



Inclusive semileptonic $B \rightarrow X_c \ell \nu$ decays at Belle (II)



Data used in $b \rightarrow c$ inclusive analyses

BaBar	$\langle E_l^n \rangle$: $n=0,1,2,3$ [PRD 69, 111104 (2004), PRD 81, 032003 (2010)] $\langle M_x^{2n} \rangle$: $n=1,2, 3$ [PRD 81, 032003 (2010)]
Belle	$\langle E_l^n \rangle$: $n=0,1,2,3$ [PRD 75, 032001 (2007)] $\langle M_x^{2n} \rangle$: $n=1,2$ [PRD 75, 032005 (2007)]
CDF	$\langle M_x^{2n} \rangle$: $n=1,2$ [PRD 71, 051103 (2005)]
CLEO	$\langle M_x^{2n} \rangle$: $n=1,2$ [PRD 70, 032002 (2004)] $\langle E_\gamma^n \rangle$: $n=1$ [PRL 87, 251807 (2001)]
DELPHI	$\langle E_l^n \rangle$: $n=1,2,3$ $\langle M_x^{2n} \rangle$: $n=1,2$ [EPJ C45, 35 (2006)]

- Newest measurement is from the year 2010!

Back in 2018...

Talk by C. Schwanda
Challenges in Semileptonic B decays 2018

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~~Newest measurement is from the year 2010!~~

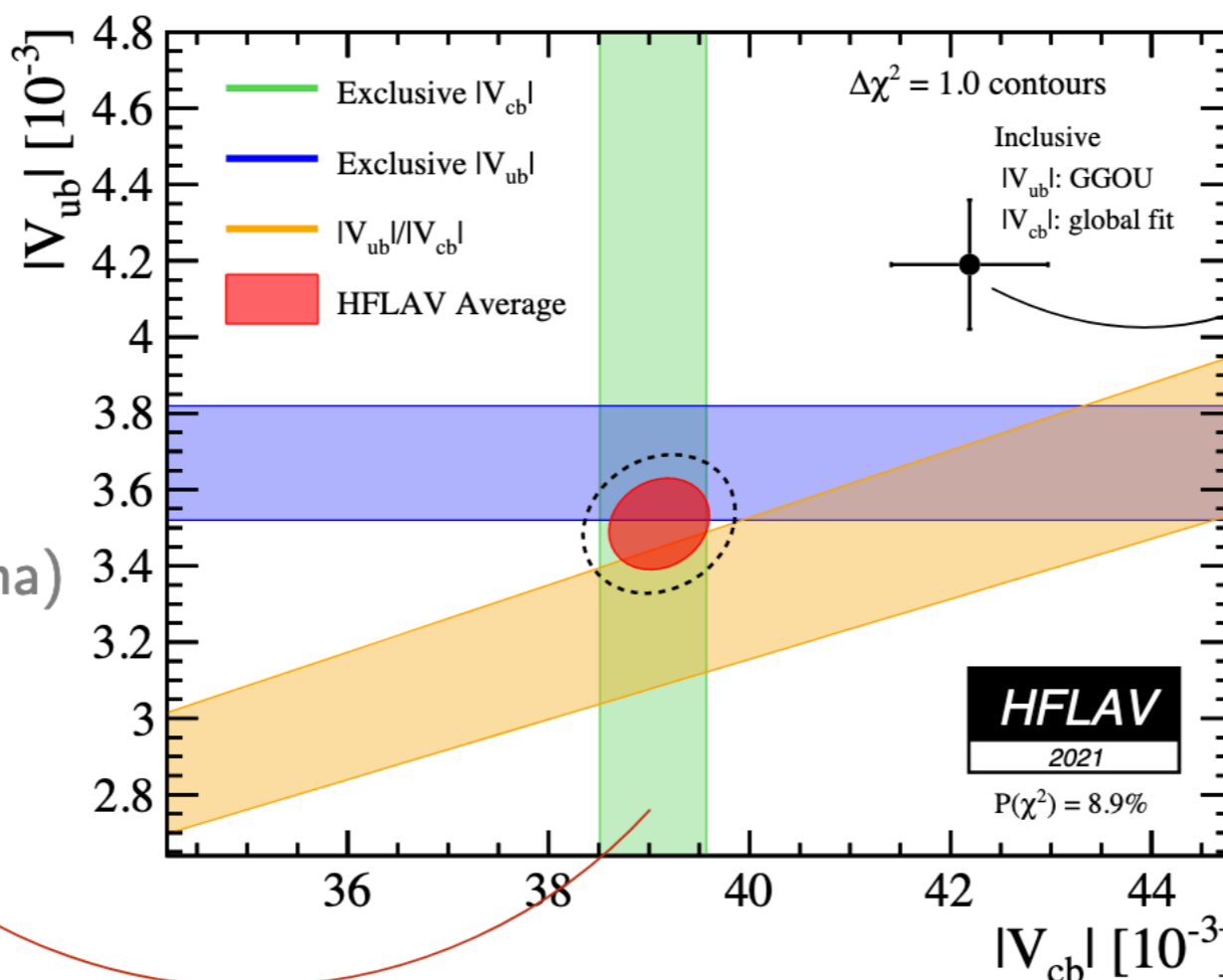
A brief recap

Manca's **awesome** introduction:



Ongoing $b \rightarrow c$ analyses at Belle(II)

Tagged and untagged
 $B \rightarrow D^{**} l \nu_e$
(Göttingen, McGill)



Tagged and untagged
 $B \rightarrow X_c l \nu_e$
Ray's talk on Wed.

Tagged and untagged
 $B \rightarrow D^* l \nu_e$
(KEK, McGill, Bonn, Vienna)

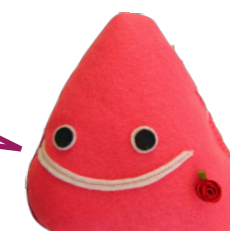
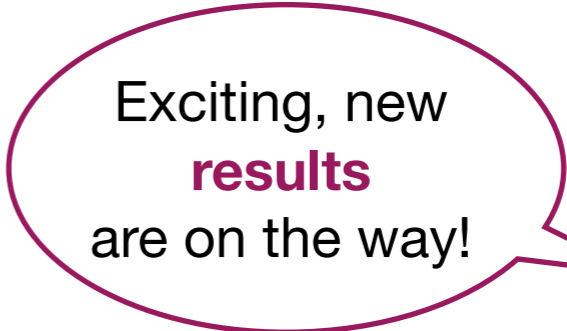
Untagged
 $B \rightarrow D l \nu_e$
(Vienna, Bonn)

Untagged: only **signal B meson** is reconstructed

Tagged: **both B mesons** are reconstructed

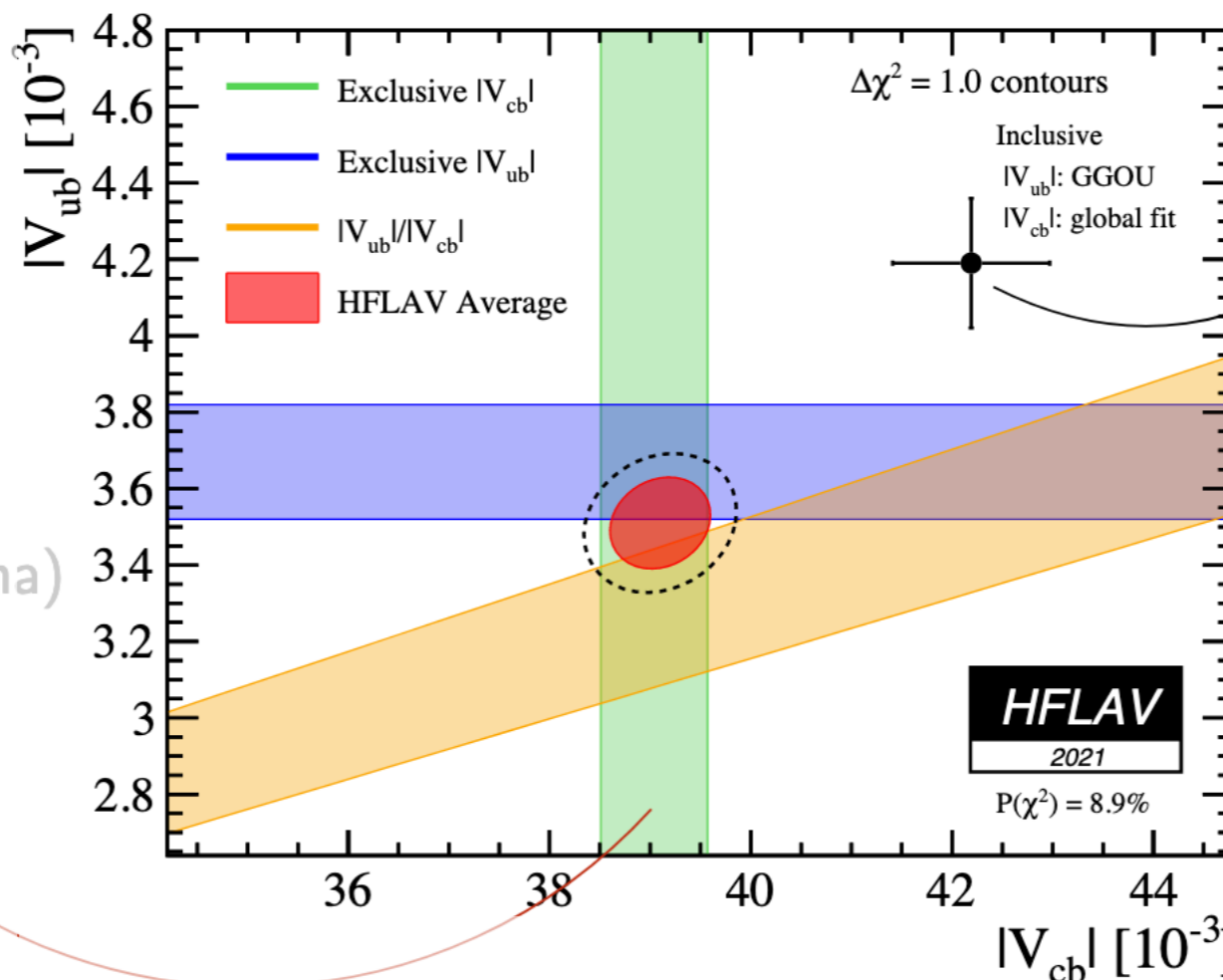
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Untagged
 $B \rightarrow D l \nu_e$
(Vienna, Bonn)

Untagged
 $\mathcal{B}(B \rightarrow X_c l \nu_e)$
(Vienna)

Tagged
 $\langle M_X^n \rangle$ moments
(Bonn)

Tagged and untagged
 $\langle q^{2n} \rangle$ moments
(Bonn, Vienna)

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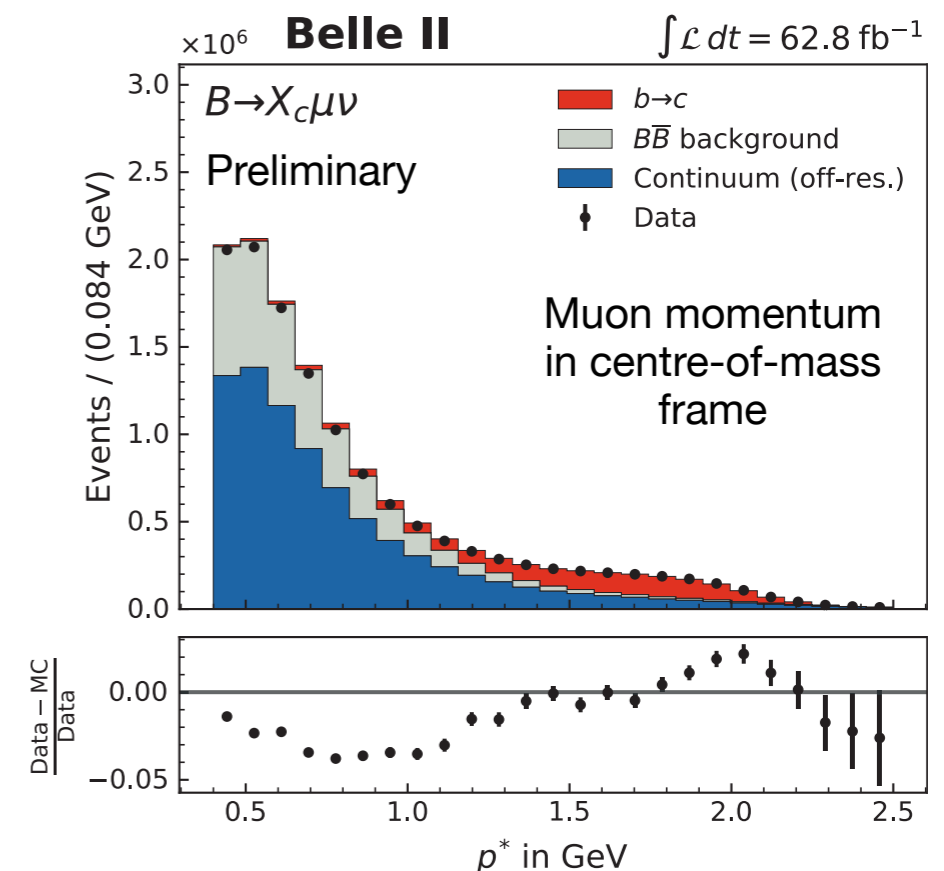
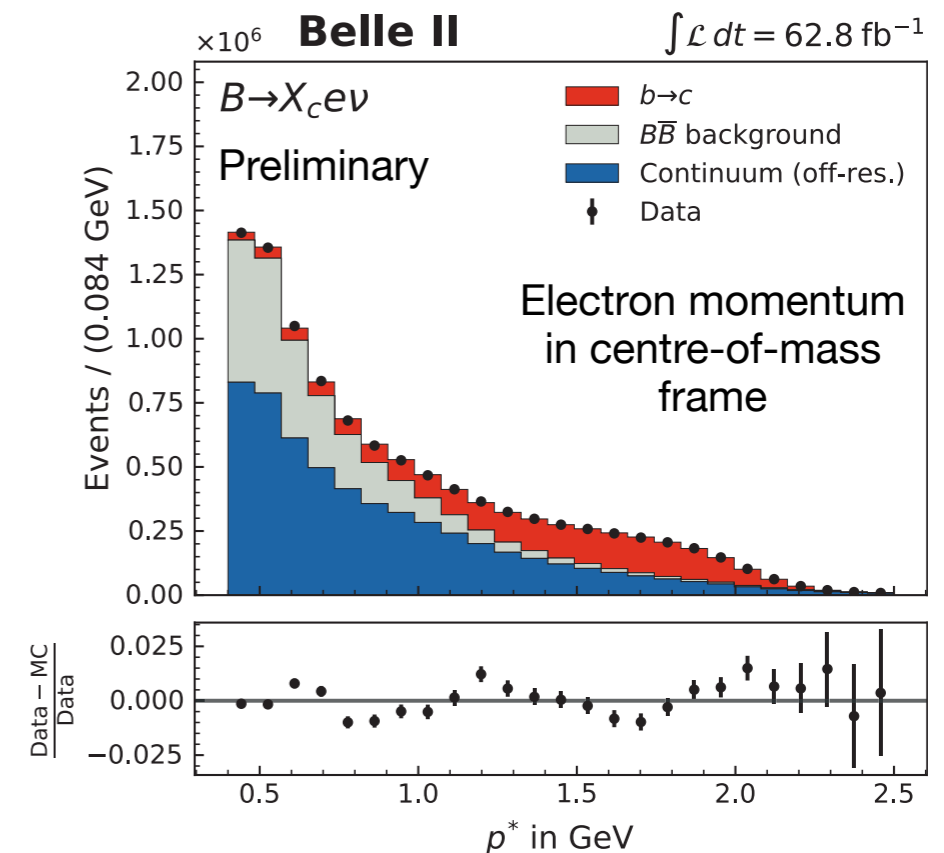
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Belle II $\mathcal{B}(B \rightarrow X_c l \nu_l)$ meas.

arXiv:2111.09405

- First presented **in summer 2021**
- Belle II data sample: 62.8 fb^{-1}
- Requires one well identified **signal lepton**
- Exploits missing mass and momentum distributions to **reject backgrounds**
- Estimate **signal yield** with binned likelihood fit in p_ℓ^*
 - separates electron and muon channels

Yield	Electron mode	Muon mode
Signal	$(1.932 \pm 0.006) \times 10^6$	$(1.501 \pm 0.007) \times 10^6$



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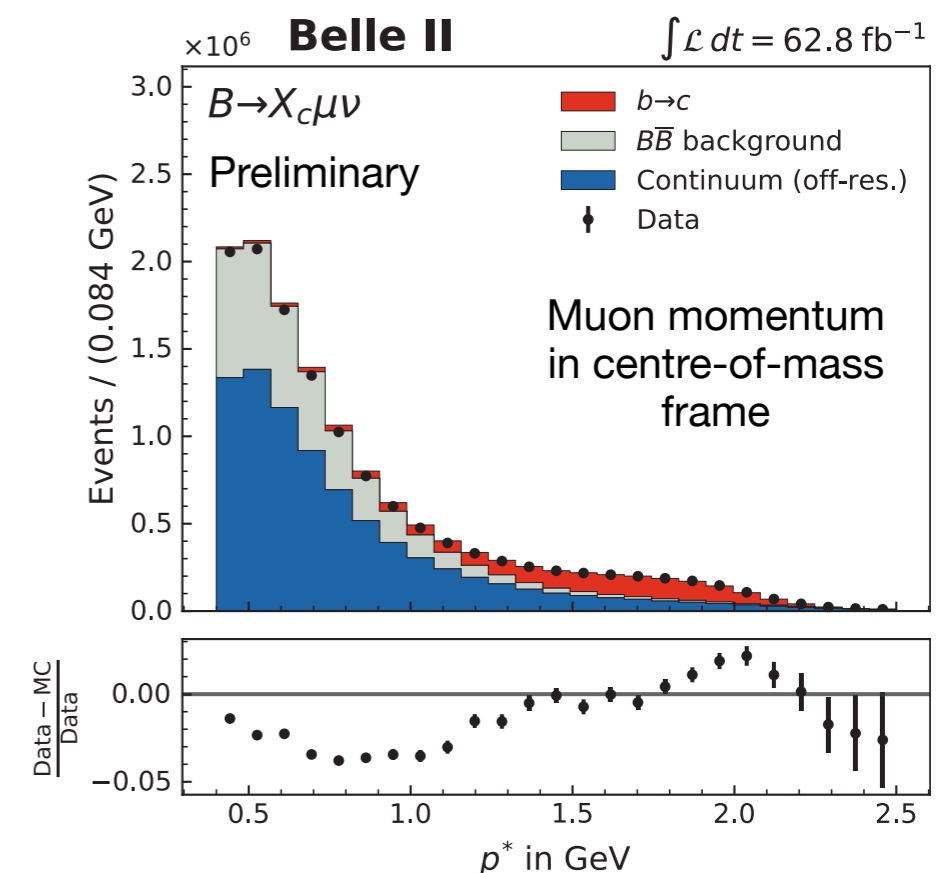
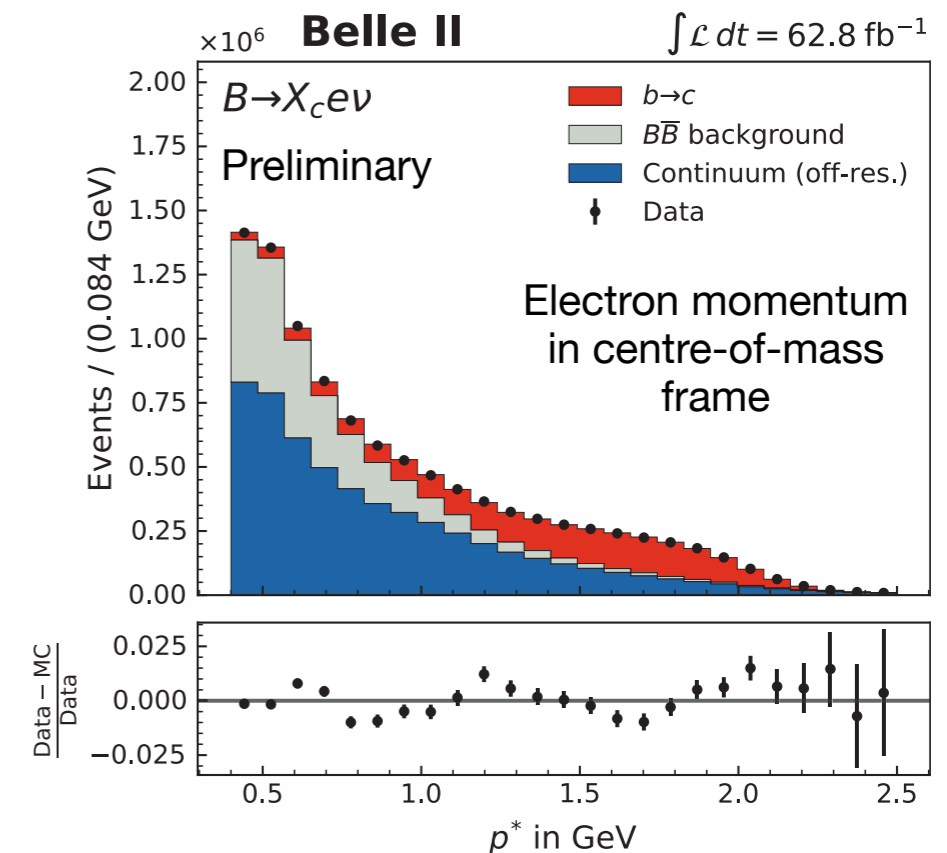
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$$\mathcal{B}(B \rightarrow X_c \ell \nu) = (9.75 \pm 0.03(\text{stat}) \pm 0.47(\text{sys}))\%$$

Leading systematics:

