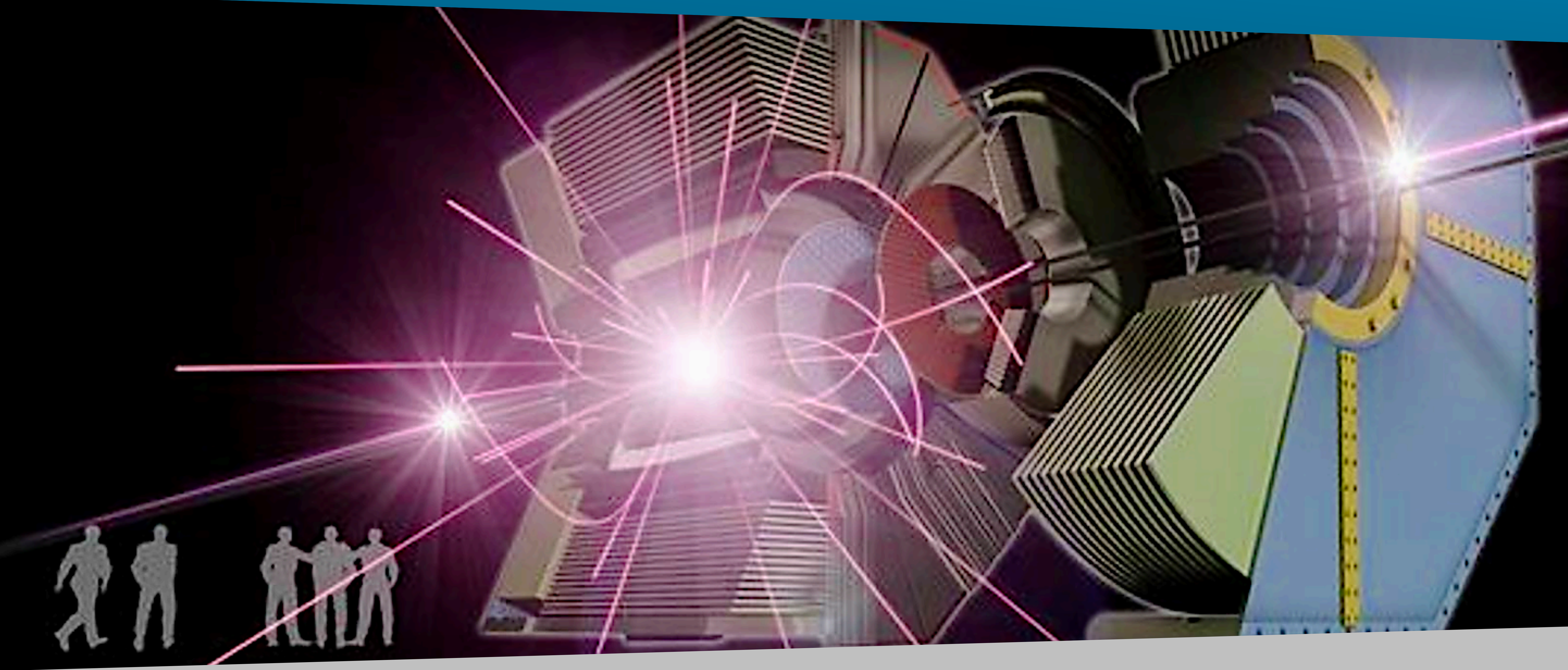
Inclusive $B \to X_u \ell \nu$ measurements at Belle (II)



Challenges in Semileptonic B Decays

Lu Cao

for the Belle & Belle II Collaborations

26 September 2024, Vienna

Inclusive $B \to X_u \ell \nu$ Studies

BELLE

Full data: 711fb^{-1}

Belle II

Run1 data: 364fb^{-1}, **ongoing**

- $\Delta\mathscr{B}(B \to X_u \ell \nu)$ and |V$_{ub}$| [PRD 104 , 012008 (2021)]
- d$\mathscr{B}$/d$x, x = M_X, M_X^2, q^2, P^{\pm}, E_l^B$ [PRL 127, 261801 (2021)]
- Simultaneous determination of excl. & incl. |V$_{ub}$| [PRL 131, 211801 (2023)]
- $\Delta\mathscr{B}$ ratio and incl. |V$_{ub}$|/|V$_{cb}$| [arXiv: 2311.00458]

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- $\Delta\mathcal{B}(B \to X_u \ell \nu)$ and |V$_{ub}$| [PRD 104 , 012008 (2021)]

presented in the last workshop@Barolo

- d$\mathcal{B}$/dx, $x = M_X, M_X^2, q^2, P^{\pm}, E_l^B$ [PRL 127, 261801 (2021)]
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Inclusive $B \to X_u \ell \nu$ Studies

BELLE

Full data: 711fb^{-1}

- $\Delta\mathcal{B}(B \to X_u \ell \nu)$ and $|V_{ub}|$ [PRD 104 , 012008 (2021)]
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- Simultaneous determination of excl. & incl. $|V_{ub}|$ [PRL 131, 211801 (2023)]
- $\Delta\mathcal{B}$ ratio and incl. $|V_{ub}|/|V_{cb}|$ [arXiv: 2311.00458]

Belle II

Run1 data: 364fb^{-1}, **ongoing**

- $\Delta\mathcal{B}(B \to X_u \ell \nu)$ and $|V_{ub}|$
- $\mathrm{d}\mathcal{B}/\mathrm{d}x, x = x_{\mathrm{Belle}} + \cos\theta_\ell$
- Weak annihilation in $B \to X_u \ell \nu$

