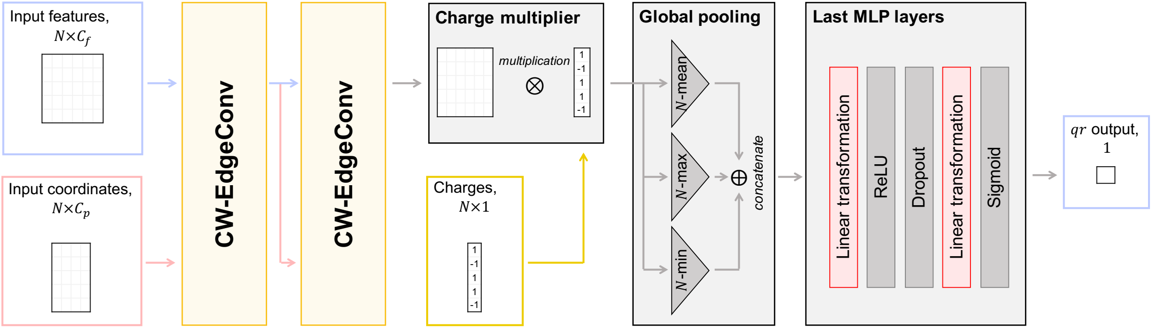
$\overline{B}^0 \to \Lambda_c^+\ X^-$
$\hookrightarrow \Lambda\ \pi^+$
$\hookrightarrow p\ \pi^-$



- effective tagging efficiency:
  $\varepsilon_{\rm eff} = \varepsilon_{\rm tag}(1-2\omega)^2 = (31.68 \pm 0.45)\%$
- only kept best candidate per category so lost potential tagging power

Improvements of flavor tagging

- feed up to 16 tracks into graph neural network
- input features of tracks: output of 13 former categories + momentum components + PID variables
- coordinates: initially point-of-closest approach to IP, then position in feature space



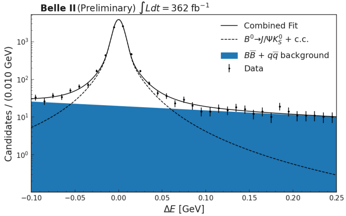
- effective tagging efficiency increases to $\varepsilon_{\text{eff}} = (37.40 \pm 0.43 \pm 0.34)\%$

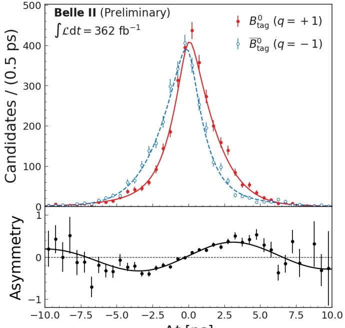
Validation of new flavor tagging algorithm

- perform CP violation measurement in golden channel $B^0 \to J/\psi K_S^0$
- reconstruct $J/\psi \to \mu^+\mu^-$ and $J/\psi \to e^+e^-$ and $K_S^0 \to \pi^+\pi^-$
- fit energy difference ΔE and calculate signal weights
- fit weighted Δt distribution $\Rightarrow$ only signal parameterization needed

$$
\begin{aligned}
S_{J/\psi K_S^0} &= 0.724 \pm 0.035 \pm 0.014 \\
C_{J/\psi K_S^0} &= -0.035 \pm 0.026 \pm 0.012
\end{aligned}
$$

- results in agreement with previous measurements
- statistical precision 7.9% / 7.1% better than with old flavor tagging algorithm
- largest systematic uncertainty from tag-side interference





Measurements of the CKM angle ϕ_2

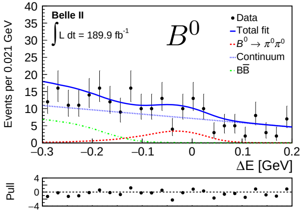
Measurement of the branching fraction and CP asymmetry of $B^0 \to \pi^0\pi^0$

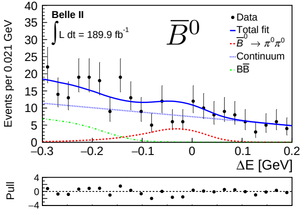
PRD 107, 112009 (2023)