$B \rightarrow X_c \ell \nu$ branching fractions + form factors



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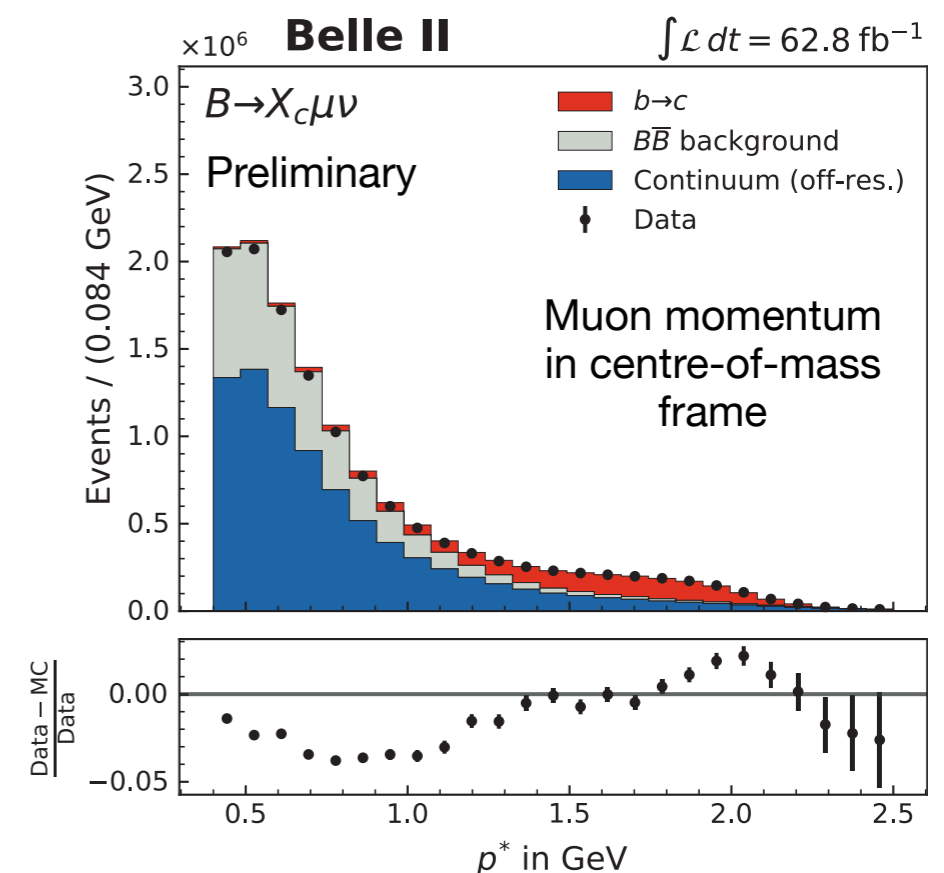
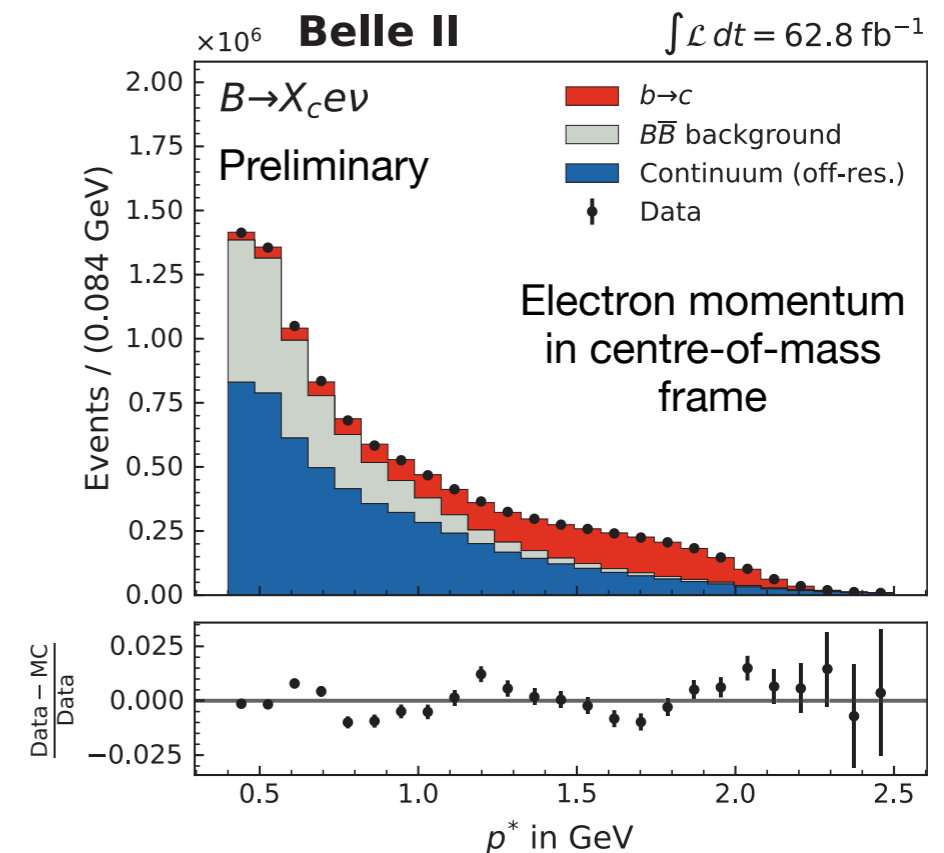
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Analysis will be **extended** to measure moments of q^2



Recipe to bake moments



9

Event-wise **Key-formula**

$$\langle q^{2n} \rangle = \frac{\sum w_i(q^2) (q_{\text{calib},i}^{2n})}{\sum_i w_i(q^2)} \times \mathcal{C}_{\text{cal}} \times \mathcal{C}_{\text{acc}}$$

Recipe to bake moments



Step #1: Subtract background

Estimate background **normalizations** by fitting M_X

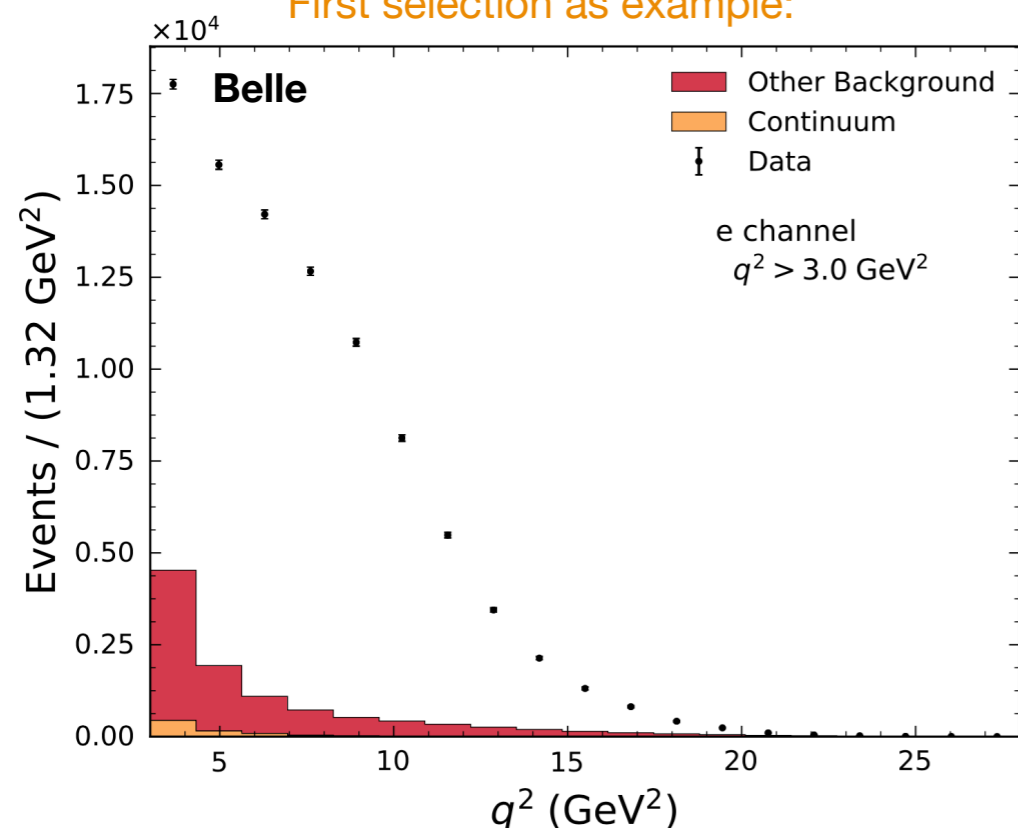
Determine sets of signal prob. weights as a **progression of threshold selections**

on q^2 (or p_ℓ for moments of M_X)

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First selection as example:



Recipe to bake moments



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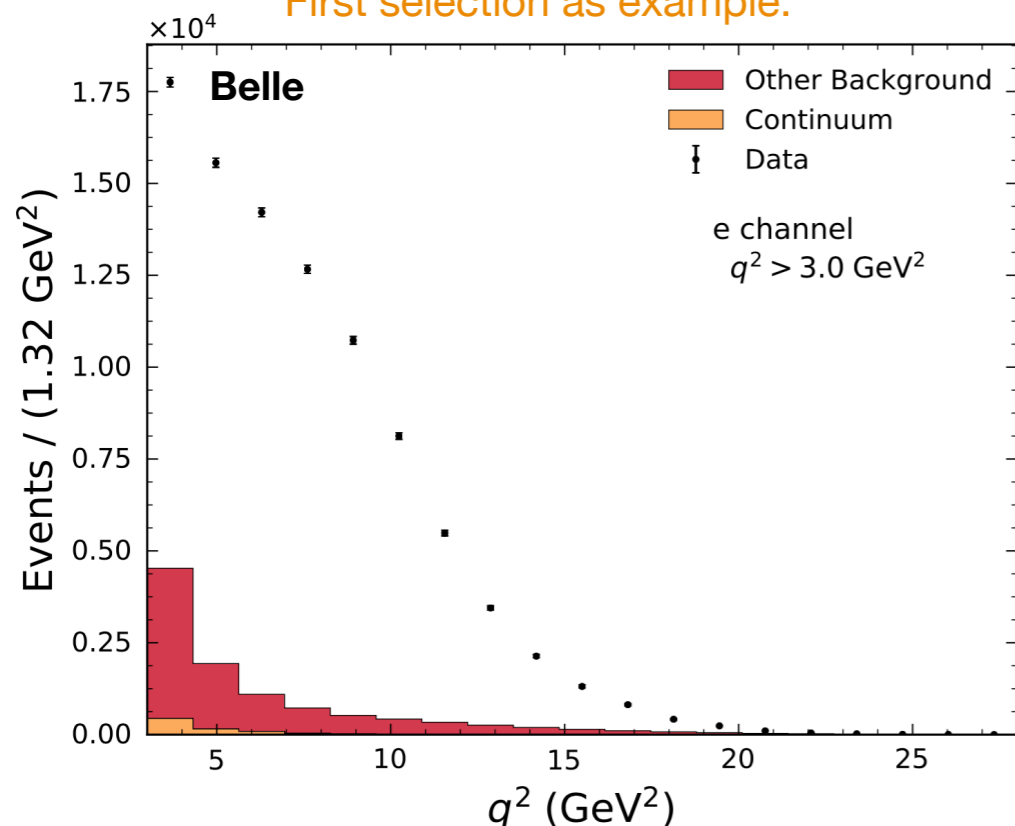
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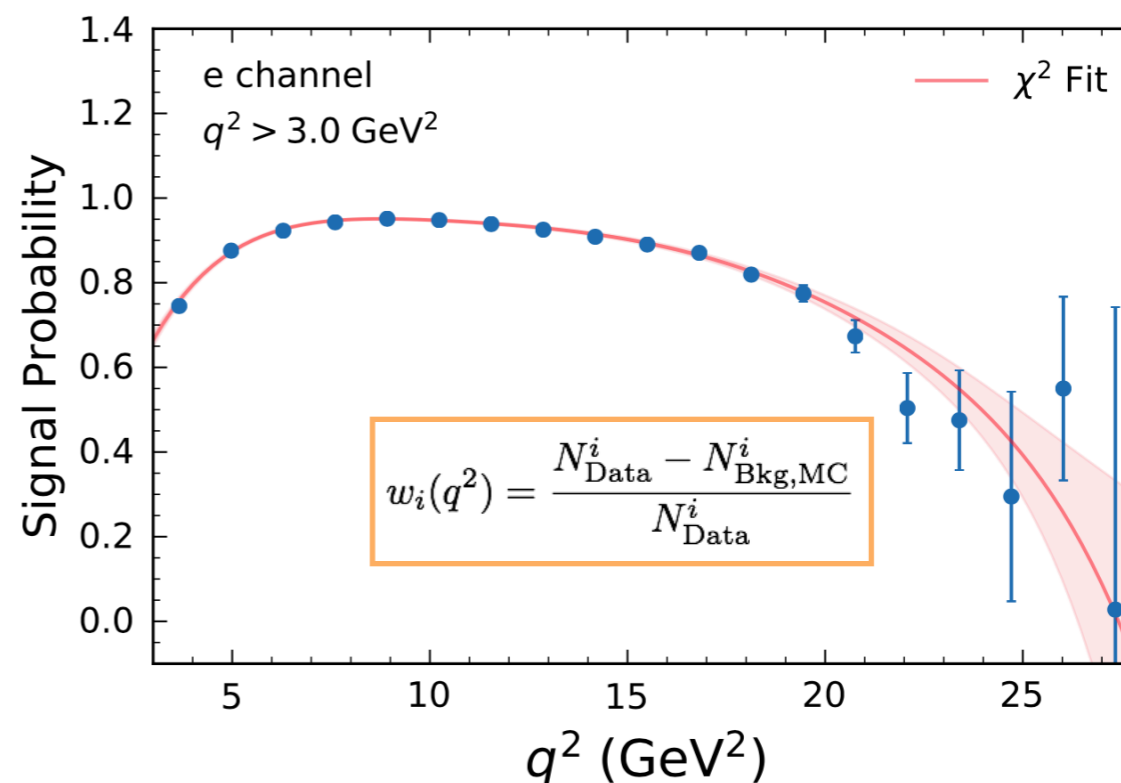
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First selection as example:



Smooth signal prob. Function $w_i(q^2)$



Recipe to bake moments



Step #1: Subtract background

Estimate background normalizations by fitting M_X

Step #2: Calibrate moments

Use **simulated data** to calibrate **reconstructed** moments

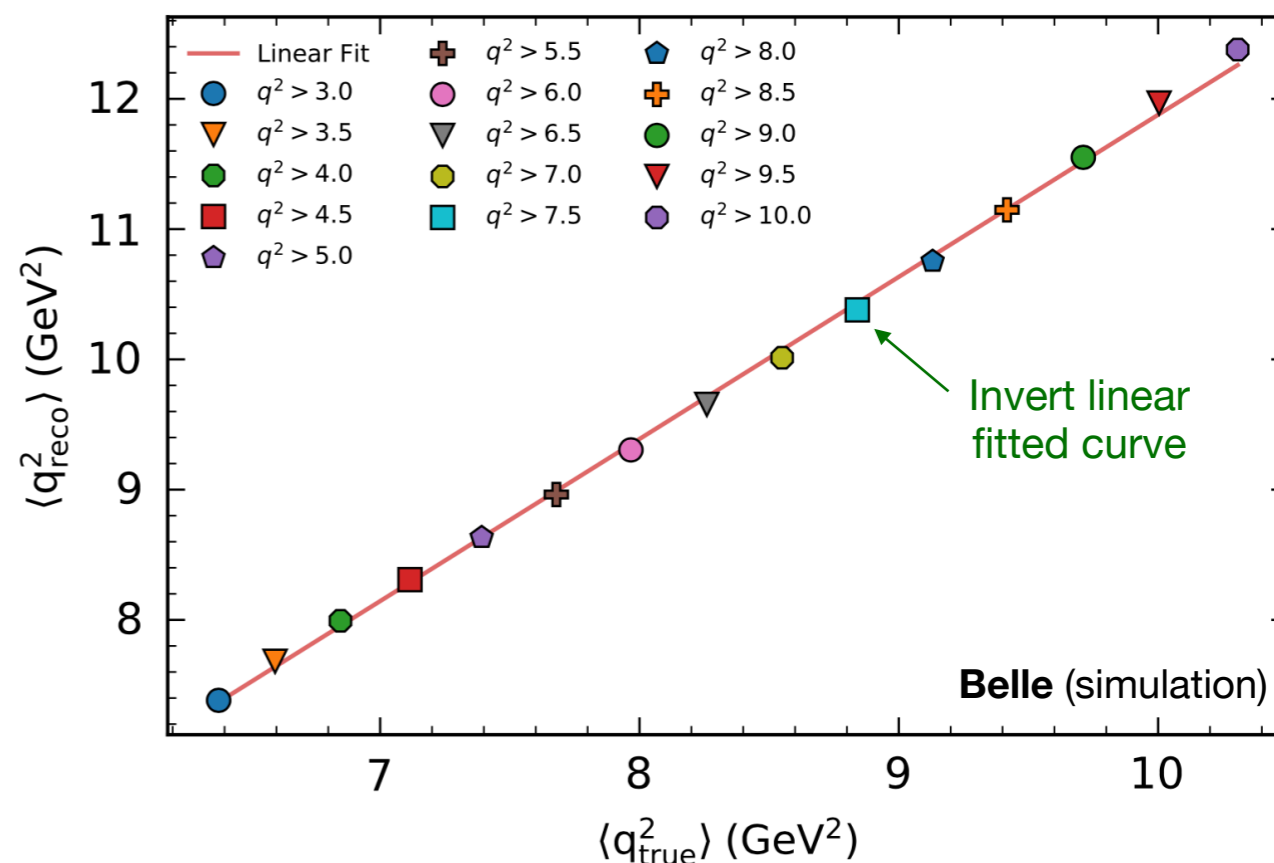
Event-wise Key-formula

$$\langle q^{2n} \rangle = \frac{\sum w_i(q^2) (q_{\text{calib},i}^{2n})}{\sum_i w_i(q^2)} \times C_{\text{cal}} \times C_{\text{acc}}$$

Linear dependence between reconstructed and true moments as a function of q^2

$$q_{\text{calib},i}^2 = (q_{\text{reco},i}^2 - c) / m$$

from simulation



Recipe to bake moments



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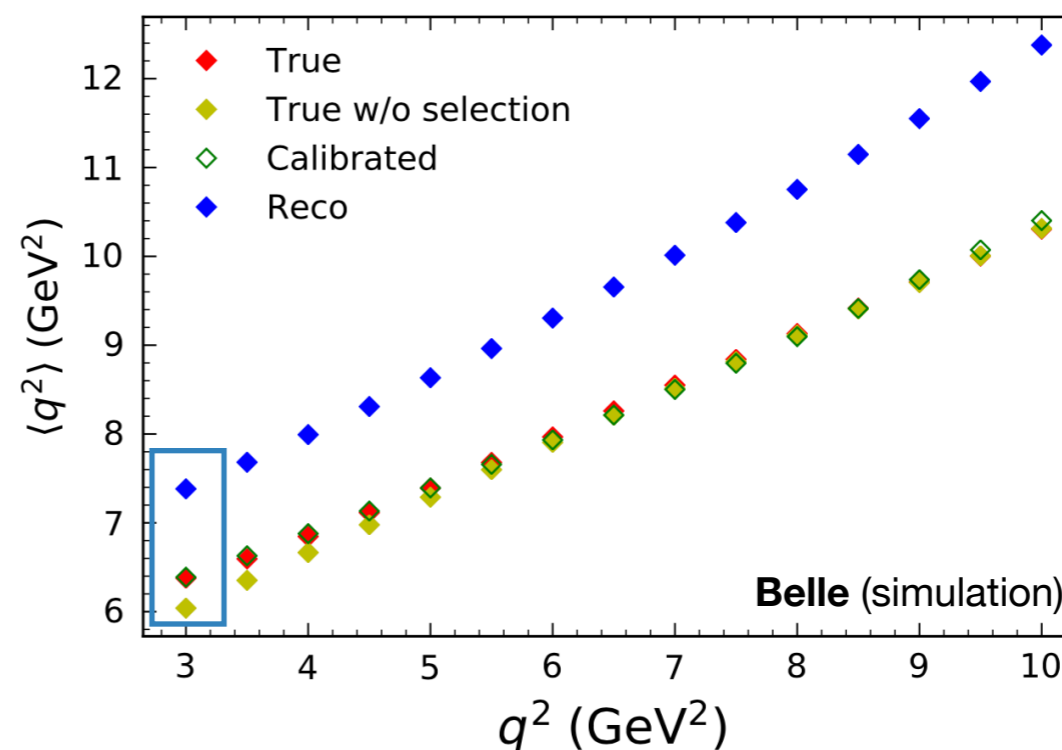
Step #3: If you fail,
try, try again

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Correct for residual calibration bias