Inclusive $B \to X_u \ell \nu$ Studies

BELLE

Full data: 711fb^{-1}

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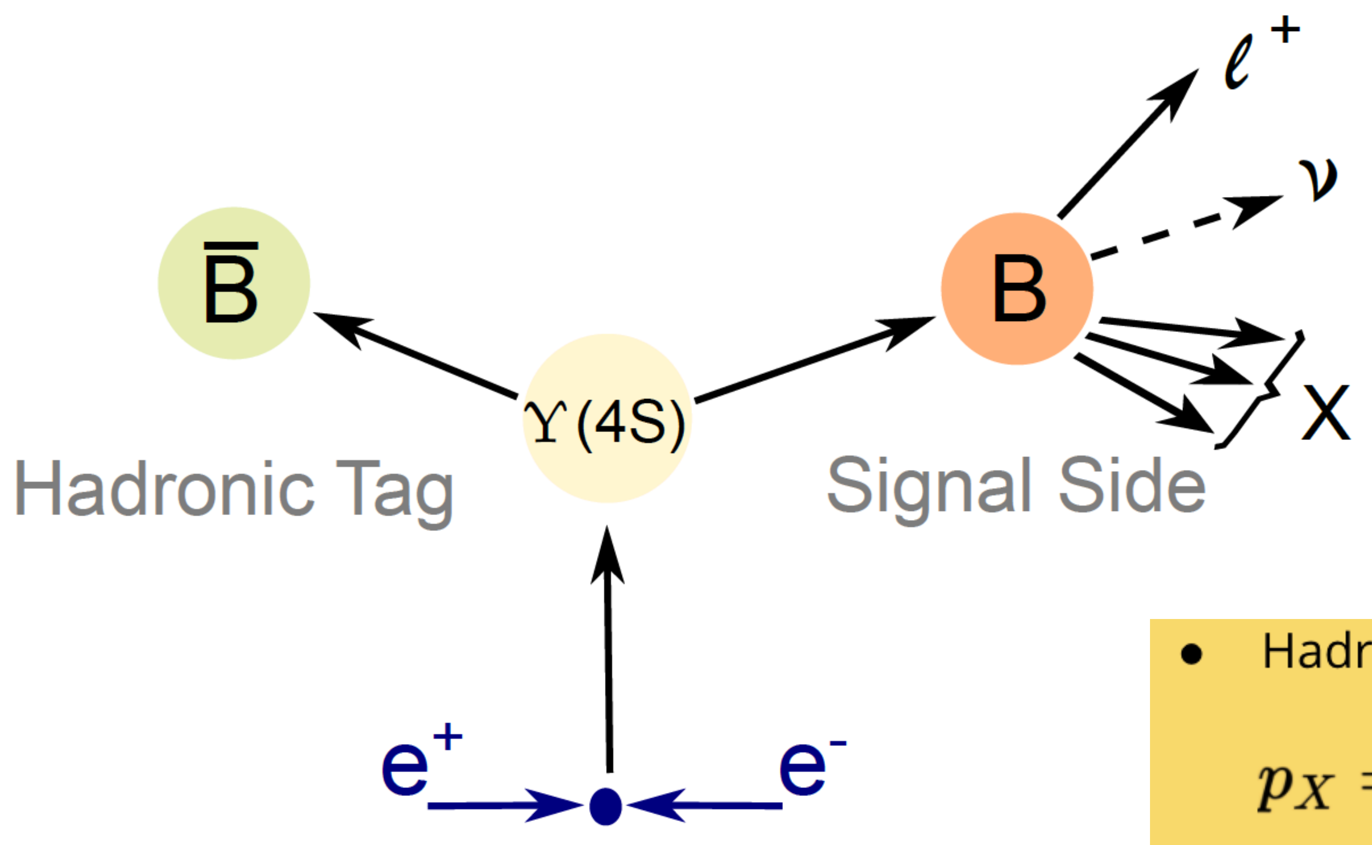
- Weak annihilation in $B \to X_u \ell \nu$

Analysis team:

Tommy Martinov, Marcel Hohmann, Munira Khan, Merle Graf-Schreiber, Martin Angelsmark, LC, Florian Bernlochner, Kerstin Tackmann

$\Delta\mathcal{B}(B \to X_u \ell \nu)$ and $|V_{ub}|$

Reconstruct Inclusive Semileptonic B Decays with Fully Known B_{tag}



Can fully assign each final state particle to either the tag or signal side

Allows to reconstruct inclusive *X*

- Hadronic system *X*:

$$p_X = \sum_i \left(\sqrt{m_\pi^2 + |\mathbf{p_i}|^2}, \mathbf{p_i}\right) + \sum_i (E_i, \mathbf{k_i})$$

- Missing mass squared:

$$MM^2 = (P_{\Upsilon(4S)} - P_{\text{tag}} - P_X - P_\ell)^2$$

- Leptonic system:

$$q^2 = (P_B - P_X)^2 = (P_l + P_\nu)^2$$

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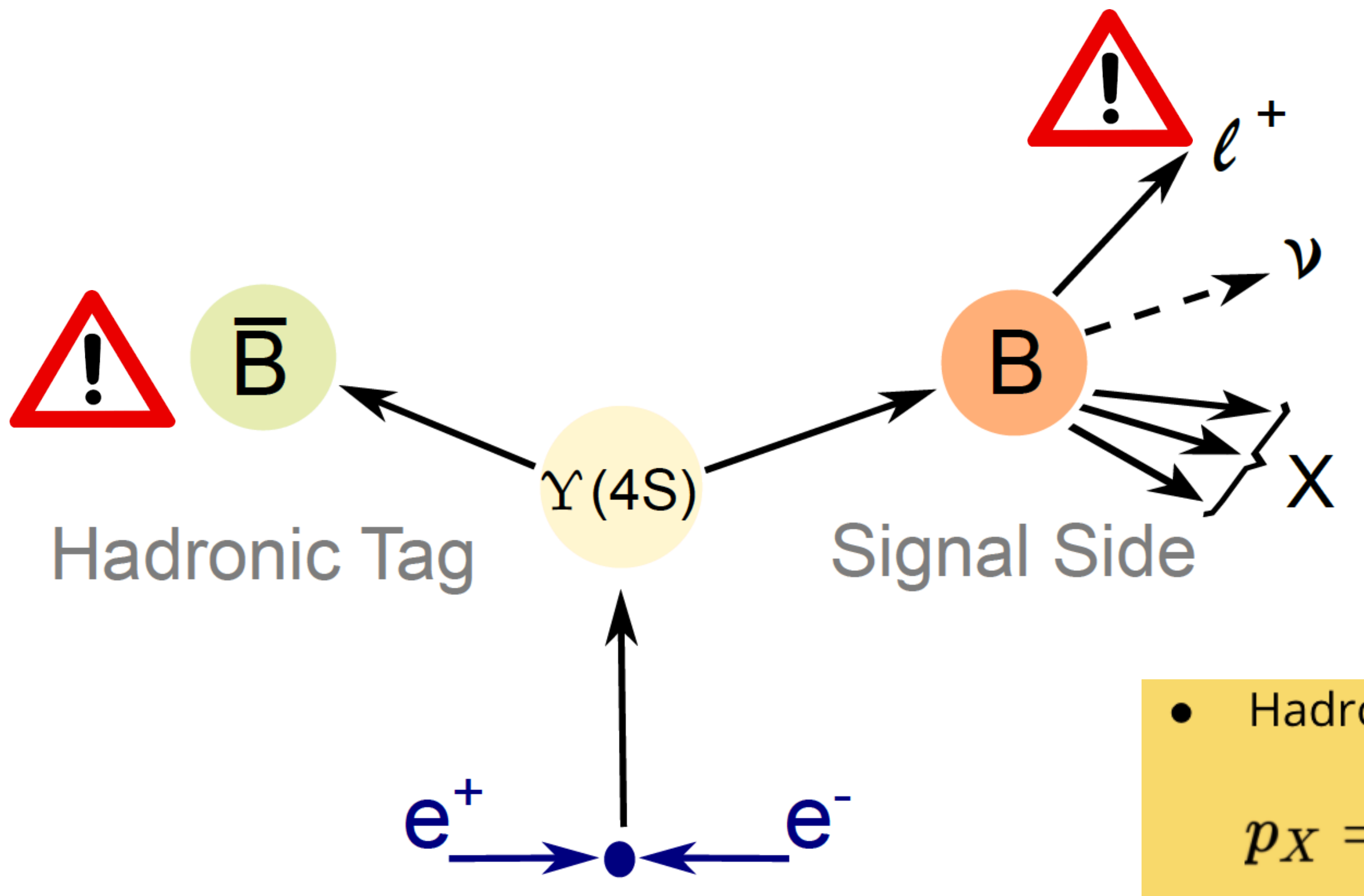

