



# Exotic and Conventional Quarkonium Physics Prospects at Belle II



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On behalf of the Belle II collaboration



Bundesministerium  
für Bildung  
und Forschung



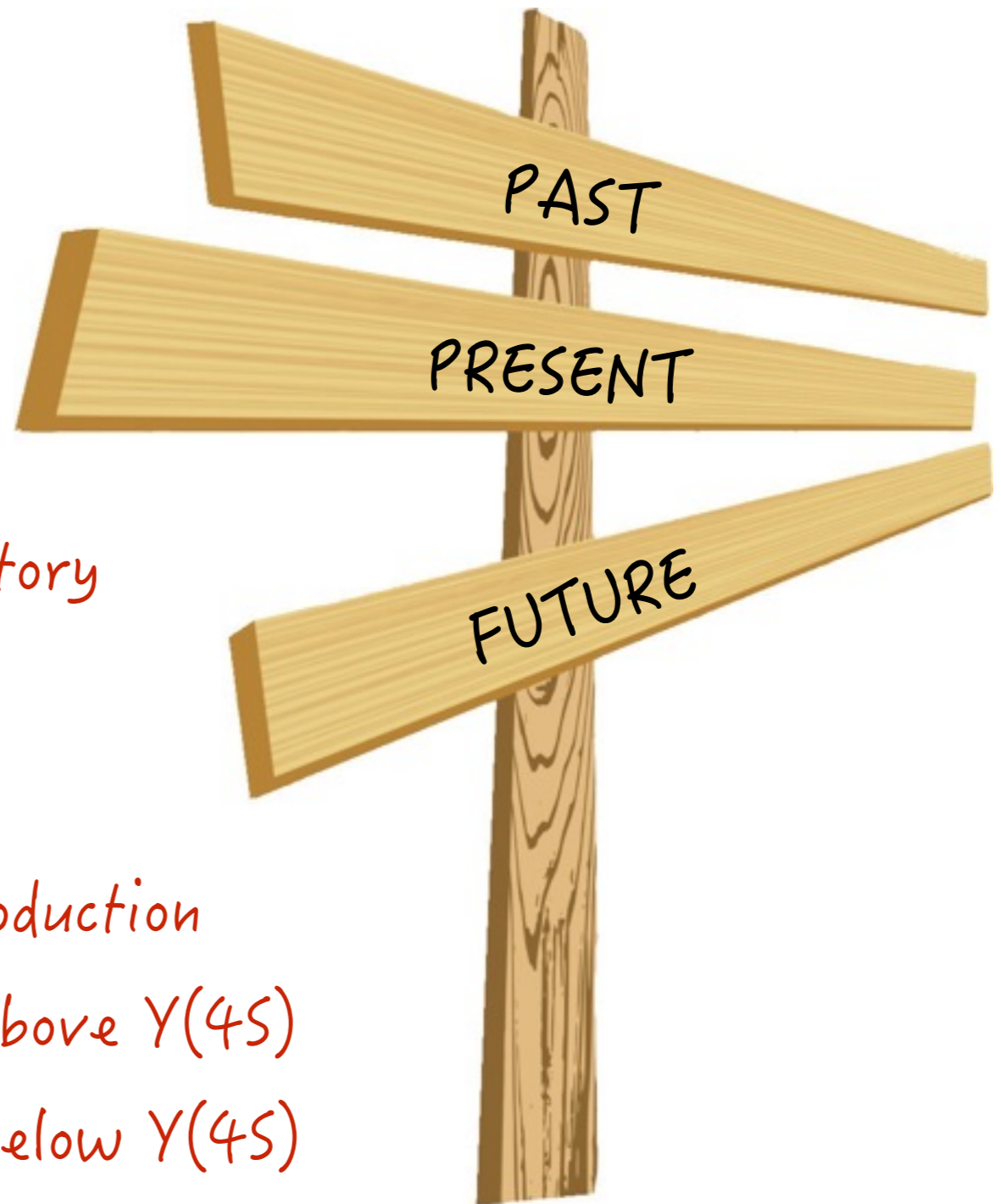
JOHANNES GUTENBERG  
UNIVERSITÄT MAINZ

📌 QUARKONIA:  
legacy of 1st generation  
B-Factories

📌 BELLE II:  
the next generation B-Factory

📌 FUTURE PROSPECTS:

- Charmonium(-like) production
- Bottomonium(-like): above  $\Upsilon(4S)$
- Bottomonium(-like): below  $\Upsilon(4S)$

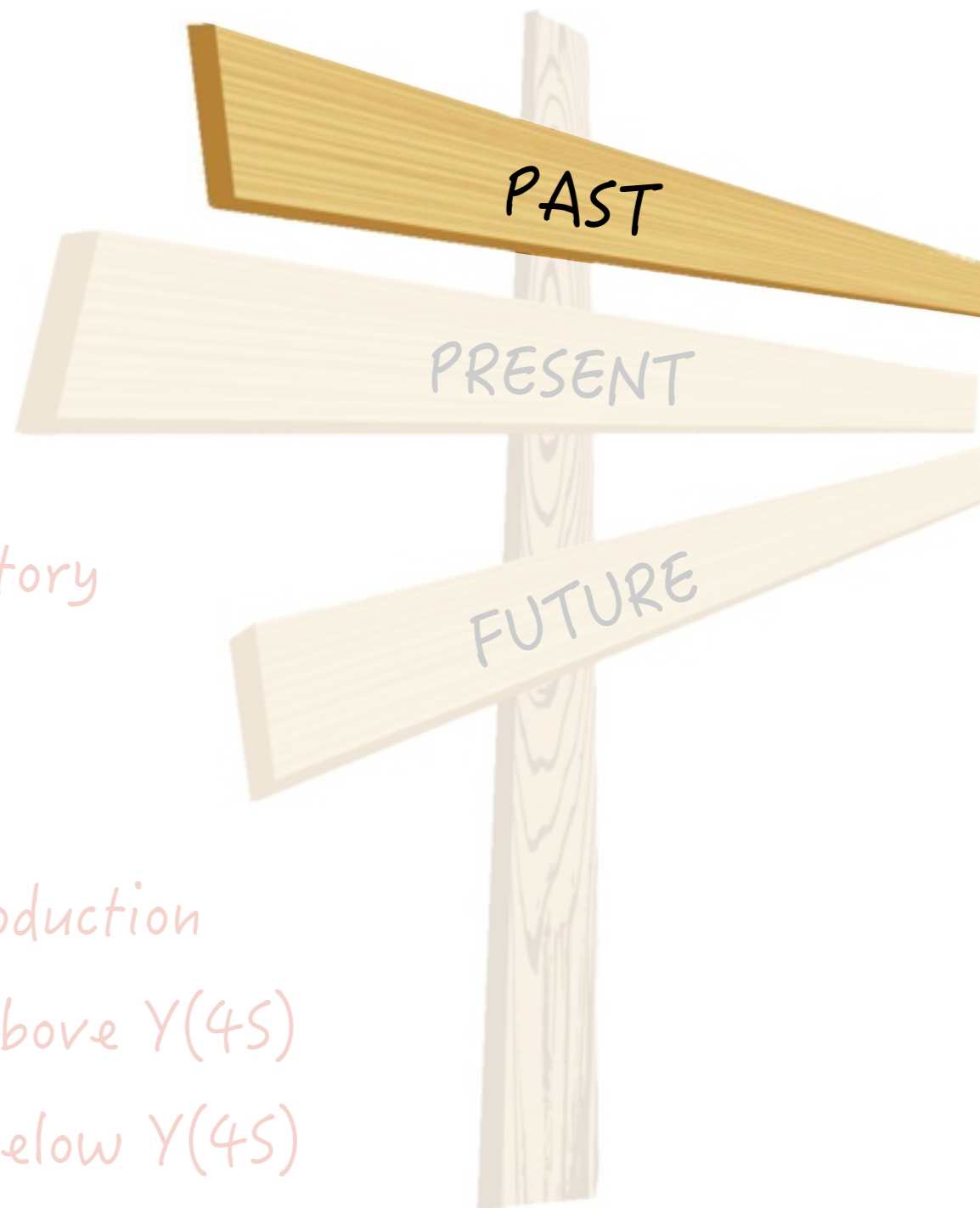


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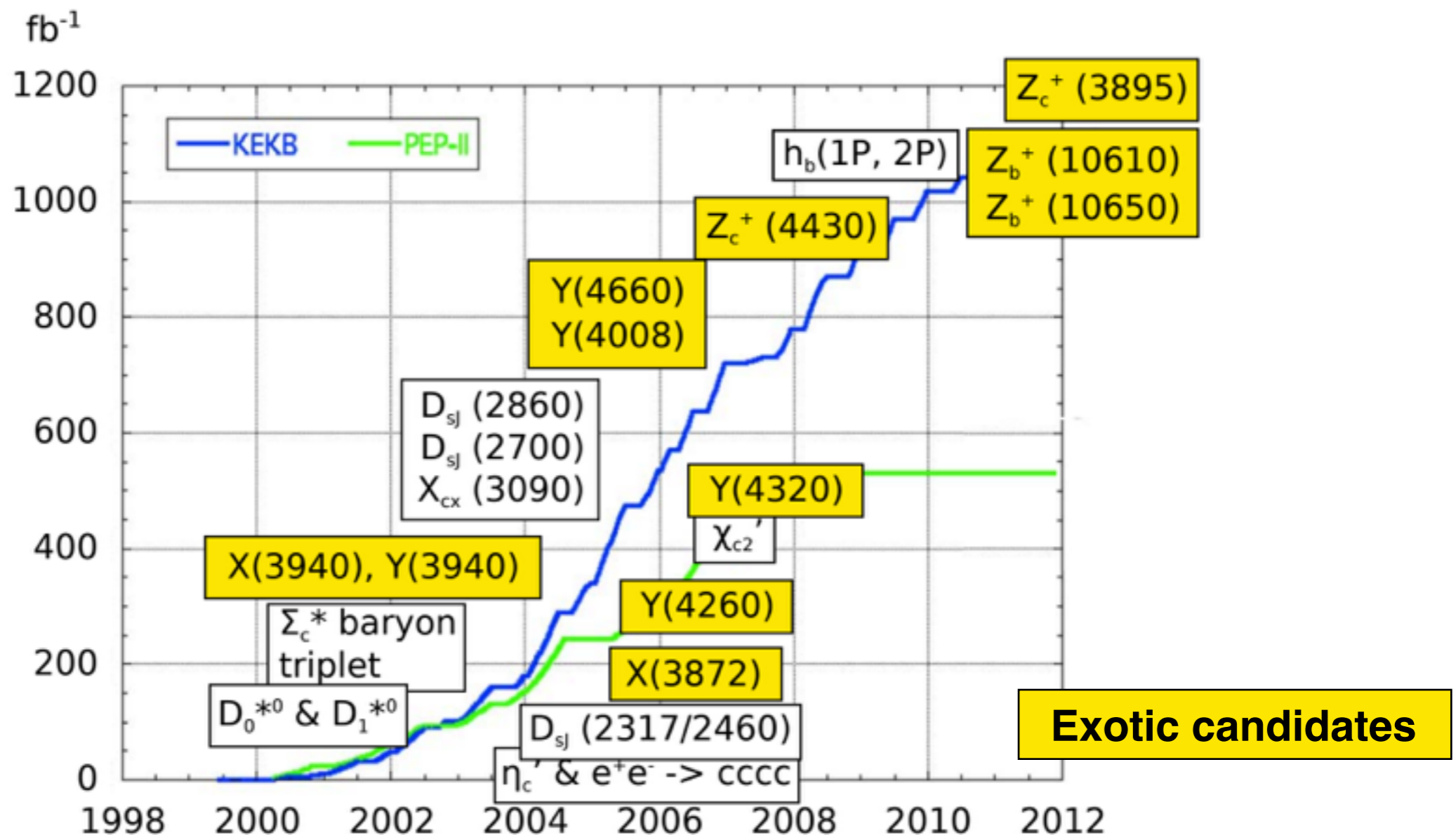
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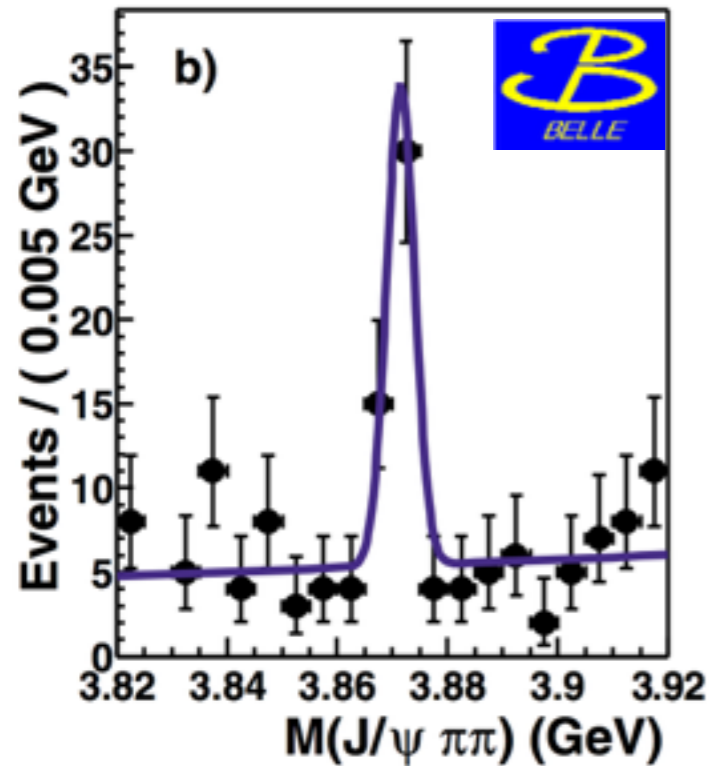
# B-Factory: a success story



- The series of discoveries started with the observation of the  $\eta_c'$  meson in  $B \rightarrow K \eta_c'$  decays
- The first exotic state was  $X(3872)$ , found in  $B \rightarrow K X(3872)$

# B-Factory: milestones

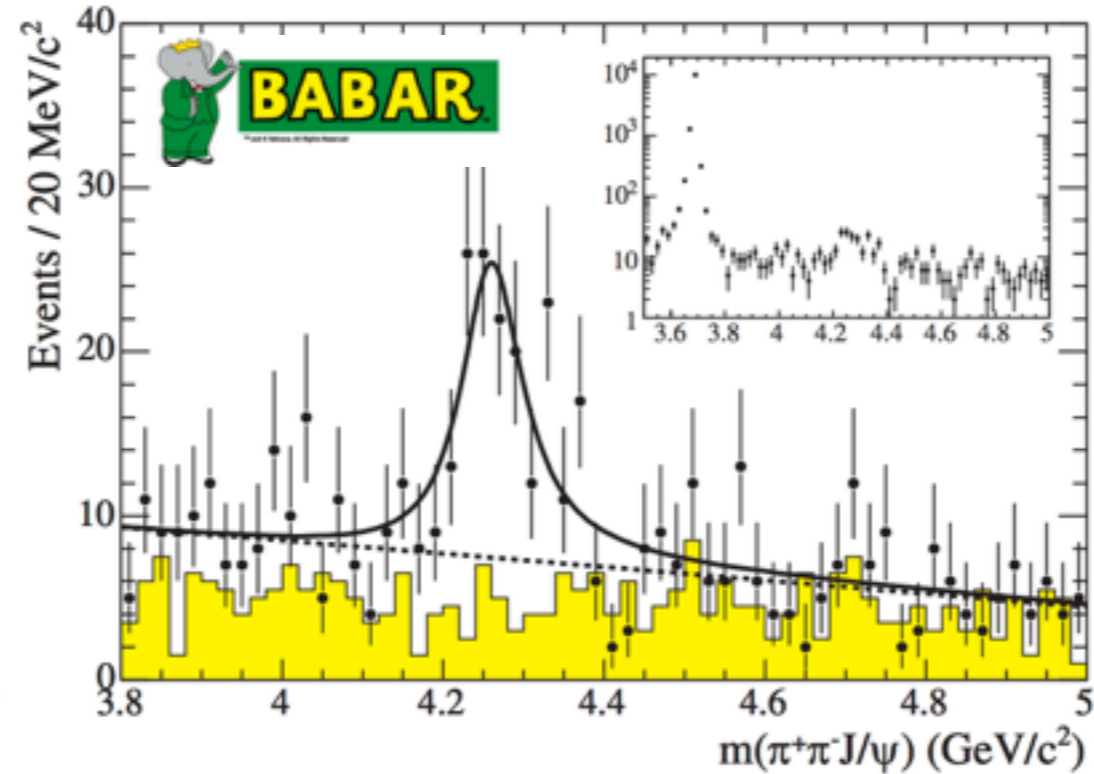
PRL 91, 262001 (2003)



**X(3872)**

$B^\pm \rightarrow K^\pm [\pi^+ \pi^- J/\psi]$

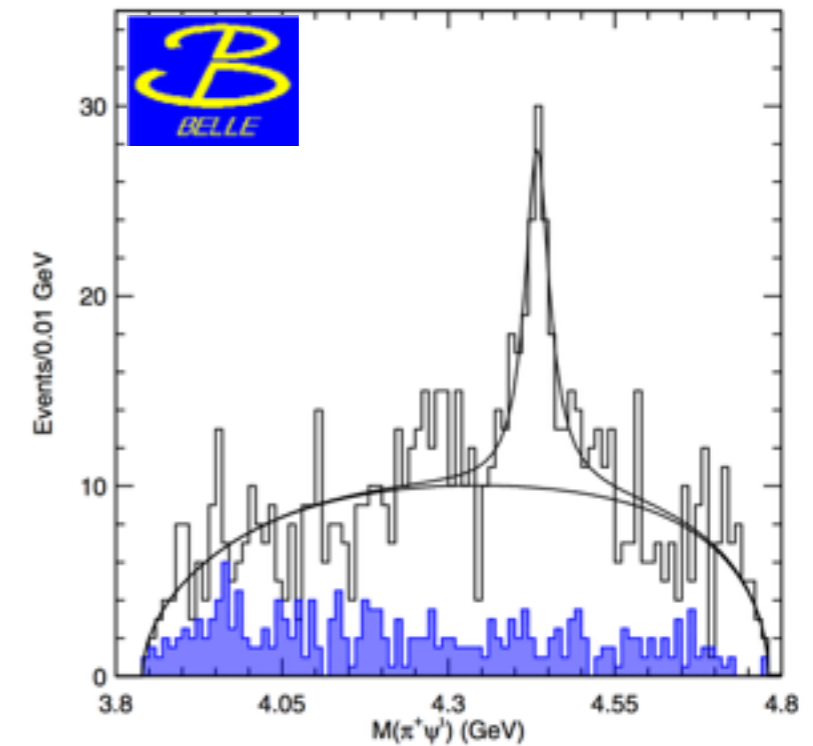
PRL 95, 142001 (2005)



**Y(4260)**

$e^+ e^- \rightarrow \gamma [\pi^+ \pi^- J/\psi]$

PRL 100, 142001 (2008)

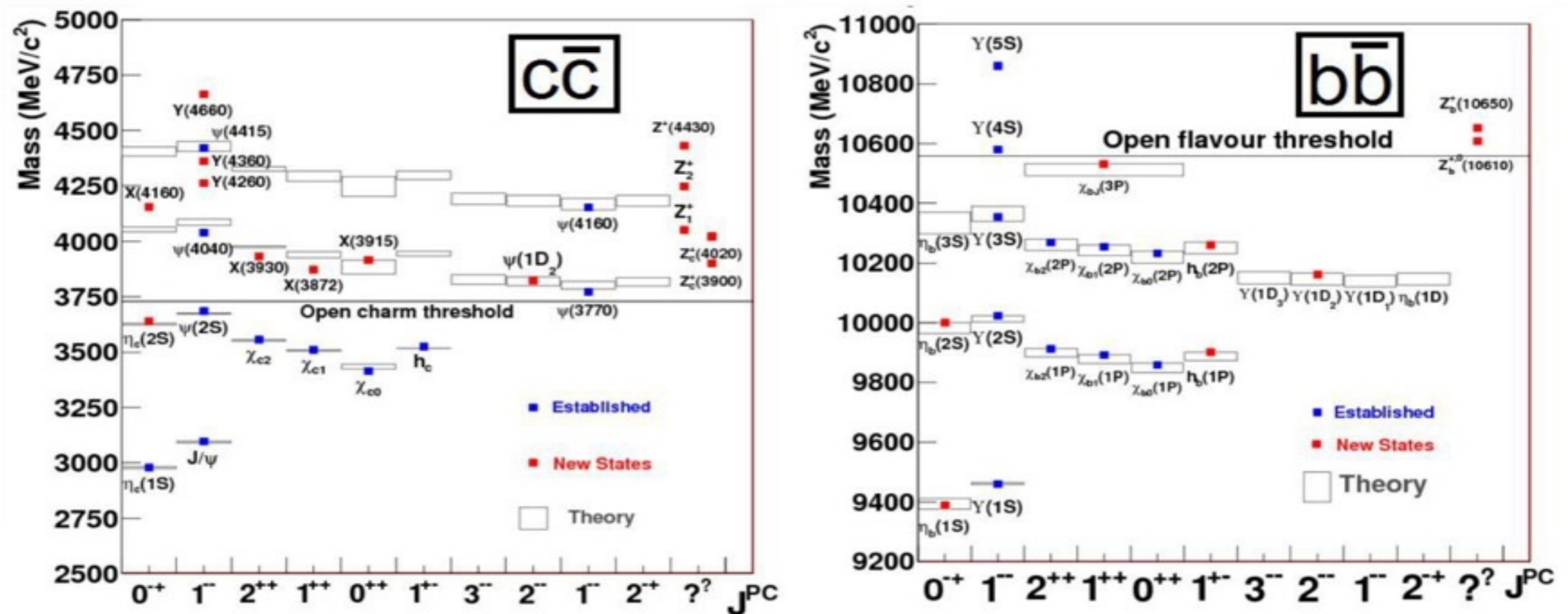


**Z(4430)**

$B \rightarrow K [\pi^\pm \psi']$

# B-Factory: legacy

- The legacy of 1<sup>st</sup> generation B-factories is a variety of quarkonium states
- Good agreement with predictions below open flavor threshold
- Many discoveries are difficult to explain with quarkonium model
- Several states have non-zero charge (cannot be  $q\bar{q}$  pairs)
- Exotic candidates, XYZ states



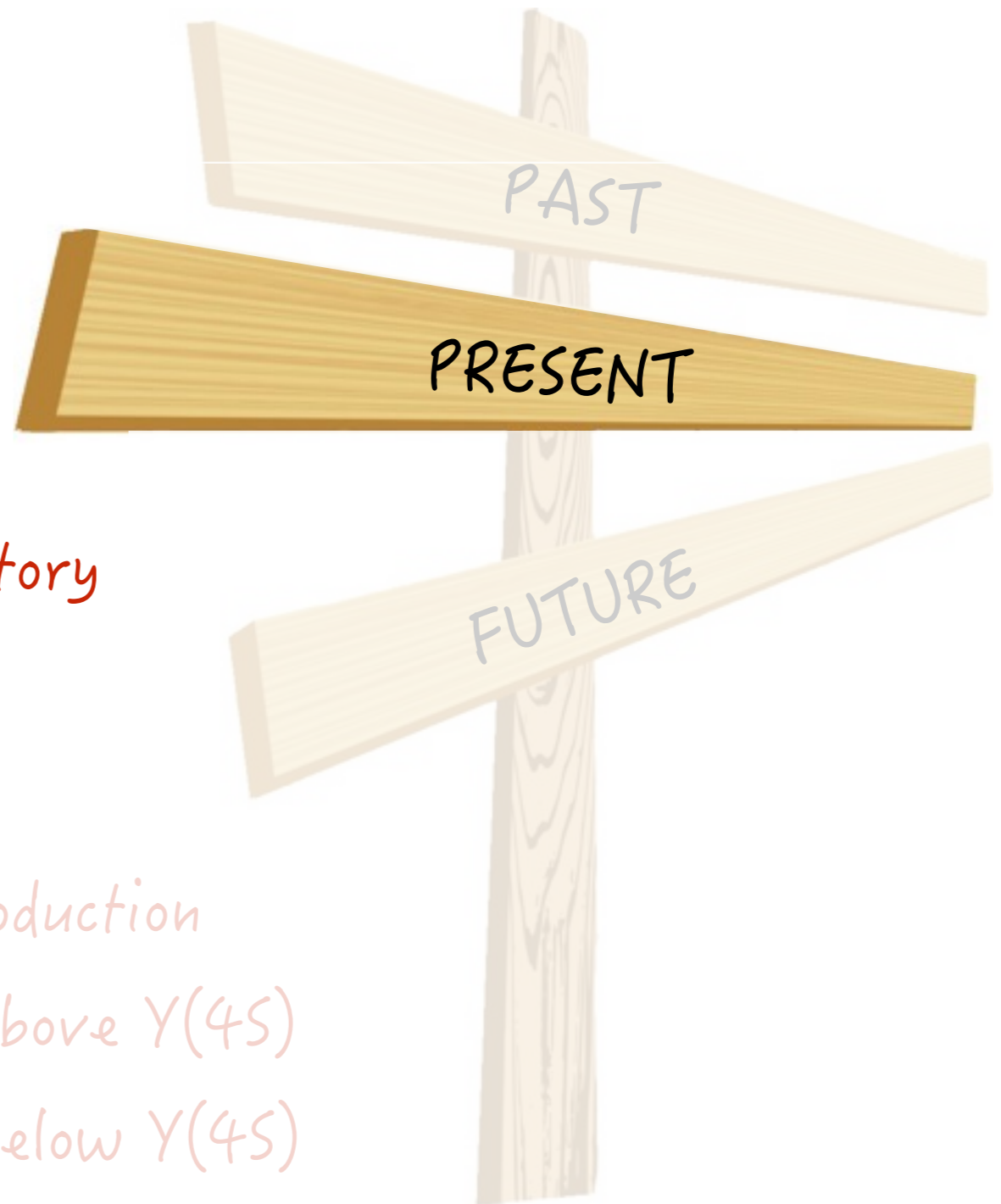
➔ Challenge for the new generation B-factories