Compare with expectation from simulated data



Recipe to bake moments



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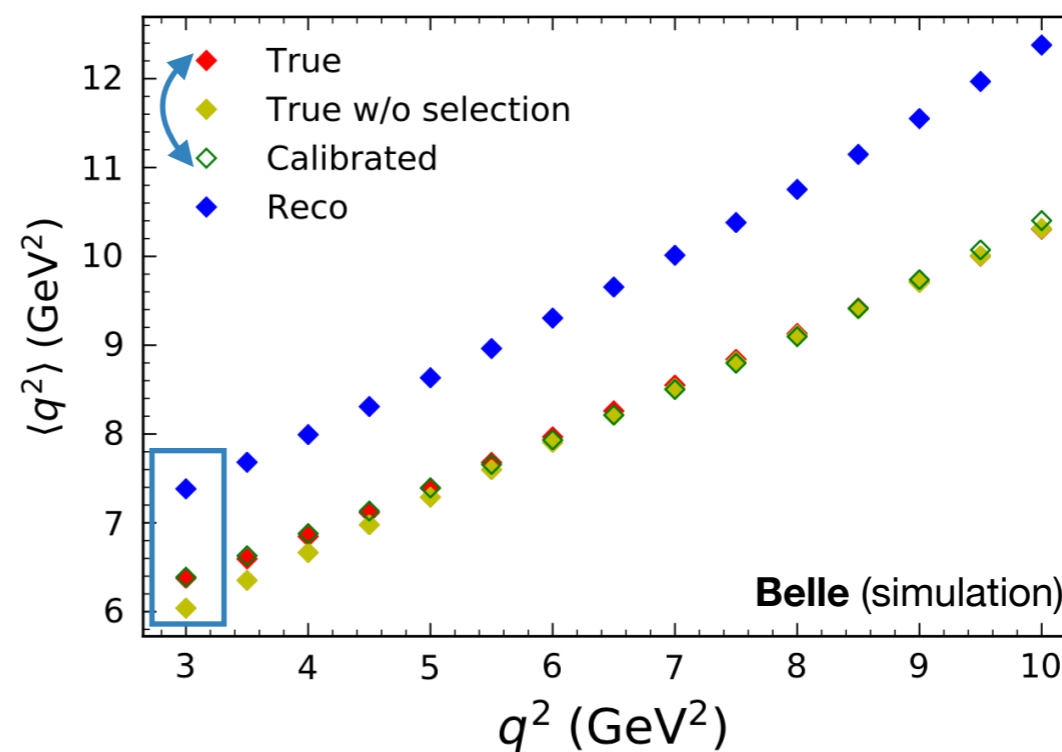
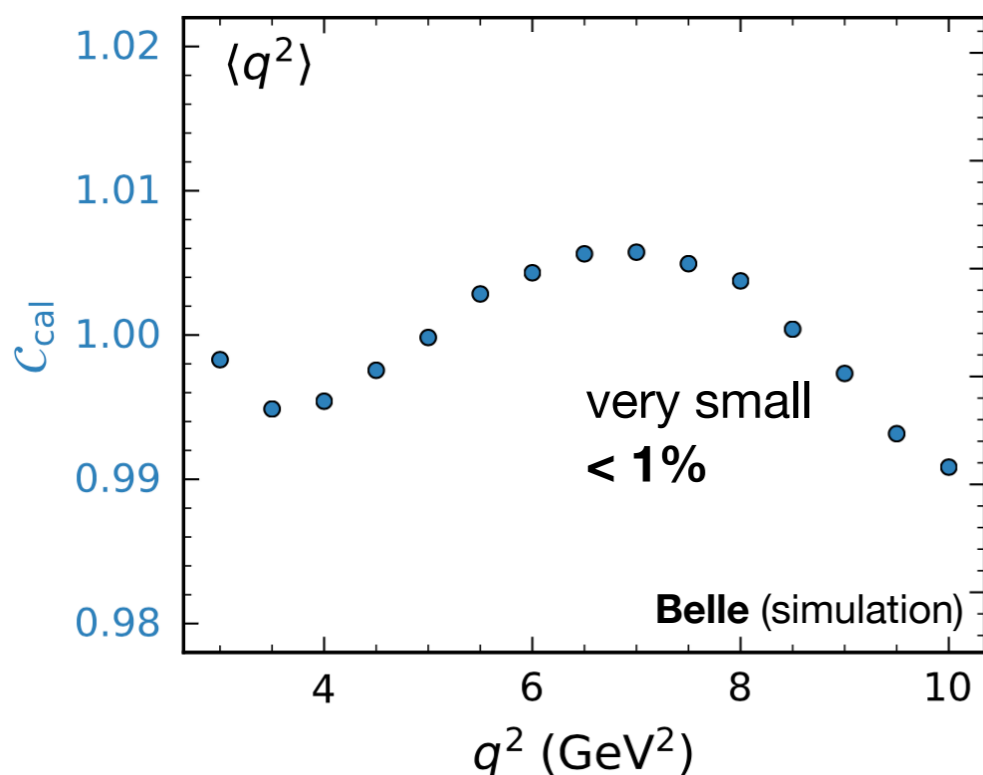
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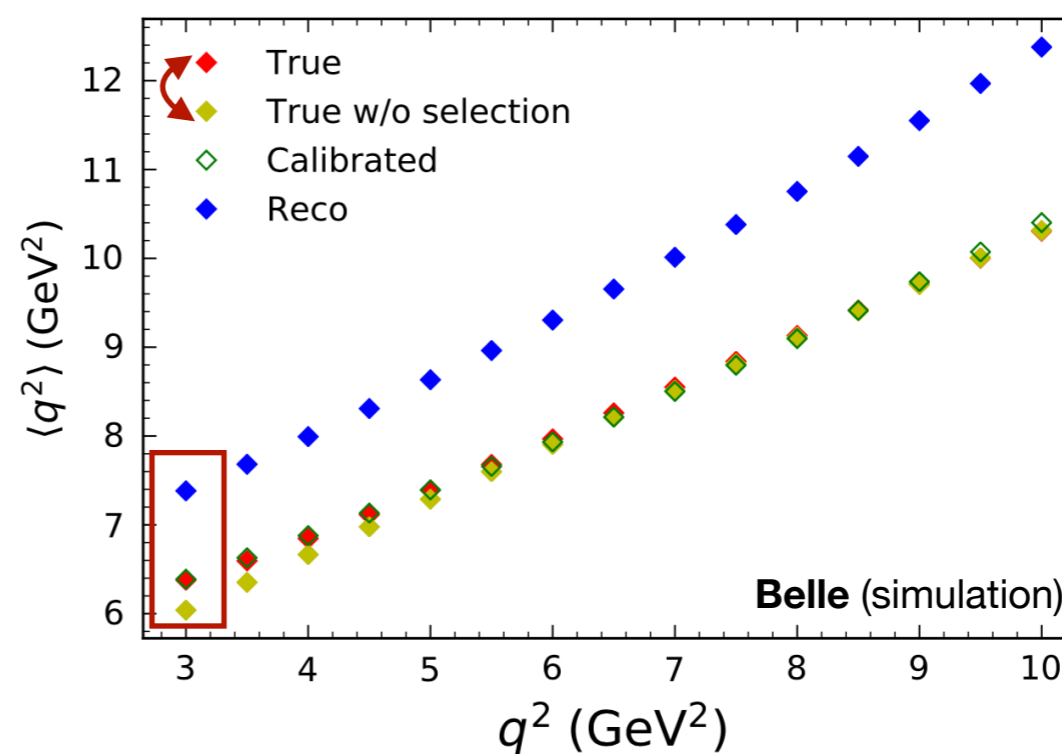
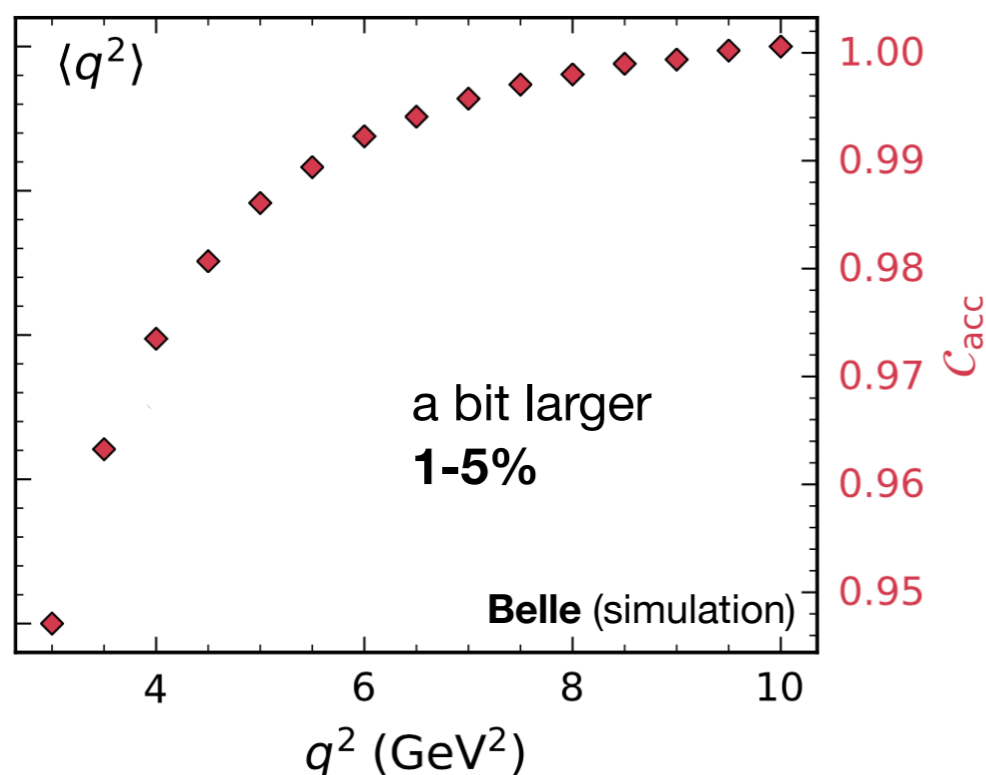
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Step #4: Correct selection bias

Correct for residual calibration bias

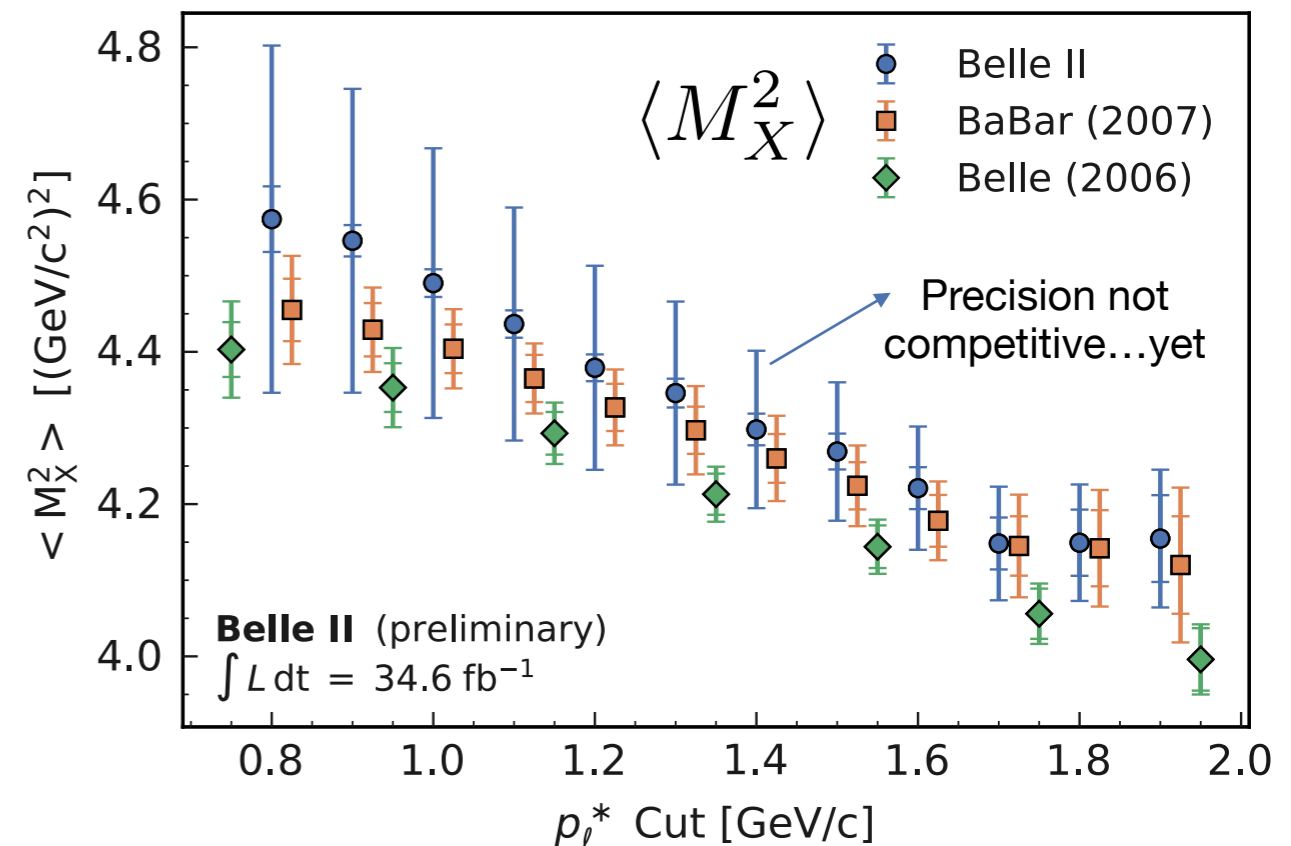
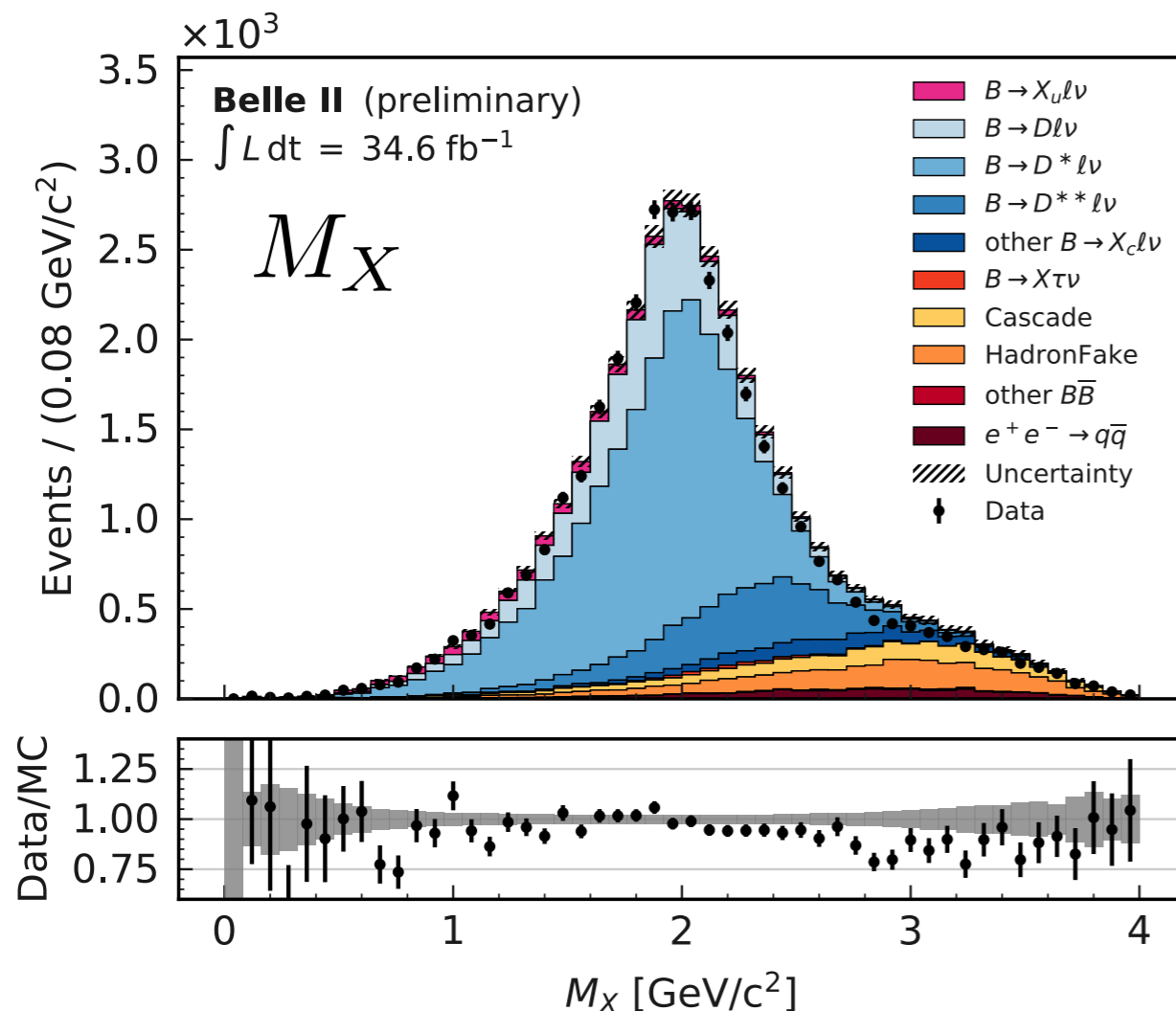
Overall event reconstruction also **biases** measured moments



Belle II $\langle M_X^n \rangle$ moments

arXiv:2009.04493

- First presented **in summer 2020**
- Belle II data sample: 34.6 fb^{-1}
- Companion B meson reconstructed using the **Full Event Interpretation (FEI)**
- Requires one **high momentum** signal lepton
- Reduces background by exploiting **inclusive kinematic variables**
- Performs calibration as a function of: Missing energy and momentum, X system multiplicity, p_ℓ
- Leading systematics: $B \rightarrow X_c \ell \nu$ **composition + modelling**

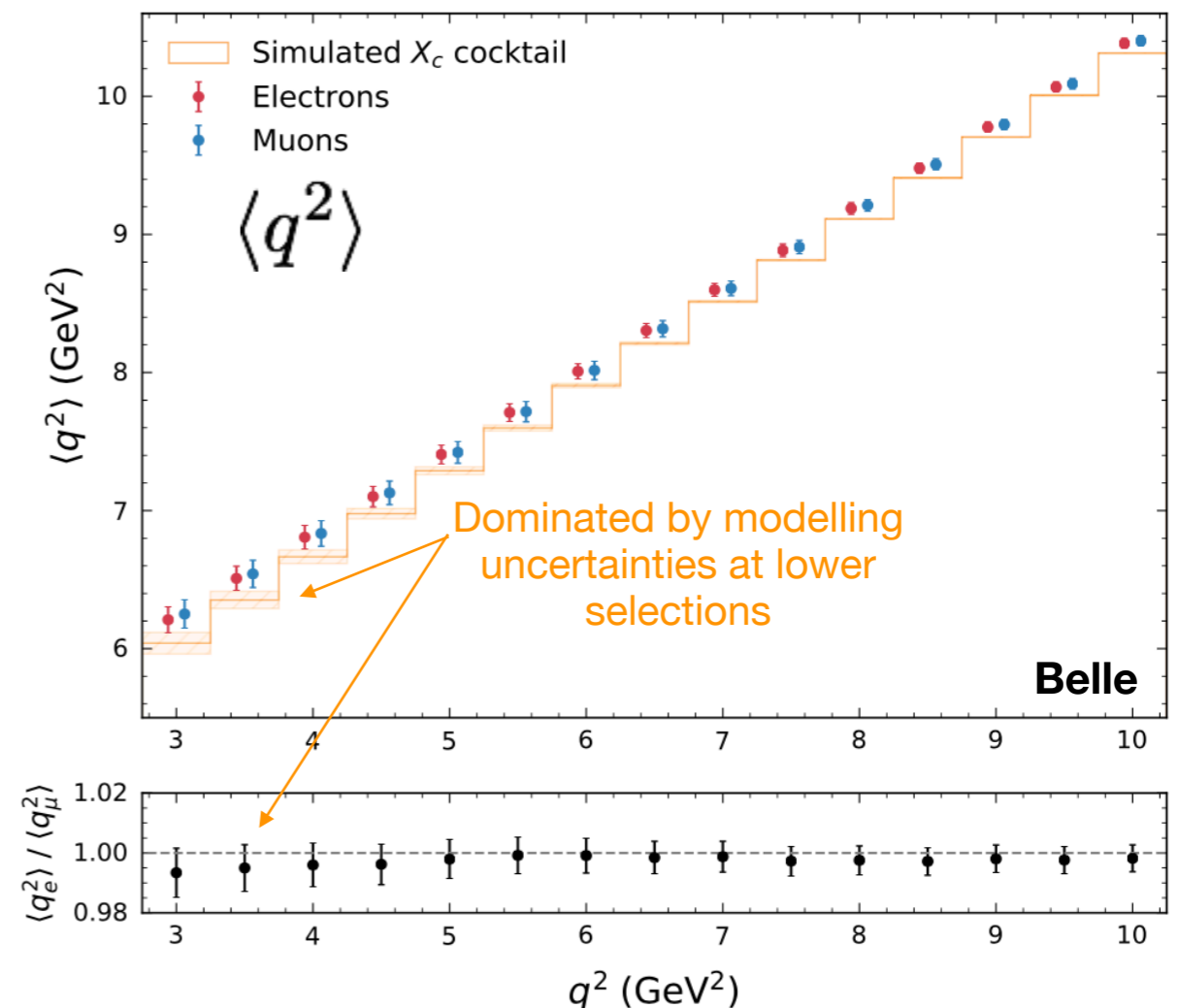
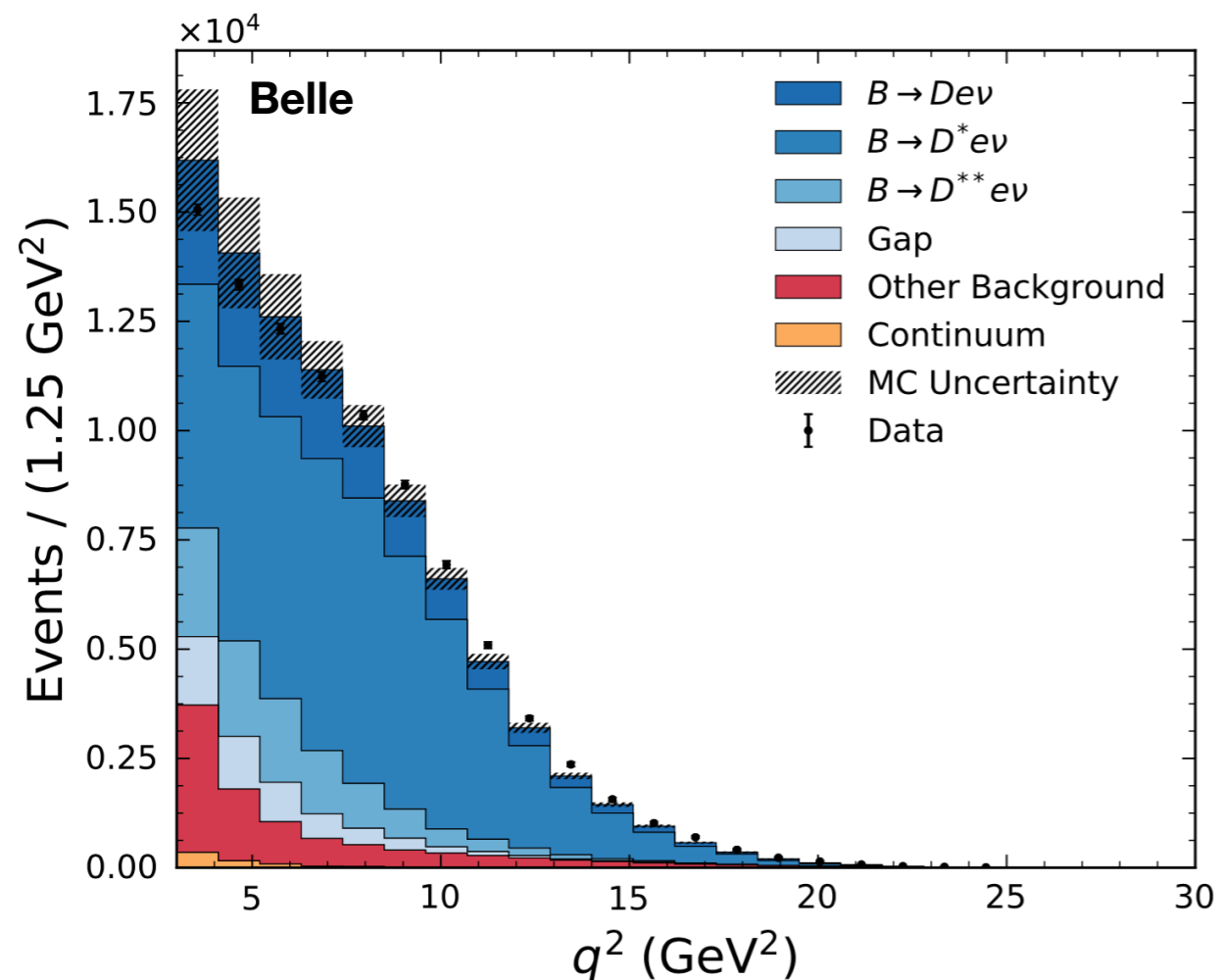


Belle $\langle q^{2n} \rangle$ moments

PRD 104, 112011 (2021)

 **Fresh idea** from [JHEP 02, 177 (2019)] (see Keri's talk)

- First presented **in spring 2021**
- Complete Belle dataset: 711 fb^{-1}
- Companion B meson reconstructed using the **Full Reconstruction (FR)**
- Selects leptons at **detector acceptance** limit: $p_e^* > 0.3 \text{ GeV}$, $p_\mu^* > 0.5 \text{ GeV}$
- Separates **electron** and **muon** channels
- Exploits missing energy and momentum to **reject backgrounds**



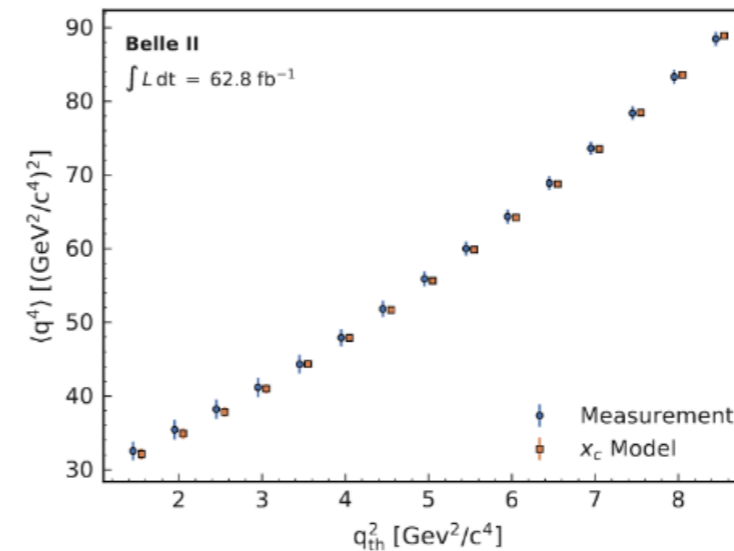
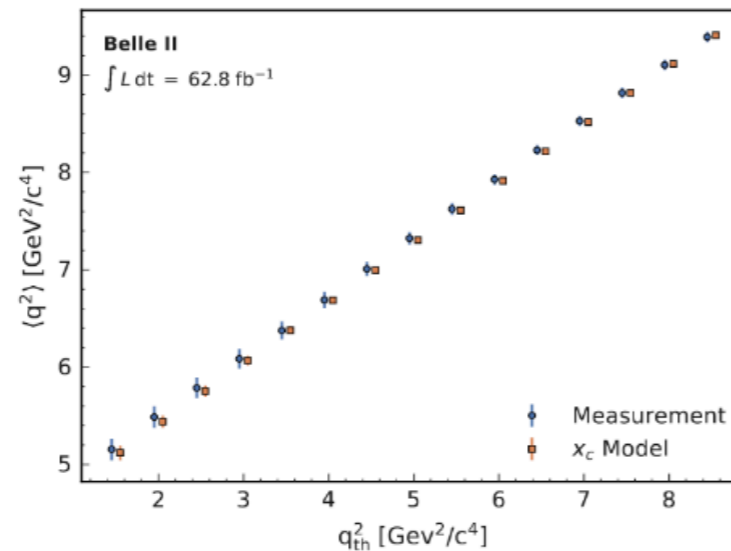
Hot off the press from Belle II!