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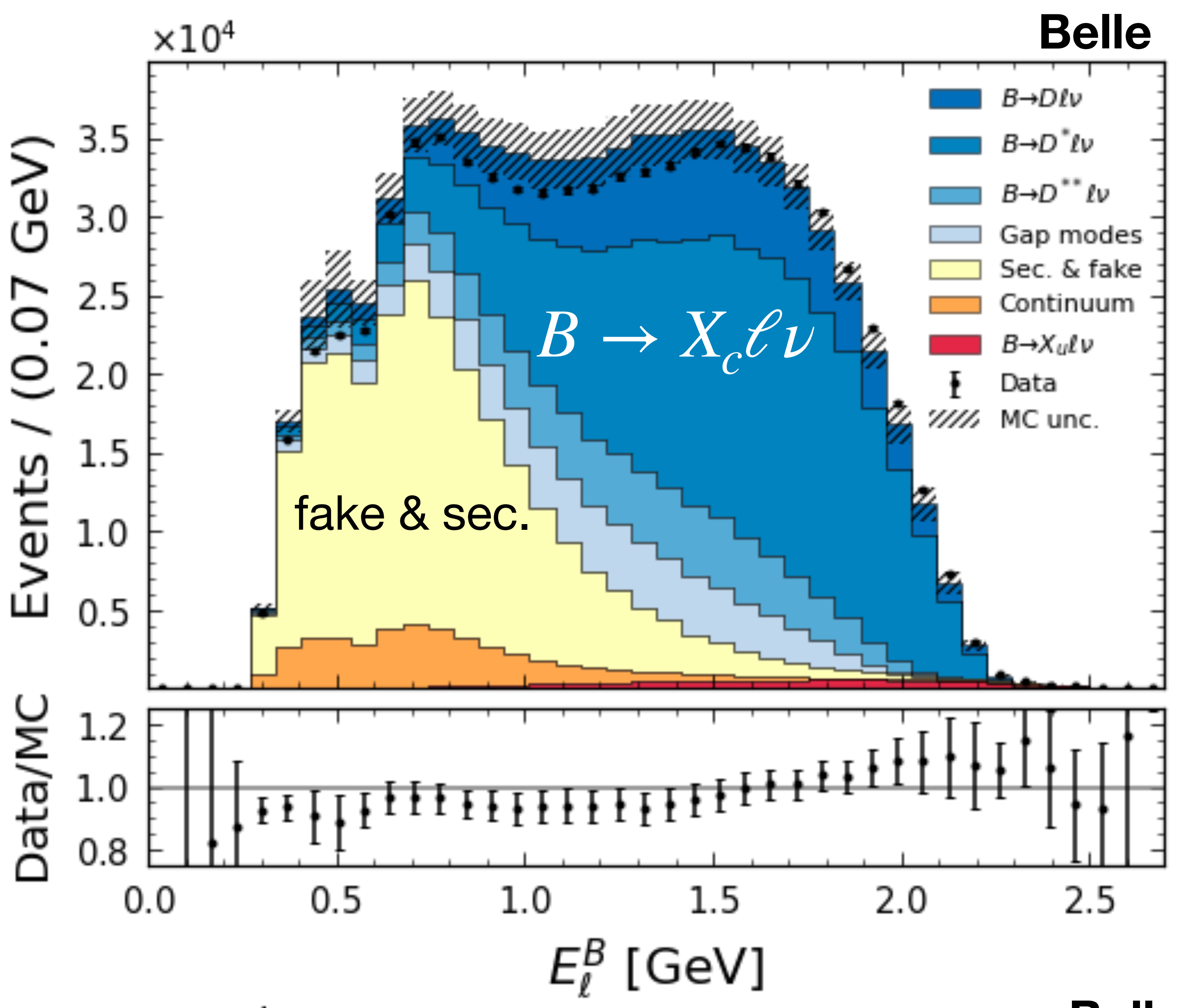
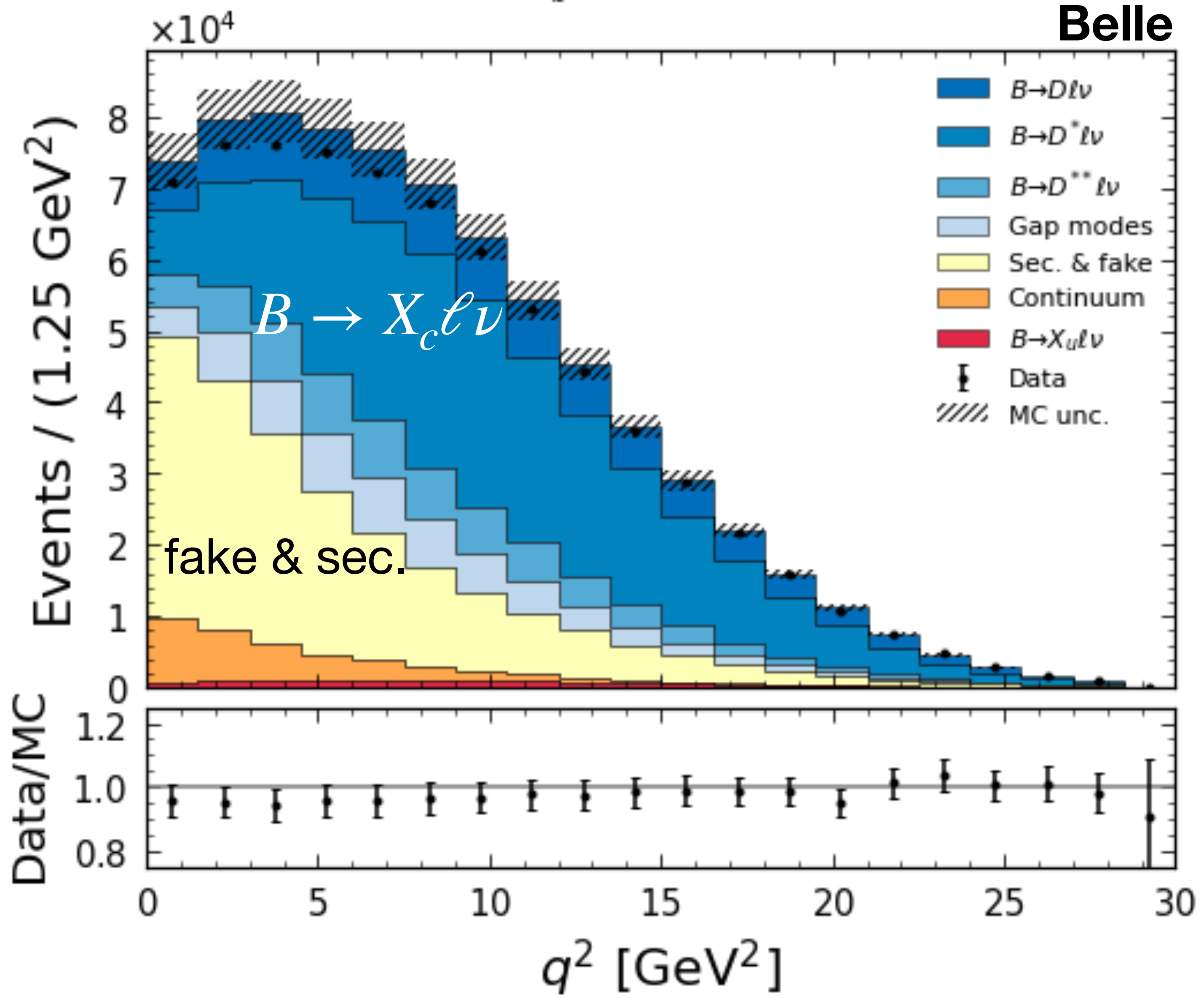