- ϕ_2 least known CKM angle
- if only tree-level $b \to u$ processes $B^0 \to \pi^+\pi^-$ would allow extraction of ϕ_2
- however significant loop contributions present so combine multiple charmless $B^0 \to hh$ modes
- challenging reconstruction since final state are just 4 photons
- signal photon selection via BDT
- another BDT to suppress continuum background
- signal yield extraction with 3D fit to beam-constrained mass $M_{\rm bc}$, energy difference ΔE, and transformed BDT output

$$\mathcal{B}(B^0 \to \pi^0\pi^0) = (1.38 \pm 0.27 \pm 0.22) \times 10^{-6}$$
$$\mathcal{A}_{CP}(B^0 \to \pi^0\pi^0) = 0.14 \pm 0.46 \pm 0.07$$

- same precision on branching fraction as Belle with only 1/3 of data
- further new $B \to \pi\pi$ results in backup
- first results on $B^0 \to \rho\rho$ in arXiv:2206.12362 and arXiv:2208.03554





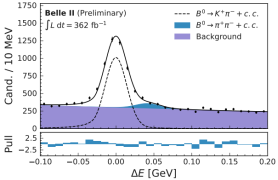
$K\pi$ puzzle

Isospin sum rule

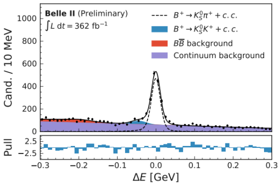
- significant difference between direct CP-violating asymmetries in $B^0 \to K^+\pi^-$ and $B^+ \to K^+\pi^0$
- large hadronic uncertainties
- isospin sum rule for $B \to K\pi$ decays

$$I_{K\pi} = \mathcal{A}_{K^+\pi^-} + \mathcal{A}_{K^0\pi^+} \frac{\mathcal{B}(K^0\pi^+)}{\mathcal{B}(K^+\pi^-)} \frac{\tau_{B^0}}{\tau_{B^+}} - 2\mathcal{A}_{K^+\pi^0} \frac{\mathcal{B}(K^+\pi^0)}{\mathcal{B}(K^+\pi^-)} \frac{\tau_{B^0}}{\tau_{B^+}} - 2\mathcal{A}_{K^0\pi^0} \frac{\mathcal{B}(K^0\pi^0)}{\mathcal{B}(K^+\pi^-)}$$

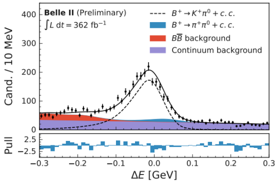
- null test of SM ($I_{K\pi} = 0$) in the limit of isospin symmetry and no electroweak penguin contributions
- Belle II with unique access to all $K\pi$ modes



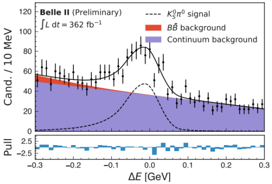
$B^0 \to K^+\pi^-$



$B^+ \to K_S^0\pi^+$



$B^+ \to K^+\pi^0$



$B^0 \to K_S^0\pi^0$

Results on $K\pi$ puzzle

$$\mathcal{B}(B^0 \to K^+\pi^-) = (20.67 \pm 0.37 \pm 0.62) \times 10^{-6}$$
$$\mathcal{A}_{CP}(B^0 \to K^+\pi^-) = -0.072 \pm 0.019 \pm 0.007$$

$$\mathcal{B}(B^+ \to K^+\pi^0) = (13.93 \pm 0.38 \pm 0.71) \times 10^{-6}$$
$$\mathcal{A}_{CP}(B^+ \to K^+\pi^0) = 0.013 \pm 0.027 \pm 0.005$$

$$\mathcal{B}(B^+ \to K^0_S\pi^+) = (24.37 \pm 0.71 \pm 0.86) \times 10^{-6}$$
$$\mathcal{A}_{CP}(B^+ \to K^0_S\pi^+) = 0.046 \pm 0.029 \pm 0.007$$

$$\mathcal{B}(B^0 \to K^0_S\pi^0) = (10.40 \pm 0.66 \pm 0.60) \times 10^{-6}$$
$$\mathcal{A}_{CP}(B^0 \to K^0_S\pi^0) = -0.06 \pm 0.15 \pm 0.05$$