Talk by W. Sutcliffe
Moriond EW 2022

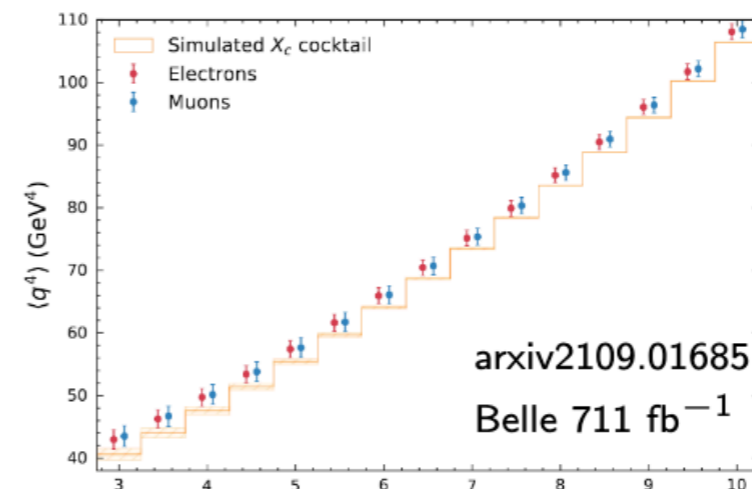
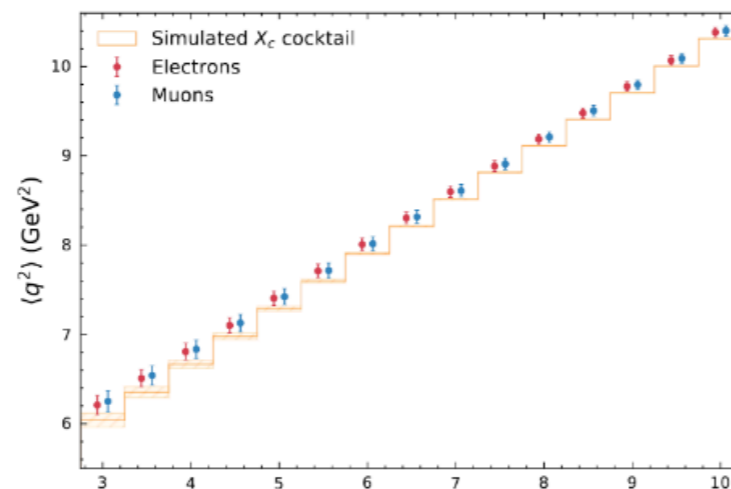
Measurements of inclusive $B \rightarrow X_c \ell \nu$ decays.

q^2 moments from $B \rightarrow X_c \ell \nu$ decays

- Here compare $n = 1, 2$ q^2 moments as a function of lower q^2 threshold between Belle II and Belle.



Paper soon!




- Expect global fits for inclusive $|V_{cb}|$ using the moments in the near future.

$B \rightarrow X_c \ell \nu$ modelling & composition

A **leading systematic** in all the discussed analyses:

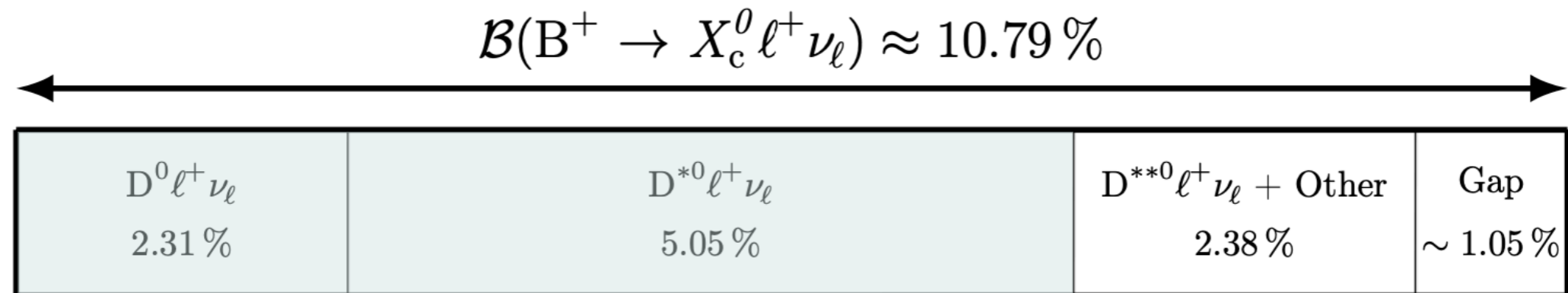
$$\mathcal{B}(B^+ \rightarrow X_c^0 \ell^+ \nu_\ell) \approx 10.79\%$$



$D^0 \ell^+ \nu_\ell$ 2.31%	$D^{*0} \ell^+ \nu_\ell$ 5.05%	$D^{**0} \ell^+ \nu_\ell + \text{Other}$ 2.38%	Gap $\sim 1.05\%$
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Decay	$\mathcal{B}(B^+)$	$\mathcal{B}(B^0)$
$B \rightarrow D \ell^+ \nu_\ell$	$(2.4 \pm 0.1) \times 10^{-2}$	$(2.2 \pm 0.1) \times 10^{-2}$
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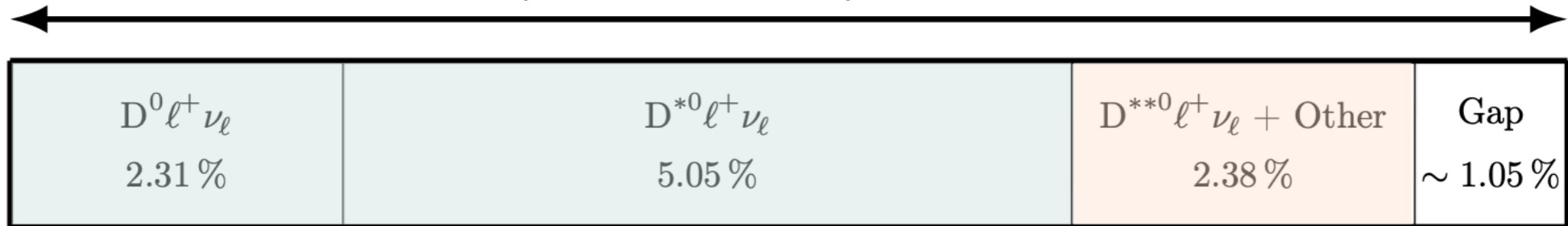


Fairly well known.
Some iso-spin tension.

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$B \rightarrow D_0^* \ell^+ \nu_\ell$	$(4.2 \pm 0.8) \times 10^{-3}$	$(3.9 \pm 0.7) \times 10^{-3}$
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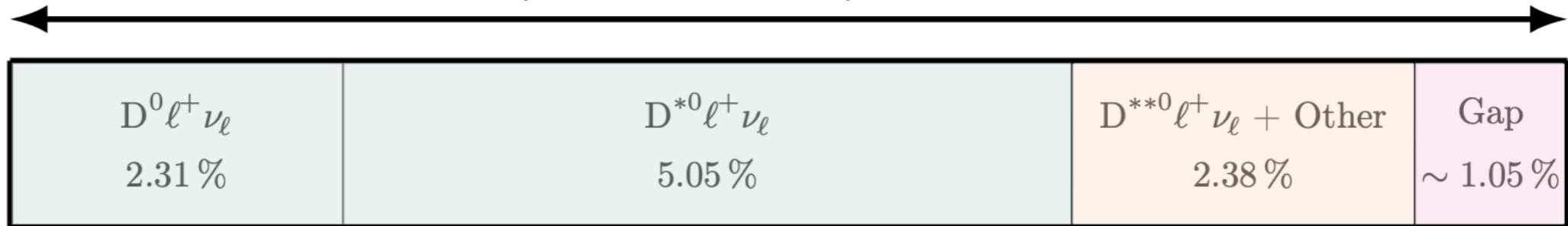


Broad states based on
3 measurements.
(BaBar, Belle, DELPHI)

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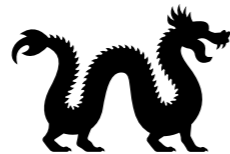
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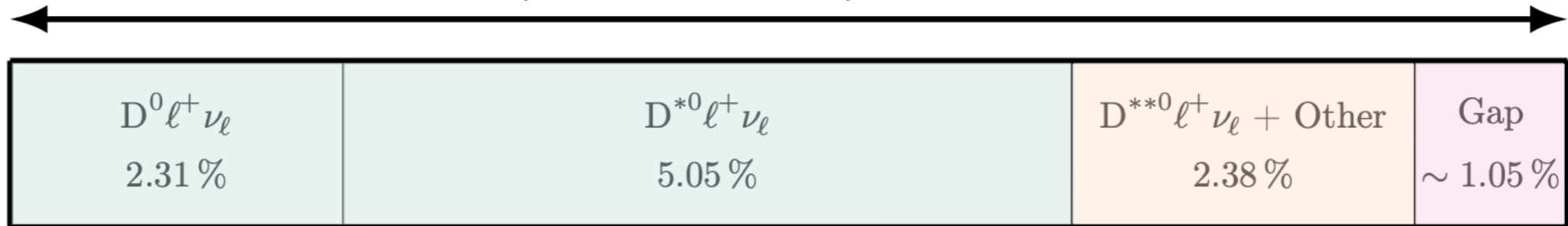


$$B \rightarrow X_c \ell \nu_\ell \quad (10.8 \pm 0.4) \times 10^{-2} \quad (10.1 \pm 0.4) \times 10^{-2}$$

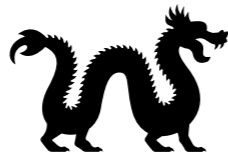
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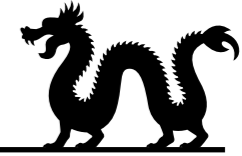
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Broad states based on
3 measurements.
(BaBar, Belle, DELPHI)



Some hints from
the BaBar result.



A tale of two 'gap' models

Model 1:

Equidistribution of all final state particles in phase space

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$B \rightarrow D^* \pi \pi \ell^+ \nu_\ell$	$(2.2 \pm 1.0) \times 10^{-3}$	$(2.0 \pm 1.0) \times 10^{-3}$
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A tale of two 'gap' models



Model 1:

Equidistribution of all final state particles in phase space

Decay	$\mathcal{B}(B^+)$	$\mathcal{B}(B^0)$
$B \rightarrow D \ell^+ \nu_\ell$	$(2.4 \pm 0.1) \times 10^{-2}$	$(2.2 \pm 0.1) \times 10^{-2}$
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Model 2:

Decay via intermediate broad D^{**} state

Decay	$\mathcal{B}(B^+)$	$\mathcal{B}(B^0)$
$B \rightarrow D_0^* \ell^+ \nu_\ell$ ($\hookrightarrow D\pi\pi$)	$(0.03 \pm 0.03) \times 10^{-2}$	$(0.03 \pm 0.03) \times 10^{-2}$
$B \rightarrow D_1^* \ell^+ \nu_\ell$ ($\hookrightarrow D\pi\pi$)	$(0.03 \pm 0.03) \times 10^{-2}$	$(0.03 \pm 0.03) \times 10^{-2}$
$B \rightarrow D_0^* \pi\pi \ell^+ \nu_\ell$ ($\hookrightarrow D^* \pi\pi$)	$(0.108 \pm 0.051) \times 10^{-2}$	$(0.101 \pm 0.048) \times 10^{-2}$
$B \rightarrow D_1^* \pi\pi \ell^+ \nu_\ell$ ($\hookrightarrow D^* \pi\pi$)	$(0.108 \pm 0.051) \times 10^{-2}$	$(0.101 \pm 0.048) \times 10^{-2}$
$B \rightarrow D_0^* \ell^+ \nu_\ell$ ($\hookrightarrow D\eta$)	$(0.396 \pm 0.396) \times 10^{-2}$	$(0.399 \pm 0.399) \times 10^{-2}$
$B \rightarrow D_1^* \ell^+ \nu_\ell$ ($\hookrightarrow D^* \eta$)	$(0.396 \pm 0.396) \times 10^{-2}$	$(0.399 \pm 0.399) \times 10^{-2}$

(Assign 100% BR uncertainty in systematics covariance matrix)

A tale of two 'gap' models



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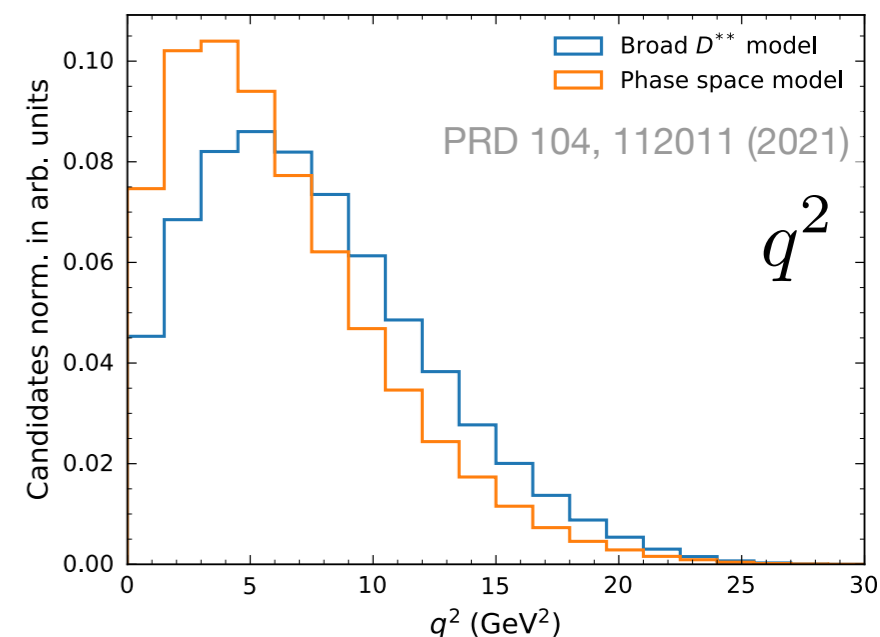
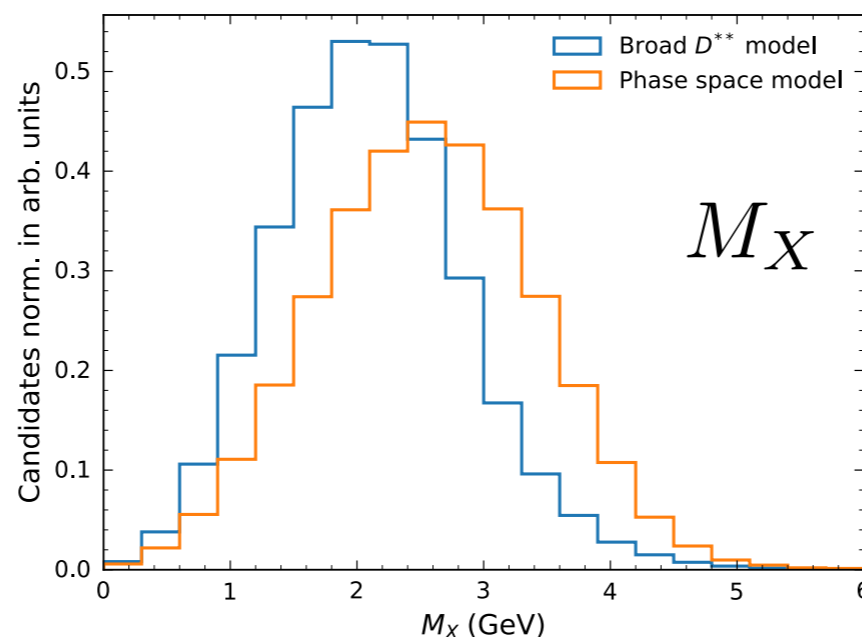
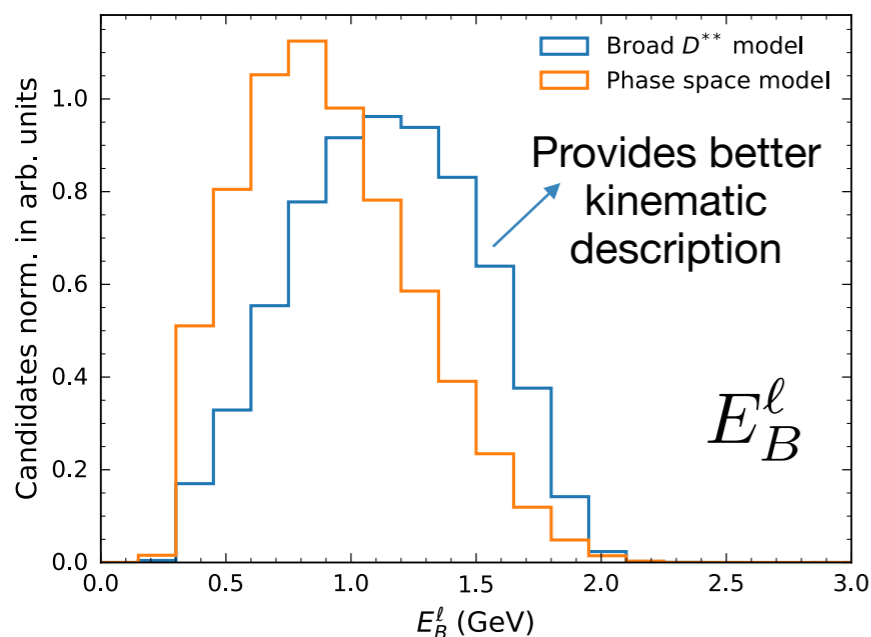
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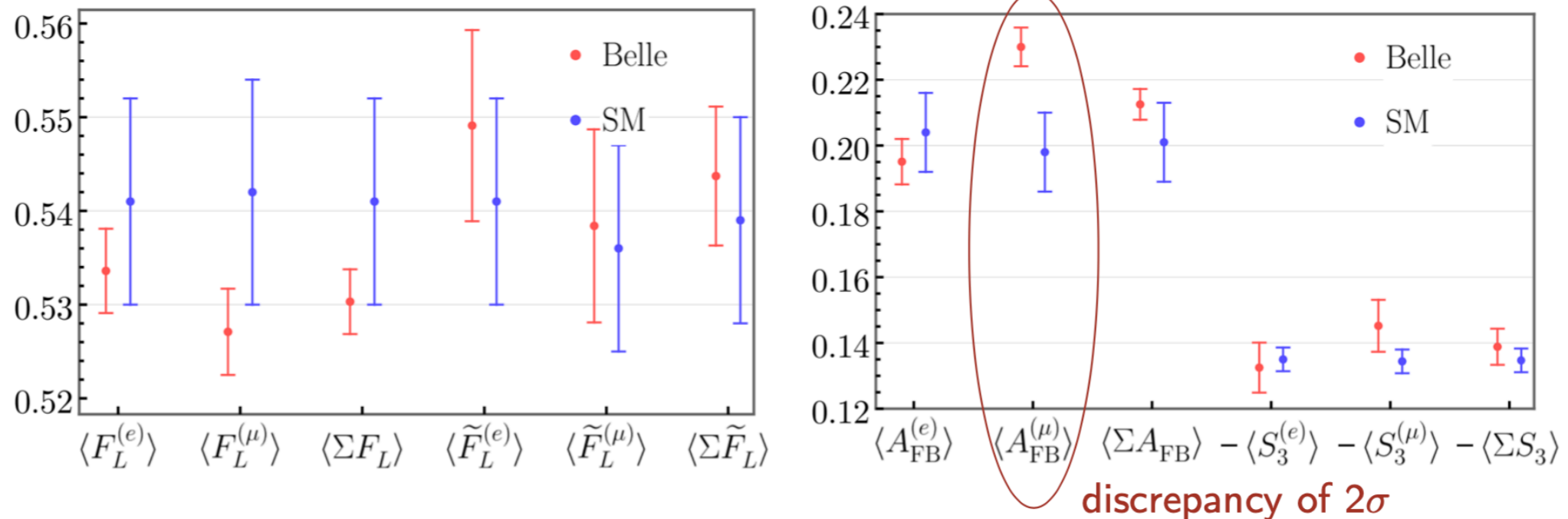


Recap: A_{FB} status

Manca's talk...again

A_{FB} from $B^0 \rightarrow D^{*-} \ell^+ \nu_\ell$ (2021)

[arXiv:2104.02094v1]



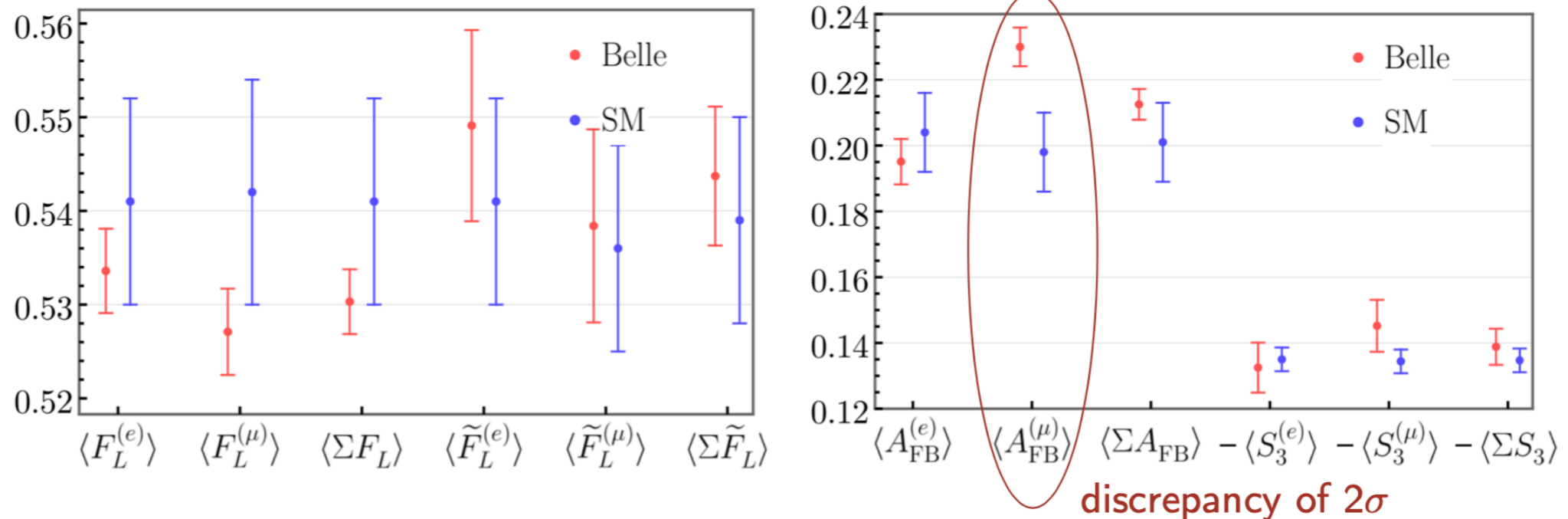
- Stable with respect to the type of fit and the systematic correlations
- Correlation matrices of the statistical uncertainties are incorrect
- ? Beyond the Standard Model physics scenario
- ? Wrong assumptions → But why no discrepancy in other parameters?
- **Further studies are needed**
- **e/μ flavours should be studied separately**