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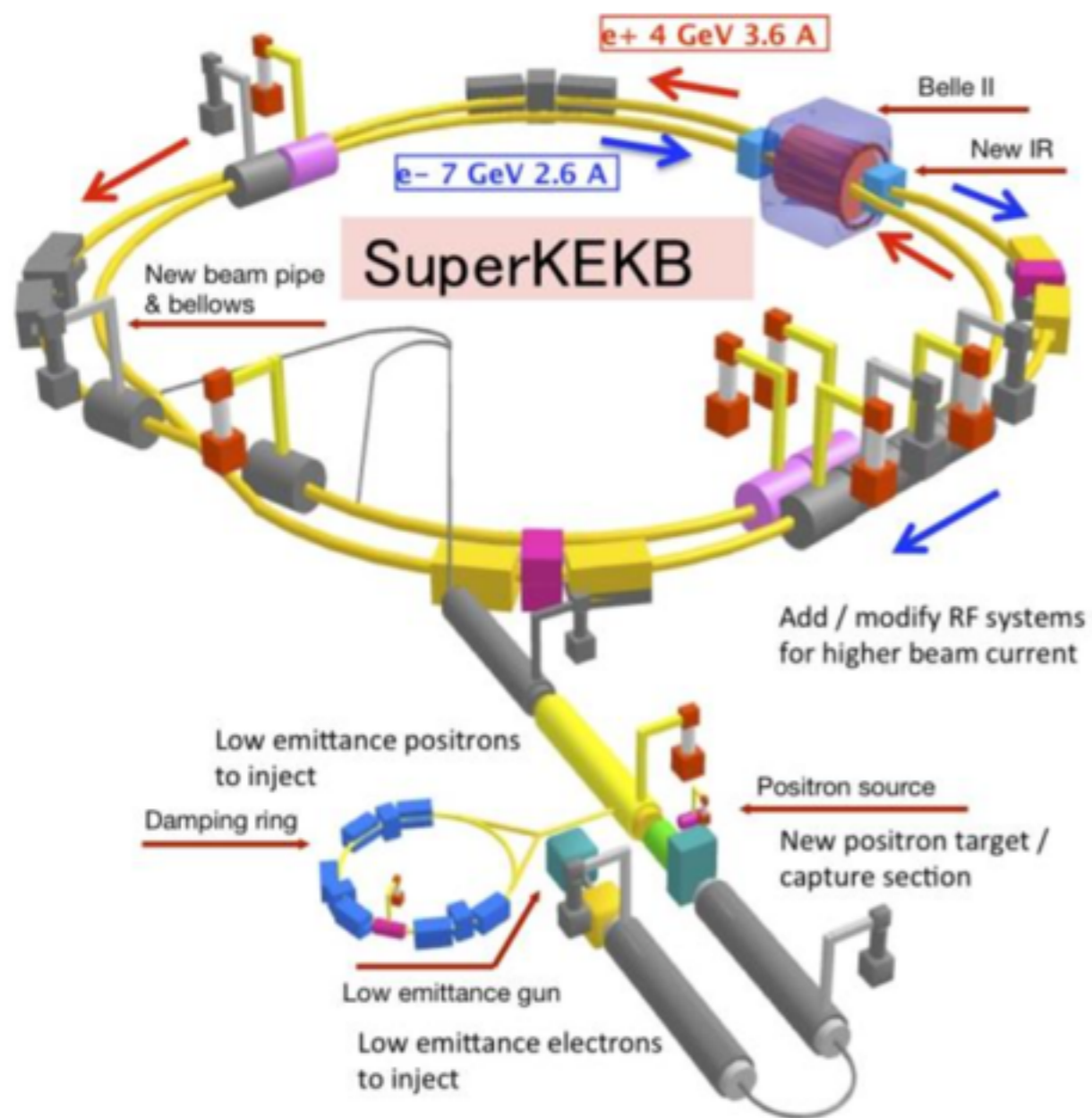
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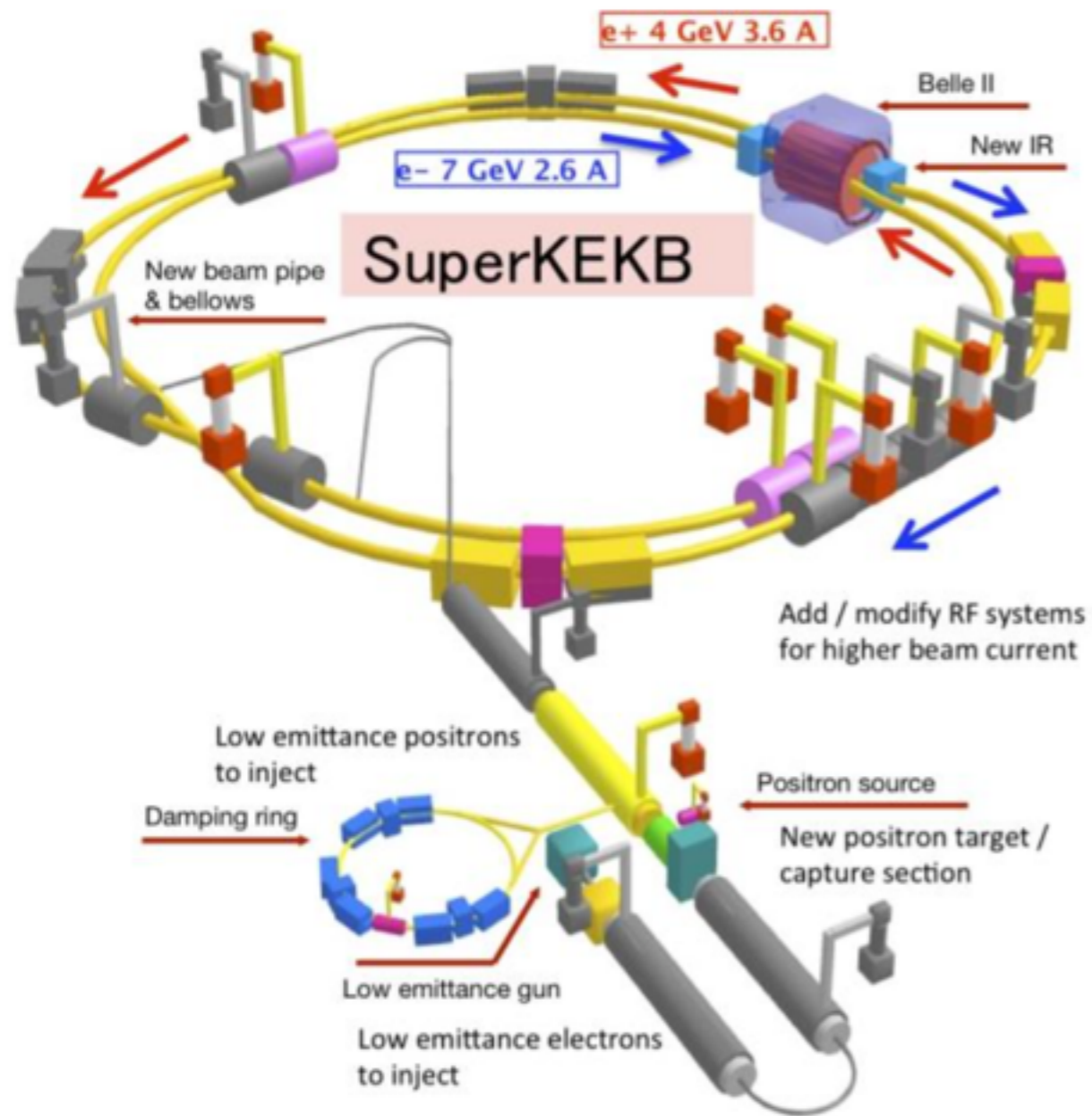
# From KEKB to SuperKEKB



- ➔ **Target integrated luminosity:**  
 **$50 \text{ ab}^{-1}$  (x50 Belle)**
- ➔ **Target peak luminosity:**  
 **$L = 8 \times 10^{35} \text{ cm}^{-2} \text{ s}^{-1}$  (x40 Belle)**

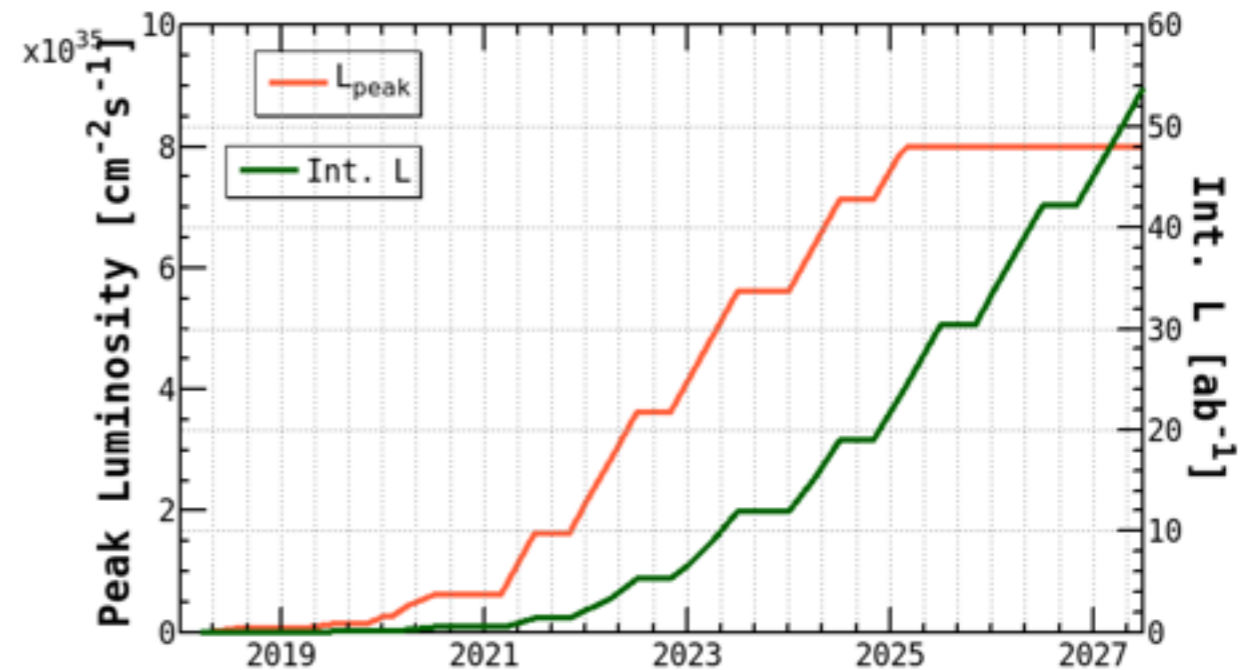


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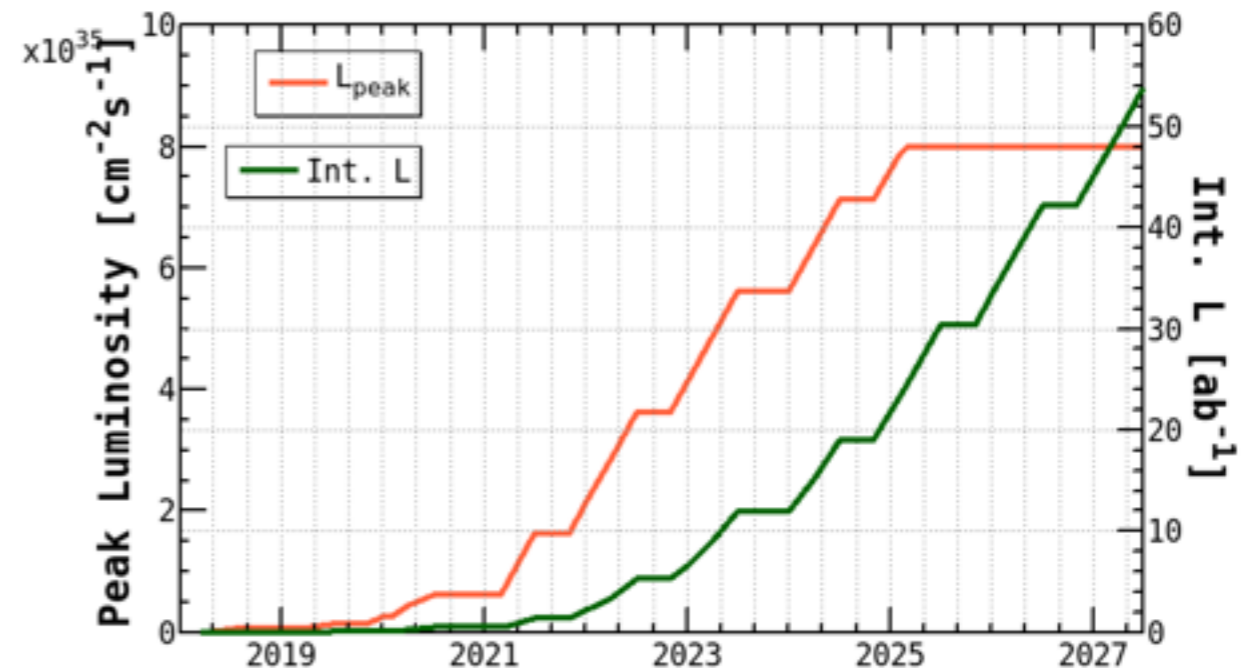


# From KEKB to SuperKEKB

*Current samples in  $fb^{-1}$  (millions of events)*

Experiment	$\Upsilon(1S)$	$\Upsilon(2S)$	$\Upsilon(3S)$	$\Upsilon(4S)$	$\Upsilon(5S)$	$\Upsilon(6S)$	$\frac{\Upsilon(nS)}{\Upsilon(4S)}$
CLEO	1.2 (21)	1.2 (10)	1.2 (5)	16 (17.1)	0.1 (0.4)	-	23%
BaBar	-	14 (99)	30 (122)	433 (471)	$R_b$ scan	$R_b$ scan	11%
Belle	6 (102)	25 (158)	3 (12)	711 (772)	121 (36)	5.5	23%
BelleII			<b>300(1200)</b>	<b><math>5 \times 10^4(5.4 \times 10^4)</math></b>	<b>1000(300)</b>	<b>100+400(scan)</b>	<b>3.6%</b>

- $\Upsilon(4S)$  physics program but not only (important especially for the bottomonium physics program)
- In general, a lot can be done in quarkonium field



# Nano beam scheme

$$L = \frac{\gamma_{e\pm}}{2er_e} \left( 1 + \frac{\sigma_y^*}{\sigma_x^*} \right) \left( \frac{I_{e\pm} \cdot \xi_{y,e\pm}}{\beta_y^*} \right) \left( \frac{R_L}{R_{\xi_y}} \right)$$

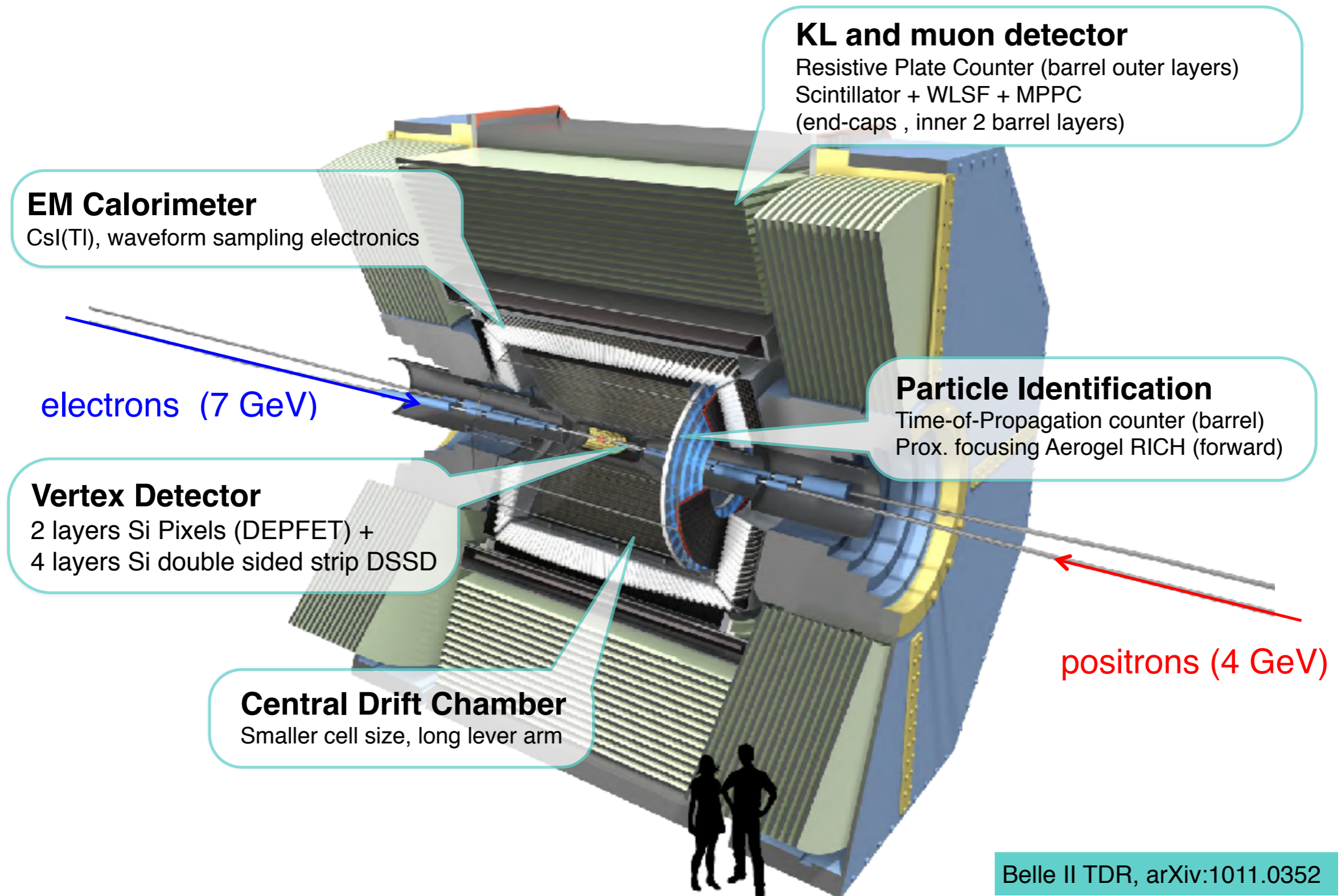
Beam aspect ratio  $\left( 1 + \frac{\sigma_y^*}{\sigma_x^*} \right)$   
 Beam current  $I_{e\pm}$   
 Vertical beta function @ IP  $\beta_y^*$   
 Beam-beam parameter  $\left( \frac{I_{e\pm} \cdot \xi_{y,e\pm}}{\beta_y^*} \right)$

	E (GeV) LER / HER	$\beta_y^*$ (cm) LER / HER	$\beta_x^*$ (mm) LER / HER	$\Phi$ (mrad)	I (A) LER / HER	L (cm <sup>-2</sup> s <sup>-1</sup> )	IP sketch
<b>KEKB</b>	3.5 / 8.0	5.9 / 5.9	120 / 120	11	1.6 / 1.2	2.1 x 10 <sup>34</sup>	
<b>SuperKEKB B</b>	4.0 / 7.0	<b>0.27 / 0.30</b> (Factor 20)	3.2 / 2.5	41.5	<b>3.6 / 2.6</b> (Factor 2)	<b>80 x 10<sup>34</sup></b>	

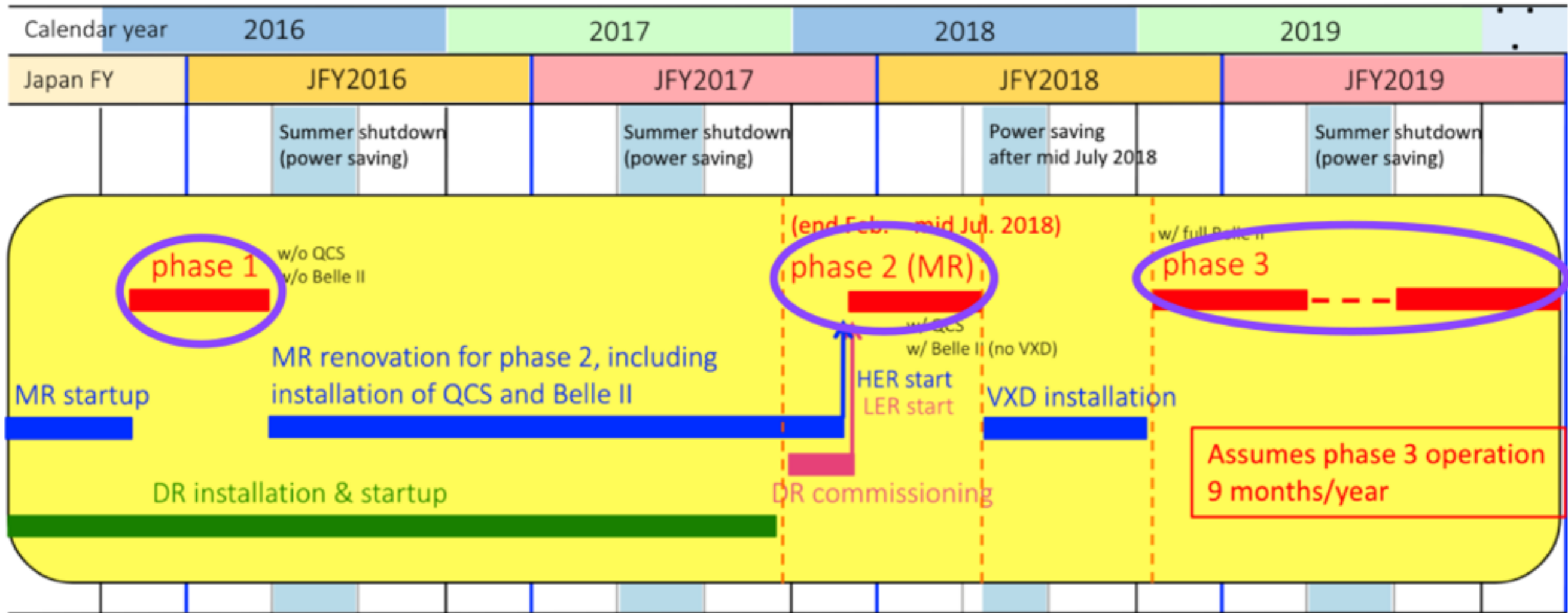
**Factor 40**

- Squeeze the beam @ IP by 1 / 20
- Double beam currents

# The Belle II detector

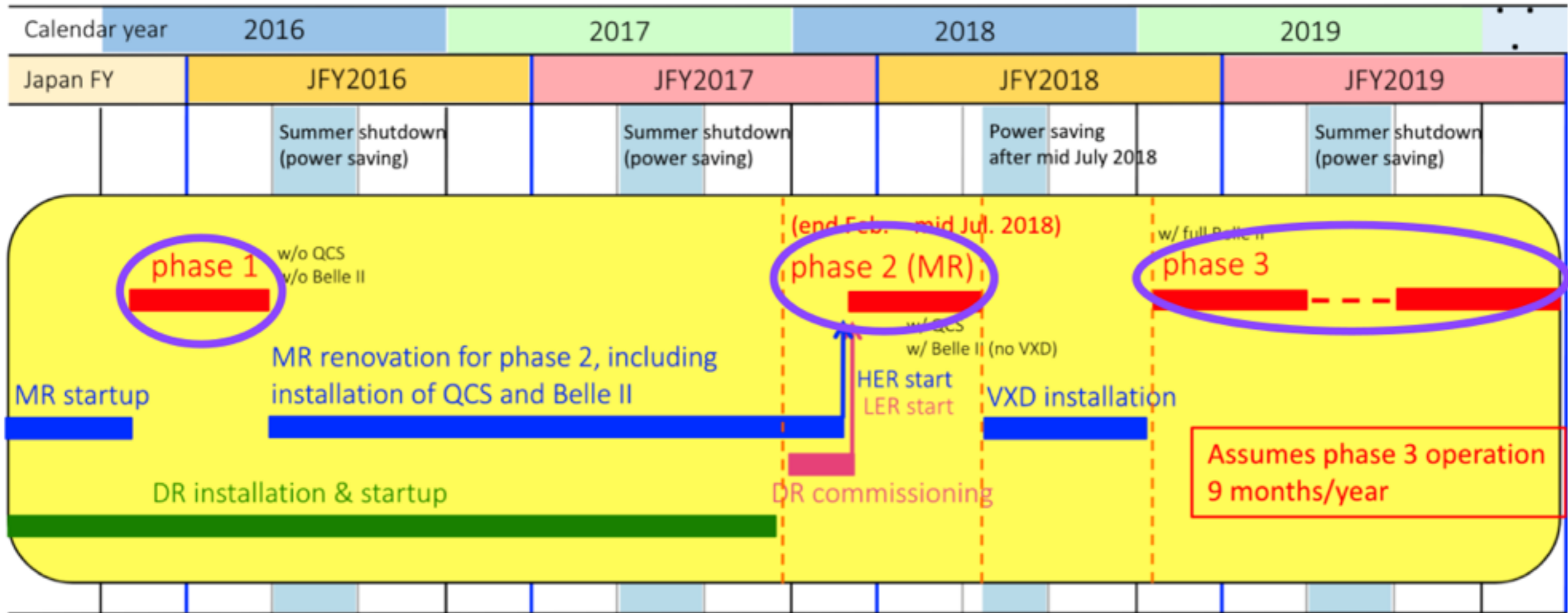


# Belle II status and schedule



- Phase 1: SuperKEKB commissioning (January - June 2016)
- Phase 2: pilot run with limited vertexing (April - July 2018)
- Phase 3: collision data taking with full Belle II detector.