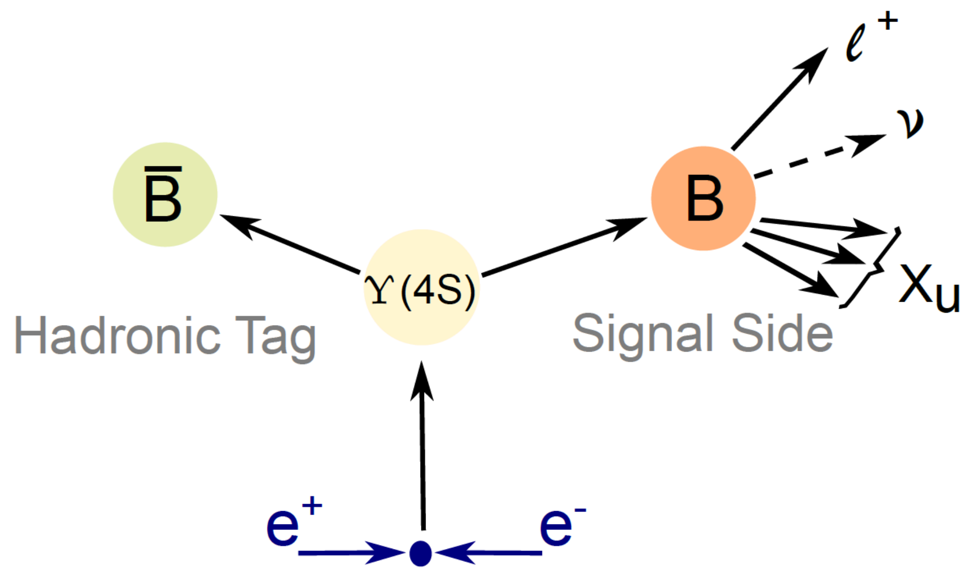
Challenge 1:

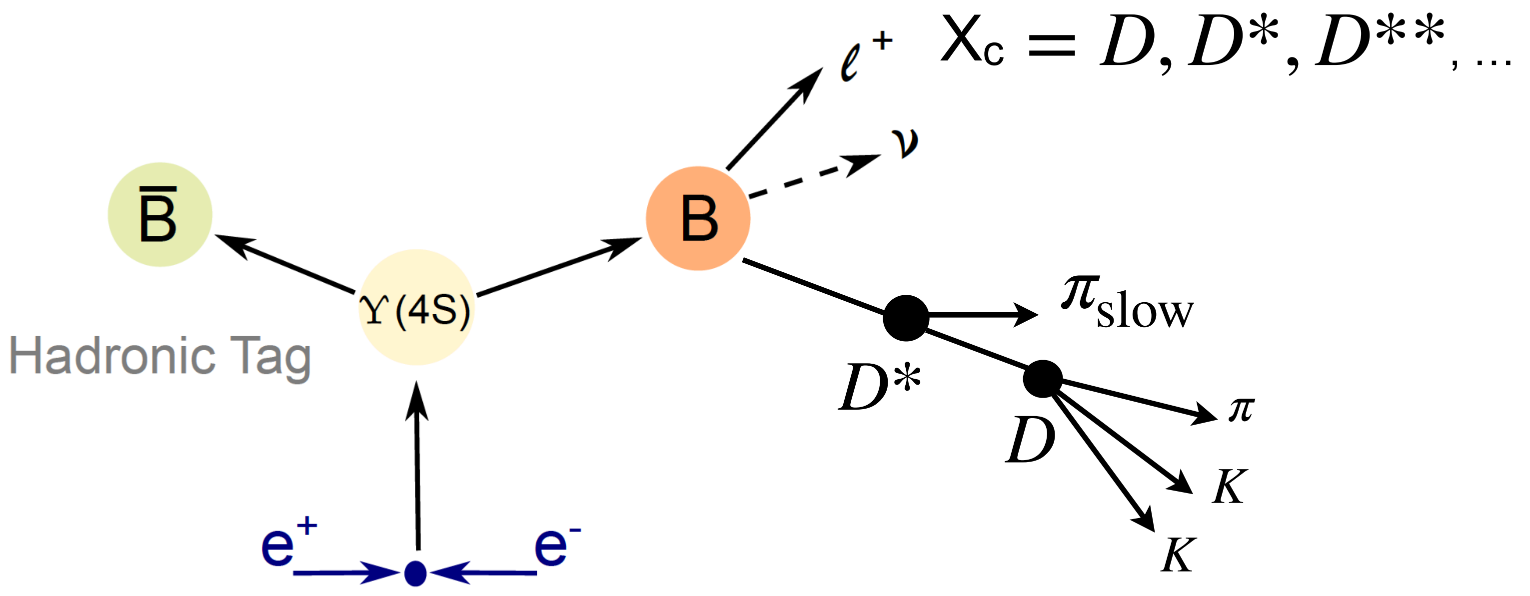
- Tagging efficiency $\epsilon \approx \mathcal{O}(0.1\%)$
- Large background for low momentum leptons: **misidentified or secondary SL decays** => $E_\ell^B > 0.8$ GeV or $q^2 > 3$ GeV²

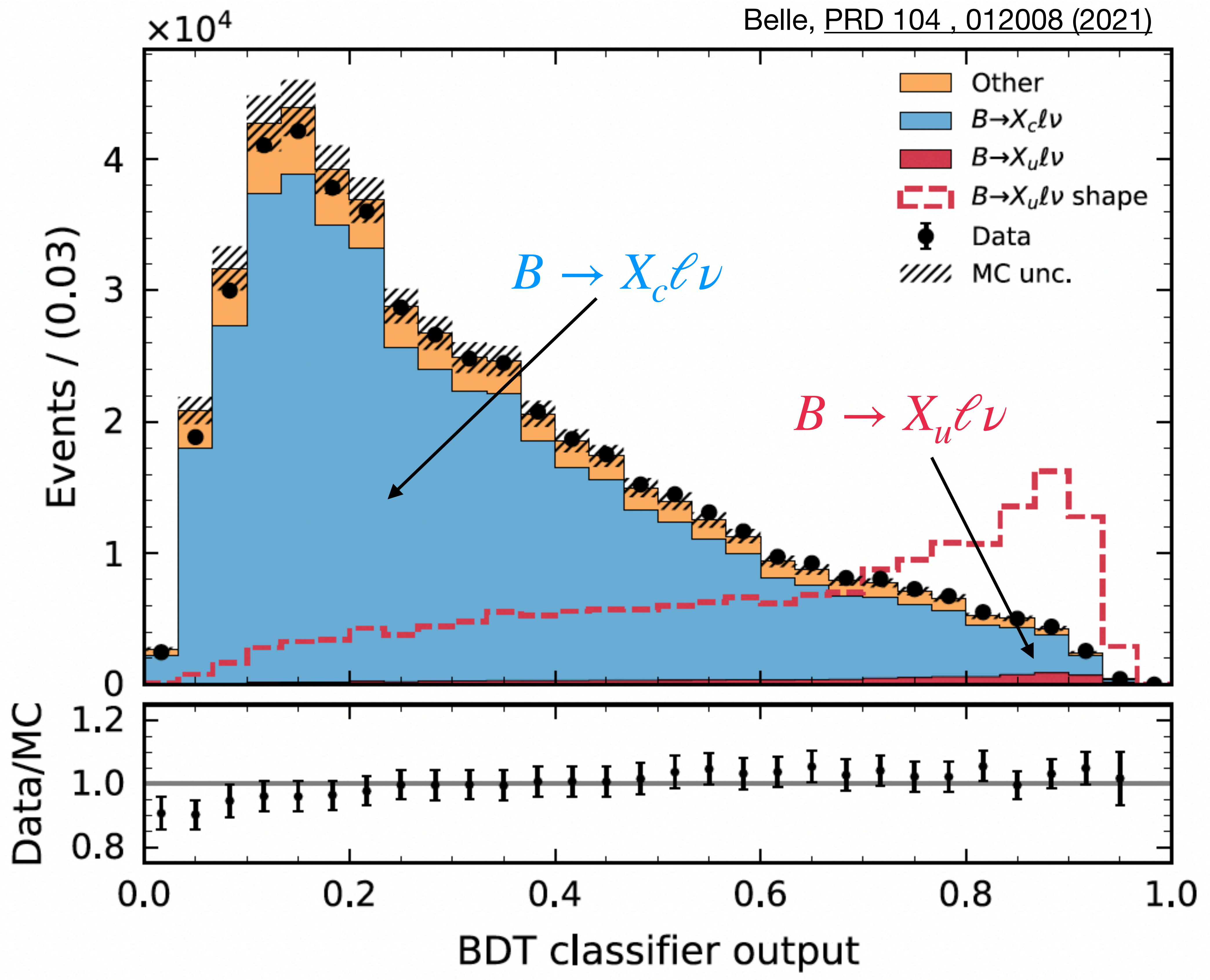
$\Delta\mathscr{B}(B \to X_u \ell \nu)$ and Inclusive |Vub|