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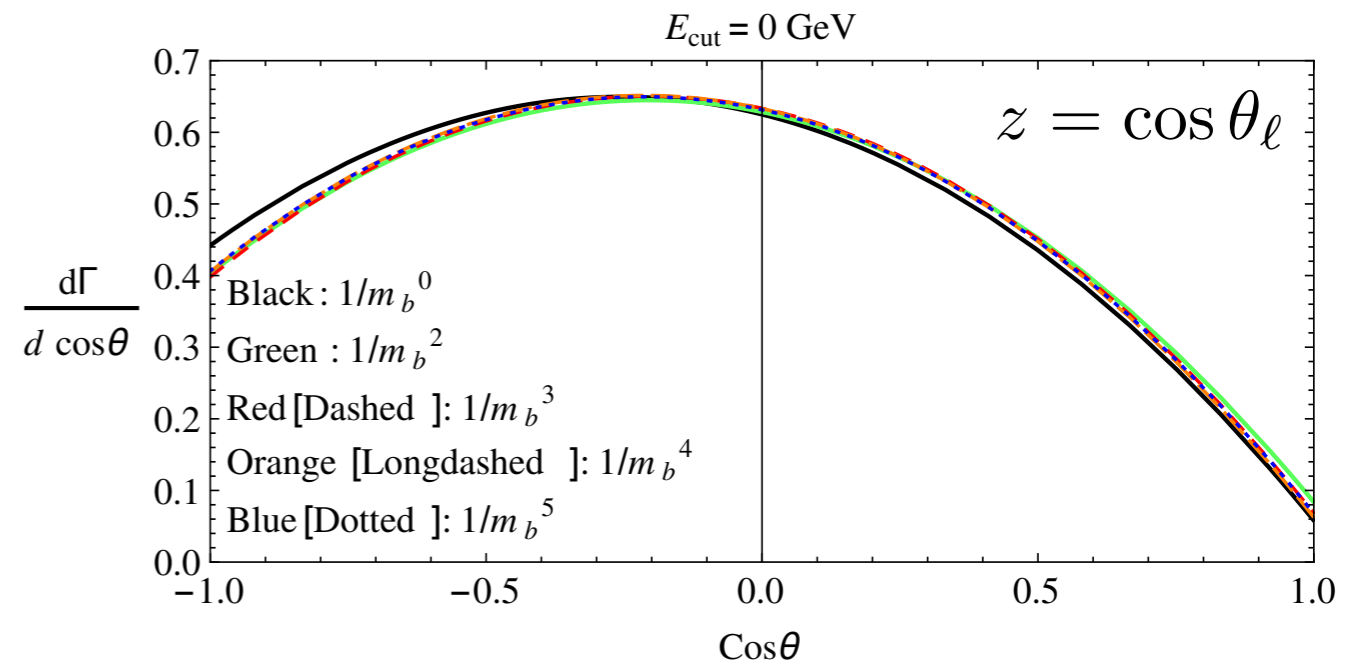
Measure A_{FB} from inclusive $B \rightarrow X_c \ell \nu$ decays for an **orthogonal, complementary study**, while also gaining **additional information** on HQE parameters. [JHEP 04 (2016) 131]

A_{FB} at Belle II

JHEP 04 (2016) 131

- Goal: Measure A_{FB} from **inclusive** $B \rightarrow X_c \ell \nu$ decays using **hadronic tagging**

$$A_{FB} = \frac{1}{\Gamma} \left(\int_{-1}^0 dz \frac{d\Gamma}{dz} - \int_0^1 dz \frac{d\Gamma}{dz} \right)$$

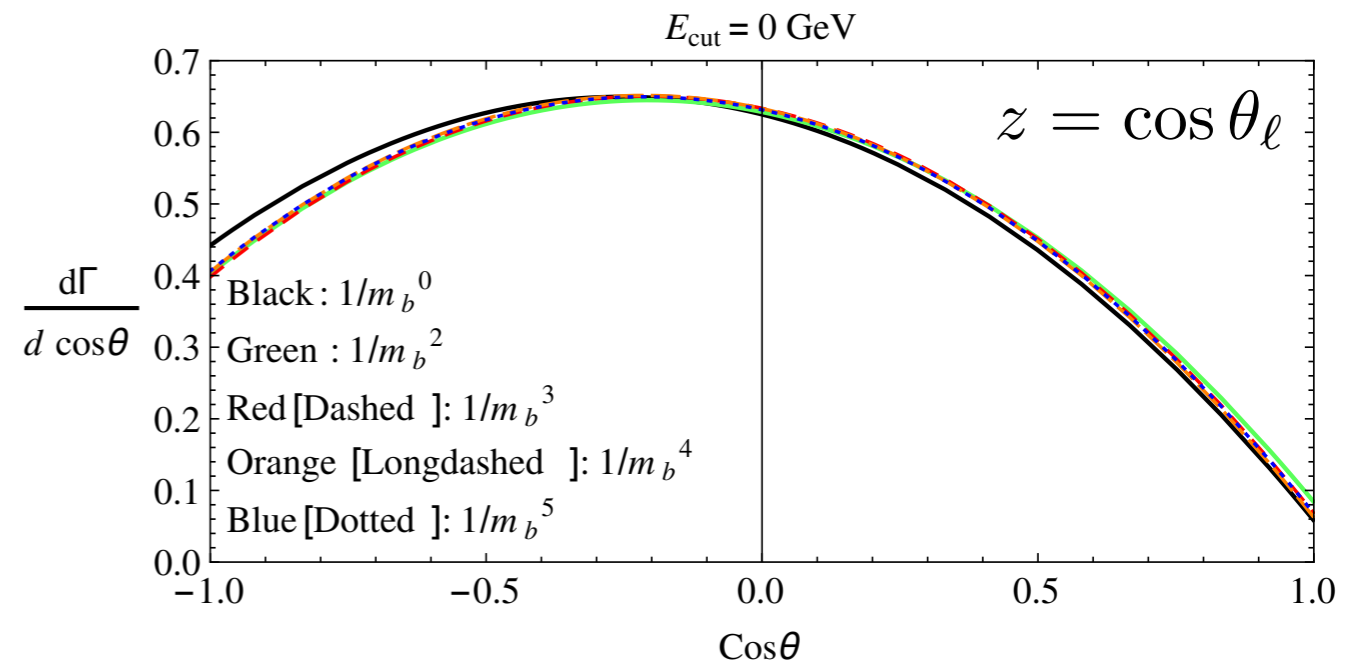


A_{FB} at Belle II

JHEP 04 (2016) 131

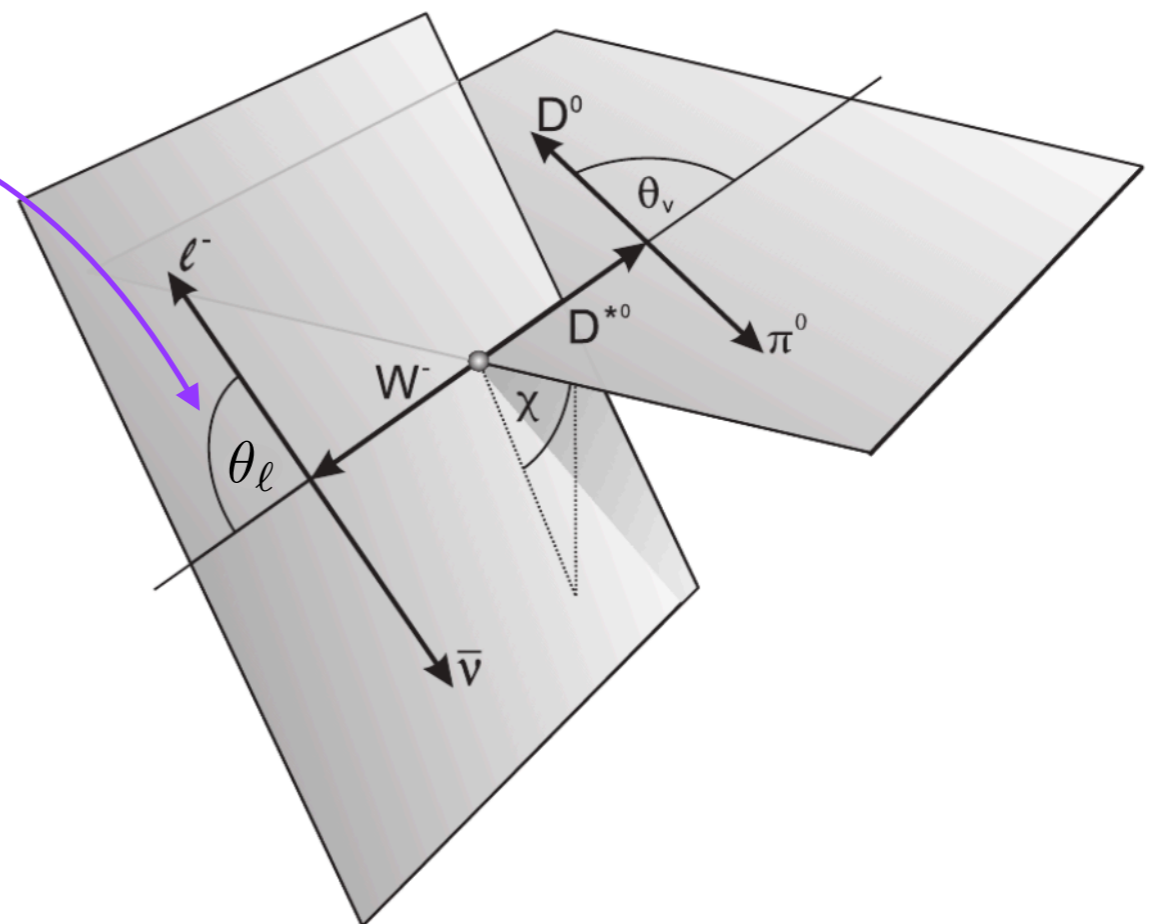
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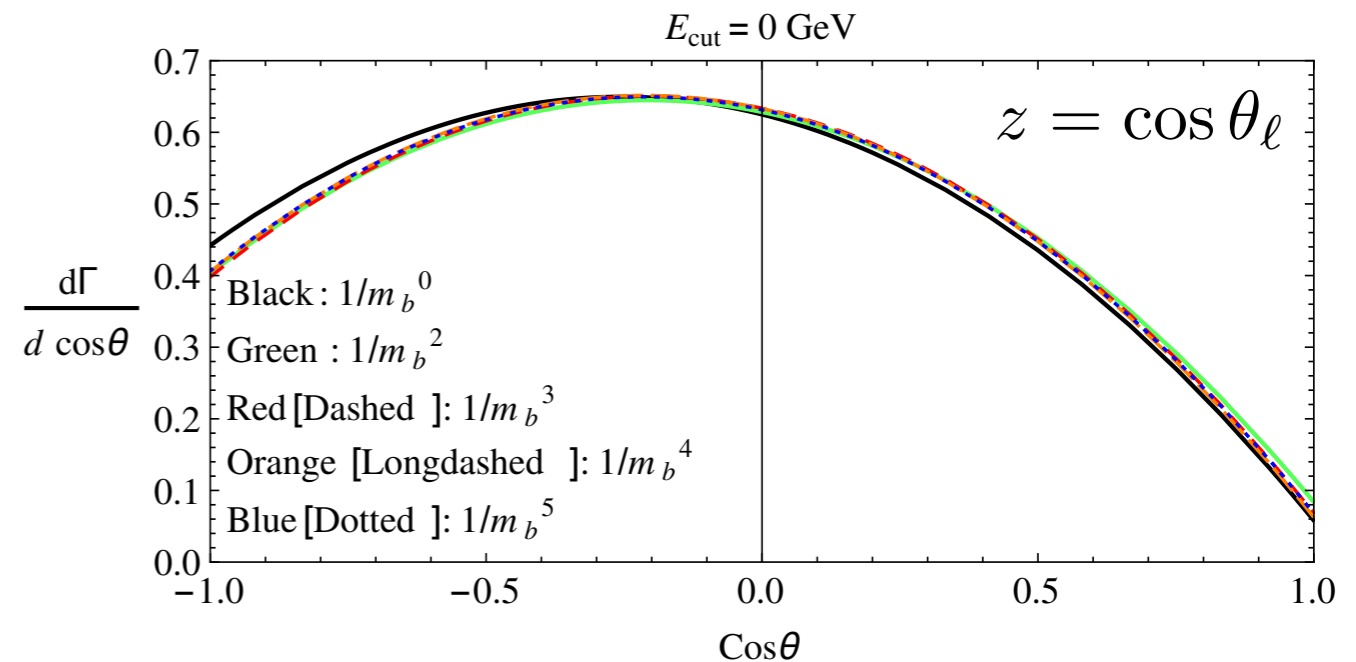


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JHEP 04 (2016) 131

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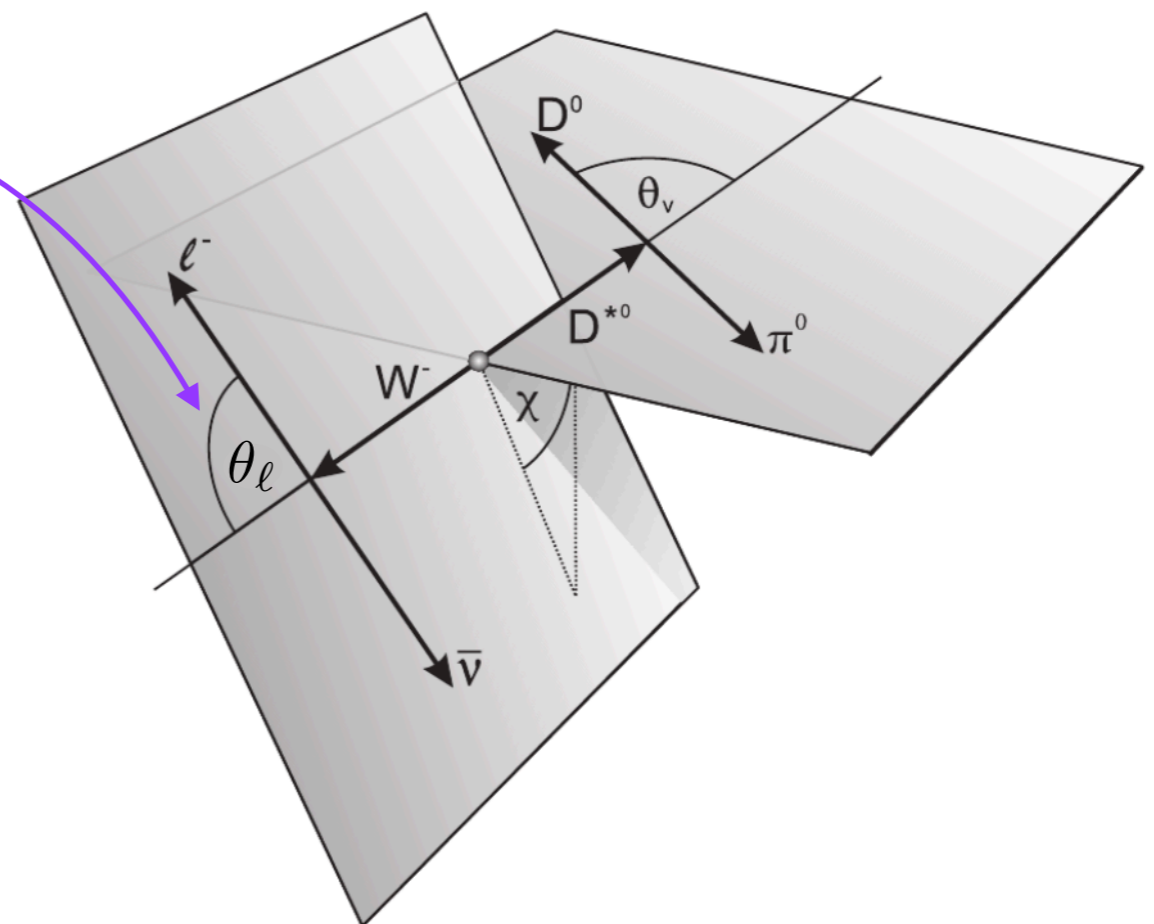
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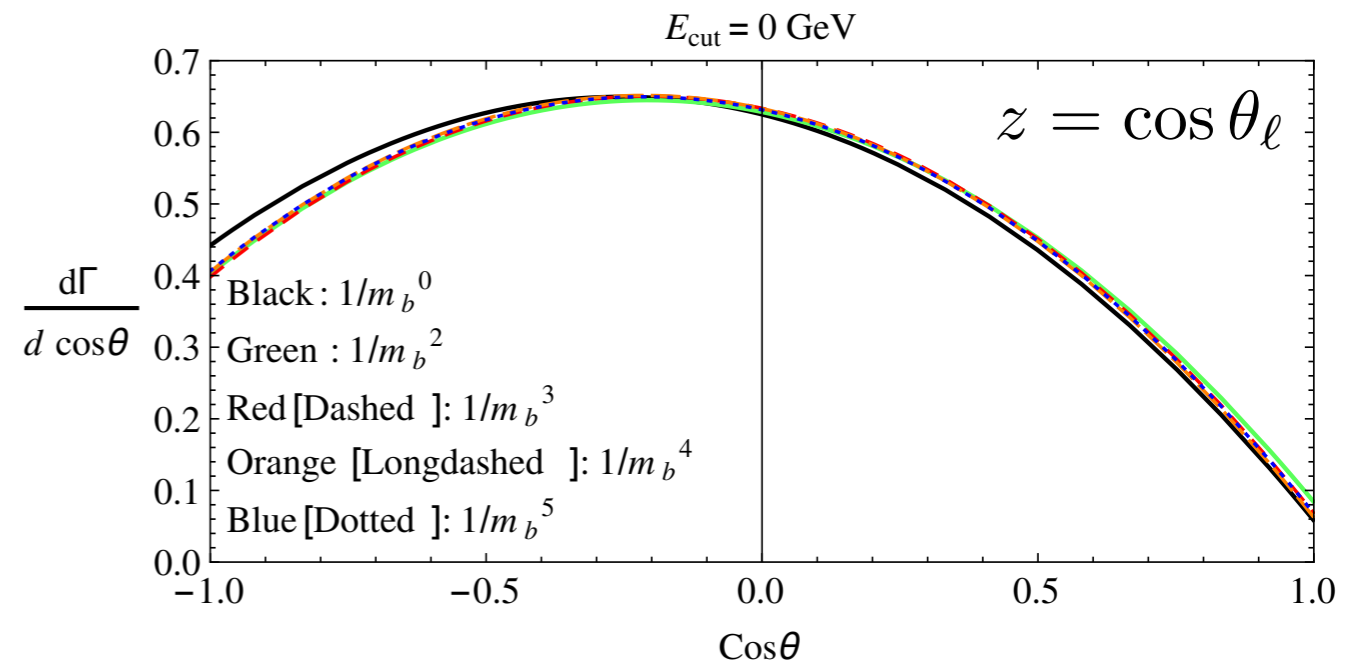


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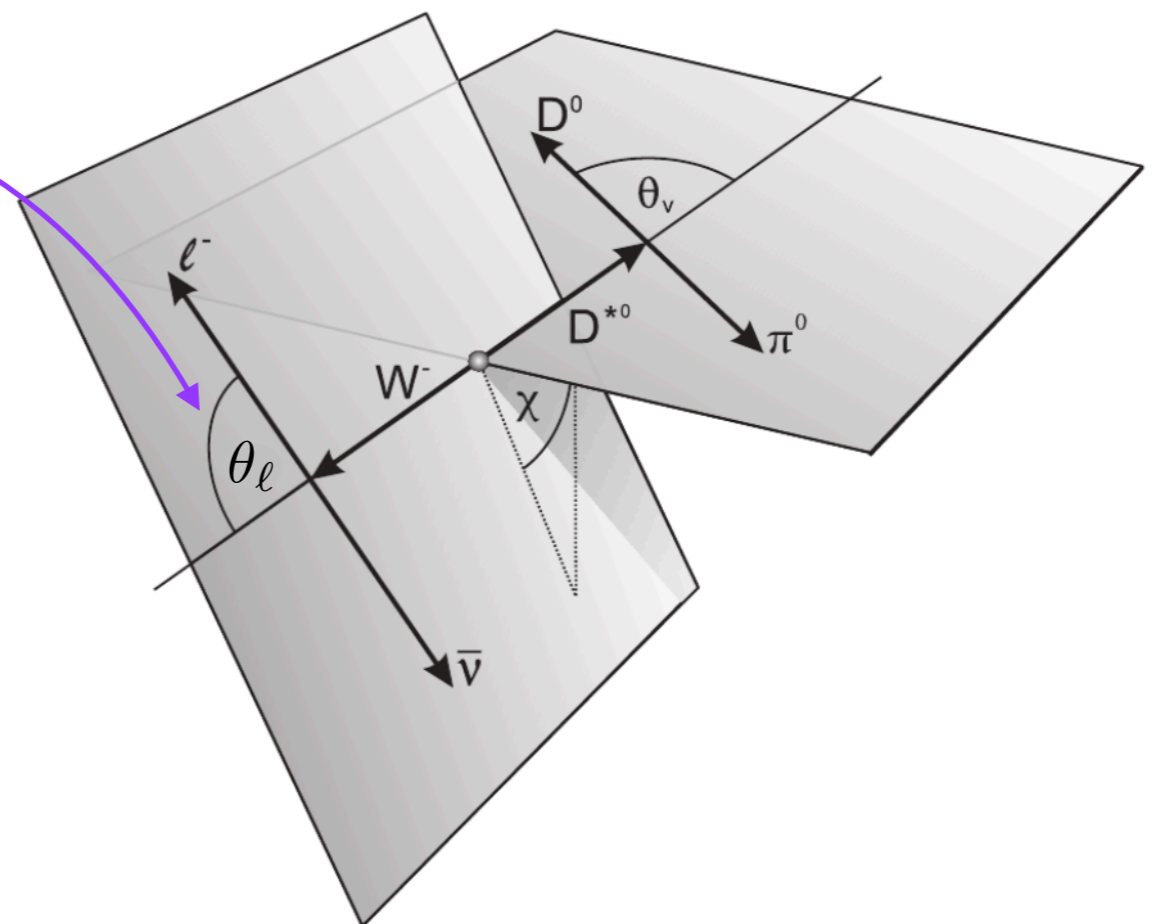
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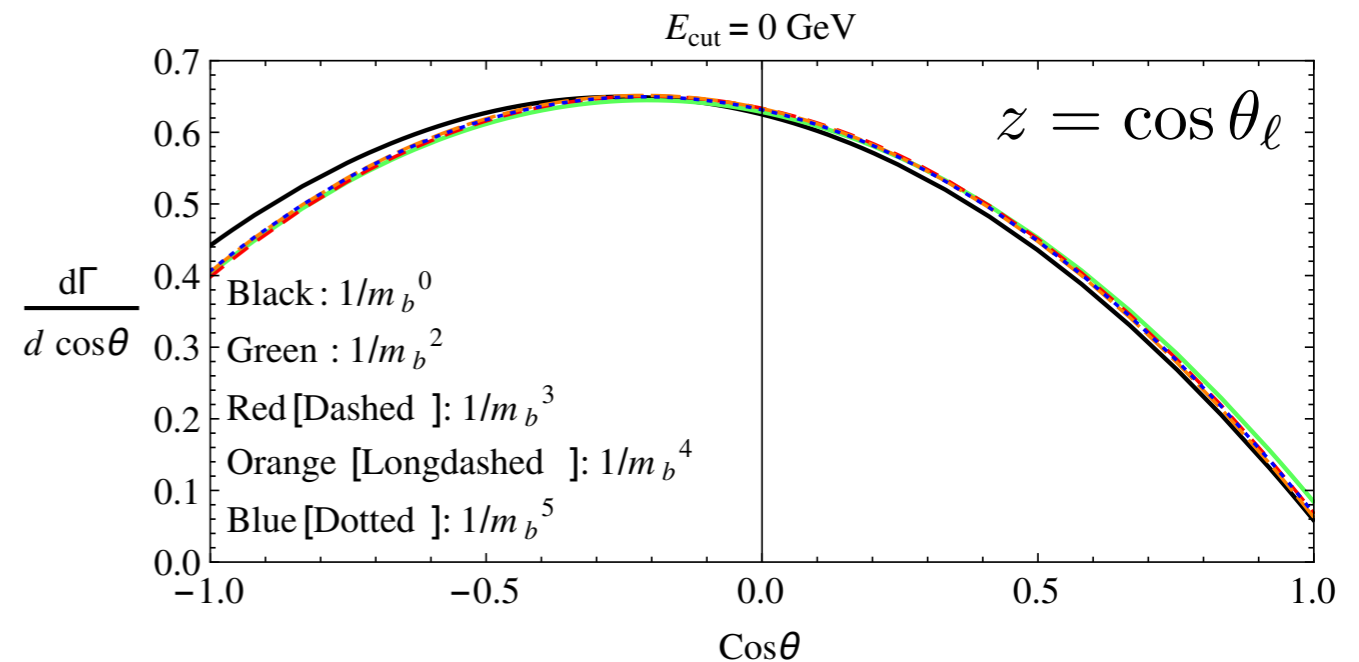