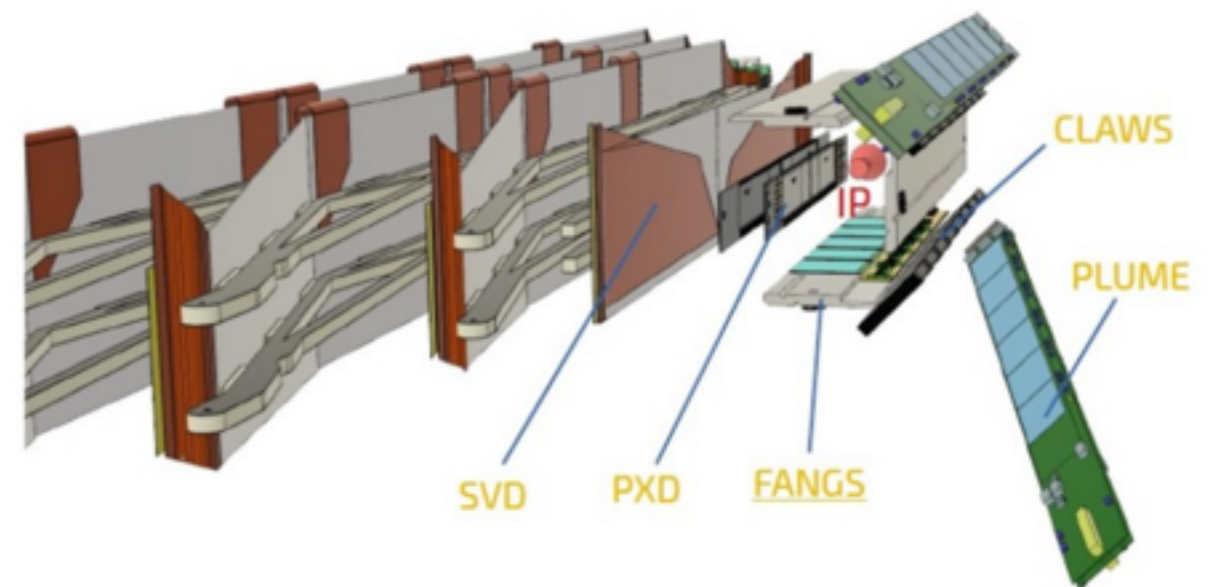
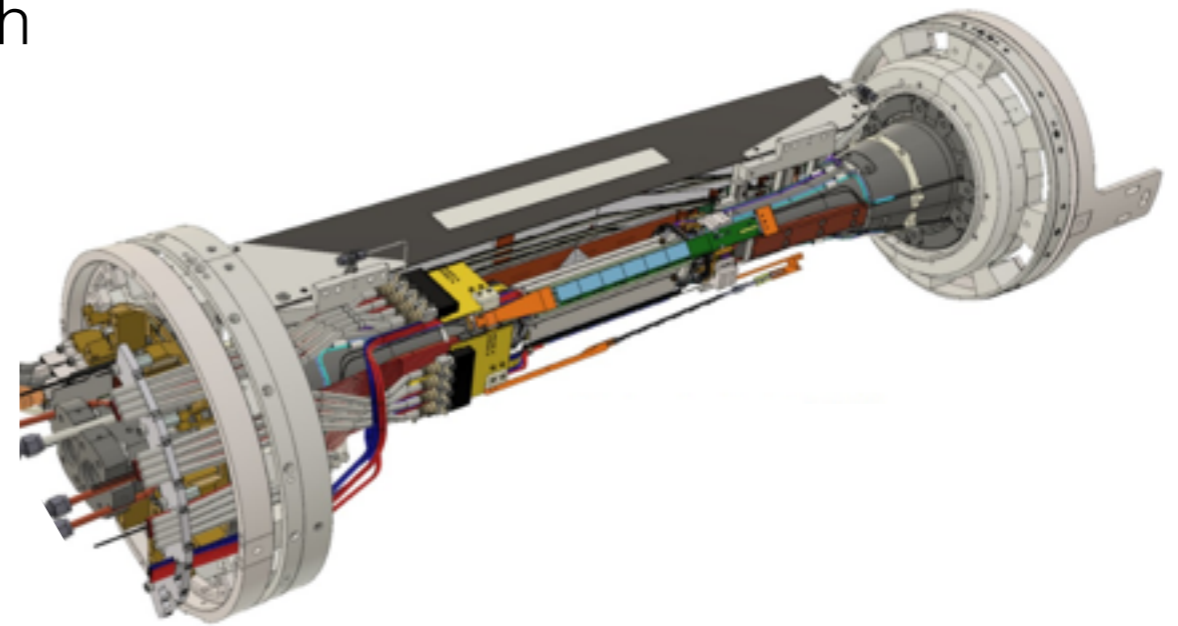
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- Phase 2: pilot run with limited vertexing (April - July 2018)
- Phase 3: collision data taking with full Belle II detector. **STARTED MARCH 2019!**

# Few more words about Phase 2

- Phase 2 lasted from April 26th to July 17th
- Pilot run with limited vertexing
  - Most of the silicon tracker replaced by background monitor detectors
  - One full octant of PXD + SVD
- Luminosity tuning had priority on data taking
  - $5.55 \times 10^{33} \text{ cm}^{-1} \text{ s}^{-2}$  maximum luminosity
  - $0.5 \text{ fb}^{-1}$  of collisions at  $Y(4S)$
- Goals
  - Verification the nano beam scheme
  - Monitoring of the background
  - Commissioning of the detector
  - Produce some physics results

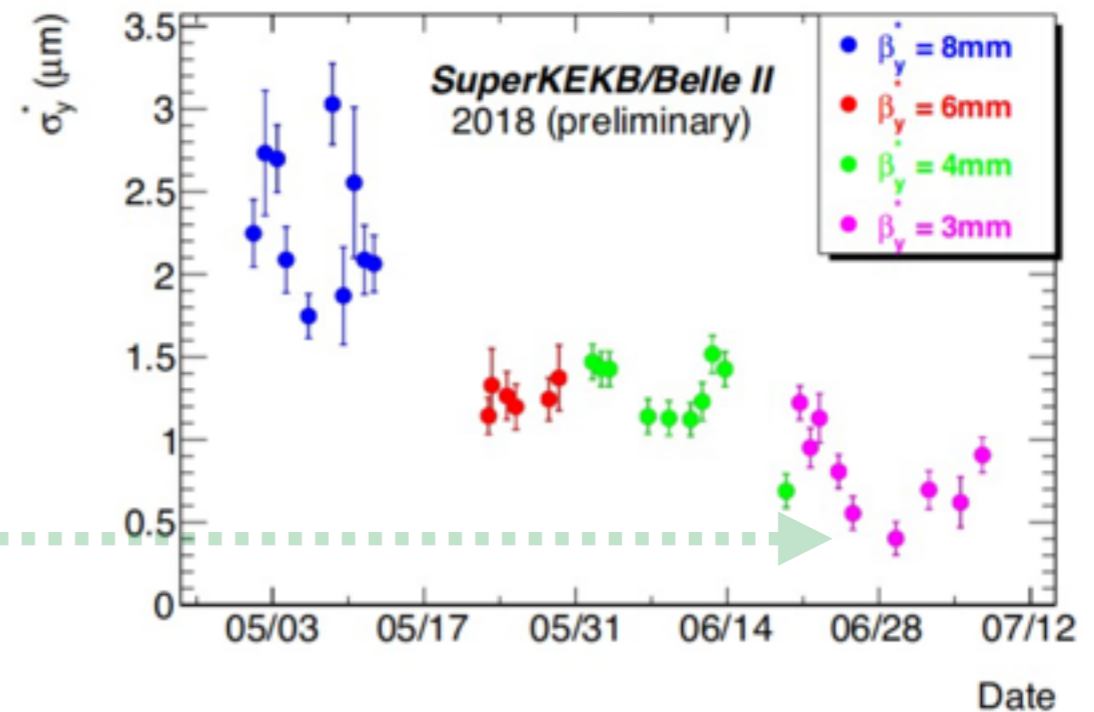


# Phase 2 highlights: beam properties verifications

- Vertical size of the beam ( $\sigma_y^*$ ):

obtained using vertical offset scans of the beams at the IP, corresponding to luminosity scans as a function of the vertical offset

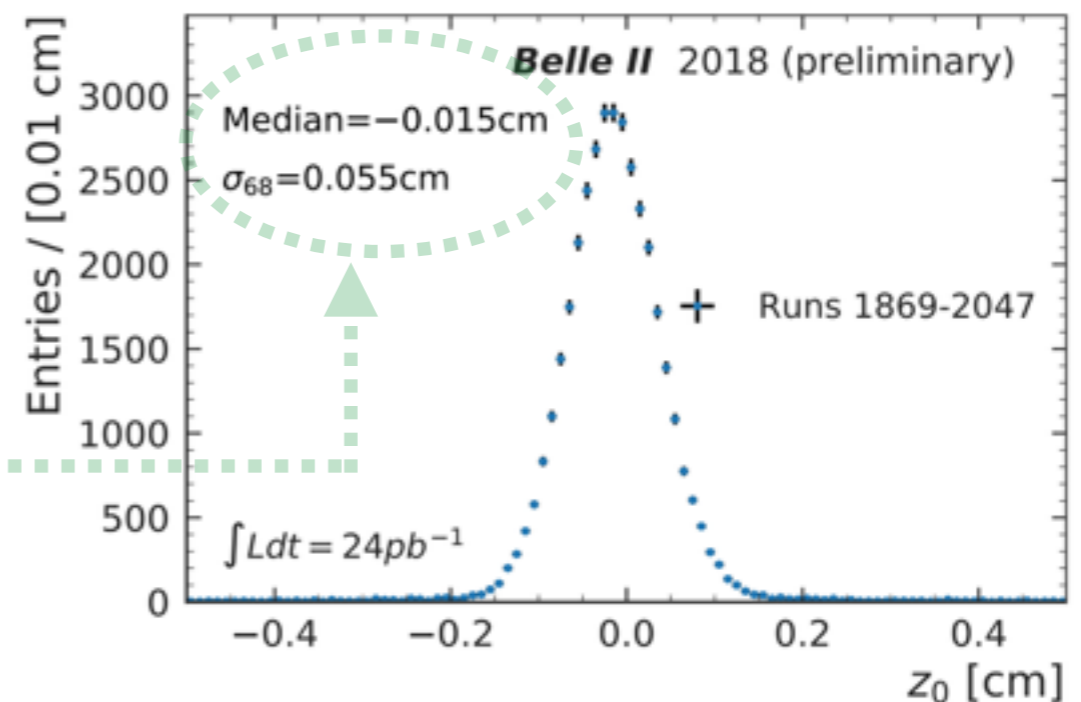
( $\rightarrow$  *less than 500 nm achieved*)



- Interaction region size ( $z_0$ ):

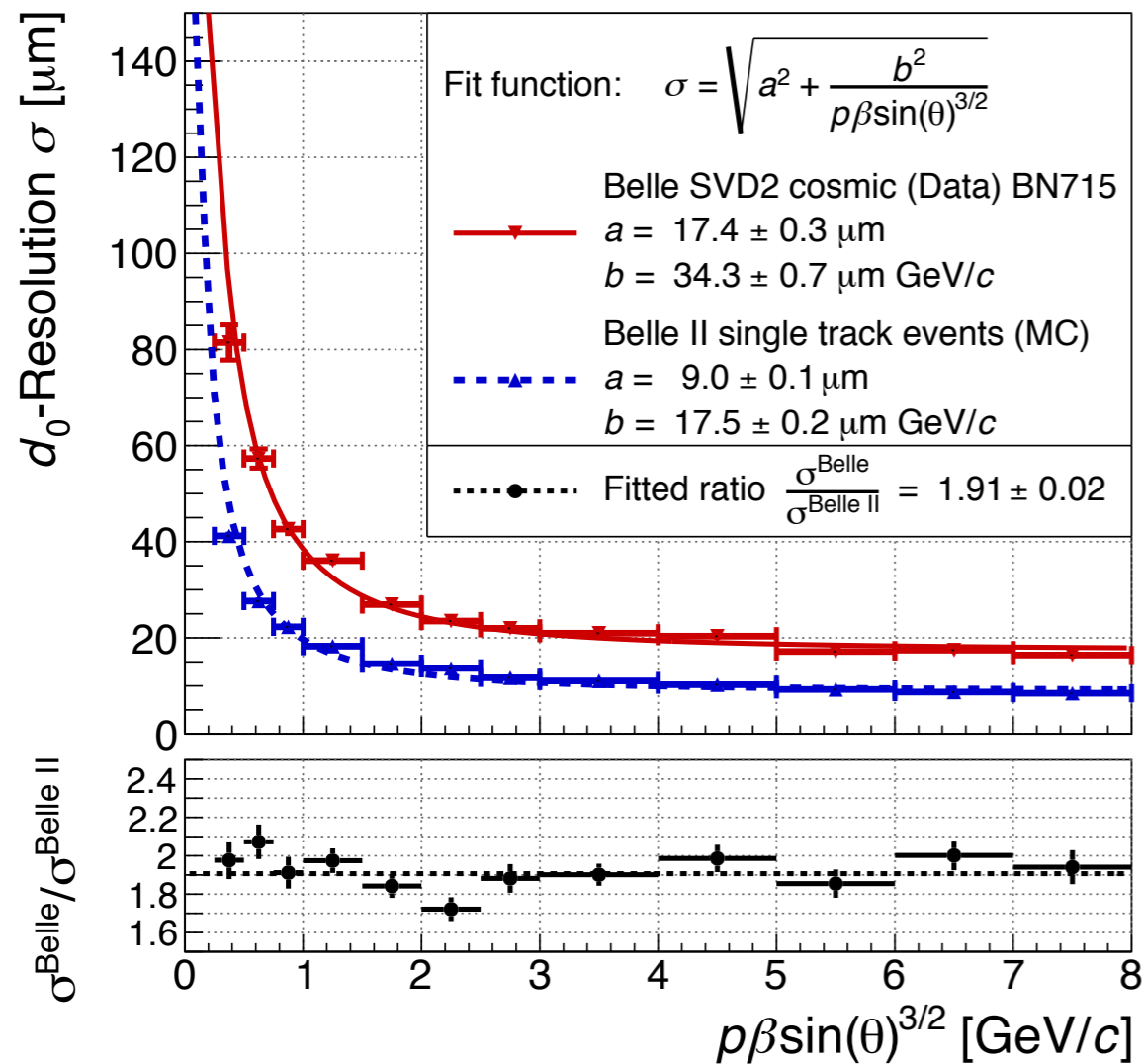
longitudinal component of the interaction vertex estimated using  $z_0$  parameter of single tracks originating from the interaction vertex

( $\rightarrow$  *Beam spot ~ 10 times smaller than KEKB*)



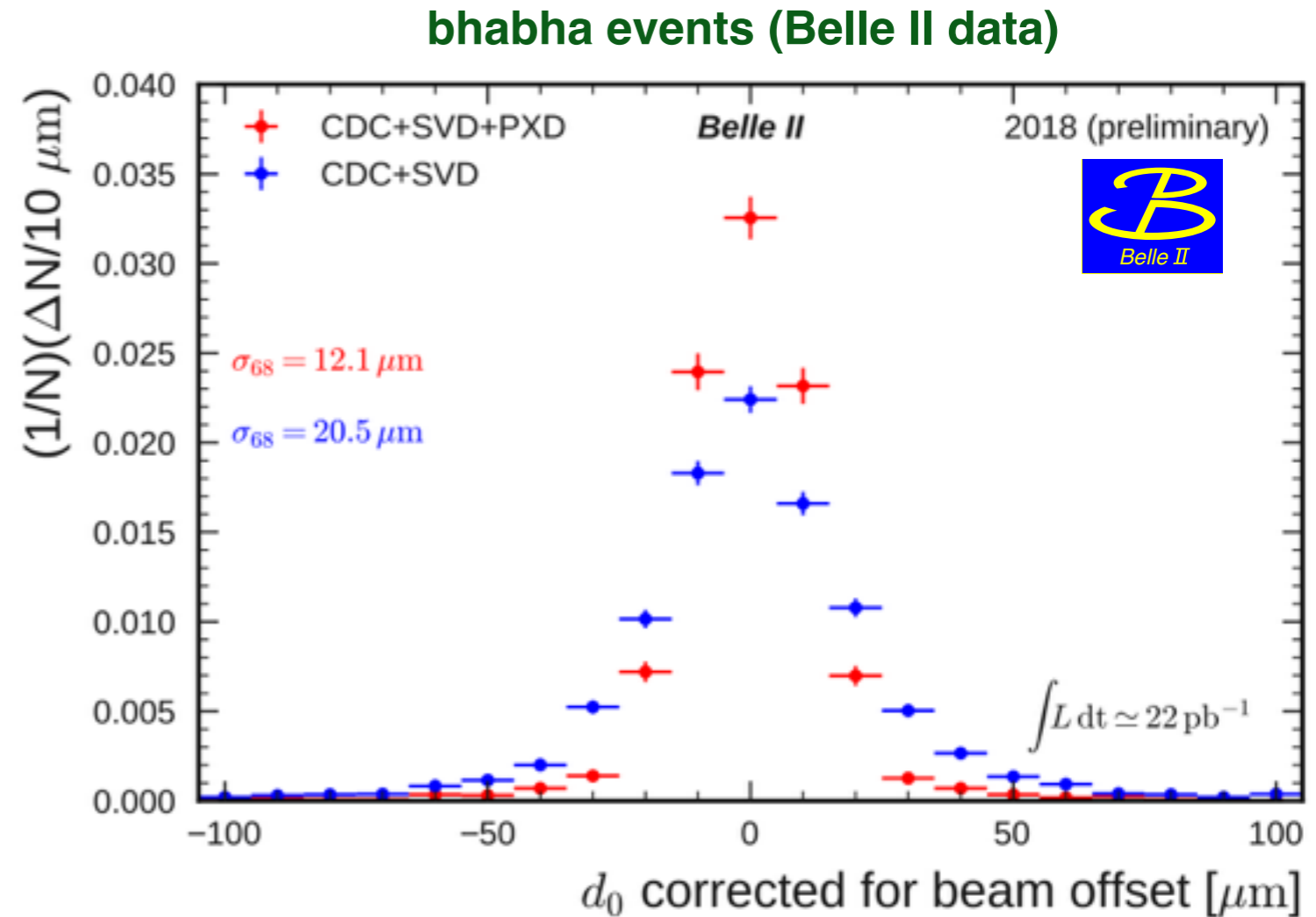
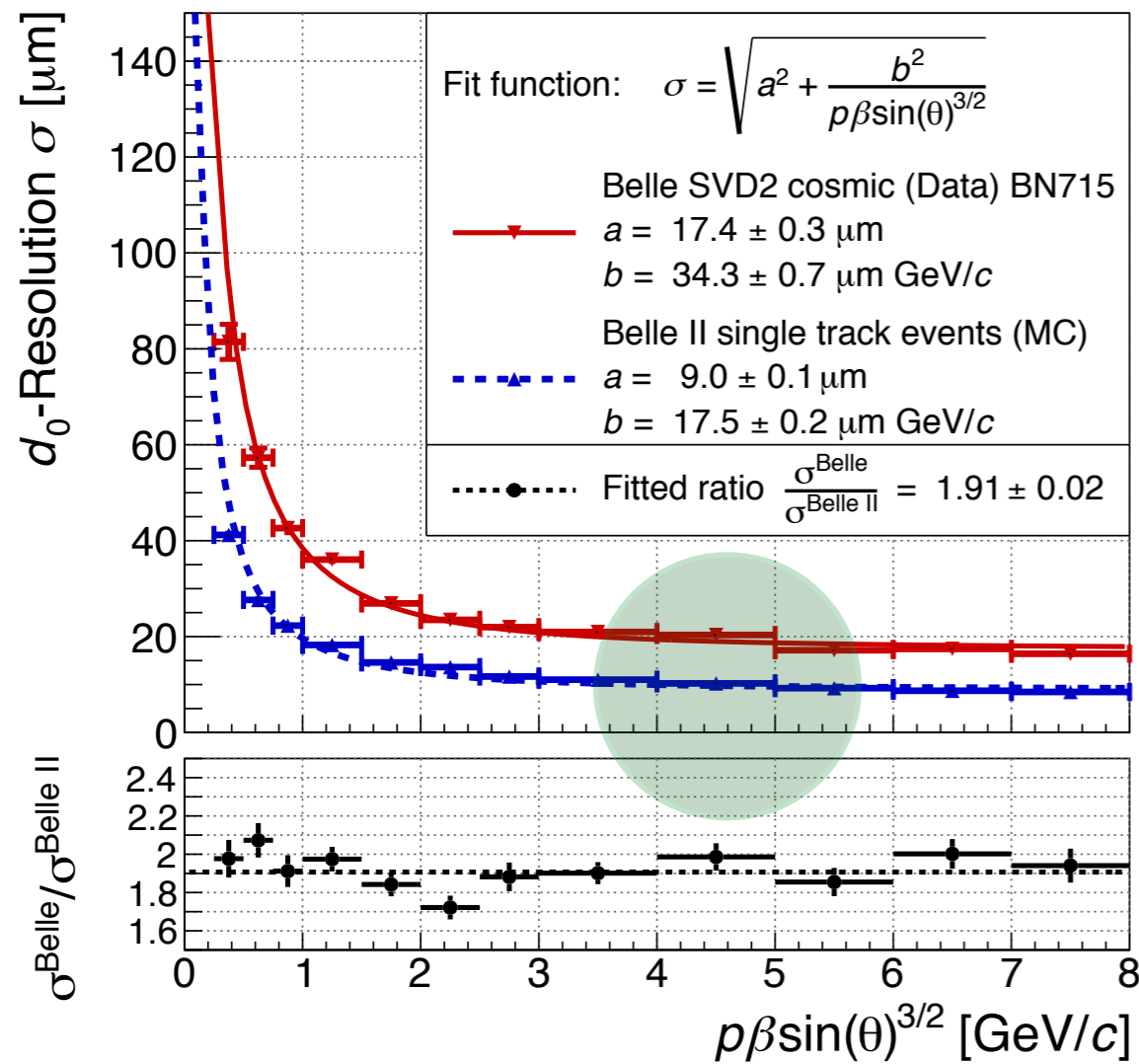