Challenge 2:

- High background from $B \to X_c \ell \nu$







Train a binary classification BDT (eXtreme Gradient Boosting) with with 11 feature variables, e.g. N_K, vertex, missing mass, etc.

Selection	$B \to X_u \ell^+ \nu_\ell$ (%)	$B \to X_c \ell^+ \nu_\ell$ (%)	Data (%)
$M_{bc} > 5.27$ GeV	84.8	83.8	80.2
$\mathcal{O}_{BDT} > 0.85$	18.5	1.3	1.6
$\mathcal{O}_{BDT} > 0.83$	21.9	1.7	2.1
$\mathcal{O}_{BDT} > 0.87$	14.5	0.9	1.1

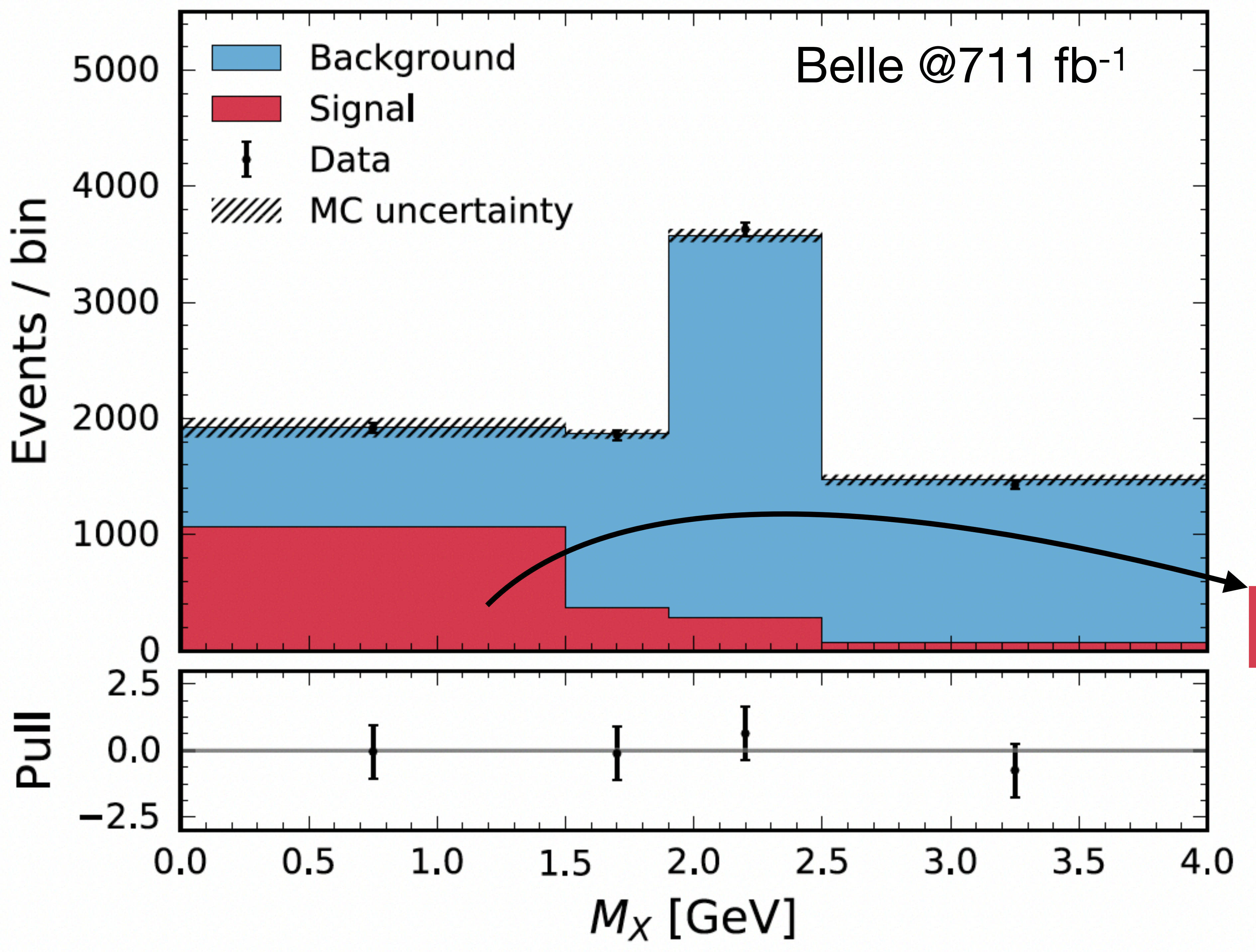
** Tagging efficiency is excluded

$\Delta\mathscr{B}(B \to X_u \ell \nu)$ and Inclusive |Vub|

Challenge 2:

- High background from $B \to X_c \ell \nu$

PRD 104 , 012008 (2021)