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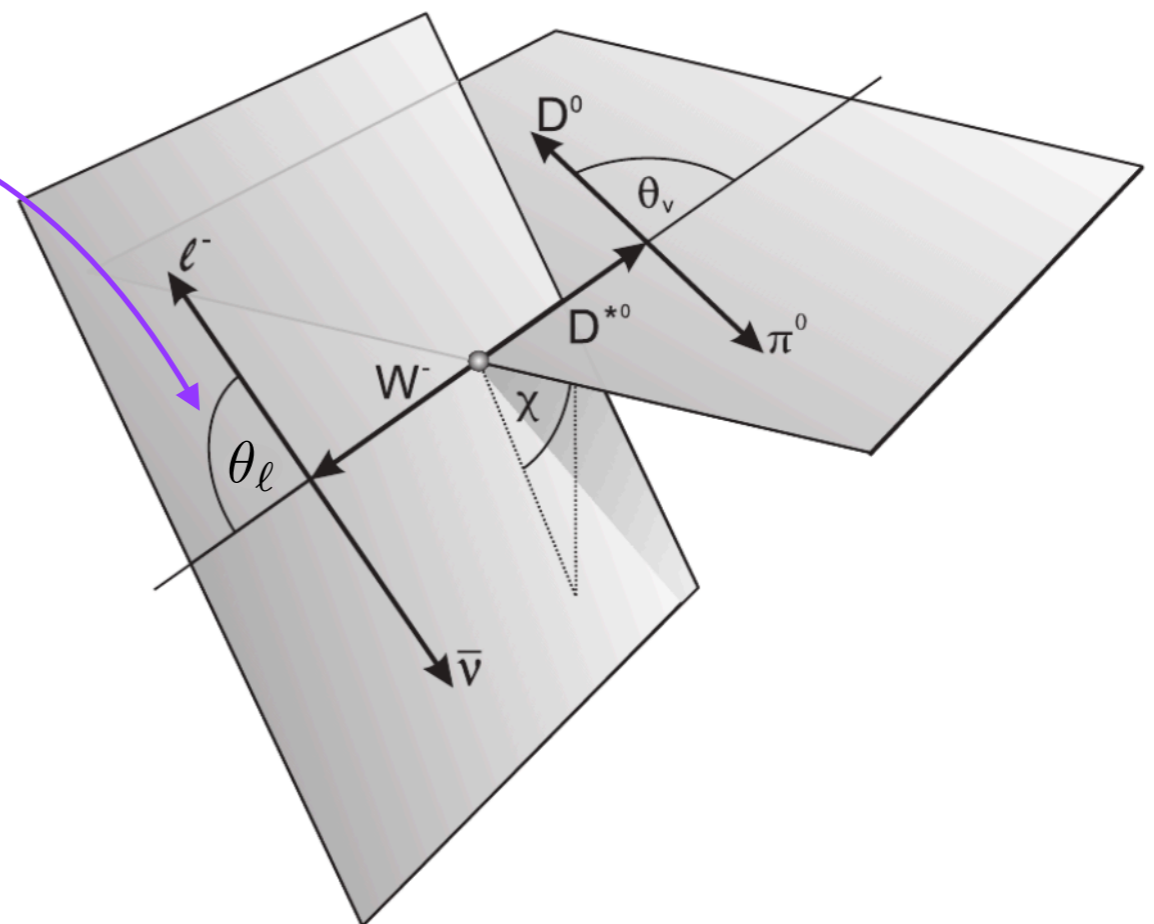
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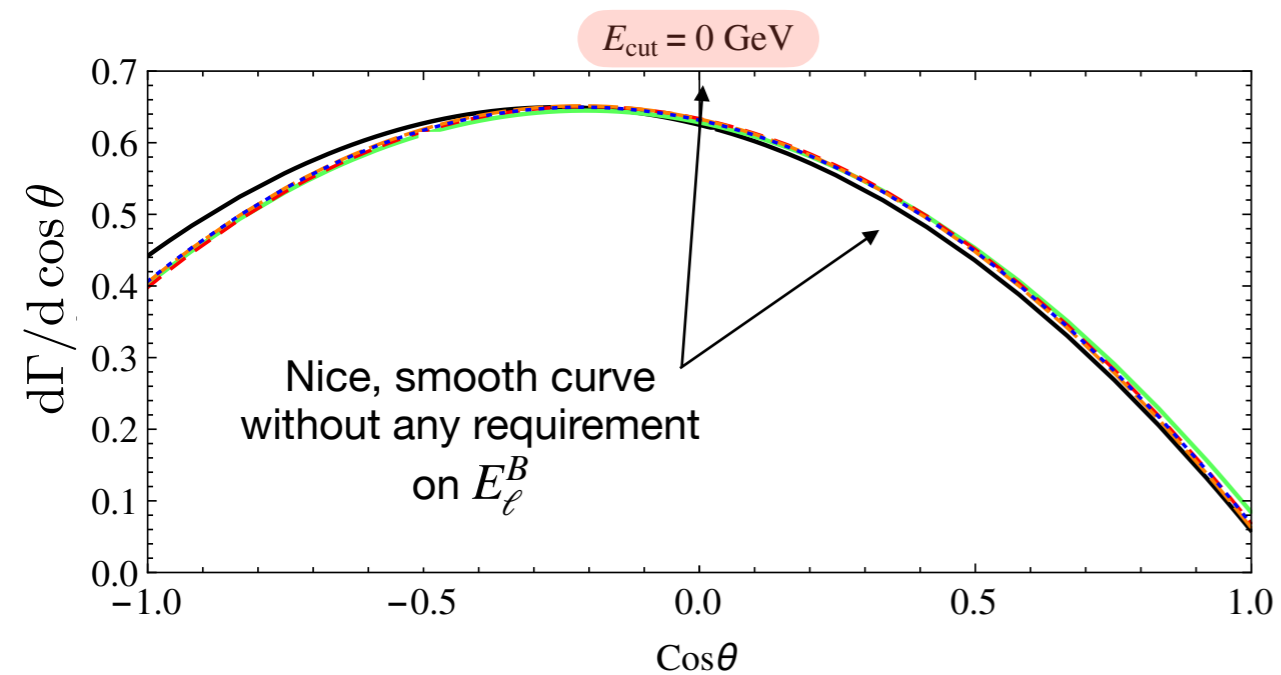
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- **Missing energy** and q^2 **easily accessible** variables with tagged approach
- Separate electron and muon channels for further **LFU tests**
- Additional information leads to **greater sensitivity** in global fits, particularly the HQE parameter $\hat{\mu}_G$

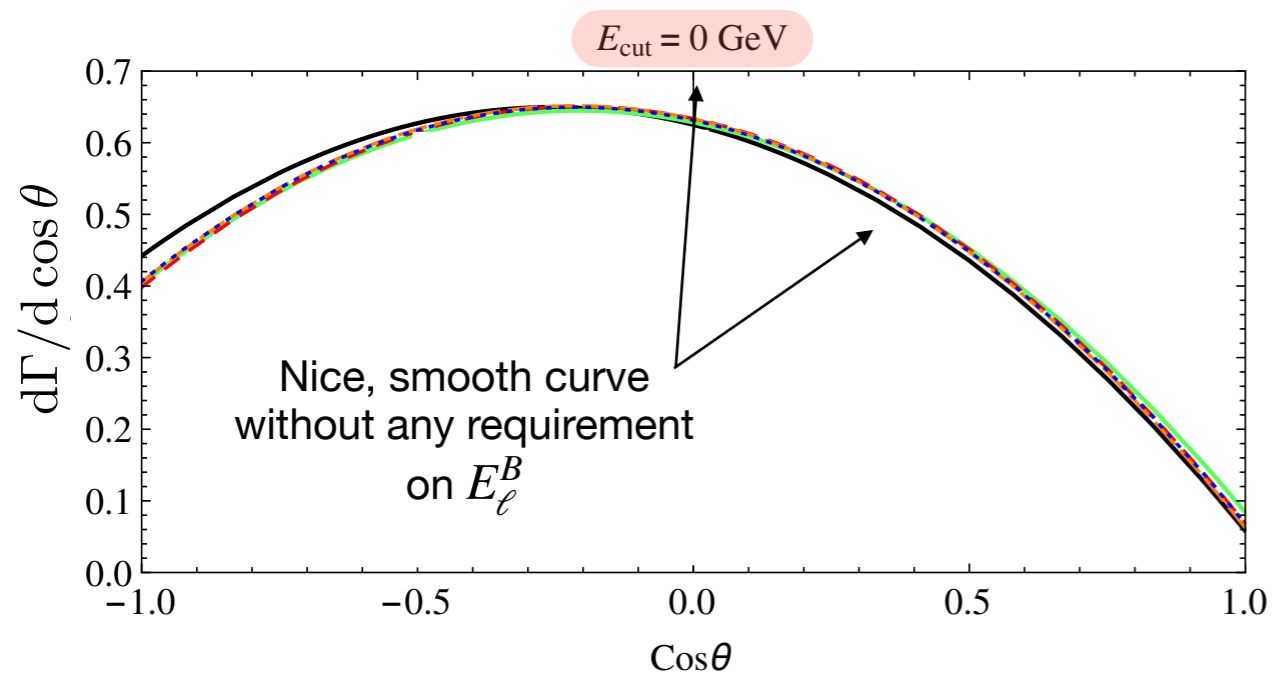


Impact of an E_ℓ requirement



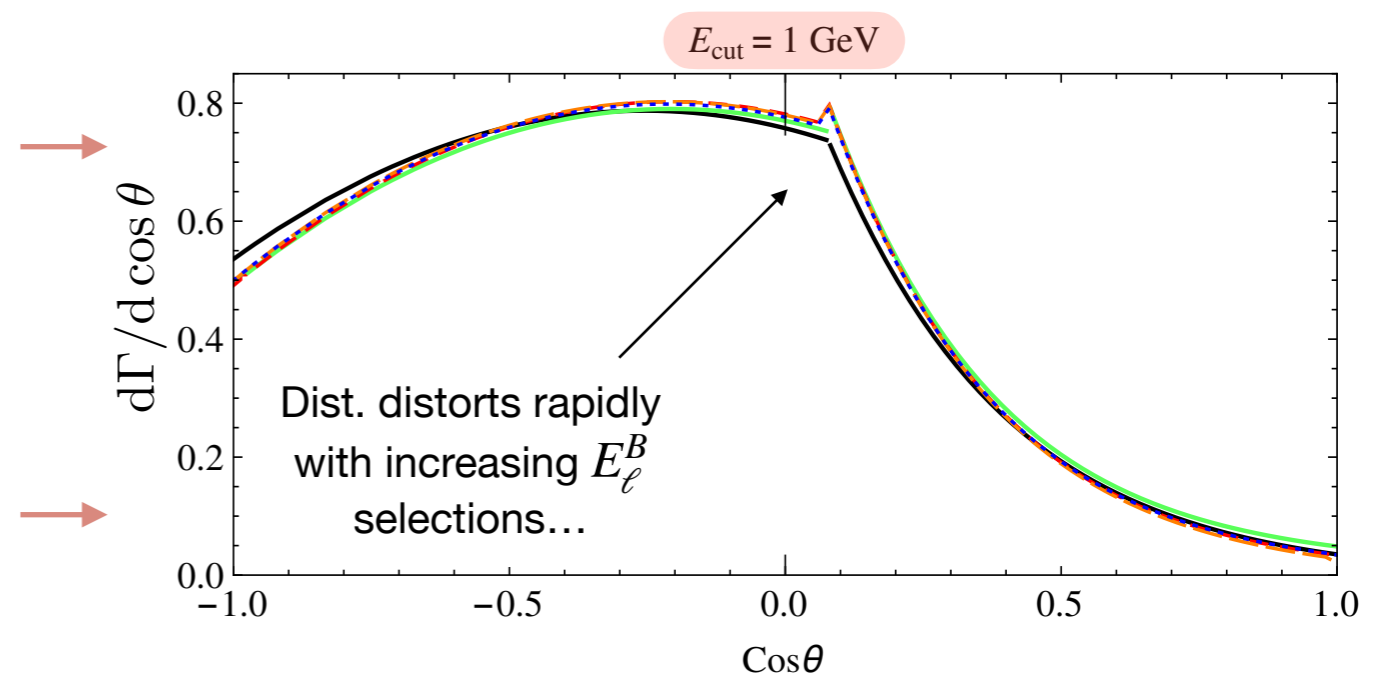
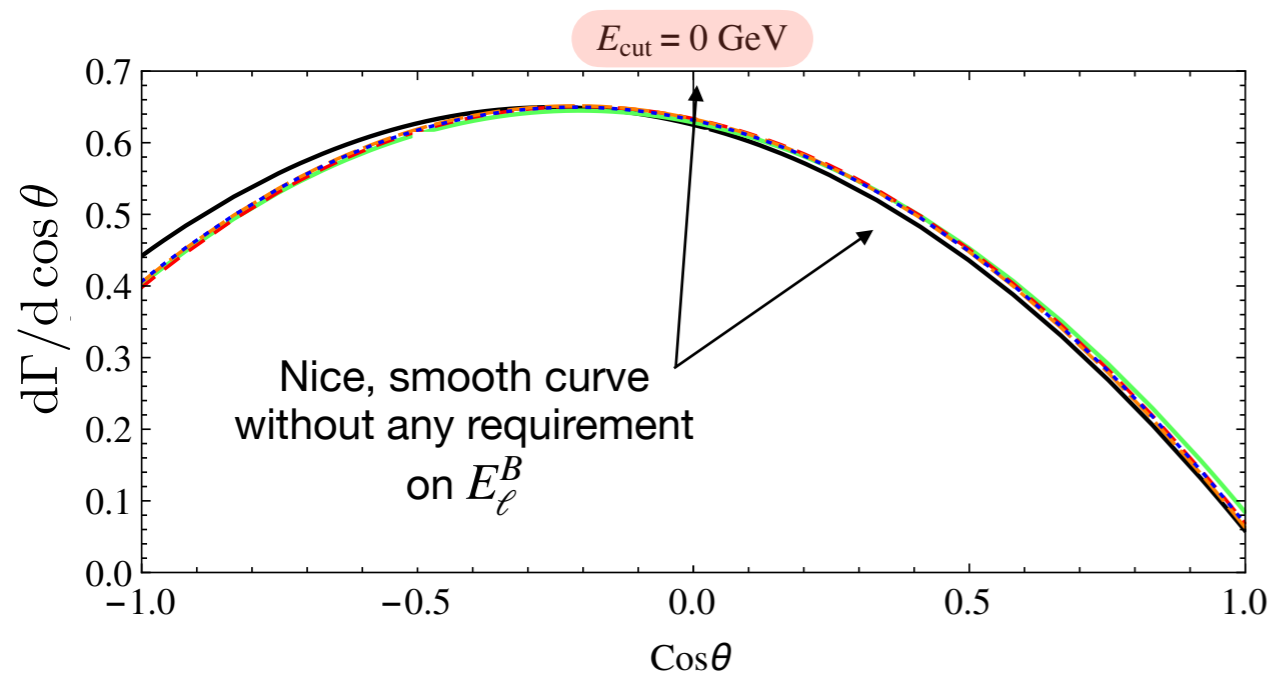
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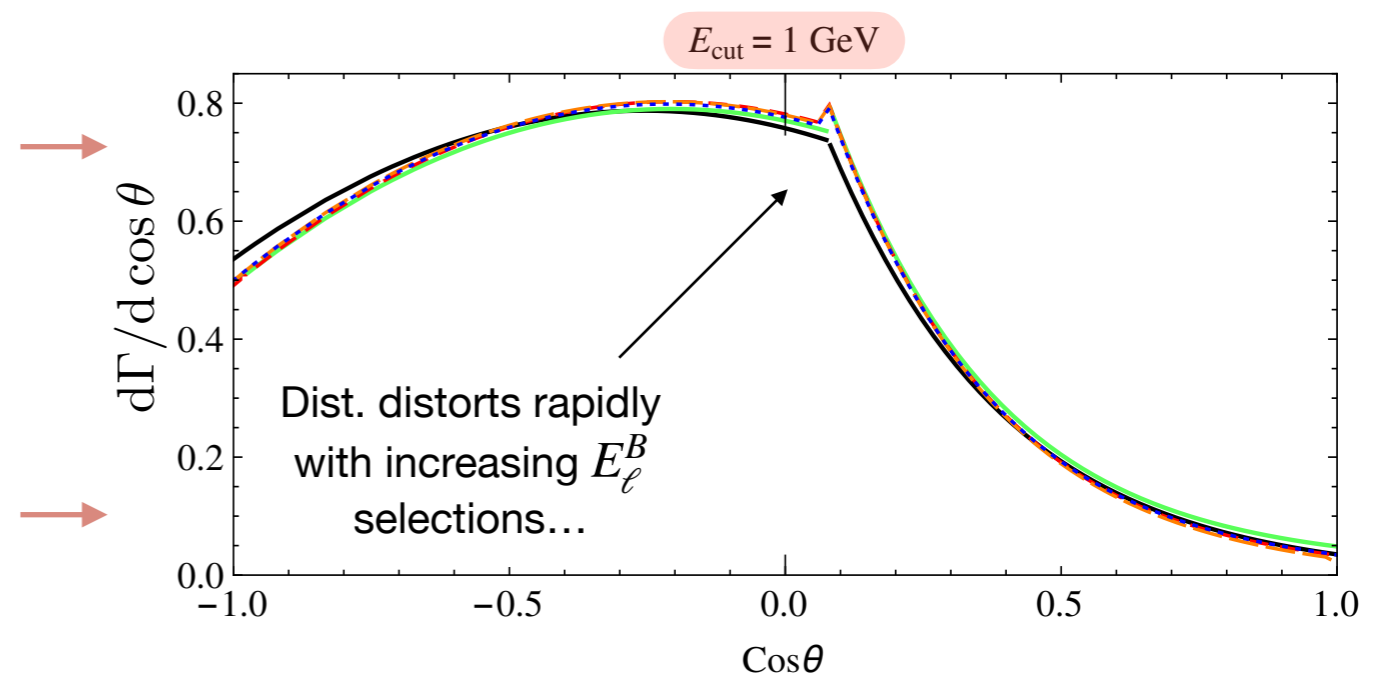
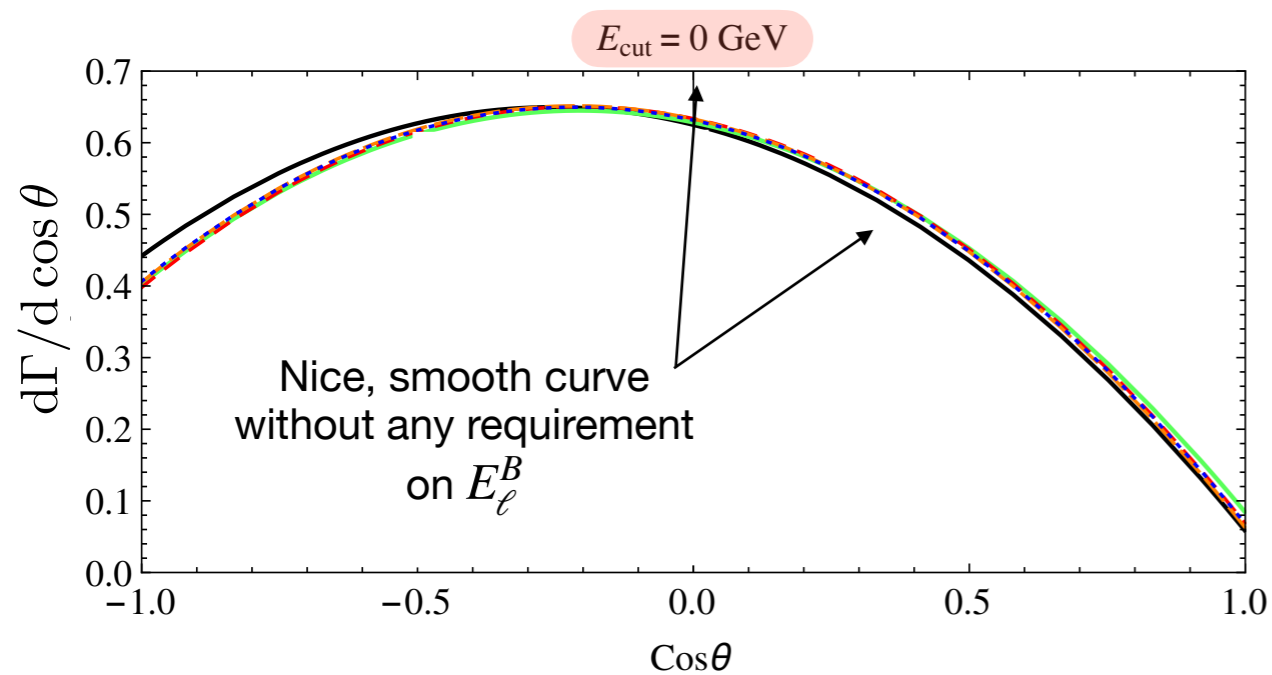
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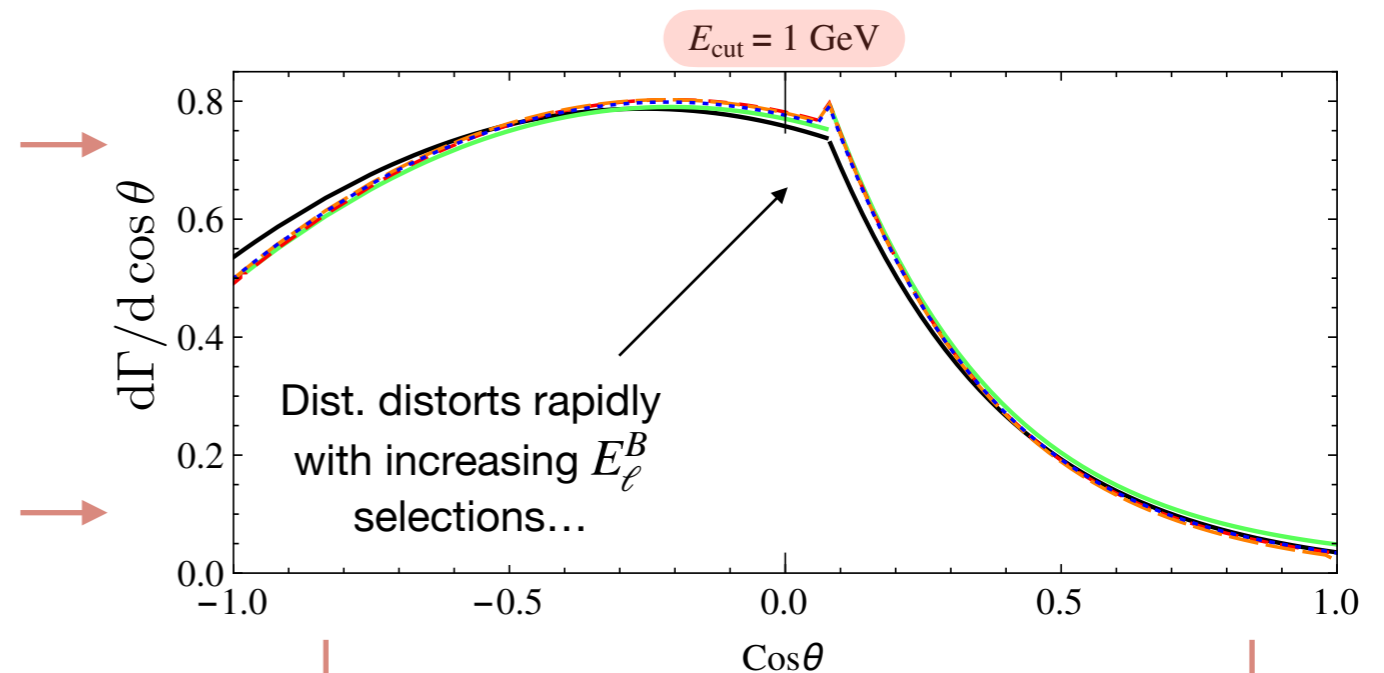
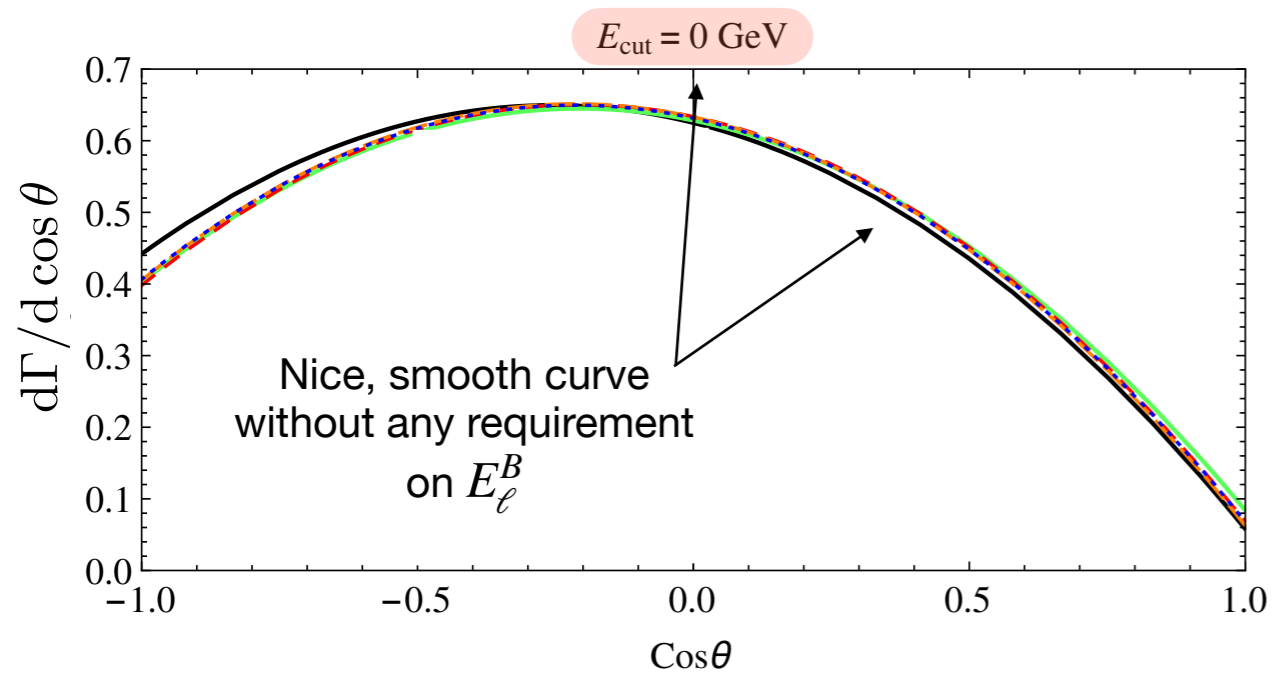
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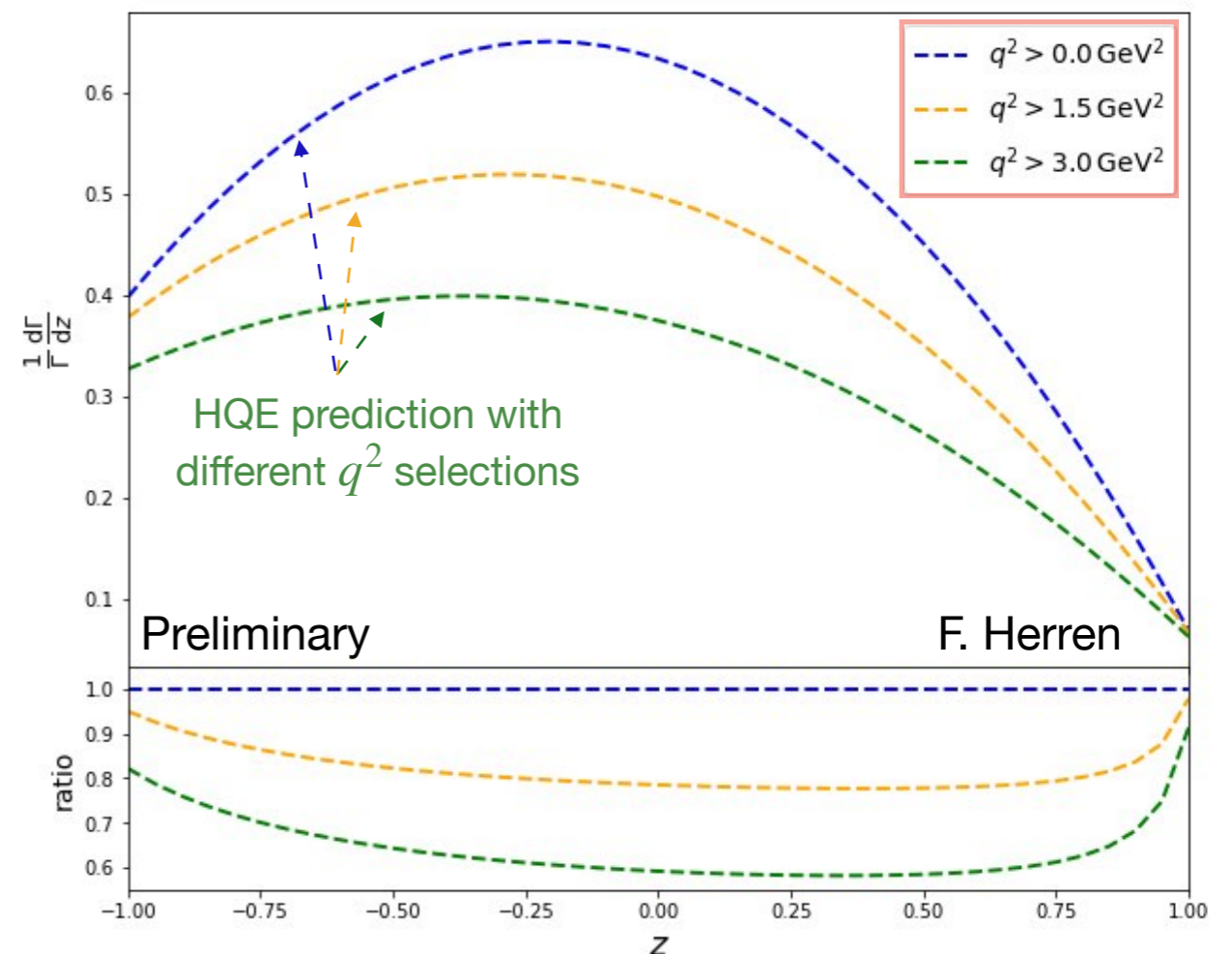
Impact of an E_ℓ requirement