# Phase 2 highlights: impact parameter resolution



- Resolution of the transverse impact parameter ( $d_0$ )
- Belle II MC events with a single muon tracks compared with results of Belle cosmic events

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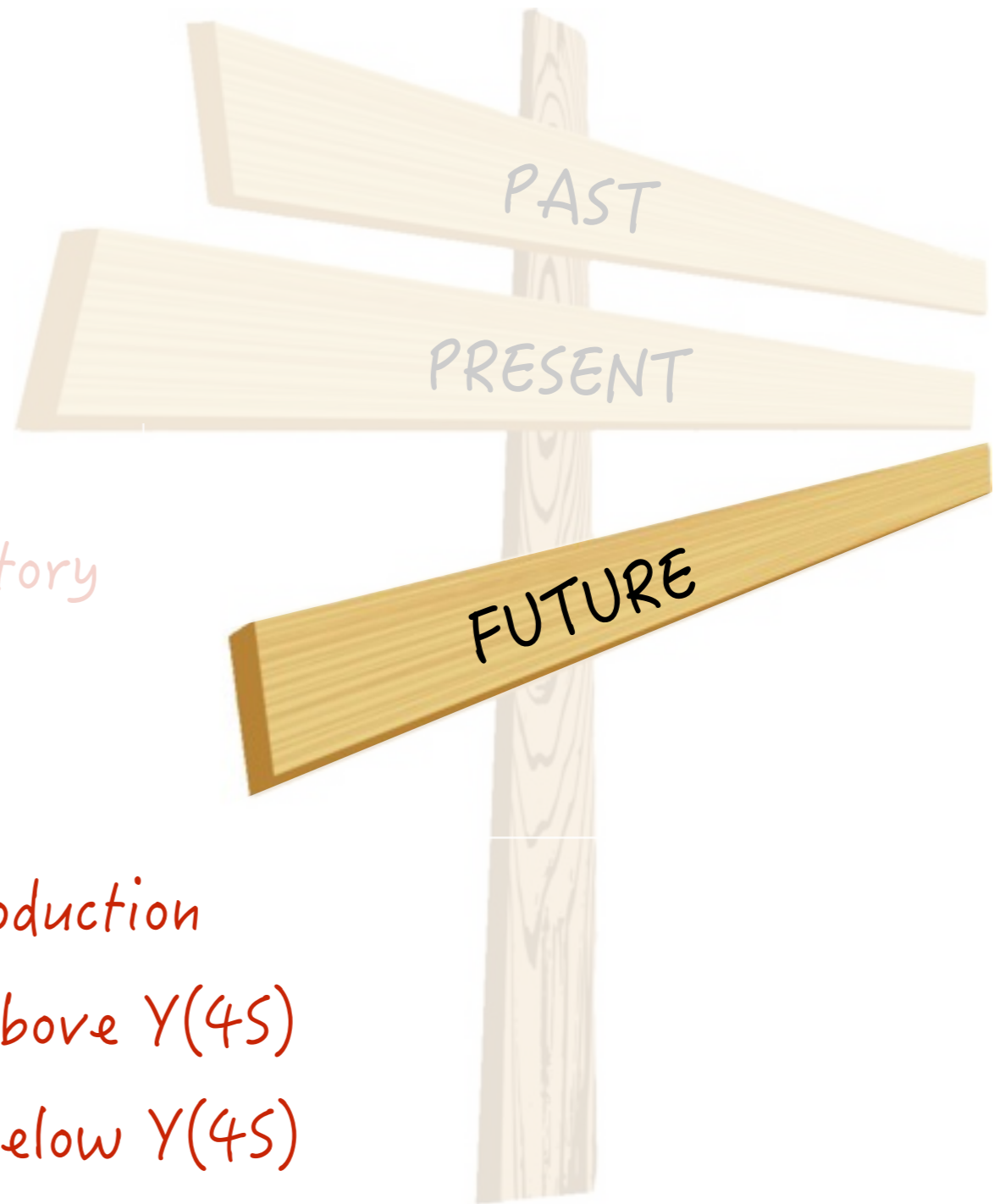


- Resolution of the transverse impact parameter ( $d_0$ ) using bhabha events
  - measured  $12.1 \mu\text{m}$ , expected  $\sim 10 \mu\text{m}$
  - PXD is crucial

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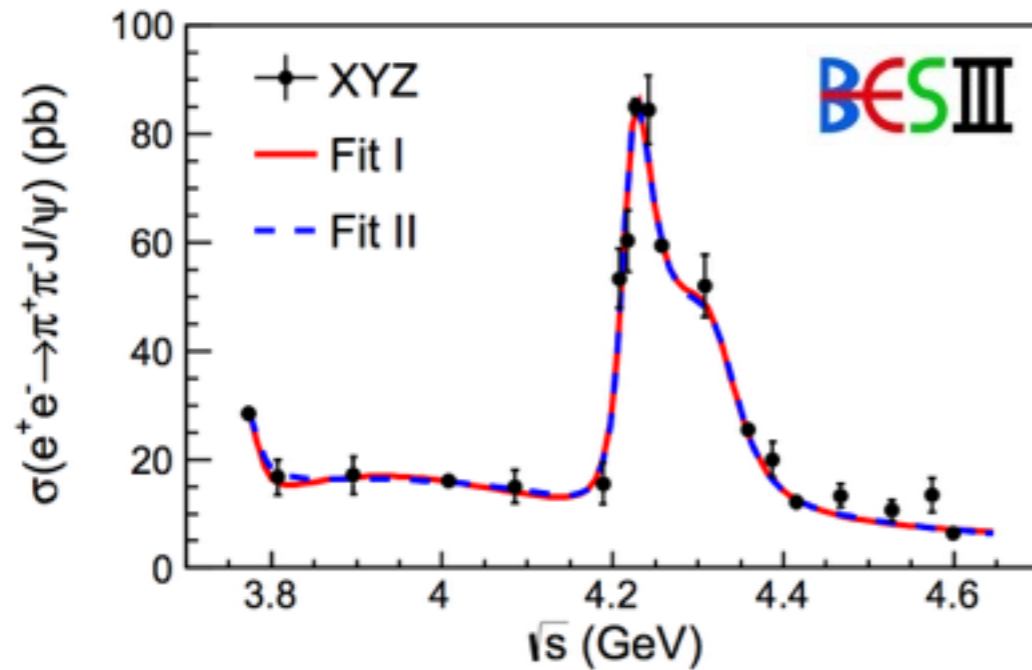
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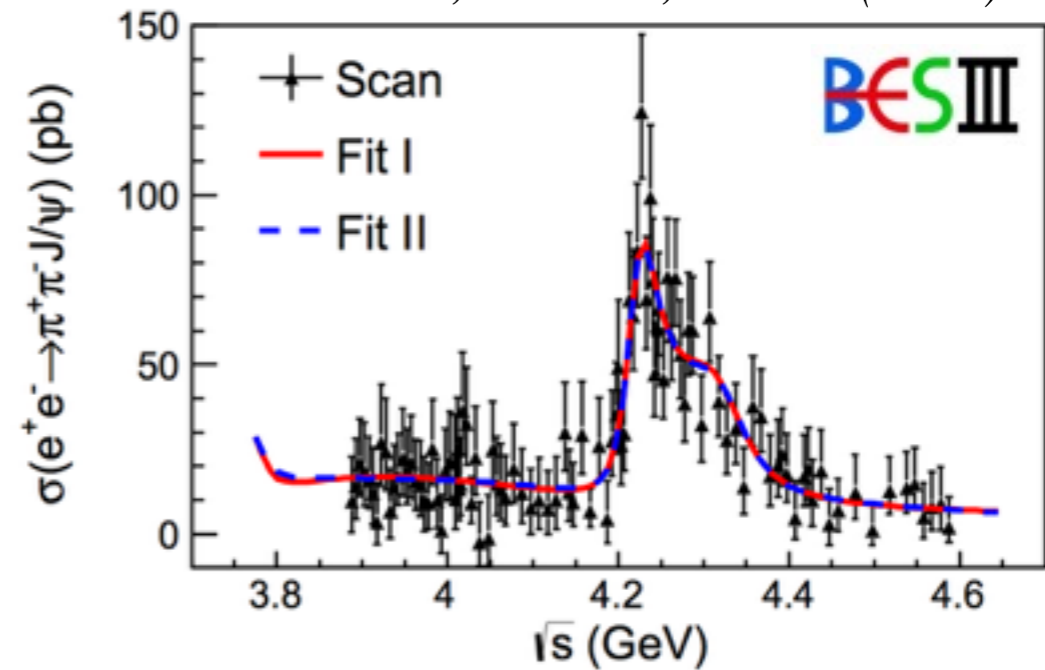


# Charmonia: ISR

- Competition from BESIII



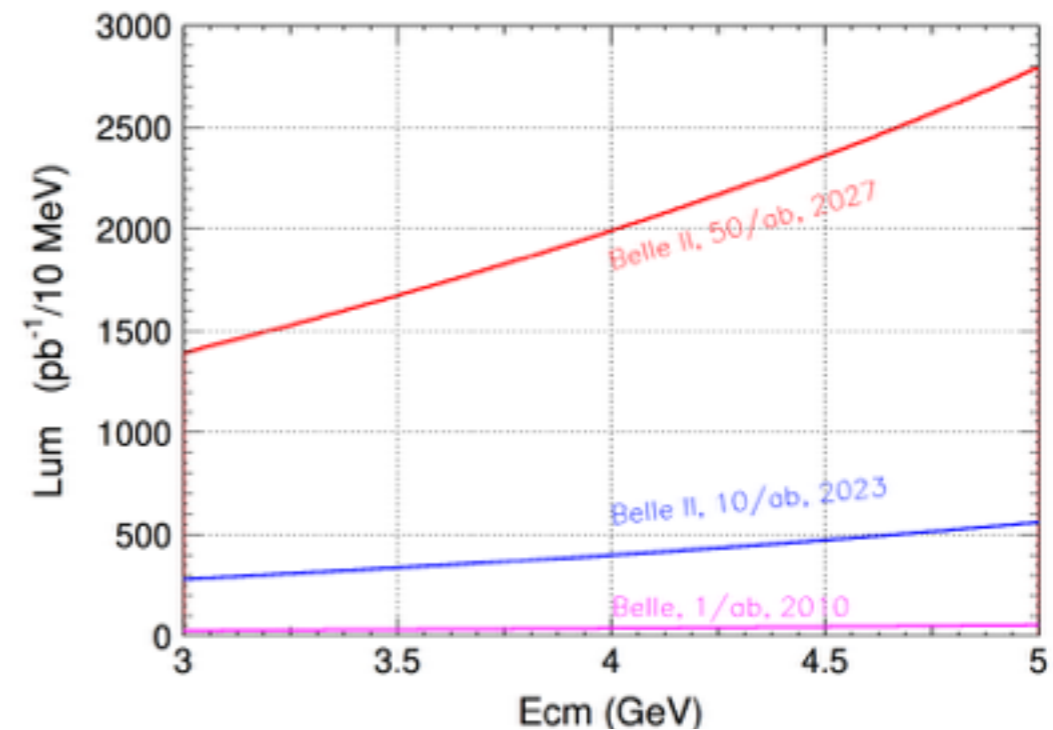
*BESIII, PRL 118, 092001 (2017)*



- Recent BESIII scan data show a complex landscape: scan of all decay channels is needed!

## ➔ Belle II

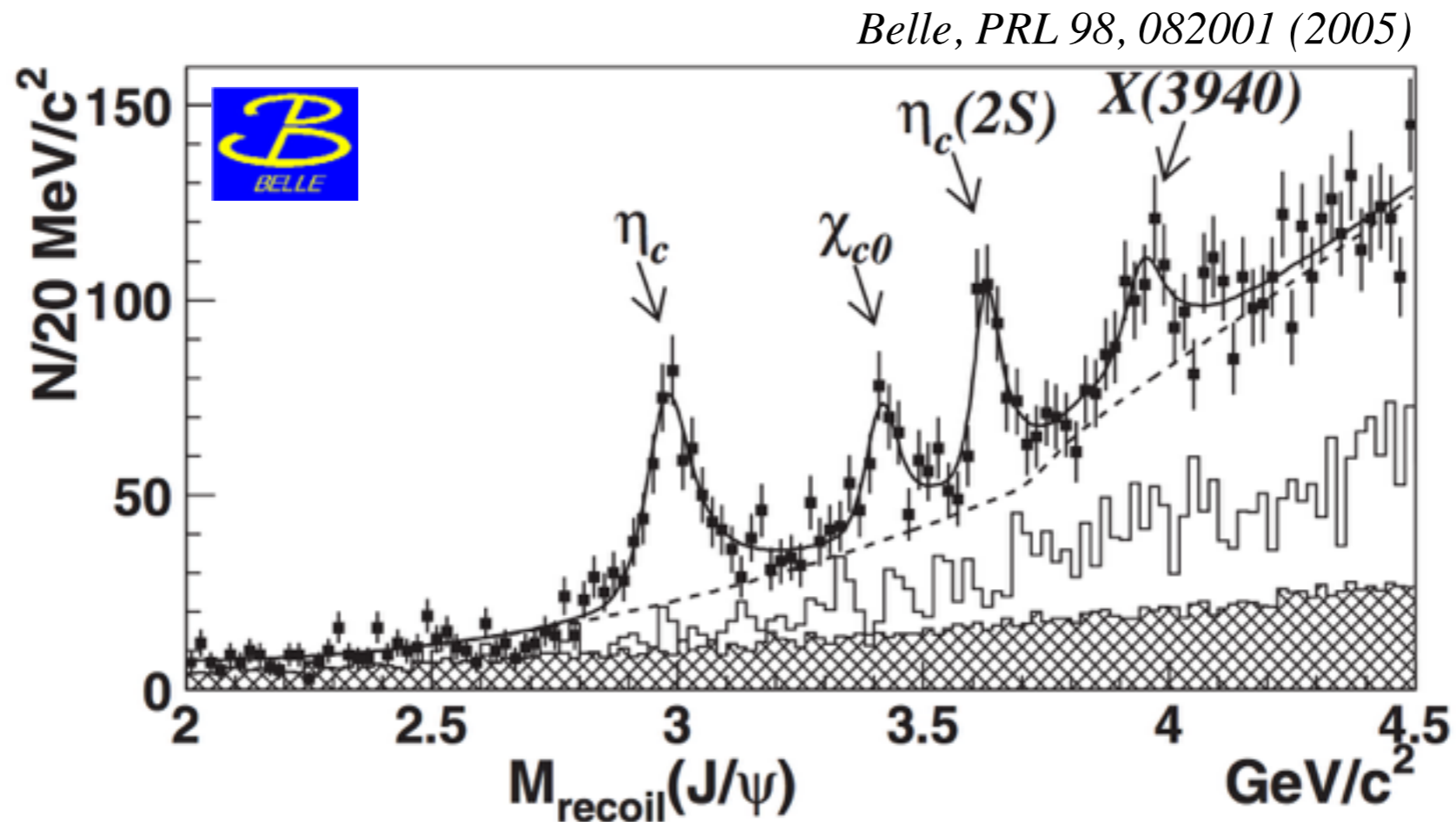
- Higher effective luminosity
- Wider mass range



# Charmonia: double charmonium production

- Observed in combinations of J=1 and J=0  
 $e^+e^- \rightarrow c\bar{c} (0+/-) c\bar{c} (1-/+)$

**only at Belle II**



## → Belle II

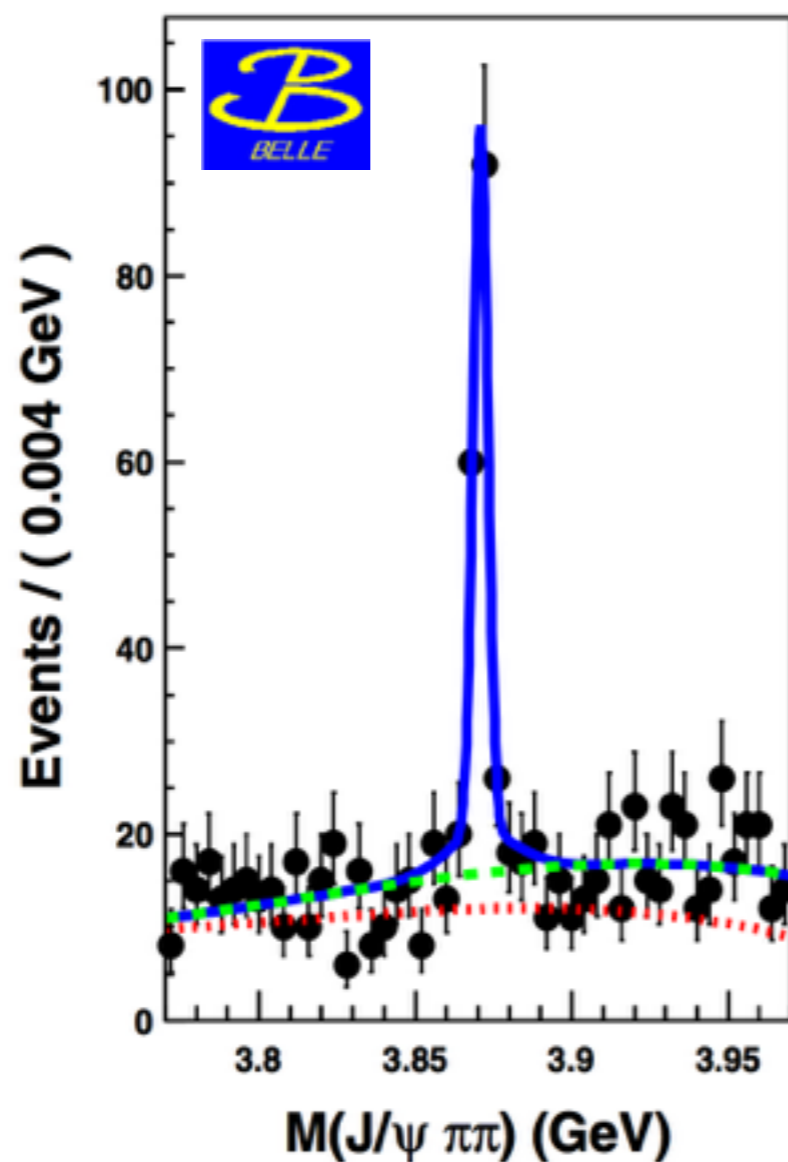
- angular distributions, production
- probe for new states

# Charmonia: $X(3872)$ width

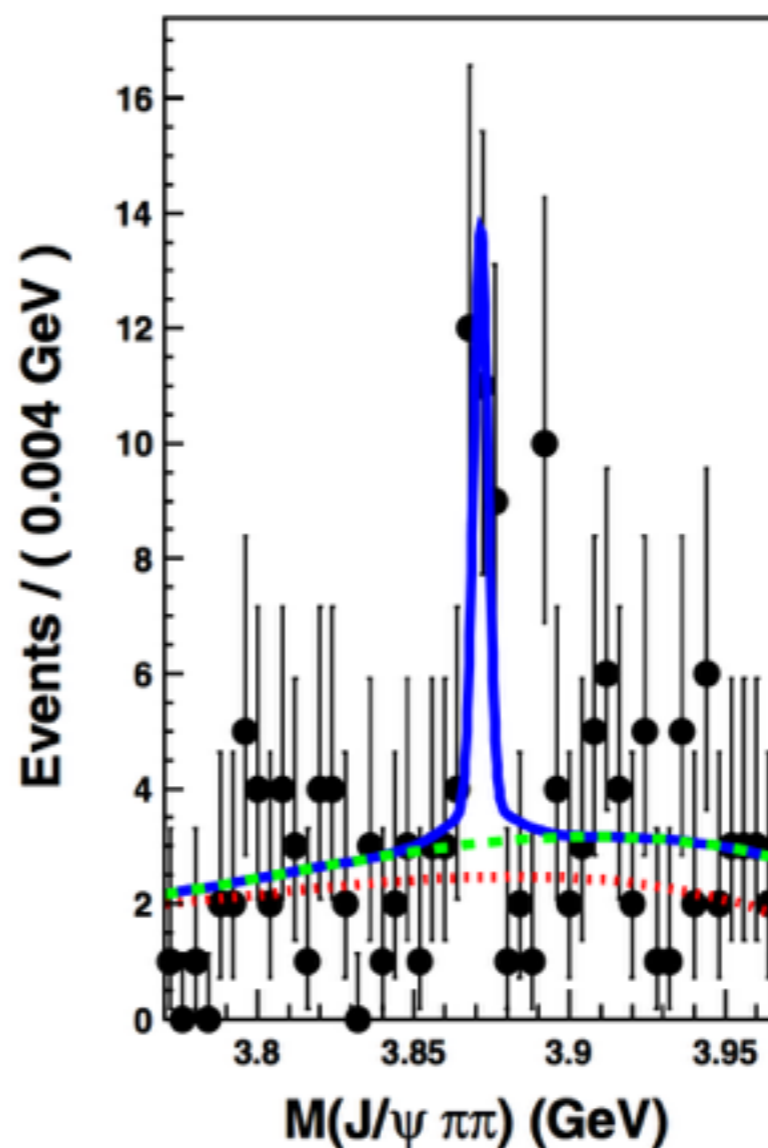
- The current upper limit on the width of the  $X(3872)$  width is the 90% C.L. of  $\Gamma_{X(3872)} < 1.2$  MeV, obtained using the mode  $B \rightarrow J/\psi \pi^+ \pi^- K$