- results agree with current world averages
- precision comparable to the current best results
- combine $B^0 \to K^0_S\pi^0$ with previous time-dependent Belle II measurement ($\sim$50% overlap of candidates) arXiv:2206.07453

$$\mathcal{A}_{CP}(B^0 \to K^0_S\pi^0) = -0.01 \pm 0.12 \pm 0.05$$

  - most precise measurement of single experiment to date
- inserting results into isospin sum rule
- competitive with world average of $I_{K\pi} = 0.13 \pm 0.11$

$$I_{K\pi} = -0.03 \pm 0.13 \pm 0.05$$

Conclusion

- various measurements of CP asymmetries in hadronic B decays
- time-dependent CP violation measurements using $B^0 \to \eta' K_S^0$, $B^0 \to \phi K_S^0$, and $B^0 \to J/\psi K_S^0$
- branching fraction and CP asymmetry measurement of $B^0 \to \pi^0 \pi^0$ towards ϕ_2
- ϕ_3 measured with GLS and GLW methods
- analyses still statistically limited $(362\ \text{fb}^{-1})$ but thanks to better resolution and improved analysis techniques precision already better than expected
  - new flavor tagging algorithm based on graph neural network increases effective tagging efficiency by 18%
- restart of data-taking planned for January $\Rightarrow$ more data coming
- in the meantime combined measurements with Belle data

Backup

$B \to \pi\pi$ results

First measurement of $B^0 \to \pi^0\pi^0$ at Belle II

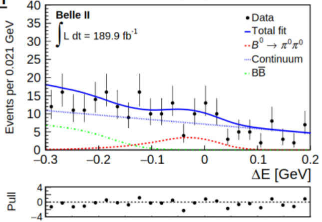
- unique channel to Belle II
- 4 photons in final state
- large background

$$\mathcal{B}(B^0 \to \pi^0\pi^0) = (1.38 \pm 0.27 \pm 0.22) \times 10^{-6}$$
$$\mathcal{A}_{CP}(B^0 \to \pi^0\pi^0) = 0.14 \pm 0.46 \pm 0.07$$

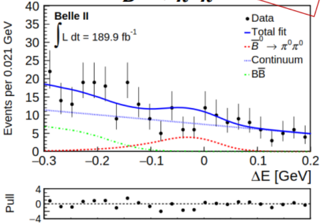
- same BR precision as Belle with 1/3 of data

$189 fb^{-1}$

$B^0 \to \pi^0\pi^0$



$\bar{B}^0 \to \pi^0\pi^0$



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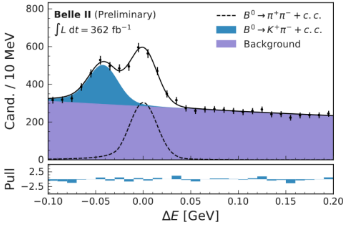
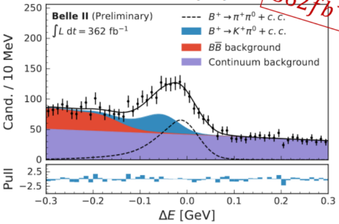
$B^0 \to \pi^+\pi^-$



$B^+ \to \pi^+\pi^0$

$362 fb^{-1}$



$$\mathcal{B}(B^0 \to \pi^+\pi^-) = (5.83 \pm 0.22 \pm 0.17) \times 10^{-6}$$
$$\mathcal{B}(B^+ \to \pi^+\pi^0) = 5.10 \pm 0.29 \pm 0.27 \times 10^{-6}$$
$$\mathcal{A}_{CP}(B^+ \to \pi^+\pi^0) = -0.081 \pm 0.054 \pm 0.008$$