Publication
in preparation



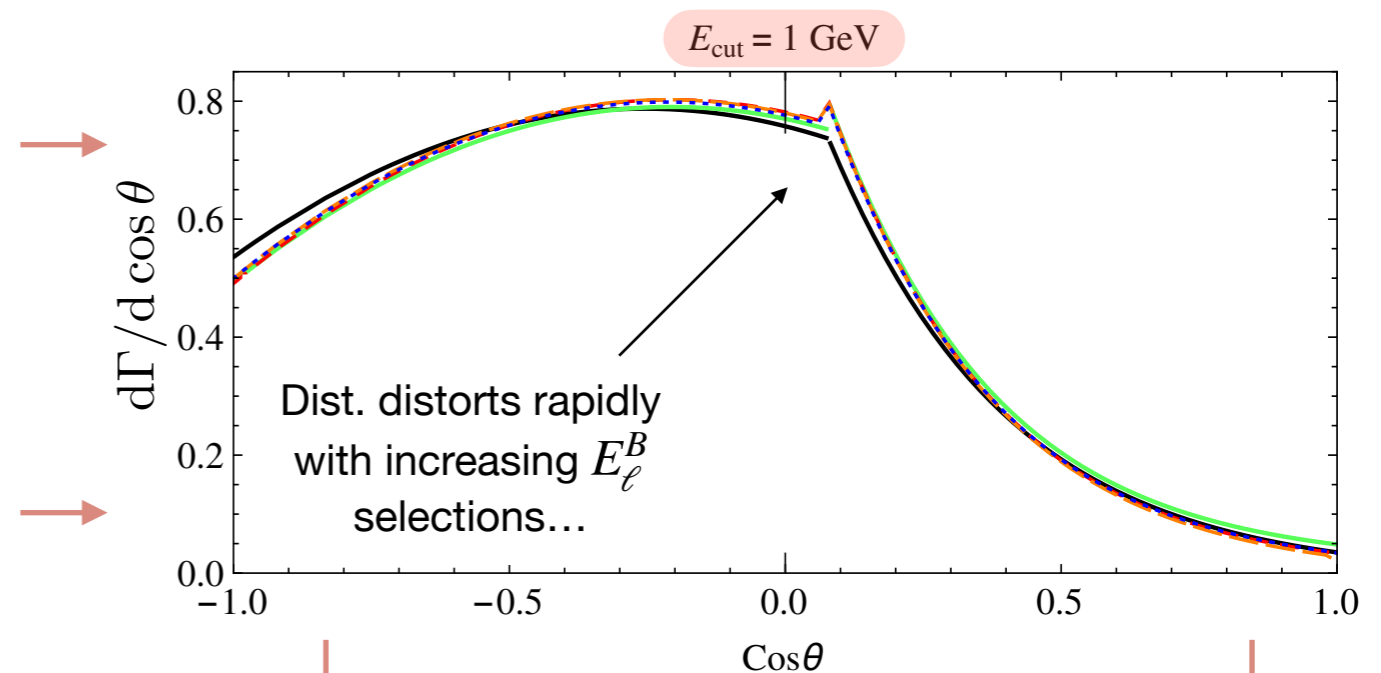
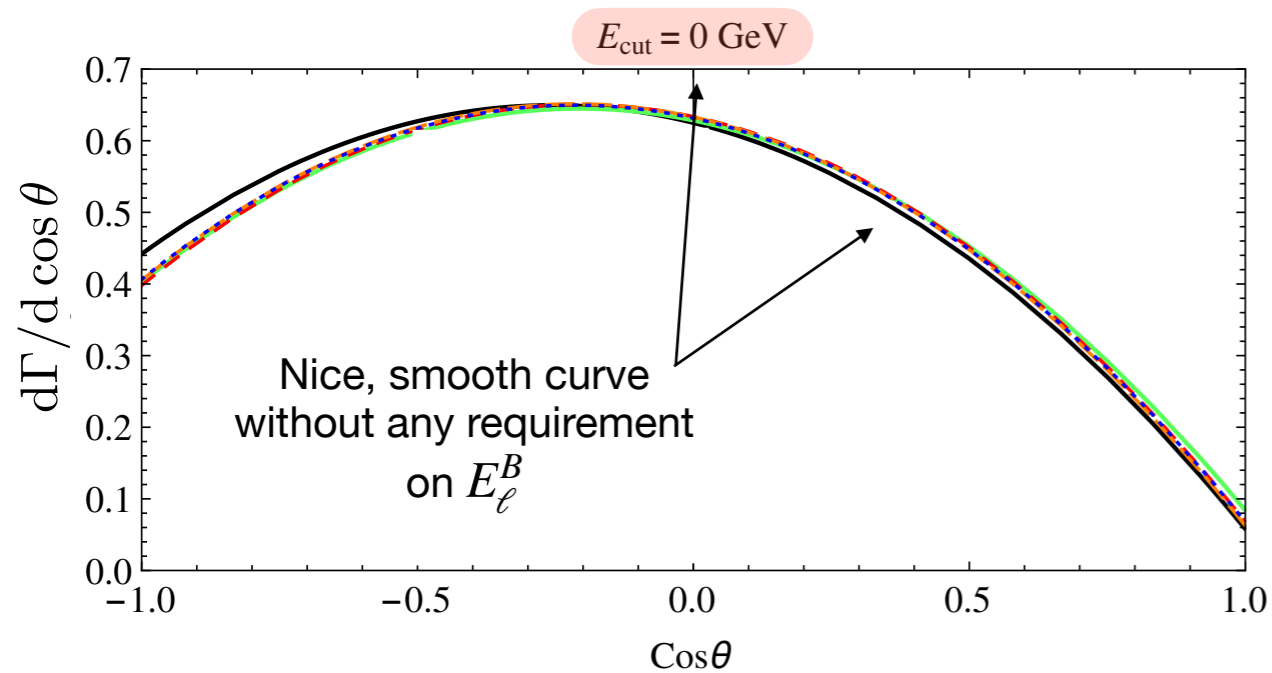
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Suggestion:
Use a q^2 selection instead



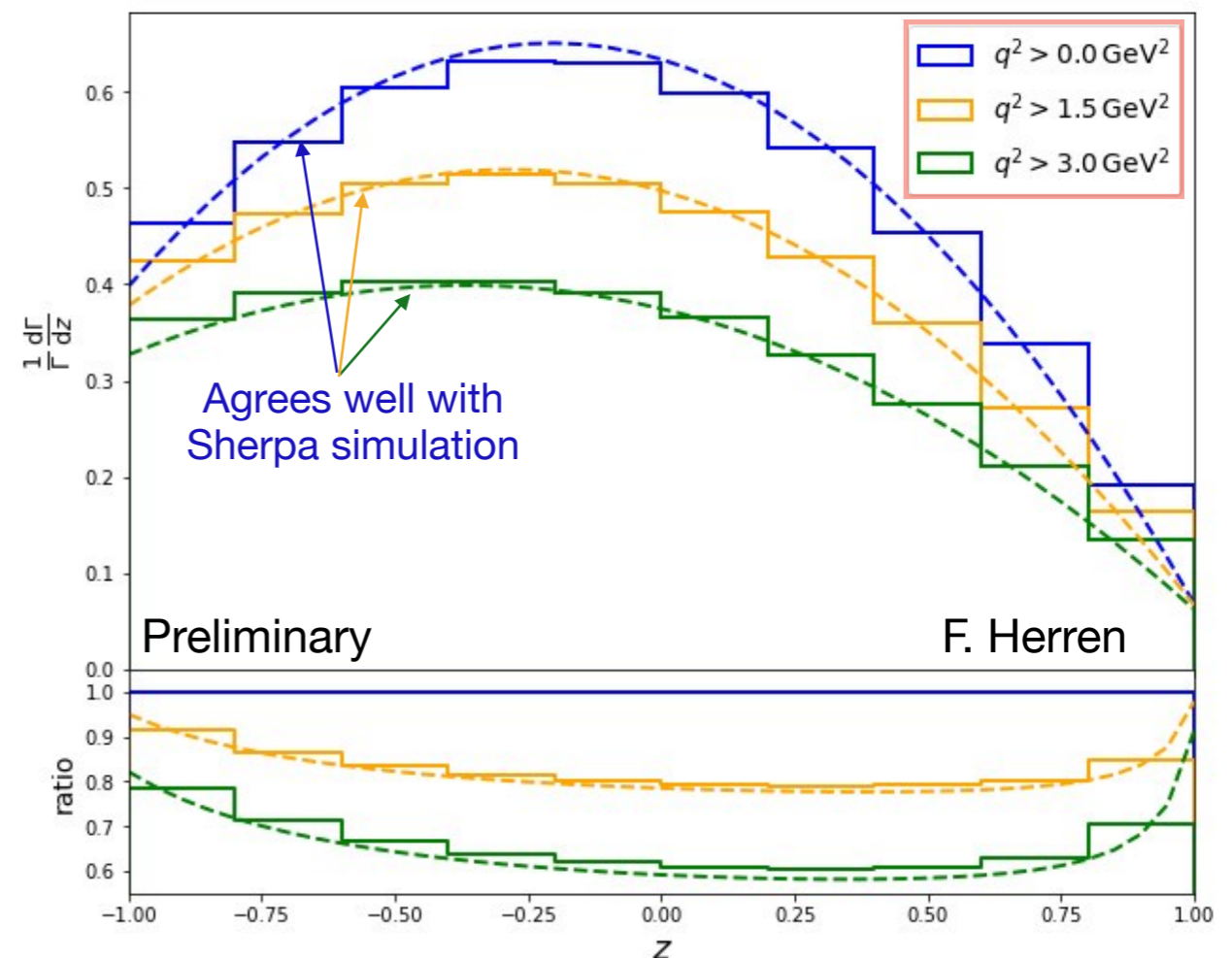
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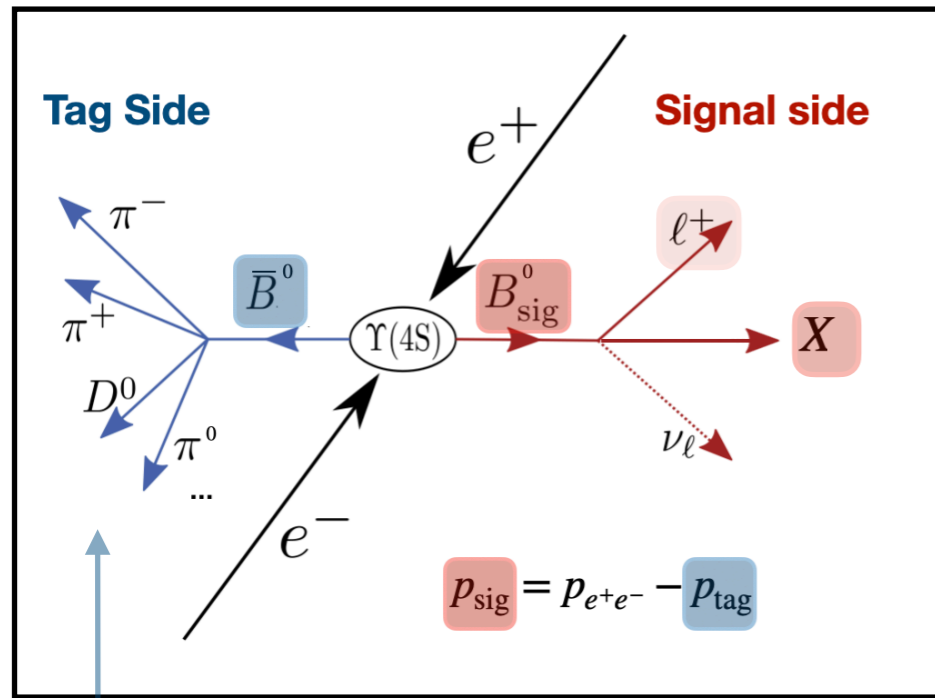
Conclusion & Outlook:



- Measurements generally used in $b \rightarrow c$ analyses are old and should be **systematically revisited** with Belle II and LHCb.
- Several **new results** on inclusive $B \rightarrow X_c \ell \nu$ decays at the B-factories: \mathcal{B} , moments of $\langle q^{2n} \rangle$ and $\langle M_X^n \rangle$...
- Modelling and composition of $B \rightarrow X_c \ell \nu$ decays are **leading systematics**. Steadily increasing dataset of Belle II will **improve our understanding** of the non-resonant contribution.
- **New observables** that could better constrain the OPE should be investigated. Analysis aiming to measure A_{FB} **already underway** —with help from theory friends! (...you?)