*Belle, PRD 84, 052004 (2011)*

$B^+ \rightarrow K^+ X(3872)$

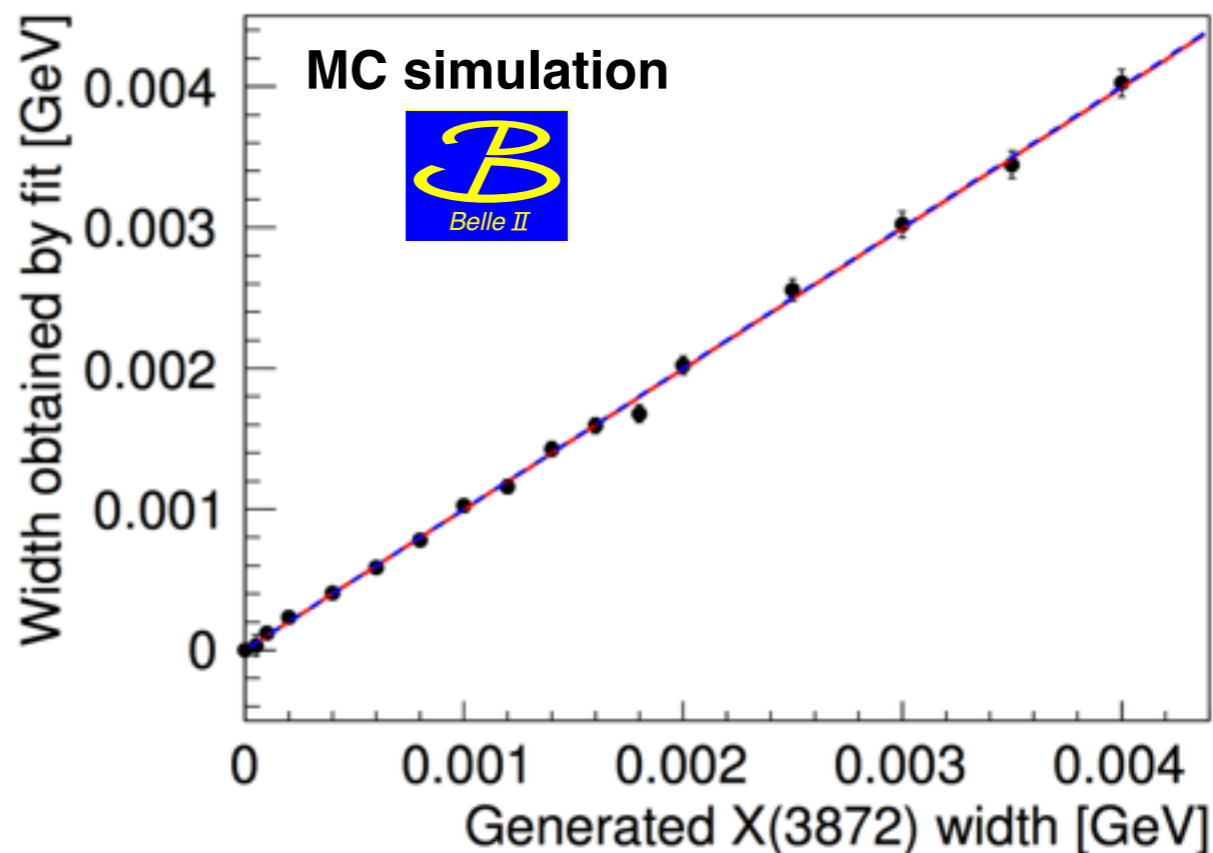
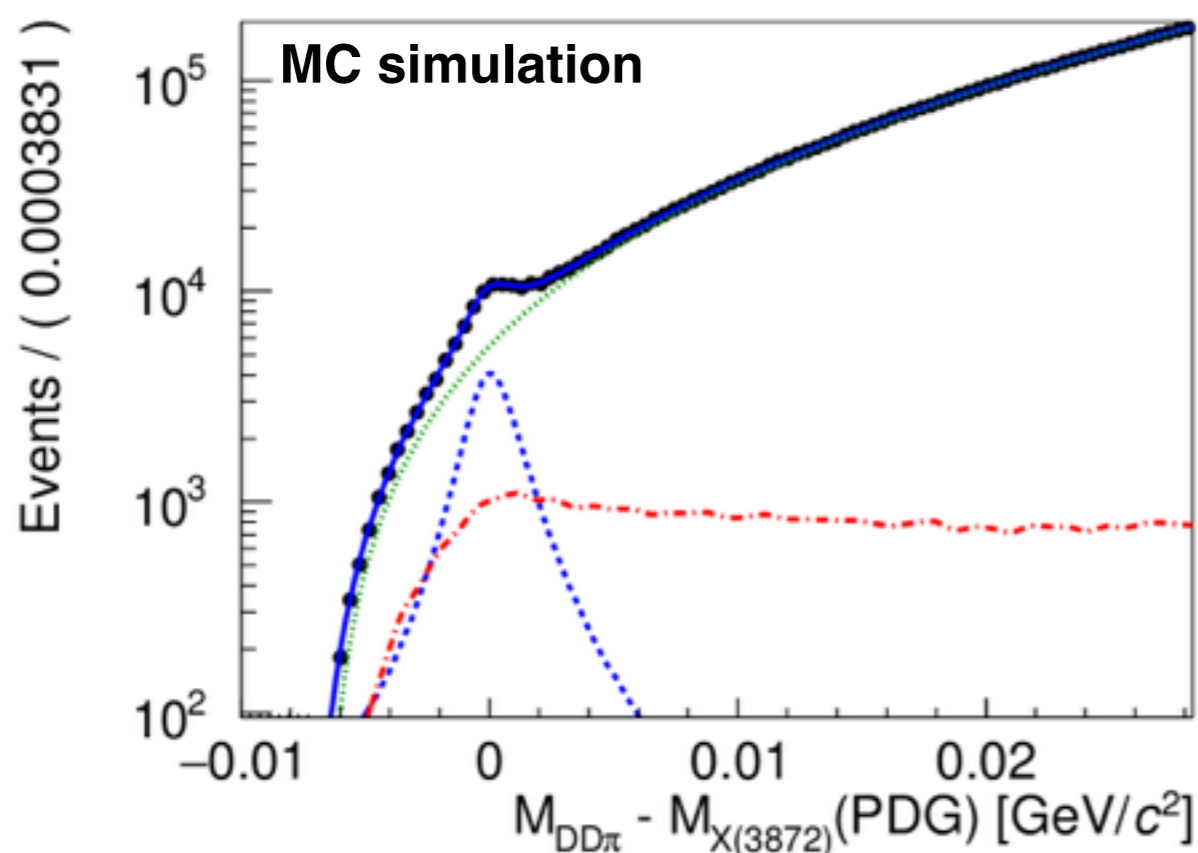


$B^0 \rightarrow K_s X(3872)$



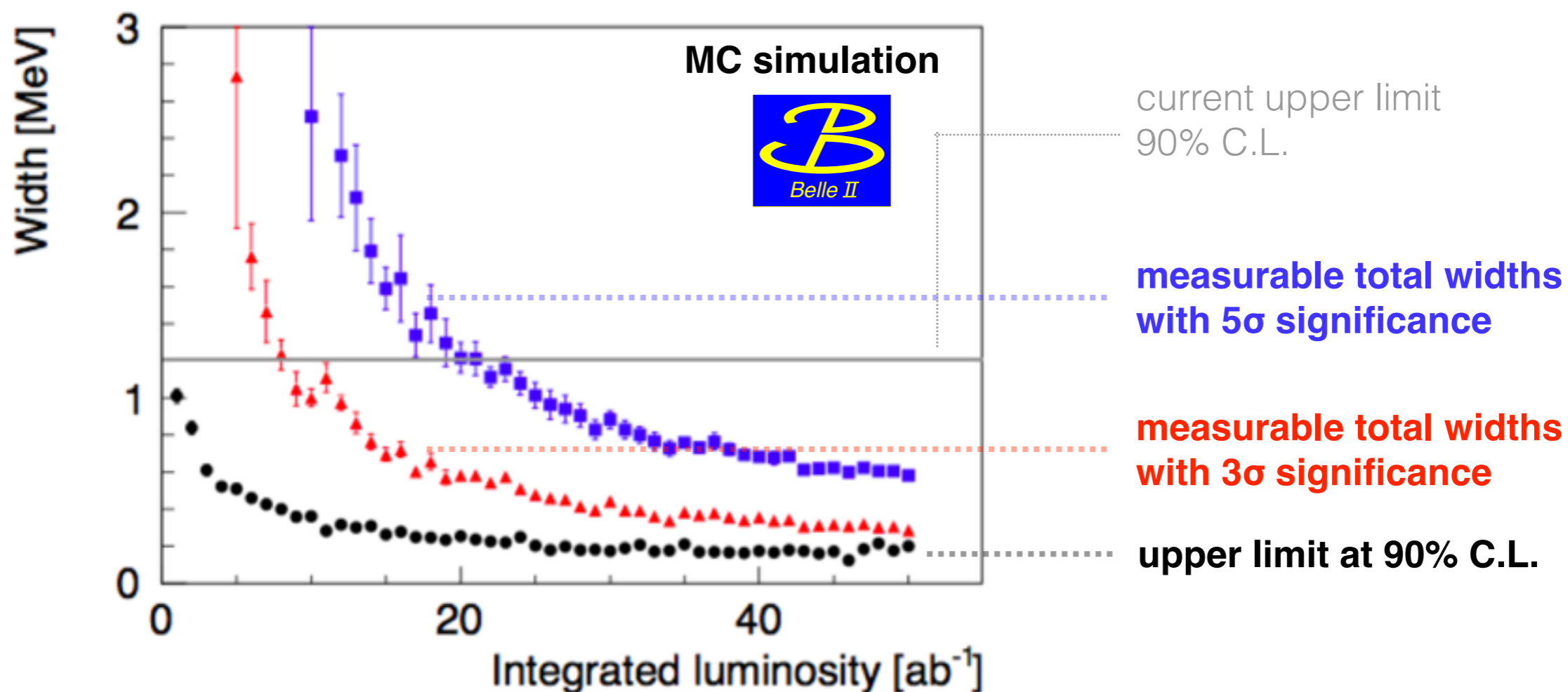
# Charmonia: $X(3872)$ width

- A way to improve the sensitivity of the  $X(3872)$  total width is to improve the mass resolution
- $B \rightarrow D^0 D^0 \pi^0$  good mode to improve the mass resolution
- The sensitivity has been estimated on MC (H. Hirata, master thesis, 2019)



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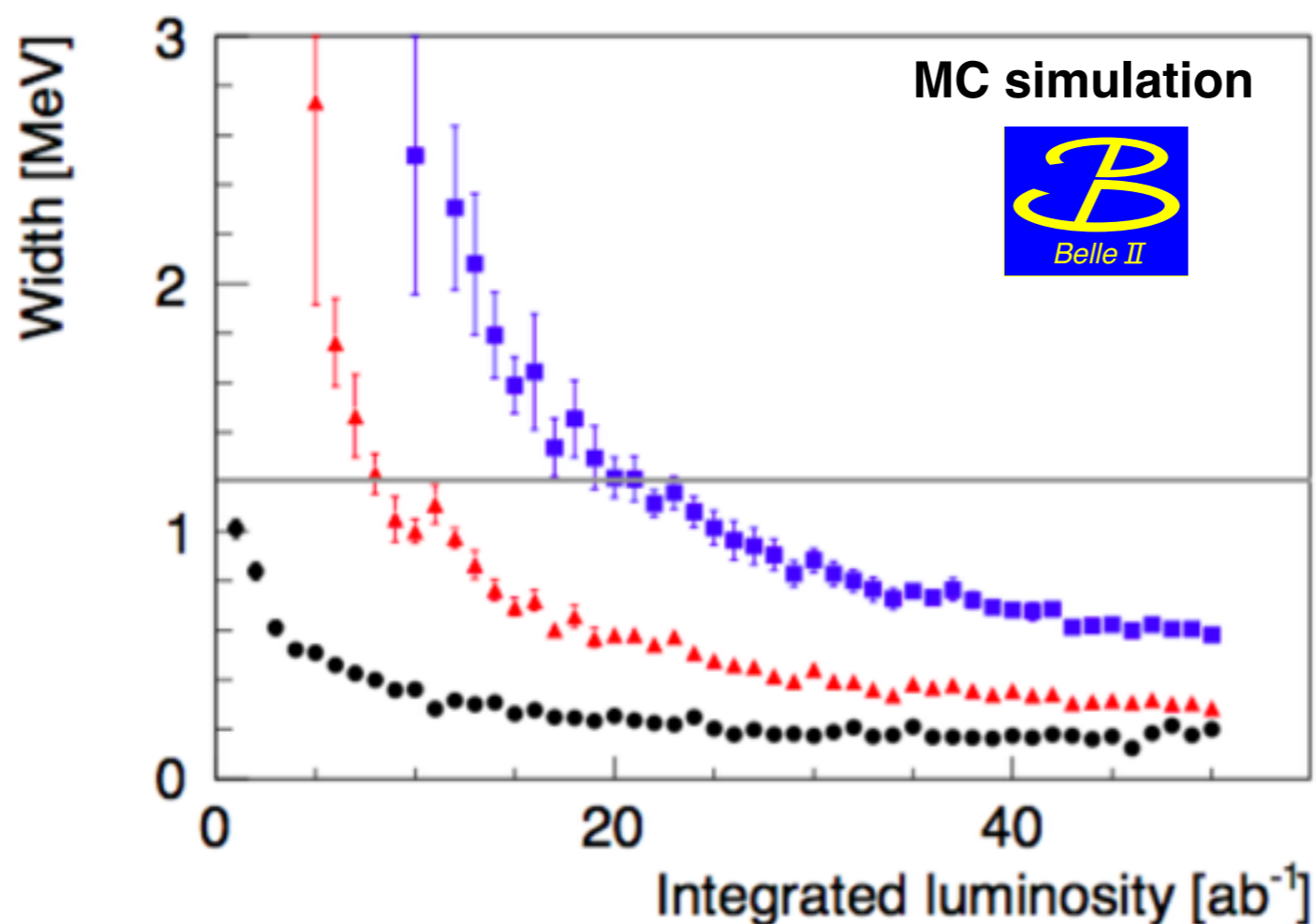
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- With the full Belle II data sample ( $50 \text{ ab}^{-1}$ ):

**90% C.L.:  $\sim 180 \text{ KeV}$**

**3s significance:  $\sim 280 \text{ KeV}$**

**5s significance:  $\sim 570 \text{ KeV}$**

# Bottomonia: motivation for non- $\Upsilon(4S)$ running

## → above $\Upsilon(4S)$ :

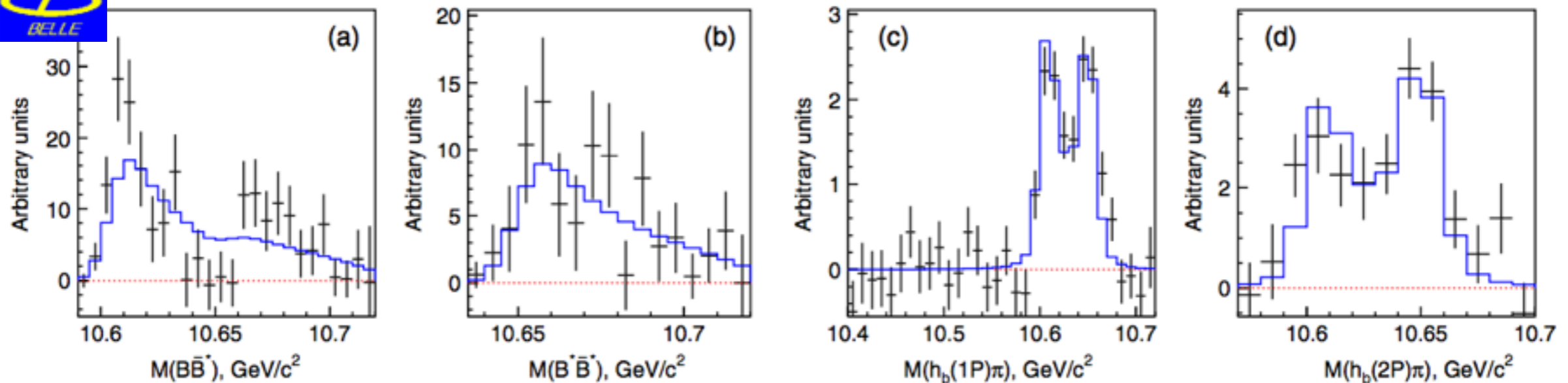
- conventional state search
  - exotica discovery
  - precision  $Z_b$  mass measurement
- 
- $1 \text{ ab}^{-1}$  @ $\Upsilon(5S)$ : also  $B_s$  physics
  - $100 \text{ fb}^{-1}$ @ $\Upsilon(6S)$  +  $\sim 400 \text{ fb}^{-1}$  scan

*Current samples in  $\text{fb}^{-1}$  (millions of events)*

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# Y(5S) runs: $Z_b$ masses (precision study)

- open question: are  $Z_b$  masses below or above  $B^{(*)}\bar{B}^*$  thresholds?
- fundamental question to understand their nature



*Phys. Rev. D 93, 074031 (2016)*

## → Belle II

- 1  $\text{ab}^{-1}$  @ Y(5S): determine if they are located above or below the open threshold

estimate of the  $Z_b$  location with respect to the thresholds:

$$\begin{aligned} \epsilon_B(Z_b) &= (0.60_{-0.49}^{+1.40} \pm i0.02_{-0.01}^{+0.02}) \text{ MeV}, \\ \epsilon_B(Z_b') &= (0.97_{-0.68}^{+1.42} \pm i0.84_{-0.34}^{+0.22}) \text{ MeV}, \end{aligned}$$

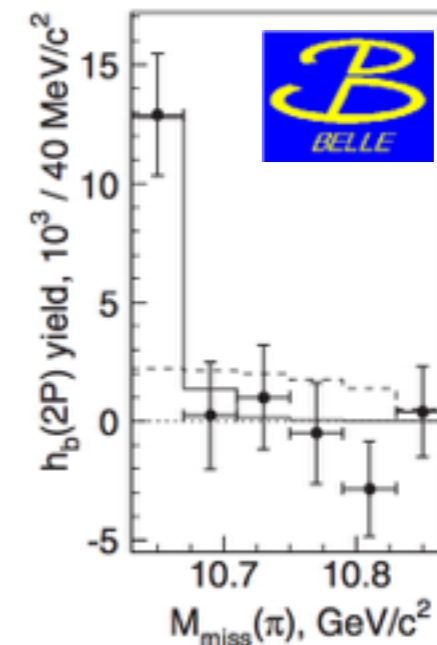
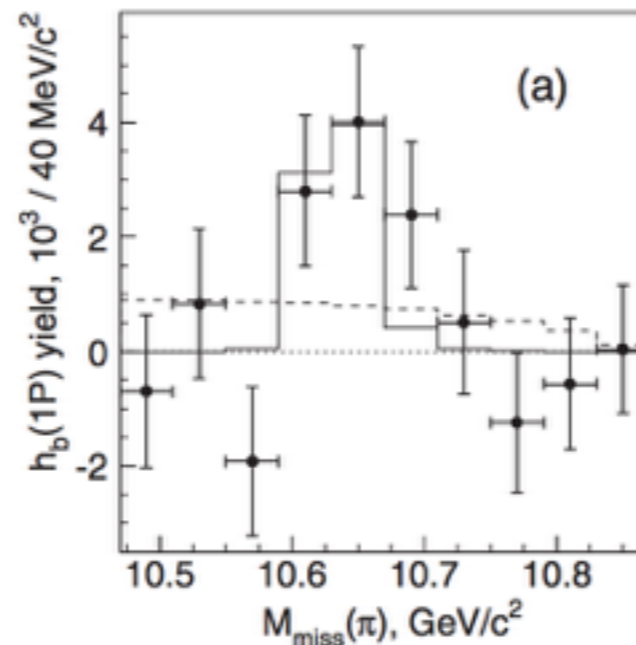
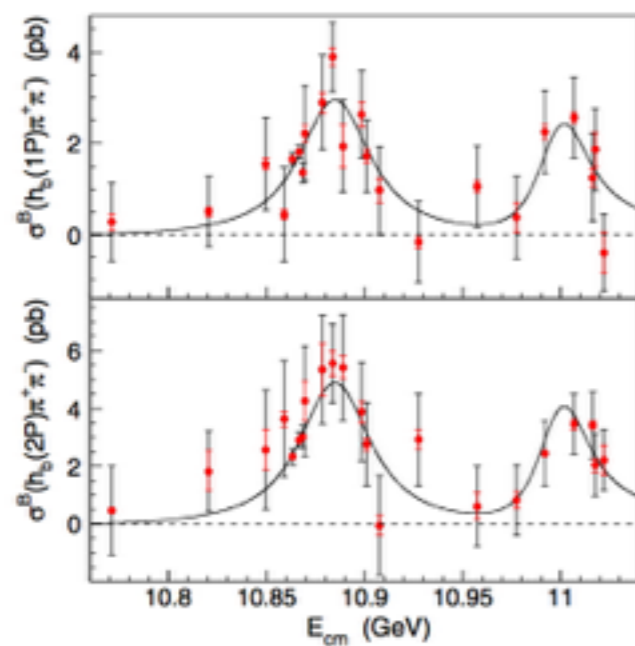
$$\epsilon_B(Z_b) \equiv M(B\bar{B}^*) - M(Z_b),$$

$$\epsilon_B(Z_b') \equiv M(B^*\bar{B}^*) - M(Z_b'),$$

# High energy scans: resolve new states ( $Z_b^\pm$ )

PRL 117, 142001 (2016)

- Belle energy scan, search for  $Y(6S) \rightarrow \pi^+ \pi^- h_b(1P,2P)$  decay
- observation of  $Z_b(106XX)$  state, but unable to resolve them



## → Belle II

- Understand  $Y(6S) \rightarrow Z_b$  decay
- $Y(6S) \rightarrow \pi^+ \pi^- h_b(1P,2P)$
- $Y(6S) \rightarrow \pi^+ \pi^- Y(1S,2S,3S)$

# Bottomonia: motivation for non- $\Upsilon(4S)$ running

## → below $\Upsilon(4S)$ :

- bottomonium studies/searches
  - new physics in decays (DM / light Higgs)
  - anti nucleon production (possible DM application)
  - baryon physics
- 
- 300 fb<sup>-1</sup> @ $\Upsilon(3S)$ : order of magnitude increase

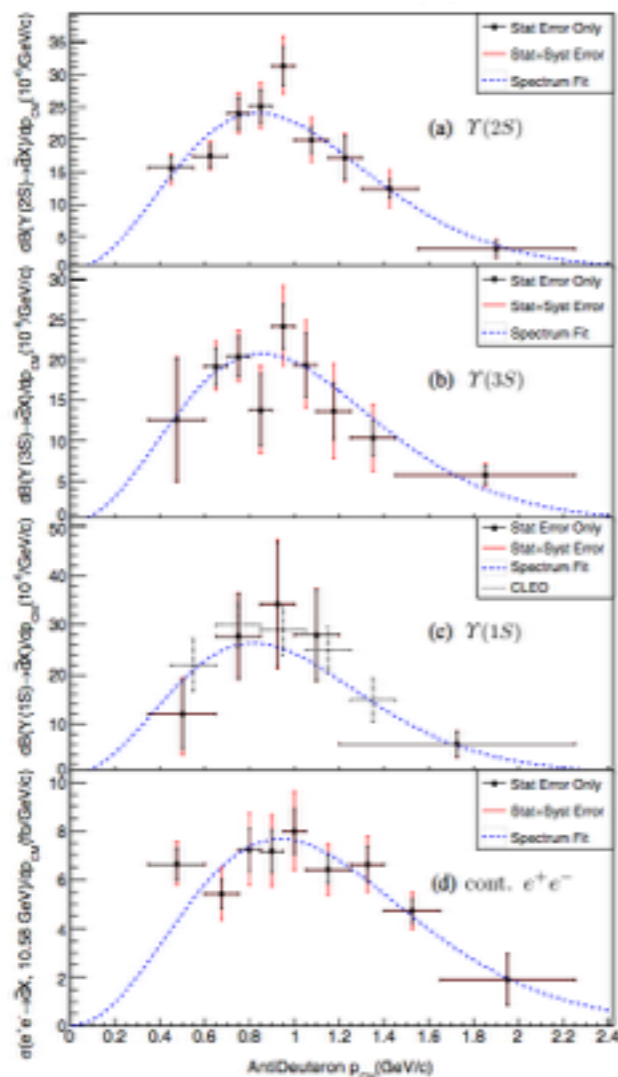
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# $\Upsilon(3S)$ runs: (Anti)deuteron

PRD 62, 043003 (2000)

Phys. Rev. D89 (2014) no.11, 111102



Process	Rate
$B(\Upsilon(3S) \rightarrow \bar{d}X)$	$(2.33 \pm 0.15^{+0.31}_{-0.28}) \times 10^{-5}$
$B(\Upsilon(2S) \rightarrow \bar{d}X)$	$(2.64 \pm 0.11^{+0.26}_{-0.21}) \times 10^{-5}$
$B(\Upsilon(1S) \rightarrow \bar{d}X)$	$(2.81 \pm 0.49^{+0.20}_{-0.24}) \times 10^{-5}$
$\sigma(e^+e^- \rightarrow \bar{d}X) [\sqrt{s} \approx 10.58 \text{ GeV}]$	$(9.63 \pm 0.41^{+1.17}_{-1.01}) \text{ fb}$
$\frac{\sigma(e^+e^- \rightarrow \bar{d}X)}{\sigma(e^+e^- \rightarrow \text{Hadrons})}$	$(3.01 \pm 0.13^{+0.37}_{-0.31}) \times 10^{-6}$

- $\bar{d}$  in cosmic rays have long been considered a probe for supersymmetric relics in the galactic halo
  - $\bar{d}$  production described with coalescence models tuned on HEP data
  - need to further constrain in the production model
- ➔ CLEO and Babar measured the  $\bar{d}$  spectrum (no dedicated PID or tracking)
- ➔ **Belle II:**
- dedicated tracking and PID
  - collect  $\sim 3 \times 10^4$   $\bar{d}$  in  $300 \text{ fb}^{-1}$
  - world's best estimate of coalescence parameter
  - search for excited nucleons ( $d^*$ )
  - $d\bar{d}$  associated production