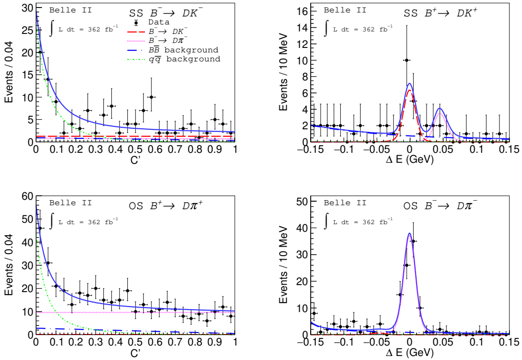
- compatible with world averages
- worlds best $BR(B^0 \to \pi^+\pi^-)$

GLS measurement of ϕ_3 arXiv:2306.02940

- $B^\pm \to DK^\pm$ and $B^\pm \to D\pi^\pm$ with $D \to K^0_S K^\pm \pi^\mp$ using Belle and Belle II data
- split into same-sign (SS), i.e., charge of B same as charge of K in $D \to K^0_S K^\pm \pi^\mp$, and opposite-sign (OS)
- seven observables: four CP asymmetries and three branching-fraction ratios

$$\mathcal{A}_m^{Dh} \equiv \frac{N_m^{Dh^-} - N_m^{Dh^+}}{N_m^{Dh^-} + N_m^{Dh^+}} \quad \mathcal{R}_m^{DK/D\pi} \equiv \frac{N_m^{DK^-} + N_m^{DK^+}}{N_m^{D\pi^-} + N_m^{D\pi^+}} \quad \mathcal{R}_{SS/OS}^{D\pi} \equiv \frac{N_{D\pi^-}^{\mathrm{SS}} + N_{D\pi^+}^{\mathrm{SS}}}{N_{D\pi^-}^{\mathrm{OS}} + N_{D\pi^+}^{\mathrm{OS}}}$$

- K-π separation based on PID likelihood
- continuum suppression via BDT
- extraction of signal yields
  - 2D fit of ΔE and transformed BDT output C'
  - simultaneous fit in 16 subsets split by B charge, OS/SS, $DK/D\pi$, Belle/Belle II
- two configurations:
  - full D phase space
  - $D \to K^{*\mp} K^\pm$ with $K^{*\pm} \to K^0_S \pi^\pm$



GLS measurement of ϕ_3 arXiv:2306.02940

full D phase space	SS	OS
$\mathcal{A}^{DK}$	-0.089 $\pm$ 0.091 $\pm$ 0.011	0.109 $\pm$ 0.133 $\pm$ 0.013
$\mathcal{A}^{D\pi}$	0.018 $\pm$ 0.026 $\pm$ 0.009	-0.028 $\pm$ 0.031 $\pm$ 0.009
$\mathcal{R}^{DK/D\pi}$	0.122 $\pm$ 0.012 $\pm$ 0.004	0.093 $\pm$ 0.013 $\pm$ 0.003
$\mathcal{R}^{D\pi}_{\text{SS/OS}}$	1.428 $\pm$ 0.057 $\pm$ 0.002	

$K^{*\pm}$ region	SS	OS
$\mathcal{A}^{DK}$	0.055 $\pm$ 0.119 $\pm$ 0.020	0.231 $\pm$ 0.184 $\pm$ 0.014
$\mathcal{A}^{D\pi}$	0.046 $\pm$ 0.029 $\pm$ 0.016	0.009 $\pm$ 0.046 $\pm$ 0.009
$\mathcal{R}^{DK/D\pi}$	0.093 $\pm$ 0.012 $\pm$ 0.005	0.103 $\pm$ 0.020 $\pm$ 0.006
$\mathcal{R}^{D\pi}_{\text{SS/OS}}$	2.412 $\pm$ 0.132 $\pm$ 0.019	

- values consistent with LHCb's results
- measurement statistically limited
- on their own results do not allow unambiguous determination of ϕ_3 but help in global fits
- largest systematic uncertainties
  - fixing shape parameters from MC
  - assumption that efficiencies of $D \to K^0_S K^- \pi^+$ and $D \to K^0_S K^+ \pi^-$ are equal