Reconstruction at B-Factories

Key-technique: hadronic tagging

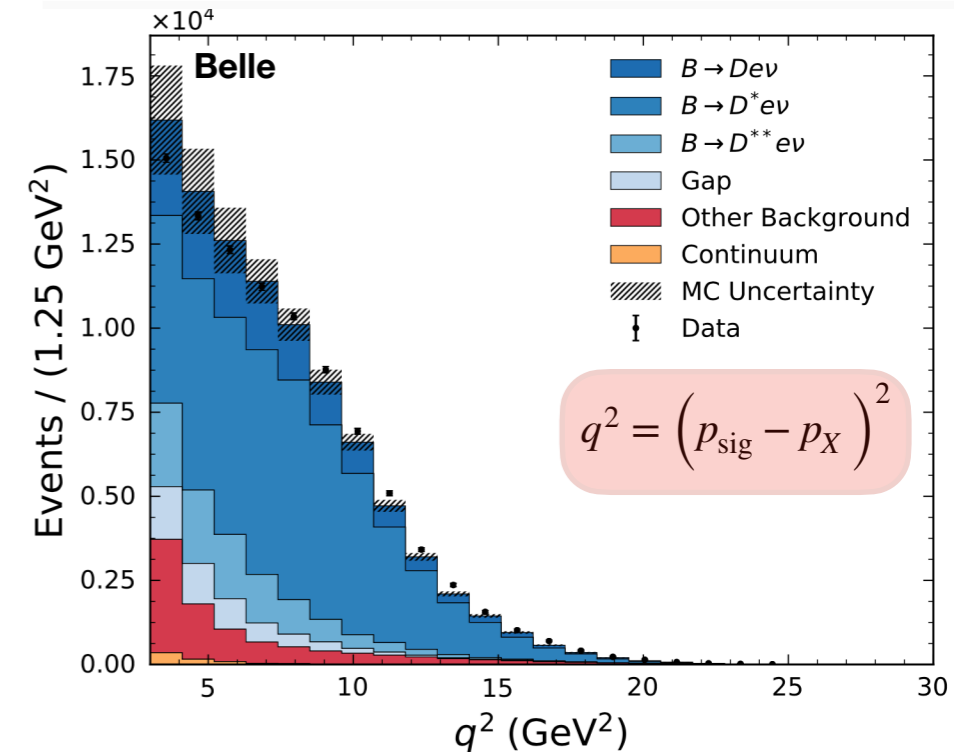
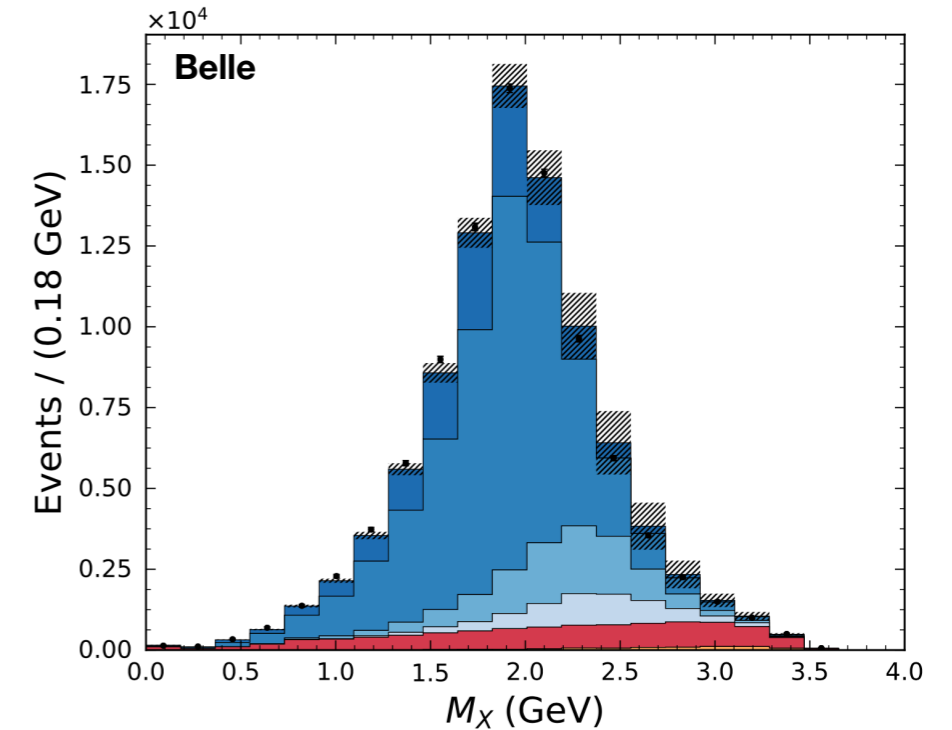


Can identify X constituents

$$M_X = \sqrt{(p_X)^\mu (p_X)_\mu}$$

Charged Tracks Neutral Clusters

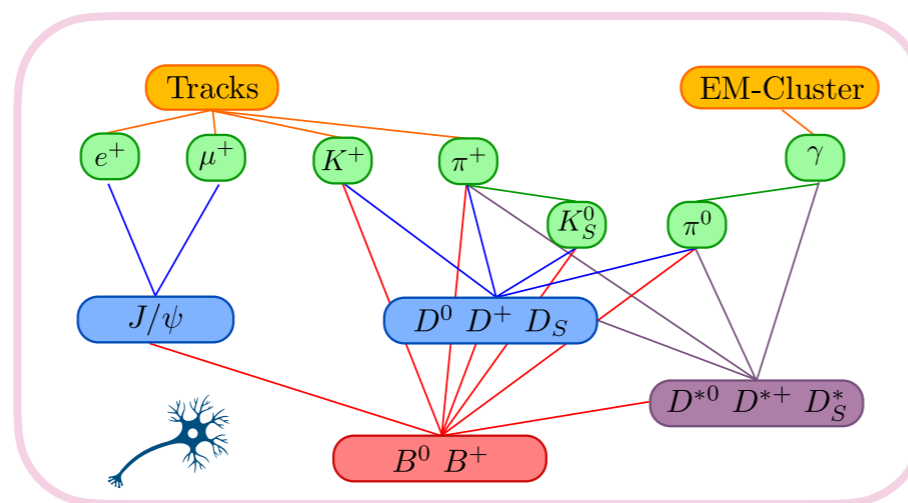
$$p_X = \sum_i \left(\sqrt{m_\pi^2 + |\mathbf{p}_i|^2}, \mathbf{p}_i \right) + \sum_j (E_j, \mathbf{k}_j)$$



Full Reconstruction =
Belle tagging algorithm

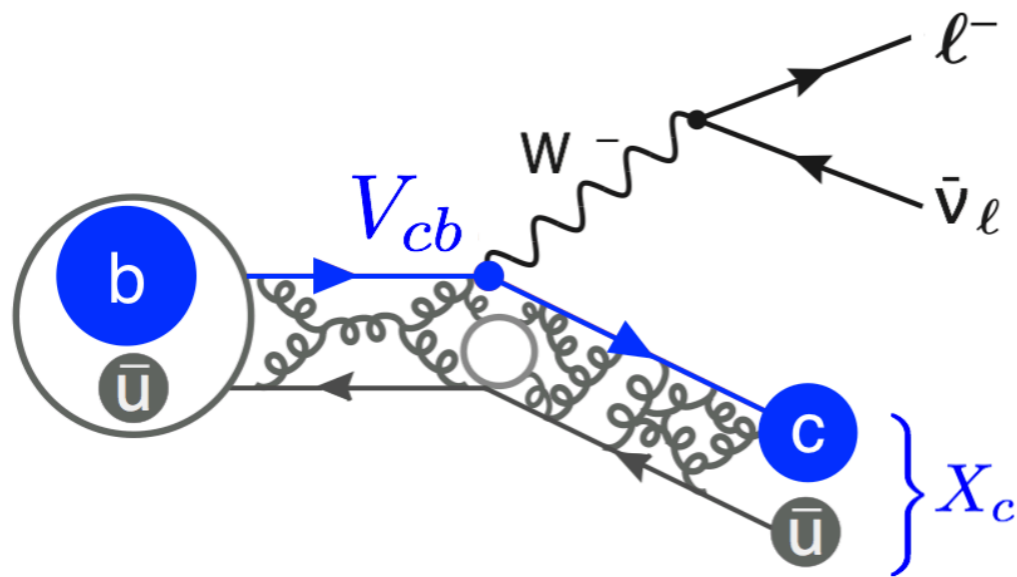
Candidates reconstructed using a **hierarchical** approach & **neural networks** in **hadronic modes**

1104 decay cascades used with an **efficiency** of **0.28% / 0.18%** for B^\pm and B^0/\bar{B}^0



$$q^2 = (p_{sig} - p_X)^2$$

Determining incl. $|V_{cb}|$



Established approach: Use hadronic mass moments, lepton energy moments etc.

$$\Gamma(B \rightarrow X_c \ell \nu_\ell) \quad \langle M_X^n \rangle \quad \langle E_\ell^n \rangle$$

to determine non-perturbative matrix elements (ME) of HQE and extract $|V_{cb}|$

$$\mathcal{B} = |V_{qb}|^2 \left[\Gamma(b \rightarrow q \ell \bar{\nu}_\ell) + 1/m_{c,b} + \alpha_s + \dots \right]$$



STOP The number ME increases if one increases expansion in orders of $1/m_{b,c}$

Novel theoretical approach introduced in [JHEP 02, 177 (2019)]

→ Exploits **reparametrization invariance** to **reduce** the # of ME, but **not true for every observable** (e.g. not for $\langle M_X \rangle$)

Holds for $\langle q^2 \rangle$ and at $1/m_b^4$ the # of ME reduces from **13** → **8(!)**

Complementary and fully **data-driven** approach!

Goal: Measure $\langle q^{2n} \rangle$ ($n = 1 - 4$) as a progression of cuts on q^2 with Belle & Belle II

Belle $\langle q^{2n} \rangle$ moments syst.

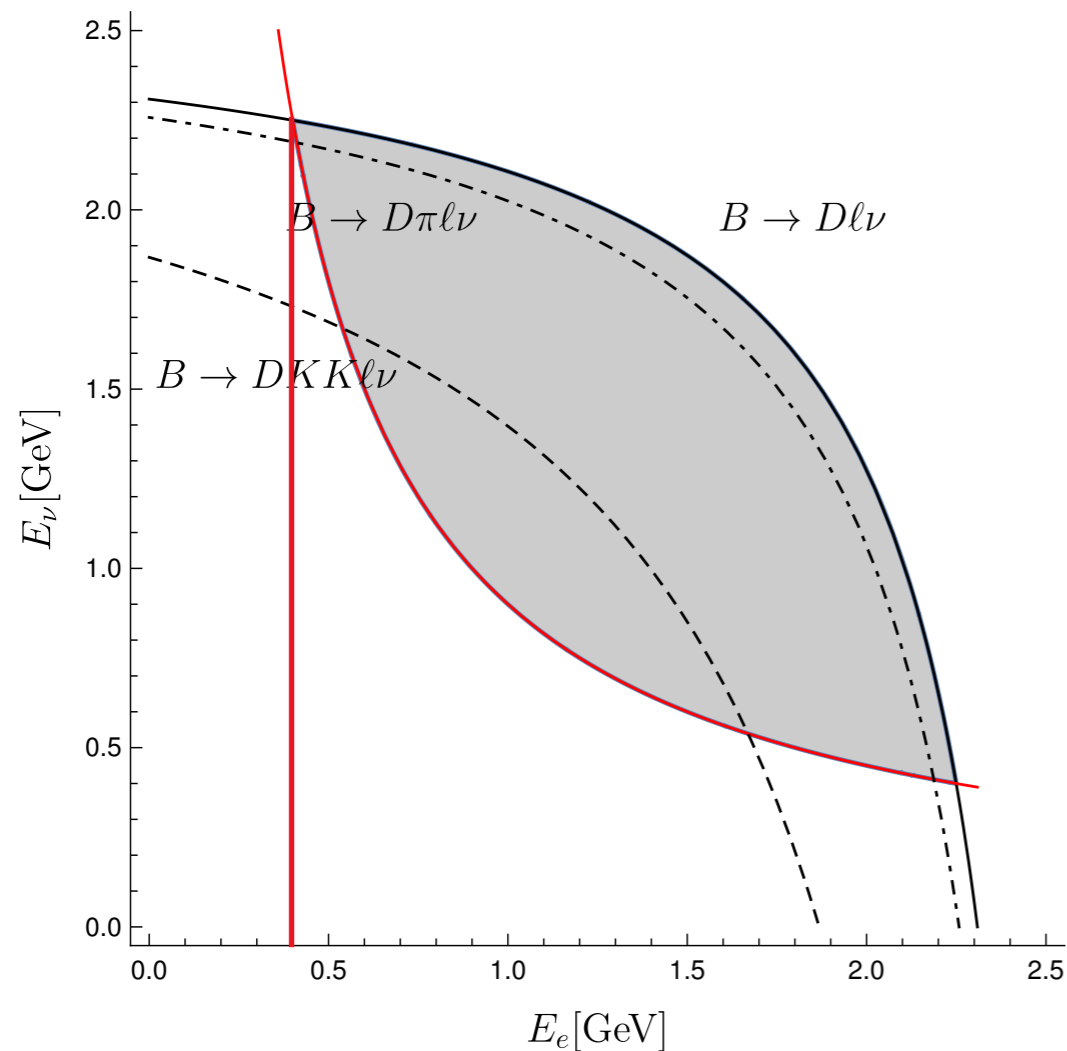
PRD 104, 112011 (2021)

q^2 selection in GeV^2	3.0	3.5	4.0	4.5	5.0	5.5	6.0	6.5	7.0	7.5	8.0	8.5	9.0	9.5	10.0
$\langle q^2 \rangle$ in GeV^2	6.25	6.54	6.83	7.13	7.42	7.72	8.02	8.32	8.61	8.91	9.21	9.51	9.80	10.09	10.40
Stat. error (data)	1.51	1.45	1.39	1.34	1.30	1.27	1.24	1.22	1.21	1.20	1.20	1.22	1.23	1.26	1.29
Bkg. subtraction	1.34	1.12	0.90	0.71	0.59	0.53	0.49	0.49	0.57	0.63	0.65	0.70	0.76	0.77	0.82
$B \rightarrow X_{u\ell\nu}$ BF	2.18	2.04	1.75	1.48	1.35	1.04	0.76	0.54	0.38	0.29	0.19	0.16	0.12	0.05	0.05
$B \rightarrow X_{c\ell\nu}$ BF	4.82	5.02	5.14	5.14	5.05	5.00	4.67	4.05	3.51	3.11	2.66	2.21	1.75	1.36	1.16
Non-resonant model	14.25	12.72	11.04	9.28	7.83	6.62	5.42	4.00	3.02	2.28	1.65	1.43	1.04	0.86	0.78
$B \rightarrow X_{c\ell\nu}$ FF	1.43	1.30	1.16	1.03	0.91	0.85	0.82	0.74	0.69	0.62	0.54	0.48	0.42	0.39	0.35
N_{tracks} res.	5.66	5.31	4.96	4.65	4.36	4.06	3.78	3.52	3.29	3.06	2.85	2.66	2.51	2.38	2.20
N_γ res.	0.39	0.38	0.34	0.31	0.30	0.28	0.30	0.31	0.32	0.28	0.27	0.26	0.25	0.27	0.29
$E_{\text{miss}} - \mathbf{p}_{\text{miss}} $ shape	1.29	1.26	1.21	1.17	1.15	1.11	1.04	1.05	1.06	1.09	1.16	1.20	1.30	1.33	1.29
q^2 scale	9.48	7.15	6.65	6.65	6.12	5.91	5.83	5.48	5.26	4.69	4.27	4.42	3.91	3.94	4.38
MC non-closure	0.19	0.11	0.12	0.11	0.11	0.05	0.05	0.06	0.08	0.07	0.11	0.04	0.04	0.06	0.02
Cal. function	0.13	0.08	0.03	0.02	0.07	0.12	0.17	0.22	0.26	0.31	0.35	0.39	0.43	0.47	0.51
Stat. bias corr.	1.32	1.27	1.23	1.19	1.16	1.13	1.10	1.08	1.07	1.06	1.06	1.06	1.07	1.09	1.11
PID eff.	0.16	0.14	0.14	0.13	0.13	0.12	0.11	0.10	0.10	0.10	0.09	0.08	0.08	0.07	0.06
Track eff.	0.44	0.42	0.39	0.36	0.34	0.31	0.29	0.27	0.25	0.23	0.21	0.20	0.18	0.17	0.15
B^0/B^\pm tag eff.	0.46	0.58	0.50	0.44	0.51	0.40	0.28	0.34	0.36	0.38	0.29	0.23	0.20	0.12	0.47
Sys. error (total)	18.99	16.65	15.03	13.62	12.22	11.19	10.17	8.86	7.97	7.06	6.30	6.09	5.44	5.27	5.50
Total rel. error in %	19.05	16.71	15.09	13.68	12.29	11.26	10.25	8.94	8.06	7.16	6.41	6.21	5.58	5.42	5.65

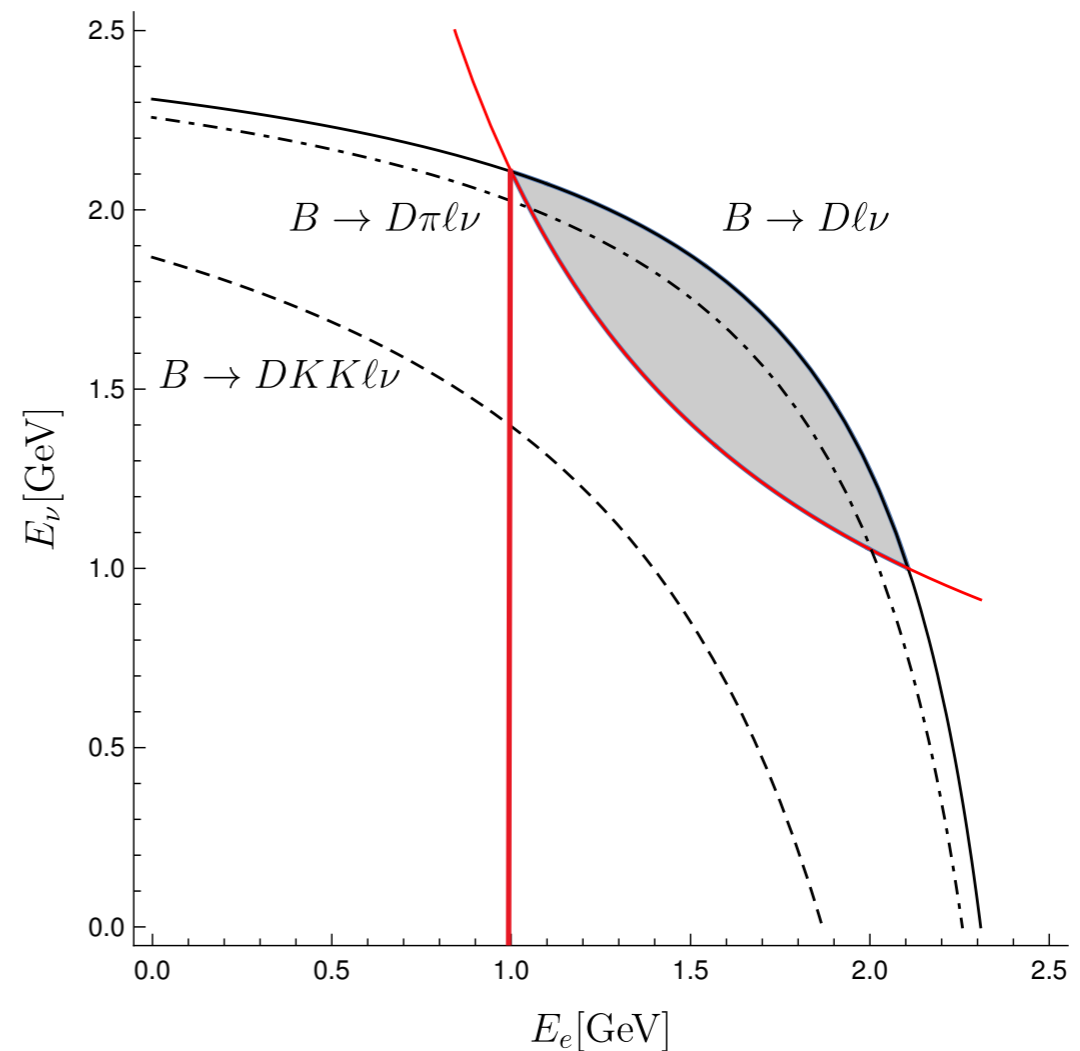
$\langle q^8 \rangle$ in GeV^8	2717.22	2963.88	3248.31	3578.45	3947.44	4384.73	4878.23	5458.95	6072.92	6780.95	7616.67	8497.60	9466.03	10603.31	11917.23
Stat. error (data)	10.35	10.07	9.78	9.47	9.19	8.89	8.63	8.36	8.19	8.05	7.94	7.91	7.94	7.95	7.99
Bkg. subtraction	5.57	5.25	4.98	4.80	4.99	5.23	5.02	5.06	5.51	5.71	5.58	6.00	5.96	5.81	6.02
$B \rightarrow X_{u\ell\nu}$ BF	11.94	11.10	9.61	7.82	7.00	5.31	3.66	2.53	1.76	1.30	0.79	0.69	0.59	0.20	0.16
$B \rightarrow X_{c\ell\nu}$ BF	21.51	22.91	23.24	23.14	22.84	22.14	20.76	18.50	16.31	14.53	12.43	10.40	8.44	6.74	5.74
Non-resonant model	49.93	45.52	40.56	35.22	30.45	26.13	21.80	16.75	13.12	10.26	7.73	6.66	5.10	4.25	3.79
$B \rightarrow X_{c\ell\nu}$ FF	4.91	4.76	4.60	4.40	4.23	4.12	4.03	3.75	3.52	3.23	2.88	2.59	2.31	2.09	1.89
N_{tracks} res.	29.72	28.51	27.15	25.82	24.47	22.99	21.54	20.09	18.76	17.40	16.09	14.89	13.83	12.86	11.73
N_γ res.	2.95	2.89	2.75	2.62	2.58	2.46	2.46	2.44	2.39	2.22	2.16	2.07	2.00	2.01	2.06
$E_{\text{miss}} - \mathbf{p}_{\text{miss}} $ shape	10.18	9.83	9.42	9.05	8.69	8.33	7.89	7.70	7.50	7.35	7.33	7.21	7.26	7.11	6.66
q^2 scale	46.61	41.26	39.53	39.00	36.70	35.23	33.82	32.22	30.11	27.83	25.47	25.28	23.04	24.16	25.90
MC non-closure	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Cal. function	0.63	0.34	0.04	0.26	0.55	0.86	1.14	1.42	1.68	1.92	2.17	2.37	2.56	2.74	2.90
Stat. bias corr.	8.21	8.00	7.79	7.59	7.38	7.18	6.99	6.83	6.67	6.54	6.43	6.35	6.29	6.27	6.27
PID eff.	0.81	0.77	0.74	0.70	0.69	0.64	0.60	0.56	0.52	0.51	0.47	0.42	0.40	0.35	0.30
Track eff.	2.25	2.16	2.05	1.95	1.84	1.72	1.60	1.49	1.38	1.27	1.16	1.06	0.98	0.89	0.81
B^0/B^\pm tag eff.	0.97	1.19	0.98	0.80	0.94	0.58	0.18	0.29	0.30	0.26	0.14	0.42	0.59	1.79	3.06
Sys. error (total)	79.98	73.90	69.18	64.95	60.23	56.09	51.86	47.19	43.06	39.32	35.61	34.06	31.06	30.93	31.65
Total rel. error in %	80.64	74.58	69.87	65.64	60.93	56.79	52.58	47.93	43.83	40.13	36.48	34.97	32.06	31.93	32.64

E_ℓ vs. q^2 selection criteria

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A requirement of $q^2 > 3.6 \text{ GeV}^2$ is equivalent to imposing a selection of $E_\ell > 0.4 \text{ GeV}$



A requirement of $q^2 > 8.43 \text{ GeV}^2$ is equivalent to imposing a selection of $E_\ell > 1.0 \text{ GeV}$