# $Y(3S)$ runs: $\Lambda$ - $\Lambda$ interaction

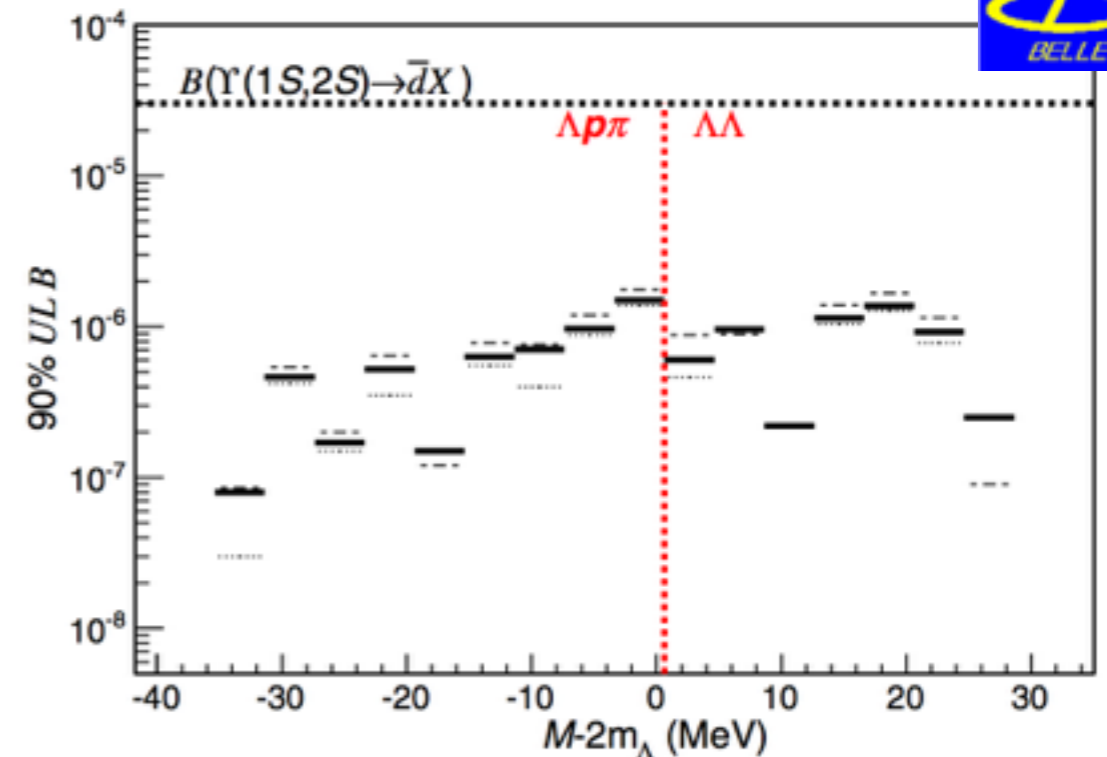
➔ From Belle:

- No sign of weakly bound H dibaryon
- Near threshold enhancement in exclusive annihilations  
 $Y(1S,2S) \rightarrow \Lambda \bar{\Lambda} X$  (still not published)

## ➔ Belle II

- search for H dibaryon in missing mass ( $Y(3S) \rightarrow \Lambda \bar{\Lambda} H + \text{hadrons}$ )
- high statistics study near threshold

*Phys. Rev. Lett. 110, 222002*



Rough extrapolation to  $300 \text{ fb}^{-1}$   $Y(3S)$   
~60 Million events with one  $\Lambda$  or  $\bar{\Lambda}$   
~3 Million events with one  $\Lambda \bar{\Lambda}$  pair

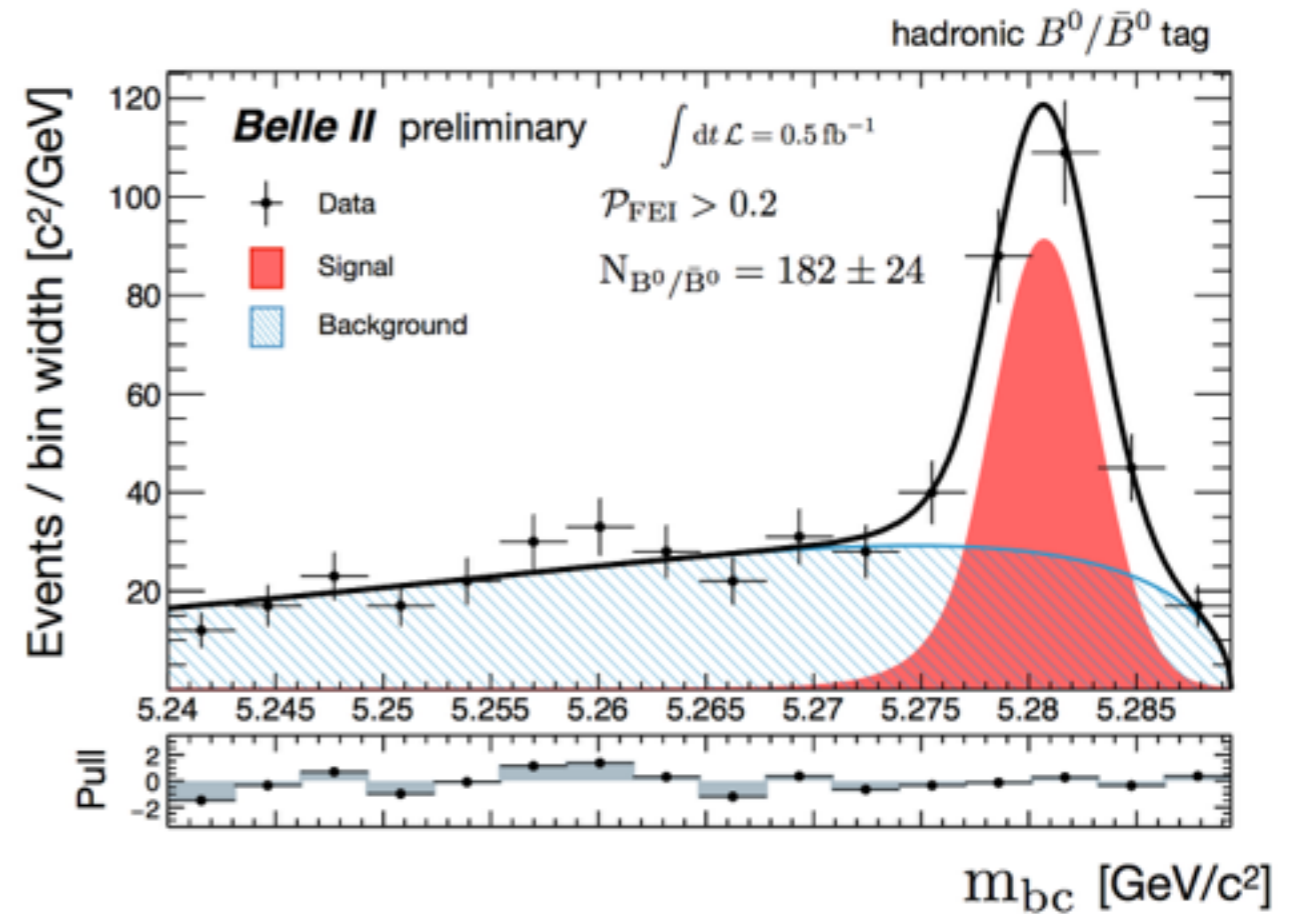
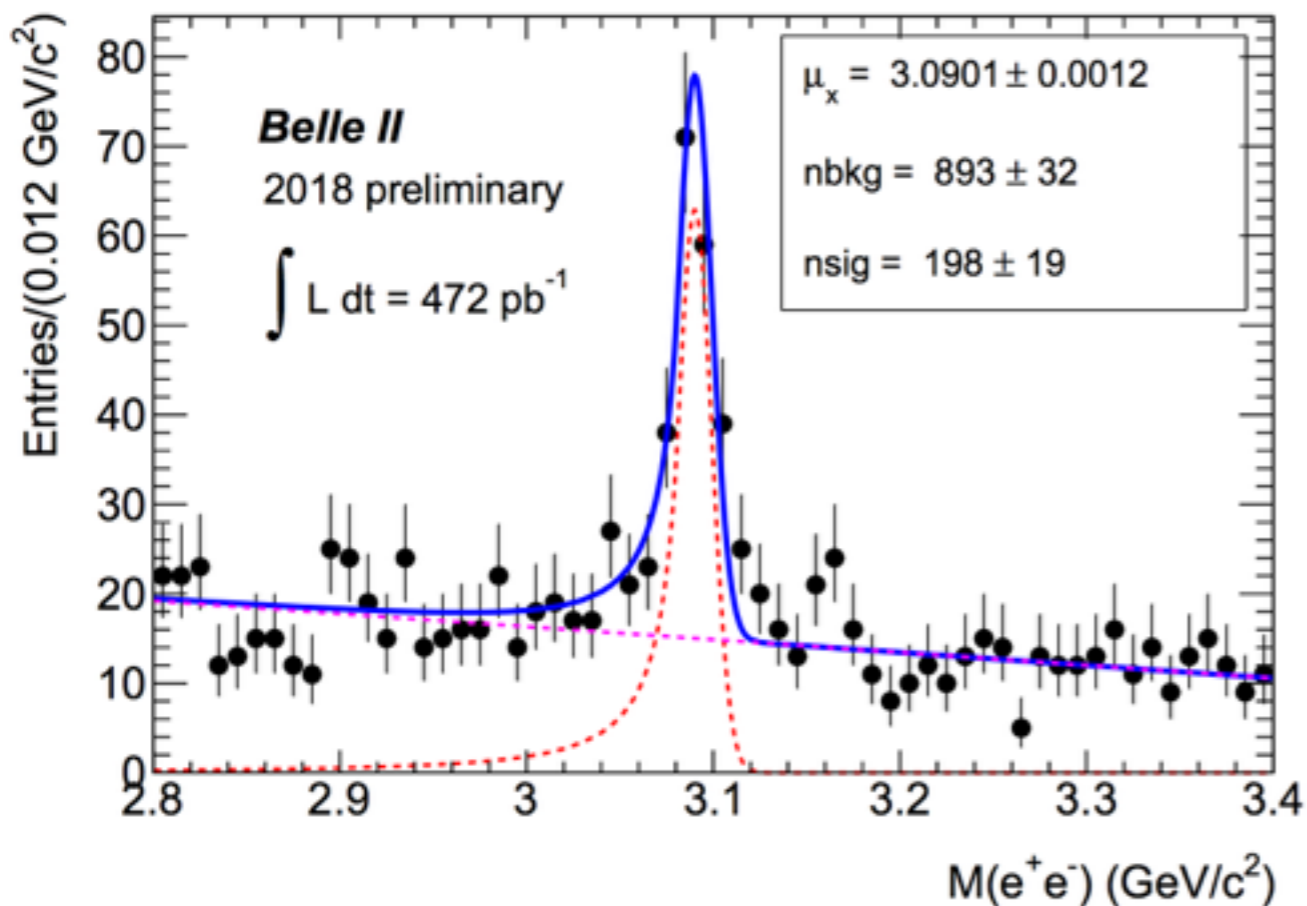
# Summary

- We have entered the post B-factory era:
  - Variety of states in the quarkonium spectroscopy
  - Exotic states
- Belle II will collect 50 times more statistics than Belle, mainly but not only @ the  $Y(4S)$  resonance
- Belle II ended a successful Phase 2 commissioning run and just entered in the physics run era
- A variety of quarkonium studies will be possible, and among them some unique topics:
  - charmonium: double charmonium
  - bottomonium: Belle II is in a unique position to address the open topics in this field



For further details about Belle II physics prospects  
see the Belle II Physics Book

arXiv:1808.10567 [hep-ex]



BACKUP

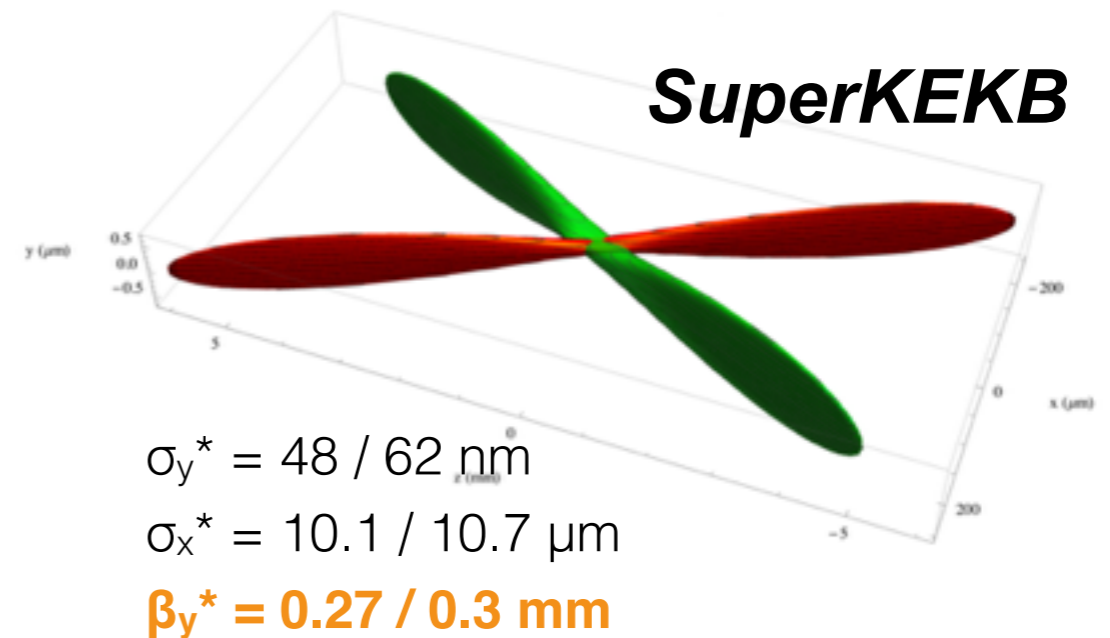
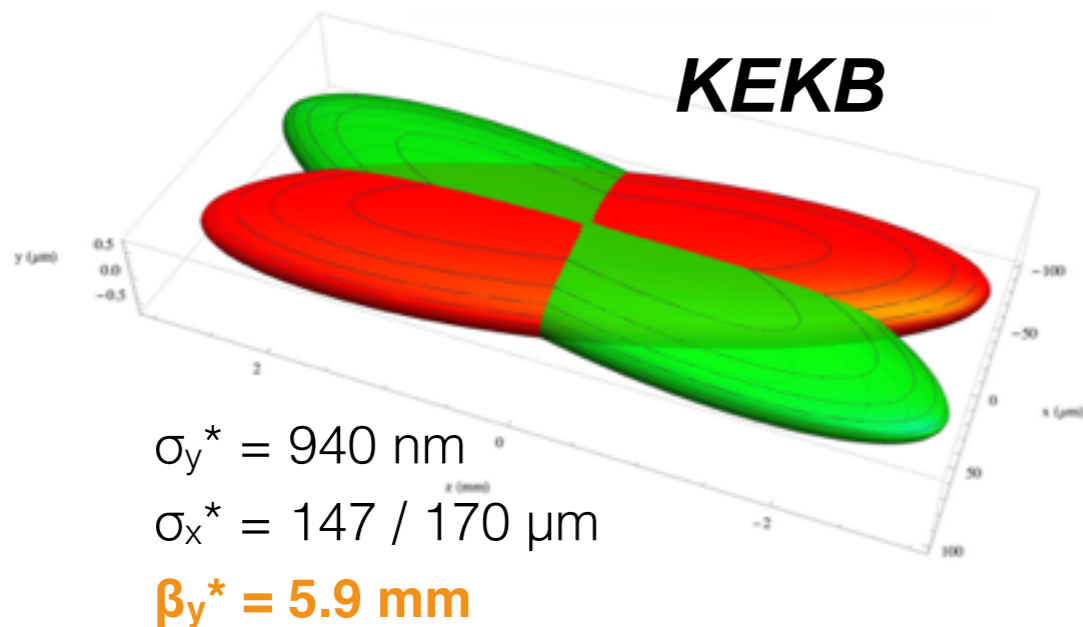
# KEKB VS SuperKEKB

parameters		KEKB		SuperKEKB		units
		LER	HER	LER	HER	
beam energy	$E_b$	3.5	8	4	7	GeV
CM boost	$\beta\gamma$	0.425		0.28		
half crossing angle	$\varphi$	11		41.5		mrad
horizontal emittance	$\epsilon_x$	18	24	3.2	4.6	nm
emittance ratio	$\kappa$	0.88	0.66	0.37	0.40	%
beta-function at IP	$\beta_x^*/\beta_y^*$	1200/5.9		32/0.27	25/0.30	mm
beam currents	$I_b$	1.64	1.19	3.6	2.6	A
beam-beam parameter	$\xi_y$	129	90	0.0881	0.0807	
beam size at IP	$\sigma_x^*/\sigma_y^*$	100/2		10/0.059		$\mu\text{m}$
Luminosity	$\mathcal{L}$	$2.1 \times 10^{34}$		$8 \times 10^{35}$		$\text{cm}^{-2}\text{s}^{-1}$

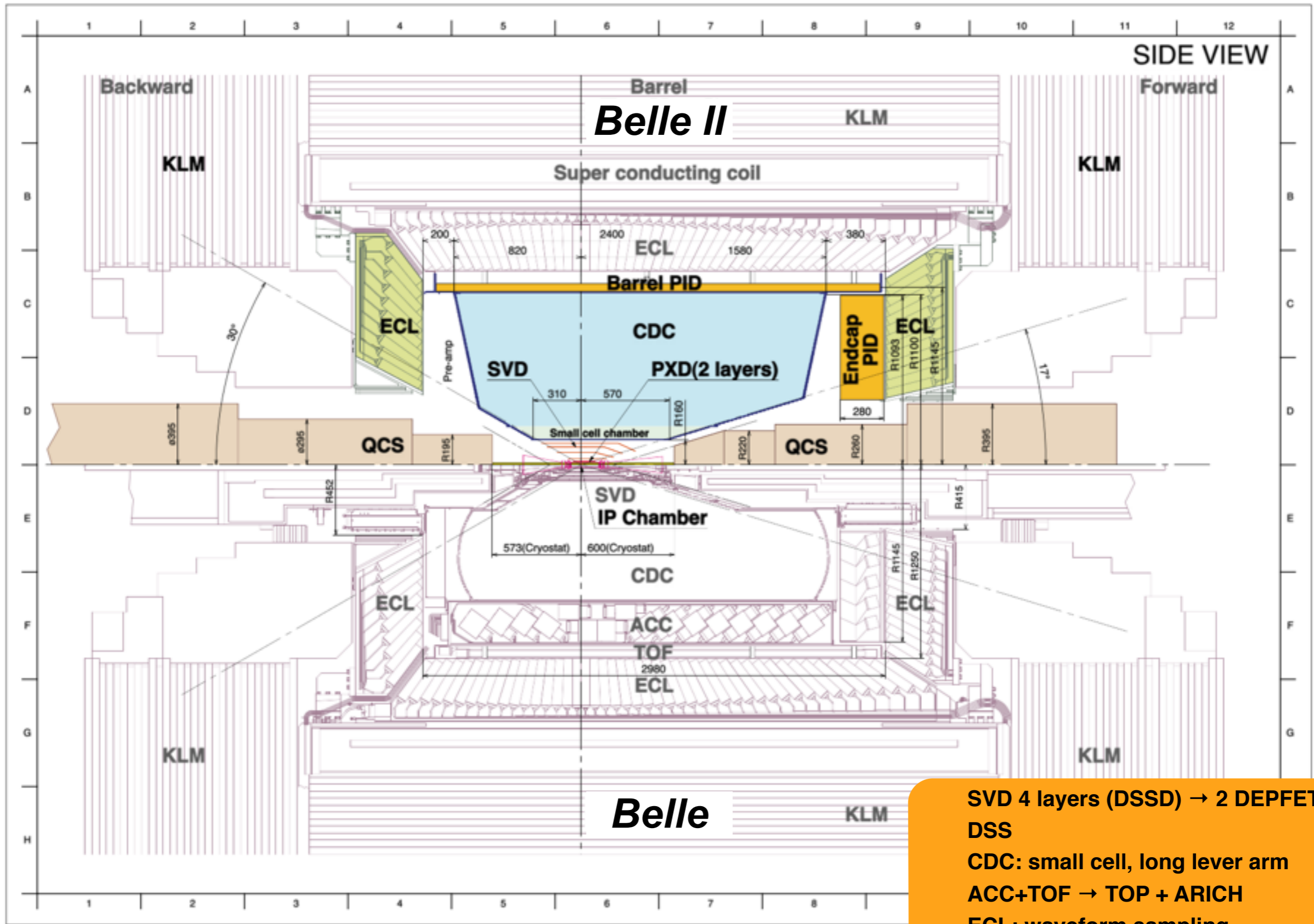
# Nano beam scheme

$$L = \frac{\gamma_{e\pm}}{2er_e} \left( 1 + \frac{\sigma_y^*}{\sigma_x^*} \right) \left( \frac{I_{e\pm} \cdot \xi_{y,e\pm}}{\beta_y^*} \right) \left( \frac{R_L}{R_{\xi_y}} \right)$$

Beam aspect ratio  $\rightarrow$   $\left( 1 + \frac{\sigma_y^*}{\sigma_x^*} \right)$   
 Beam current  $\rightarrow$   $I_{e\pm}$   
 Beam-beam parameter  $\rightarrow$   $\xi_{y,e\pm}$   
 Vertical beta function @ IP  $\rightarrow$   $\beta_y^*$

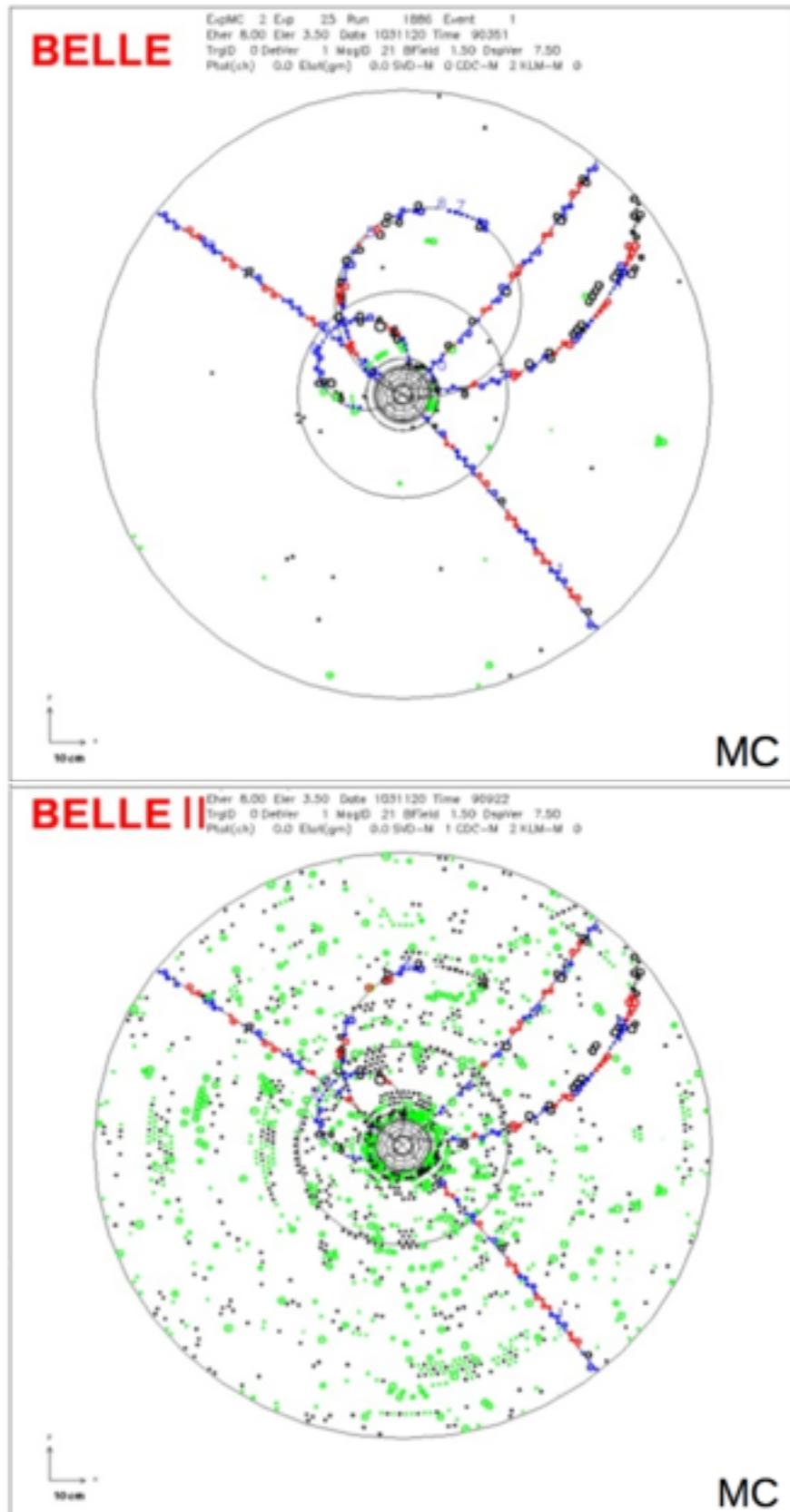


# From Belle to Belle II



**SVD 4 layers (DSSD) → 2 DEPFET + 4 DSS**  
**CDC: small cell, long lever arm**  
**ACC+TOF → TOP + ARICH**  
**ECL: waveform sampling**  
**KLM: RPC → Scintillator+SiPM**

# New challenges

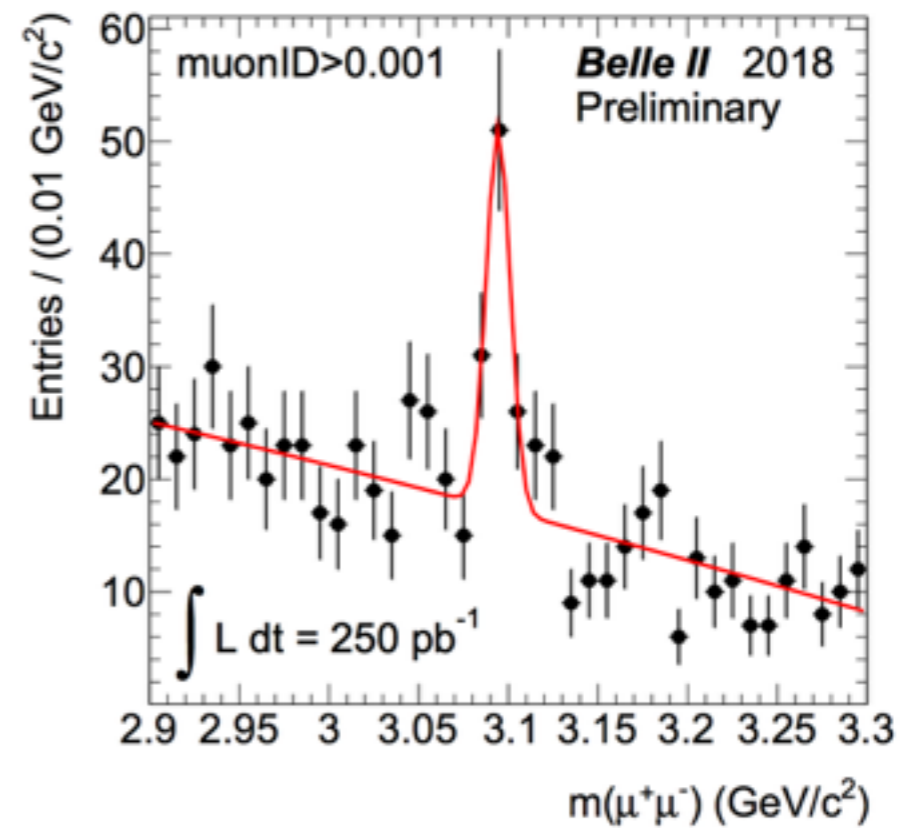
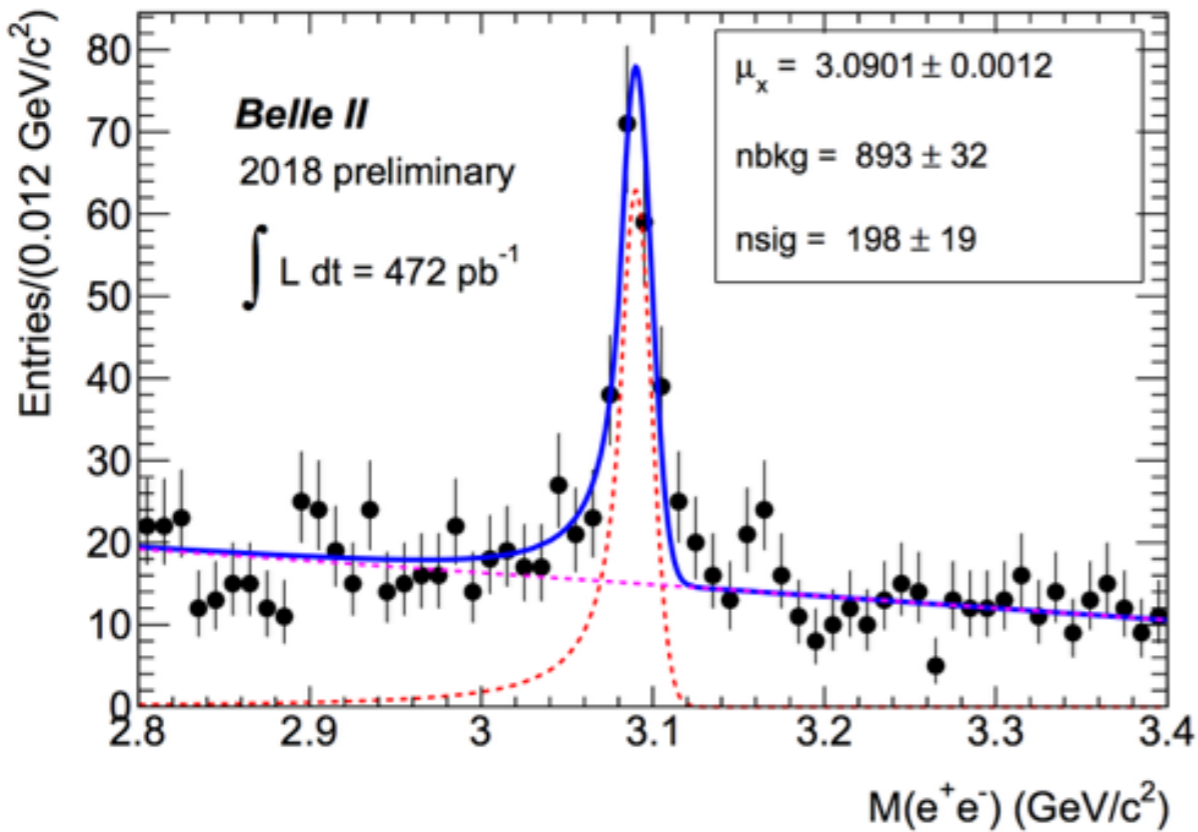
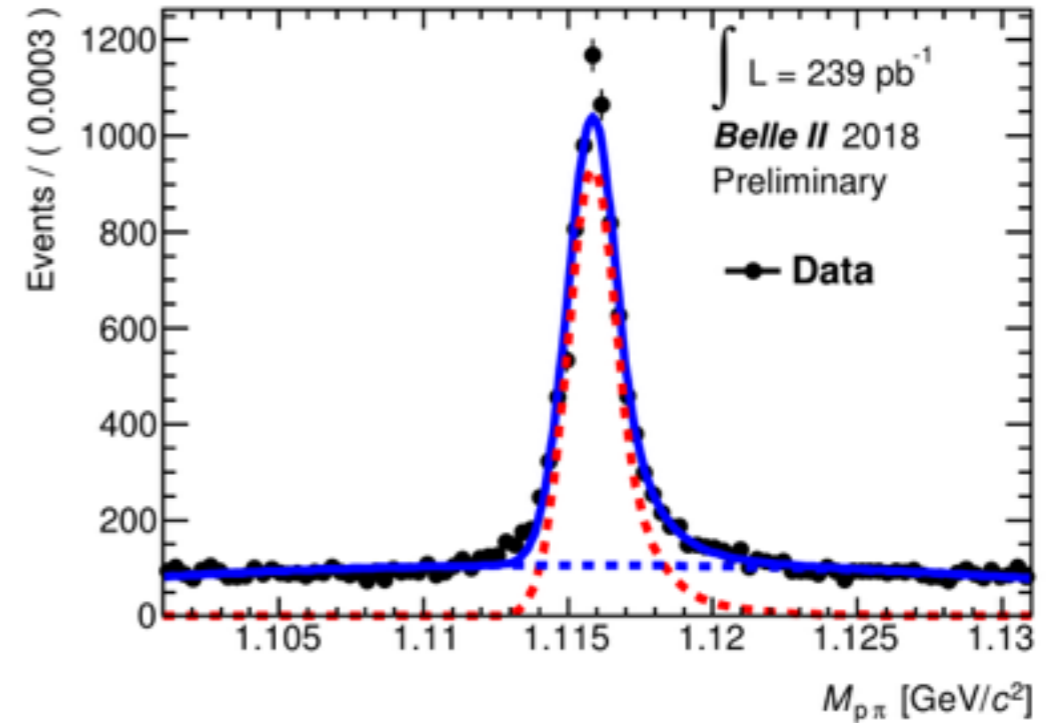
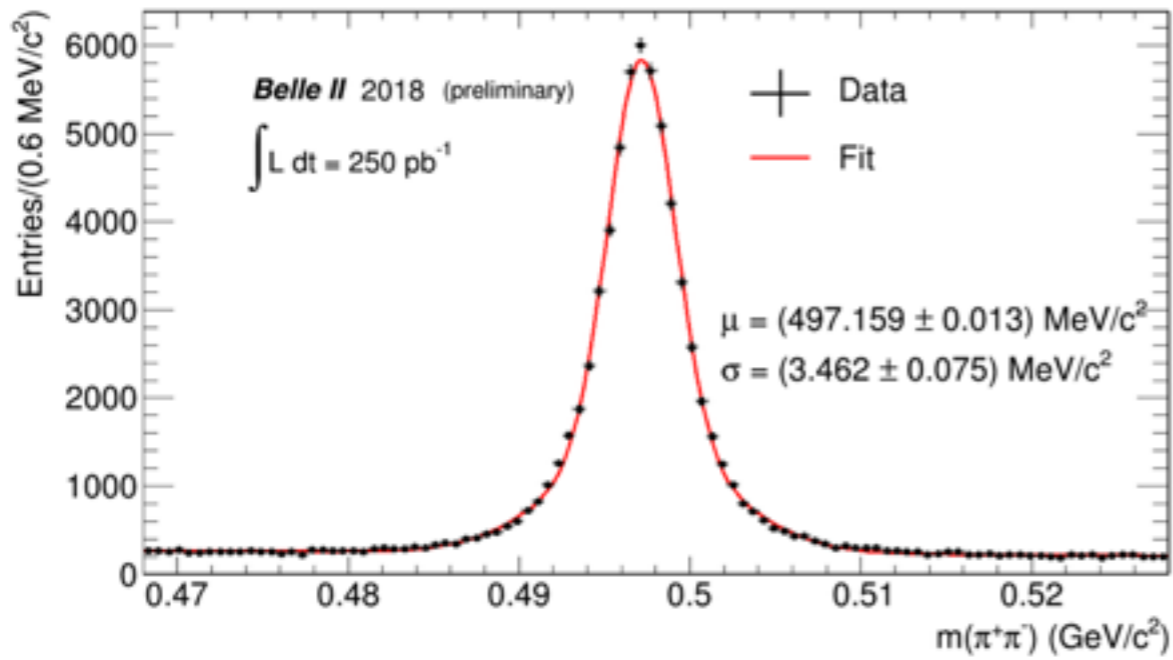


➔ x40 luminosity:

- x40 produced signal events
- Higher background (detector occupancy, fake hits, radiation damage)
- Higher event rate (trigger rate, DAQ, computing)

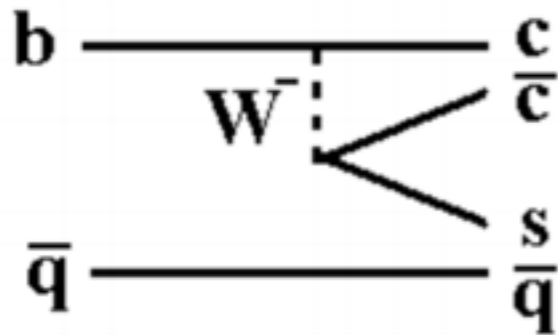
➔ Important to have a dedicated phase for background studies, detector response and alignment

# Tracking

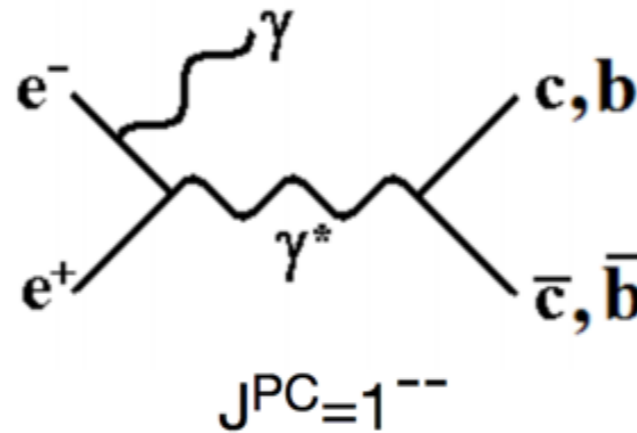


# Quarkonium production at B-factory

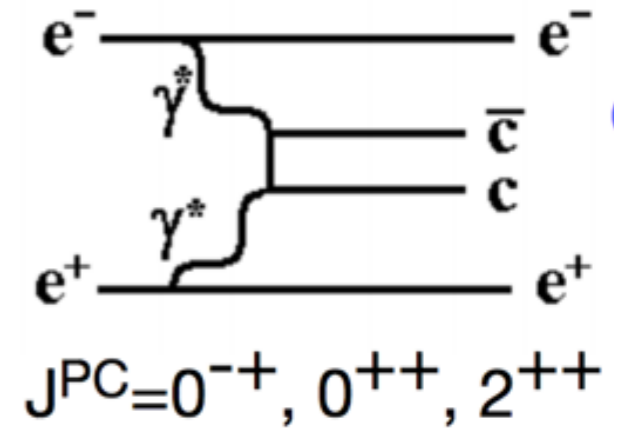
B decays



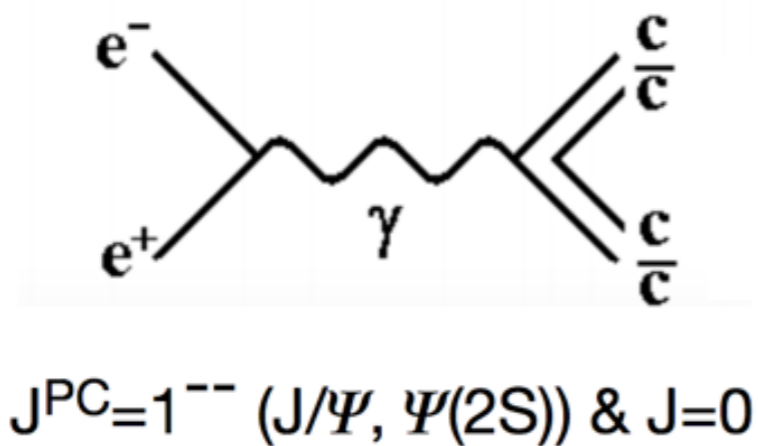
Initial State Radiation(ISR)



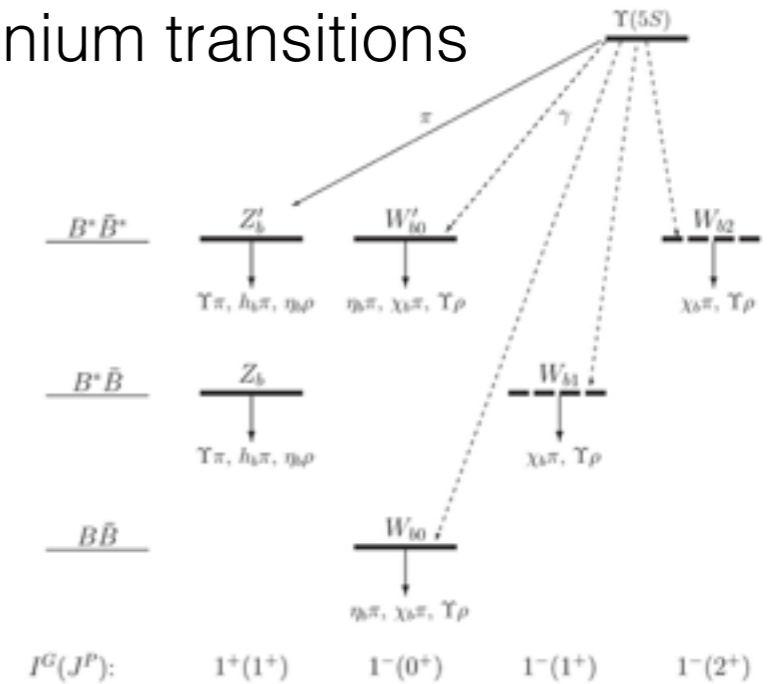
Two  $\gamma$  interaction



Double charmonium production

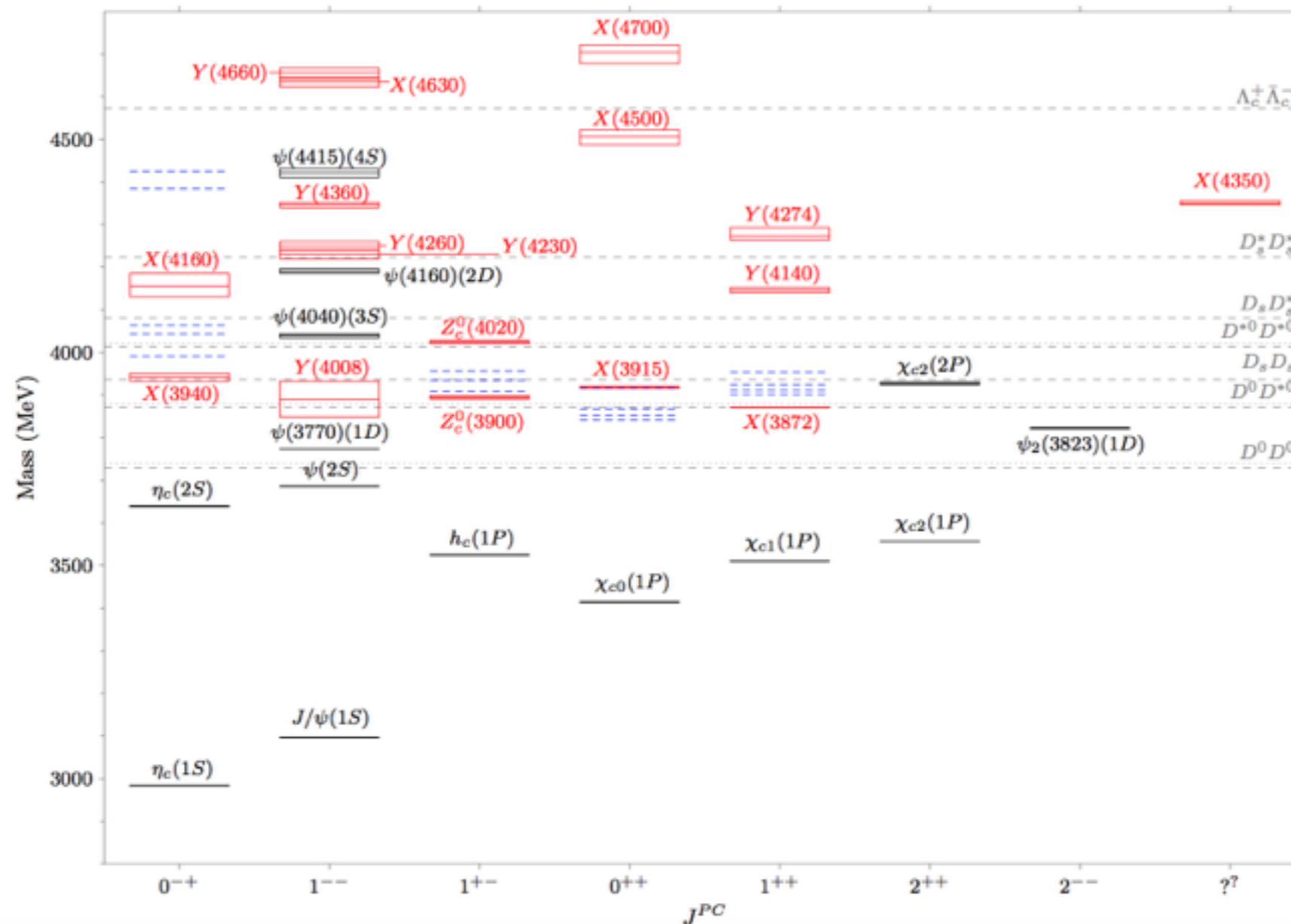


Quarkonium transitions





# Charmonia: overview



- Competition from LHCb (B decays) and BESIII (scans for  $1^{- -}$  states)
- Exploit different production methods

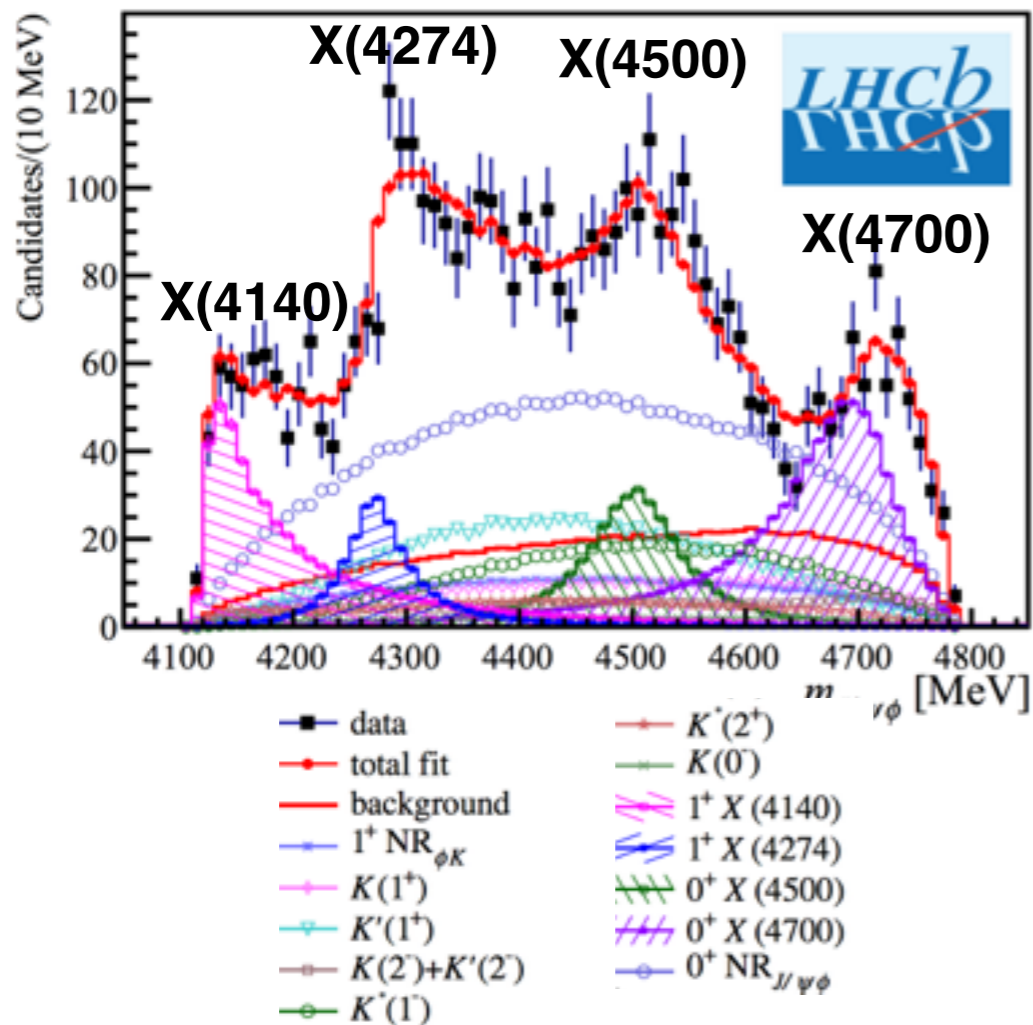
# Charmonia: B decay

- Competition from LHCb

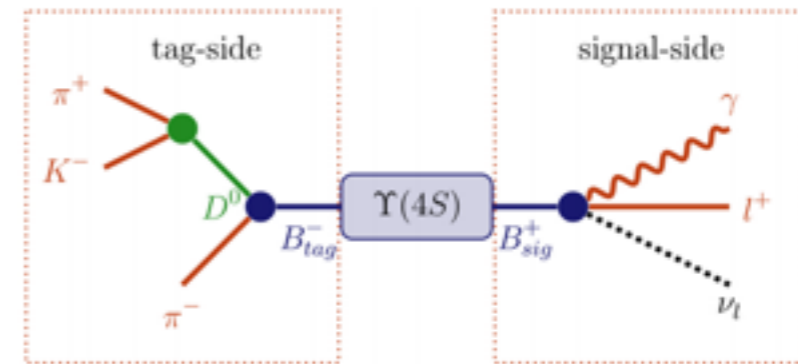
*Phys. Rev. Lett. 118, 022003 (2017)*

*Phys. Rev. D 95, 012002 (2017)*

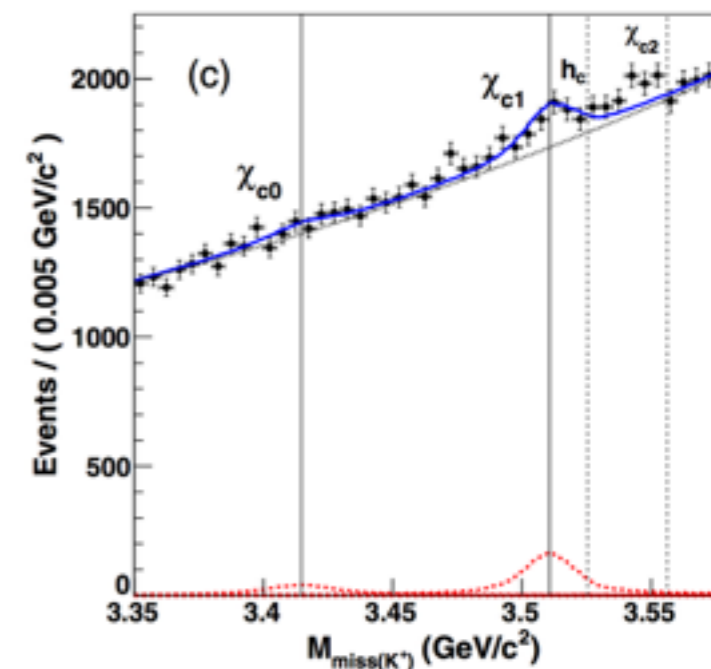
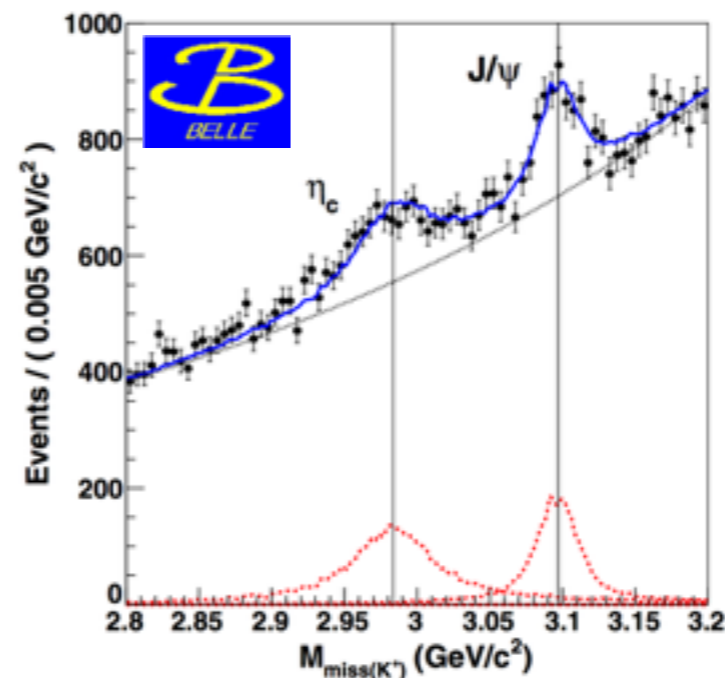
LHCb amplitude analysis of  $B \rightarrow J/\psi \phi K$



- $e^+e^-$  B-factories only: → **Belle II**



*PRD 97, 012005 (2018)*



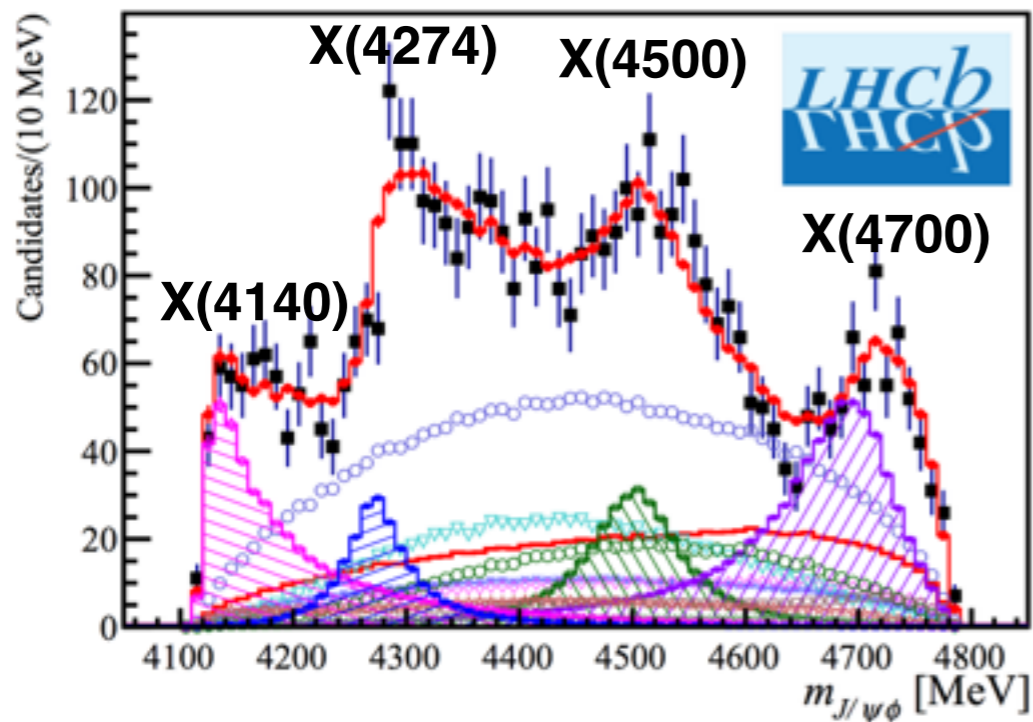
# Charmonia: B decay

- Competition from LHCb

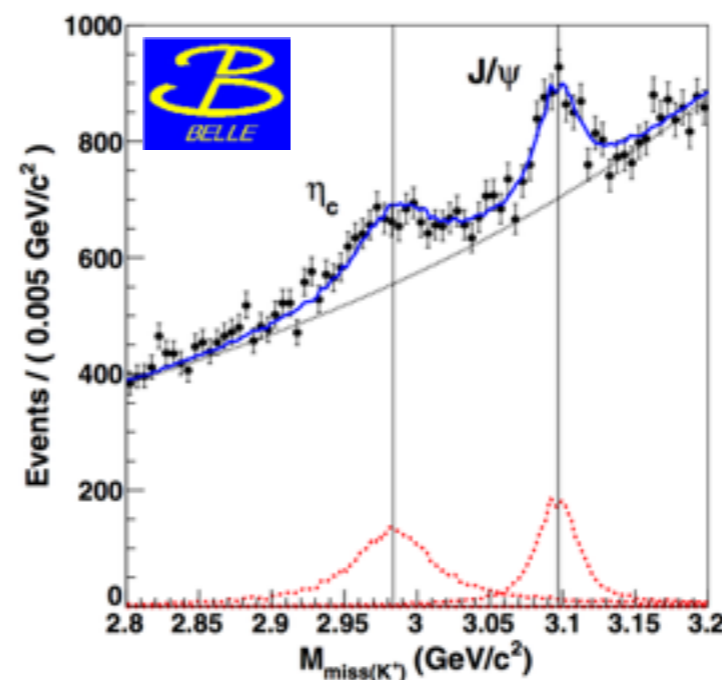
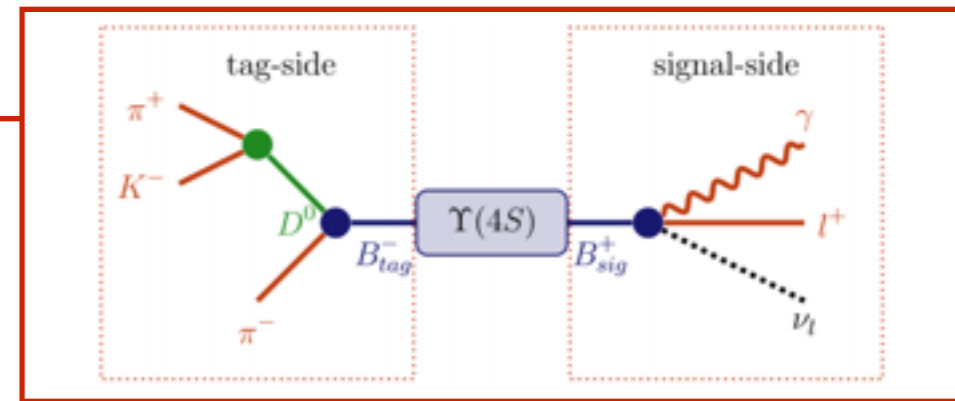
*Phys. Rev. Lett. 118, 022003 (2017)*

*Phys. Rev. D 95, 012002 (2017)*

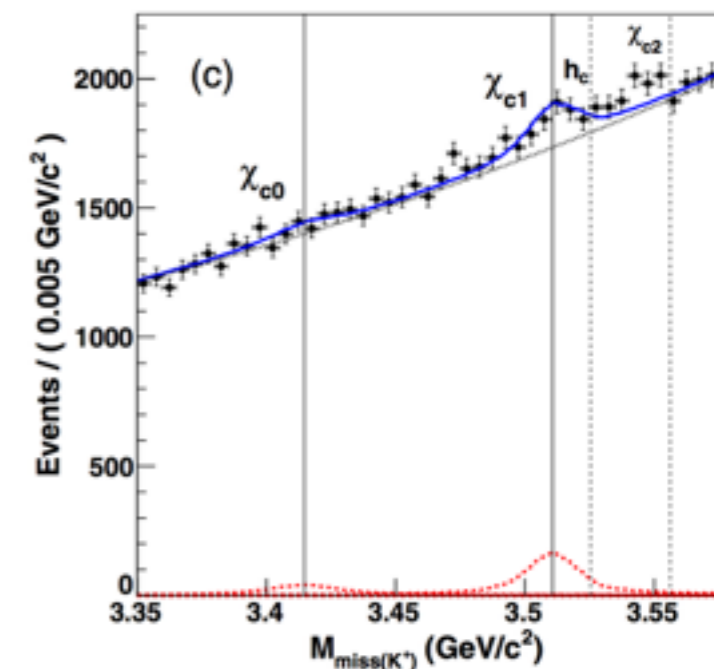
LHCb amplitude analysis of  $B \rightarrow J/\psi \phi K$



- $e^+e^-$  B-factories only: **→ Belle II**



*PRD 97, 012005 (2018)*



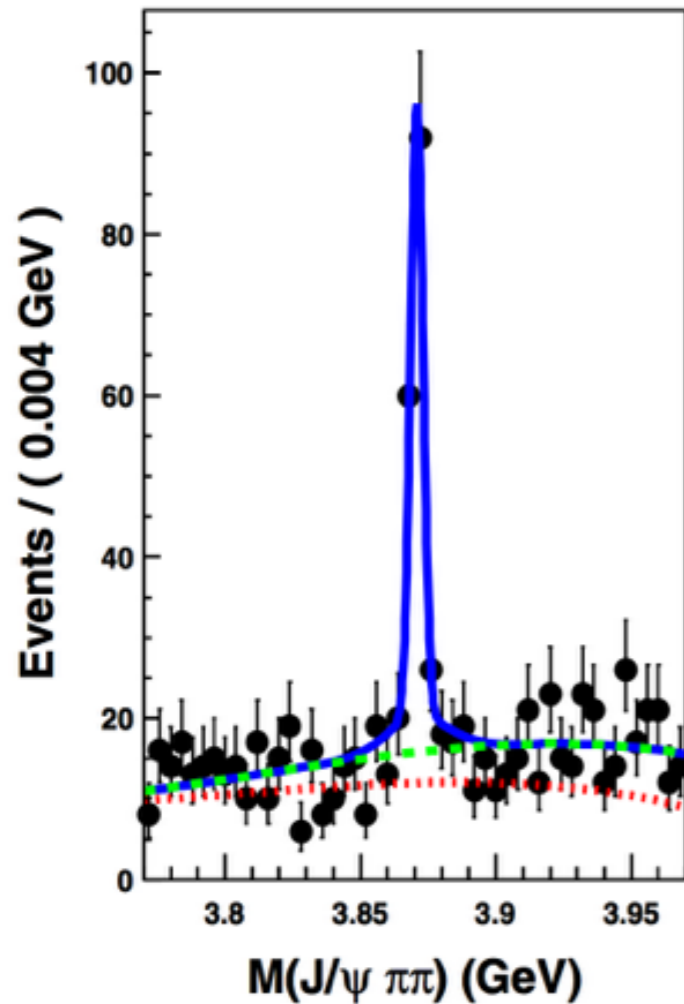
Competitive with LHCb exclusive reconstruction only for:

- hadronic transitions with  $\pi^0, \eta, \omega$  in final state
- states decaying with large multiplicities

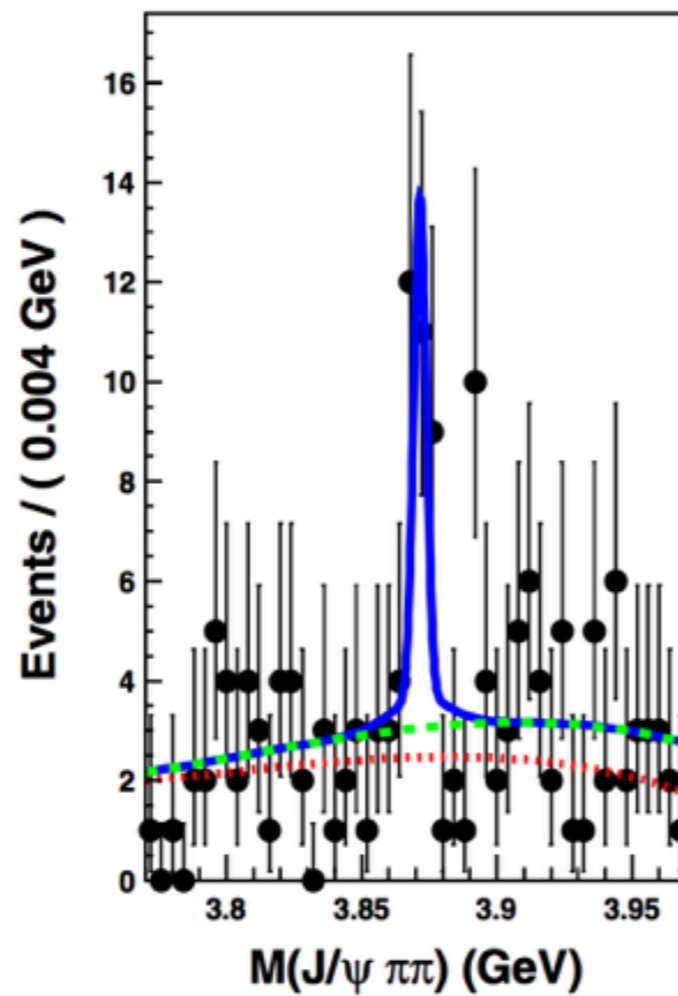
# Charmonia: $X(3872)$ width

- The current upper limit on the width of the  $X(3872)$  width is the 90% C.L. of  $\Gamma_{X(3872)} < 1.2$  MeV, obtained using the mode  $B \rightarrow J/\psi \pi^+ \pi^- K$

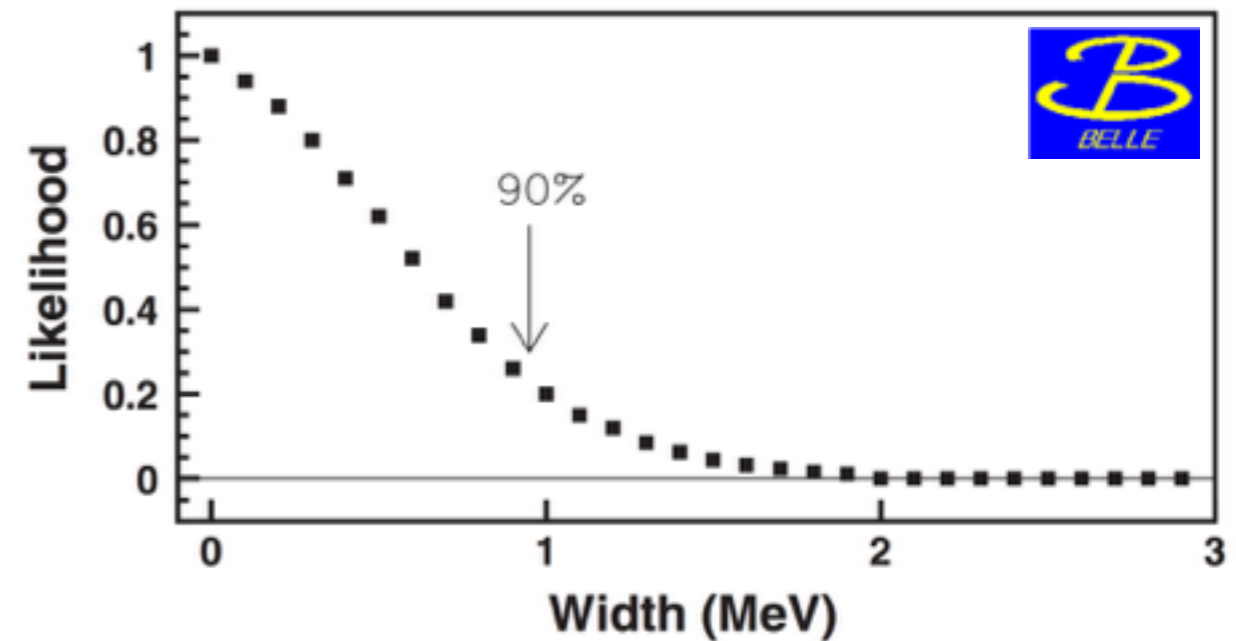
$B^+ \rightarrow K^+ X(3872)$



$B^0 \rightarrow K_S X(3872)$



*Belle, PRD 84, 052004 (2011)*



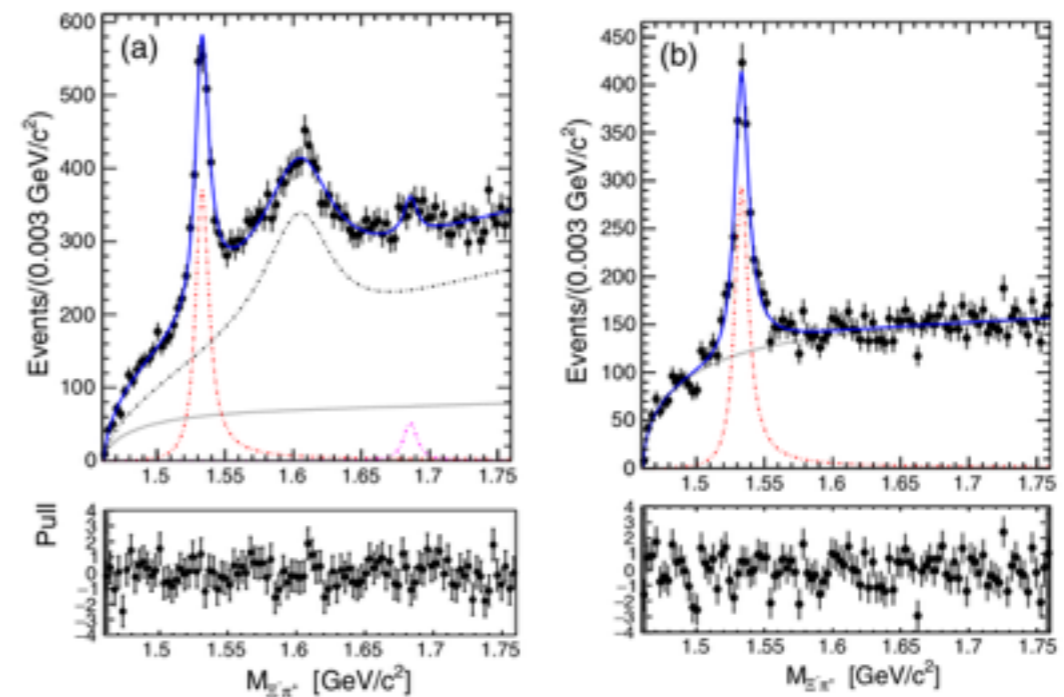
# Prospects in baryon spectroscopy

- Baryon spectroscopy is in general more complicated than quarkonia but exotic candidates exist even in the first excited states
- Excited spectrum is not well understood
- Belle still actively publishing

*Belle, PRL 9122, 072501 (2019)*

## → Belle II

- measure quantum numbers for excited charmed baryons
- search for excited baryons in charmed baryon decays
- search for exotic candidate states



# Coalescence model for anti-deuteron production

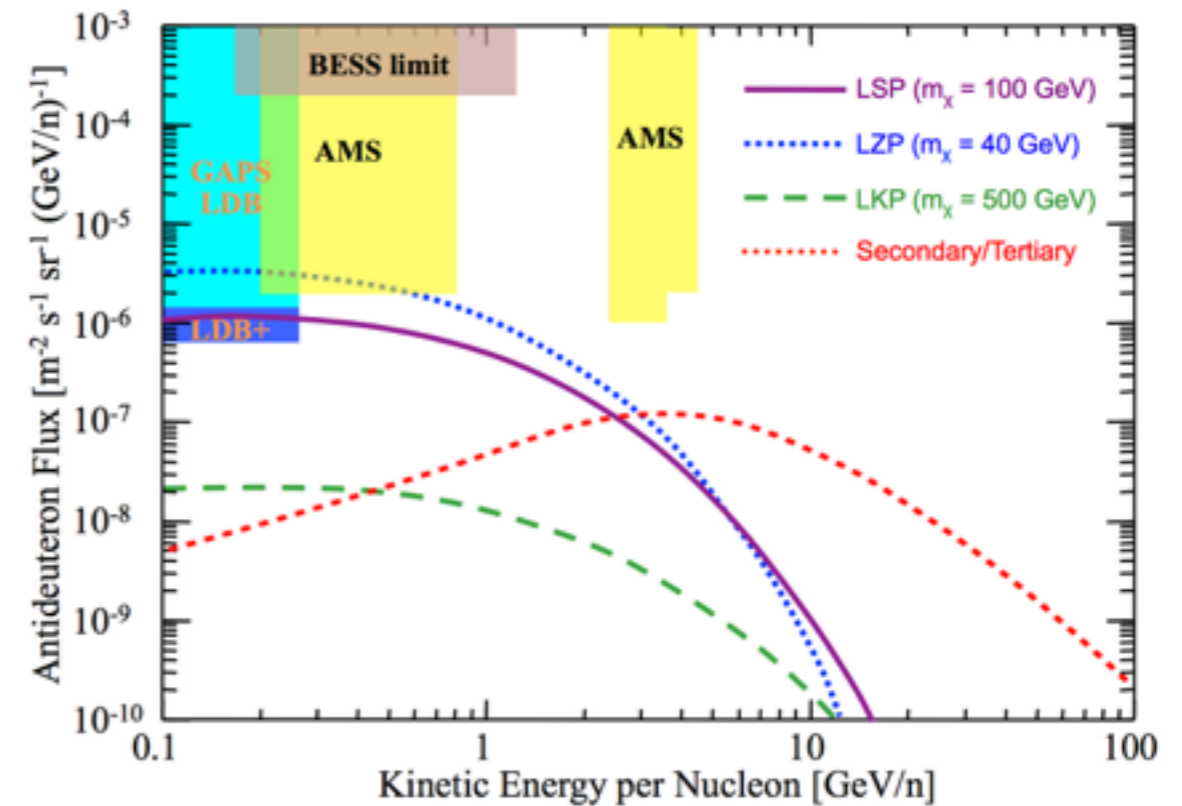
Anti-deuteron production is described by p-n coalescence models tuned on the HEP data

Most recent data are from Alice

- Large final state
- MC-driven correction

Donato, Fornengo, Salati, PRD 62, 043003 (2000)

Aramaki et al. Phys. Rept. 618 (2016) 137



$$\frac{dN_{\bar{d}}}{dT_{\bar{d}}} = \frac{p_0^3}{6k_{\bar{d}}} \frac{m_{\bar{d}}}{m_{\bar{p}}m_{\bar{n}}} \left. \frac{dN}{dT_{\bar{p}}} \right|^{**} \left. \frac{dN}{dT_{\bar{n}}} \right|^{**}$$

where  $T_i = E_i - m_i$  is the kinetic energy of  $i = \bar{d}, \bar{p}, \bar{n}$  and the  $|^{**}$  notation recalls that the  $\bar{p}$  and  $\bar{n}$  spectra must be evaluated at  $T_{\bar{p}} = T_{\bar{d}}/2$  and  $T_{\bar{n}} = T_{\bar{d}}/2$ , respectively (as dictated by Eq. (2.14)). In deriving Eq. (2.19), we have clearly assumed  $m_{\bar{p}} = m_{\bar{n}} = m_{\bar{d}}/2$ .