Belle @711 fb-1

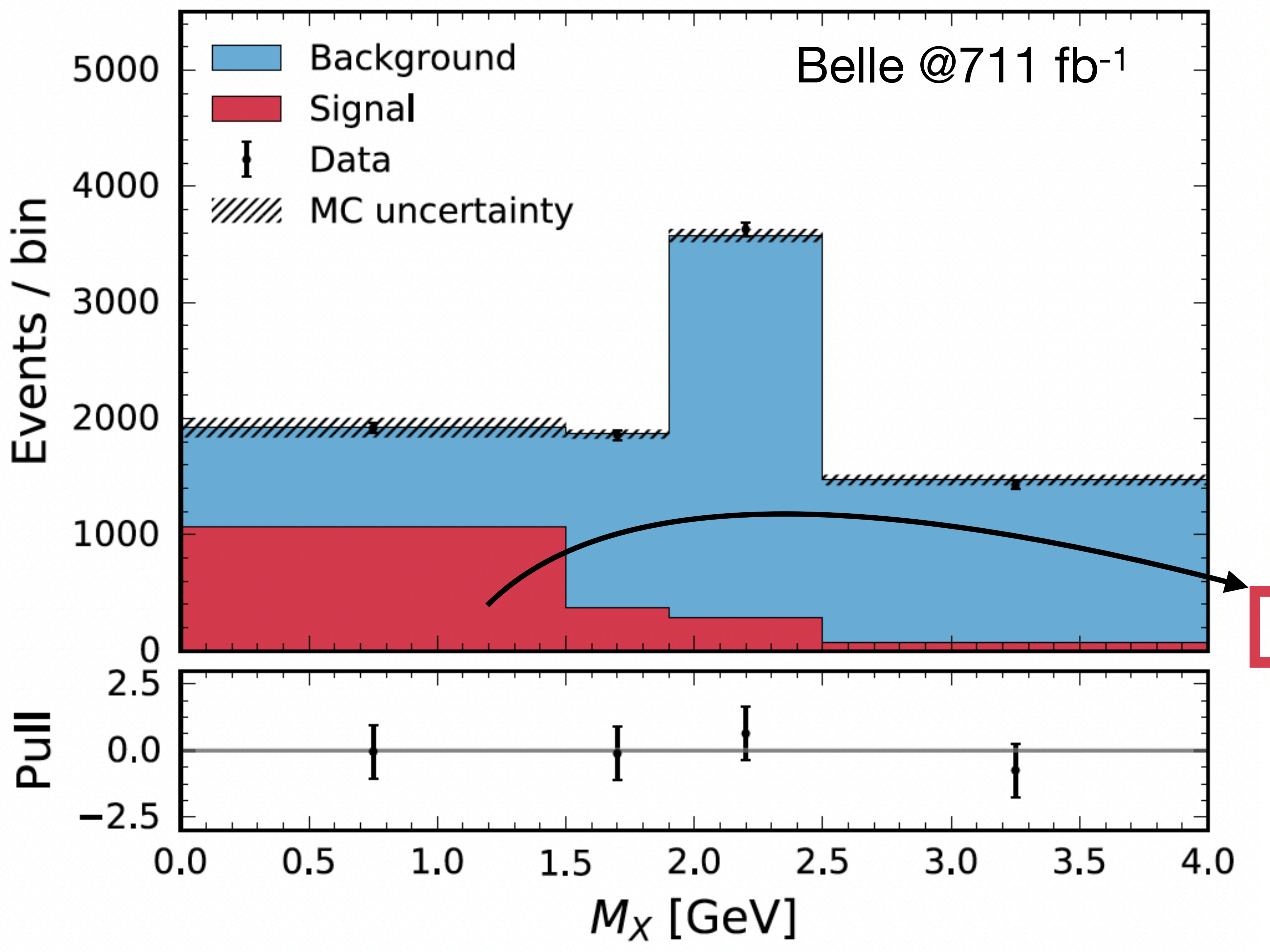
Phase-space region	Additional selection (GeV)	Fit variable(s)	$\hat{\eta}_{\rm sig}$
$M_X < 1.7$ GeV, $E_\ell^B > 1$ GeV	...	M_X fit	$1558 \pm 69 \pm 71$
$M_X < 1.7$ GeV, $E_\ell^B > 1$ GeV	$M_X < 1.7$	E_ℓ^B fit	$1285 \pm 68 \pm 139$
$M_X < 1.7$ GeV, $q^2 > 8$ GeV^2, $E_\ell^B > 1$ GeV	$M_X < 1.7$	q^2 fit	$938 \pm 99 \pm 100$
$E_\ell^B > 1$ GeV	$M_X < 1.7$	E_ℓ^B fit	$1303 \pm 69 \pm 138$
$E_\ell^B > 1$ GeV		$M_X : q^2$ fit	$1801 \pm 81 \pm 123$

$\Delta\mathcal{B}(B \to X_u \ell \nu)$ and Inclusive $|V_{ub}|$

Challenge 2:

- High background from $B \to X_c \ell \nu$

PRD 104 , 012008 (2021)



Belle @711 fb^{-1}

Phase-space region	Additional selection (GeV)	Fit variable(s)	$\hat{\eta}_{sig}$
$M_X < 1.7$ GeV, $E_\ell^B > 1$ GeV	...	M_X fit	$1558 \pm 69 \pm 71$
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$E_\ell^B > 1$ GeV	$M_X < 1.7$	E_ℓ^B fit	$1303 \pm 69 \pm 138$
$E_\ell^B > 1$ GeV		$M_X : q^2$ fit	$1801 \pm 81 \pm 123$

$N_{bkg} = 7031 \pm 164$

- **Better tagging performance with FEI (~1.5 times higher eff.)**
- **Further improvements form DNN (Tensorflow) to suppress backgrounds**

Belle II @364 fb^{-1}, estimated yields after all selections in MC:

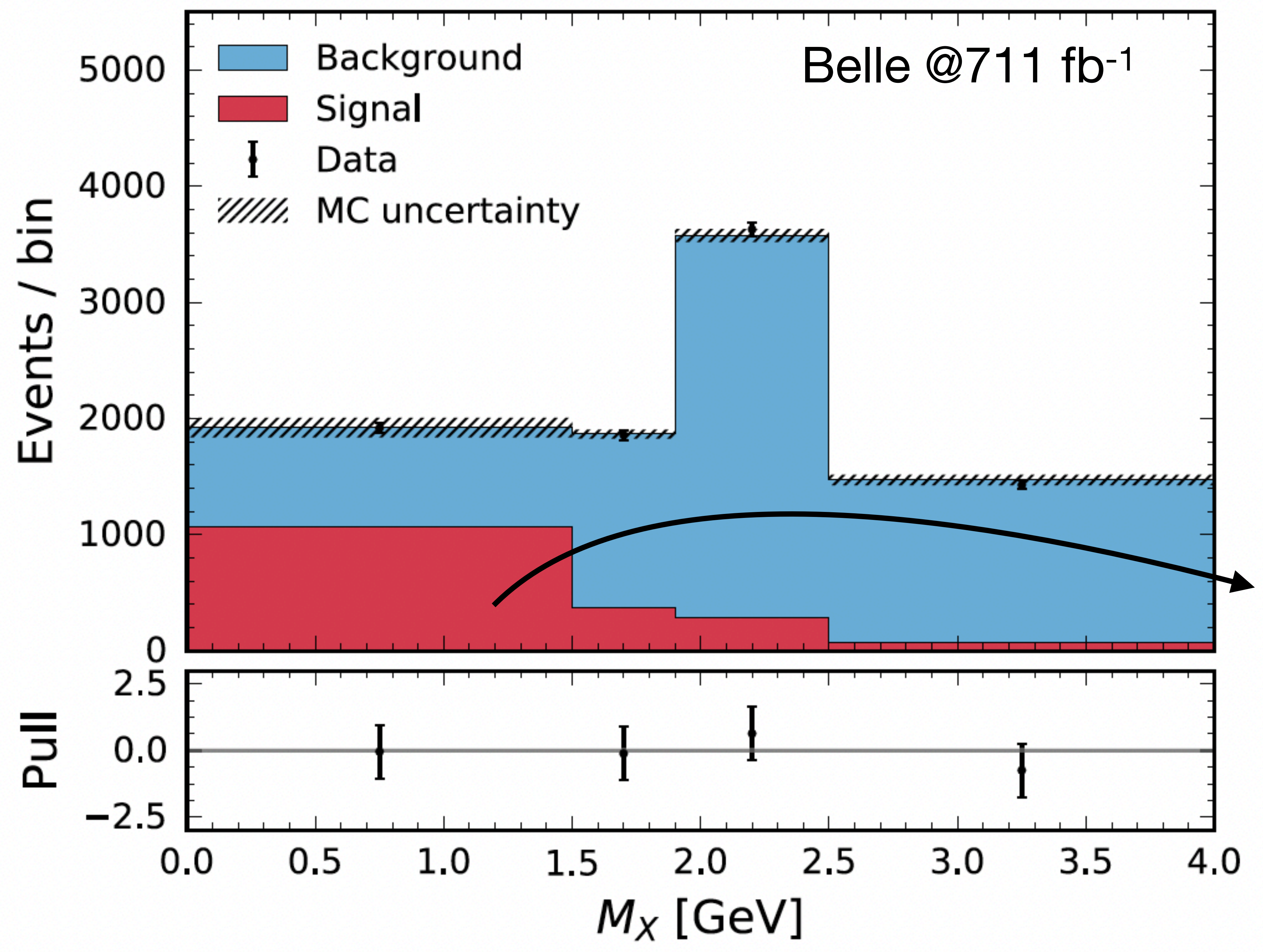
$N_{sig} = 1817$, $N_{bkg} = 4343$

$\Delta\mathcal{B}(B \to X_u \ell \nu)$ and Inclusive |Vub|

Challenge 2:

- High background from $B \to X_c \ell \nu$

PRD 104 , 012008 (2021)



Belle @711 fb^{-1}

Phase-space region	Additional selection (GeV)	Fit variable(s)	$\hat{\eta}_{\rm sig}$
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$E_\ell^B > 1$ GeV		M_X:q^2 fit	$1801 \pm 81 \pm 123$

Phase space cuts	Acceptance $\varepsilon_{\Delta\mathcal{B}}$
$p_\ell^B > 1.0$ GeV	86.7%
$p_\ell^B > 1.0$ GeV, $M_X < 1.7$ GeV	56.5%
$p_\ell^B > 1.0$ GeV, $M_X < 1.7$ GeV, $q^2 > 8$ GeV^2	31.5%
$p_\ell^B > 2.1$ GeV	19.0%

Partial region focus on the lepton endpoint will be also measured

$\Delta\mathscr{B}(B \to X_u \ell \nu)$ and Inclusive |V_ub|

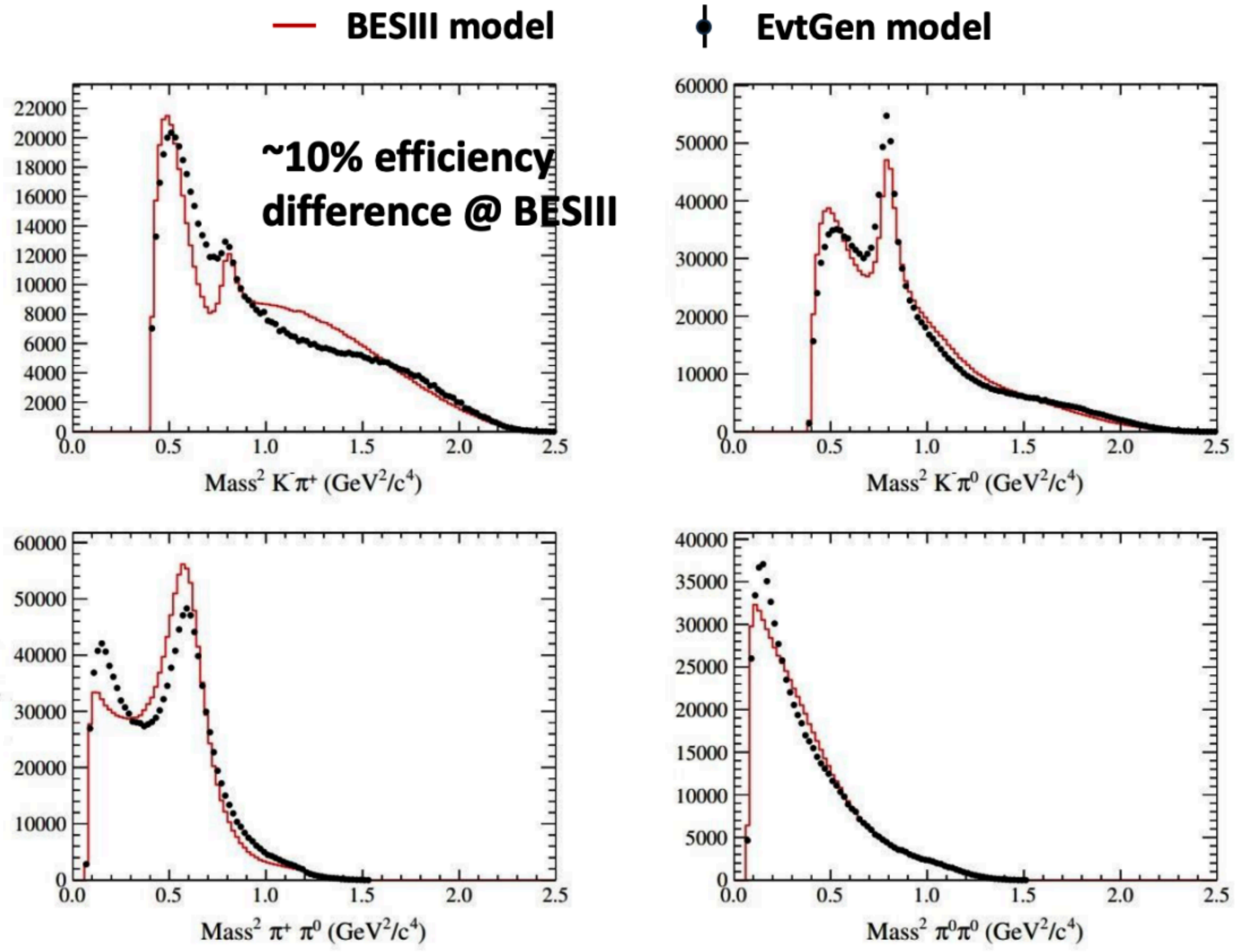
Challenge 3:

- Poorly modelled gap modes and charm multi-body decays

	Decay	$\mathscr{B}(B^+)$	$\mathscr{B}(B^0)$
D,D*	$B \to D\ell^+\nu_\ell$	$(2.35 \pm 0.06) \cdot 10^{-2}$	$(2.18 \pm 0.06) \cdot 10^{-2}$
	$B \to D^*\ell^+\nu_\ell$	$(5.41 \pm 0.11) \cdot 10^{-2}$	$(5.03 \pm 0.11) \cdot 10^{-2}$
D**	$B \to D_1\ell^+\nu_\ell$	$(6.5 \pm 1.1) \cdot 10^{-3}$	$(6.1 \pm 1.0) \cdot 10^{-3}$
	$B \to D_0^*\ell^+\nu_\ell$	$(4.2 \pm 0.8) \cdot 10^{-3}$	$(3.9 \pm 0.7) \cdot 10^{-3}$
	$B \to D_1'\ell^+\nu_\ell$	$(2.9 \pm 0.8) \cdot 10^{-3}$	$(2.6 \pm 0.7) \cdot 10^{-3}$
	$B \to D_2^*\ell^+\nu_\ell$	$(2.9 \pm 0.3) \cdot 10^{-3}$	$(2.7 \pm 0.3) \cdot 10^{-3}$
Non-res.	$B \to D\pi\pi\ell^+\nu_\ell$	$(0.7 \pm 0.9) \cdot 10^{-3}$	$(0.6 \pm 0.8) \cdot 10^{-3}$
	$B \to D^*\pi\pi\ell^+\nu_\ell$	$(2.2 \pm 1.0) \cdot 10^{-3}$	$(2.0 \pm 1.0) \cdot 10^{-3}$
	$B \to D_sK\ell^+\nu_\ell$	$(0.30 \pm 0.13) \cdot 10^{-3}$	-
	$B \to D_s^*K\ell^+\nu_\ell$	$(0.29 \pm 0.19) \cdot 10^{-3}$	-
Gap	$B \to D\eta\ell^+\nu_\ell$	$(6.5 \pm 6.5) \cdot 10^{-3}$	$(6.3 \pm 6.3) \cdot 10^{-3}$
	$B \to D^*\eta\ell^+\nu_\ell$	$(6.5 \pm 6.5) \cdot 10^{-3}$	$(6.3 \pm 6.3) \cdot 10^{-3}$
	$B \to X_c\ell^+\nu_\ell$	$(11.04 \pm 0.17) \cdot 10^{-2}$	$(10.26 \pm 0.16) \cdot 10^{-2}$

** Averages are taken from HFLAV2021

$D^0 \to K^-\pi^+\pi^0\pi^0$



$\Delta\mathcal{B}(B \to X_u \ell \nu)$ and Inclusive $|V_{ub}|$

Challenge 3:

- Poorly modelled gap modes and charm multi-body decays

Data-driven correction based on $B \to X_c \ell \nu$ enriched sideband

- Exclude N_K from MVA training
- Split data into $N_K = 0$ for signal extraction and $N_K > 0$ for cross-extrapolate $B \to X_c \ell \nu$ normalisations

$\Delta\mathscr{B}(B \to X_u \ell \nu)$ and Inclusive $|V_{ub}|$

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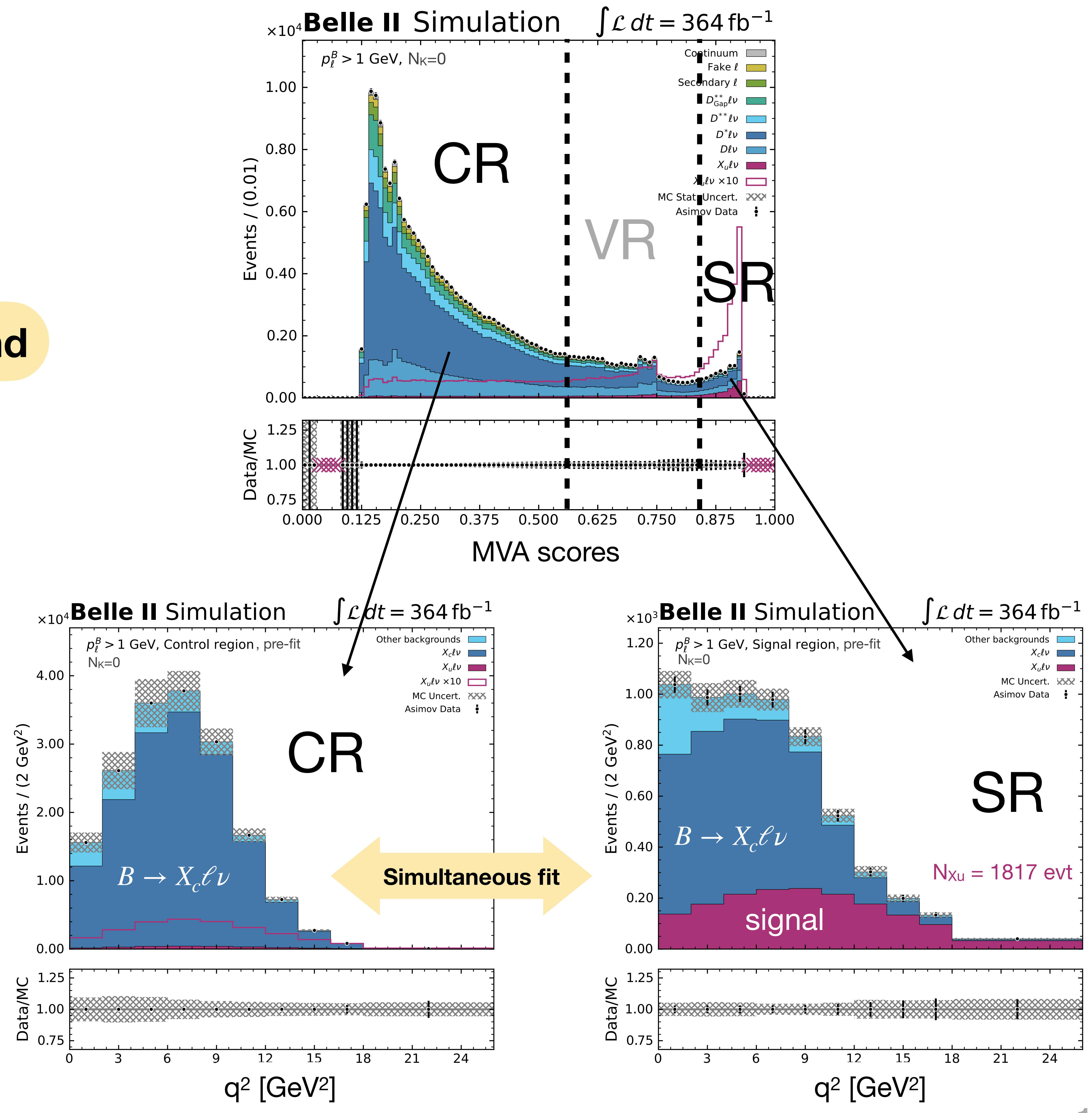
Data-driven correction based on $B \to X_c \ell \nu$ enriched sideband



- Exclude N_K from MVA training
- Split data into $N_K = 0$ for signal extraction and $N_K > 0$ for cross-extrapolate $B \to X_c \ell \nu$ normalisations
- Further split into 3 regions based MVA scores, and 3 fit templates in each region
- Simultaneous fit control region (CR) and signal region (SR), the high statistics of $B \to X_c \ell \nu$ in CR push the shape corrections to SR
- All systematics included as source-wise nuisances parameters and shared in CR & SR
- Validation region (VR) is used as pseudo-SR for checking the method

$\Delta\mathscr{B}(B \to X_u \ell \nu)$ and Inclusive $|V_{ub}|$

Challenge 4:

- Seeking advanced signal modelling

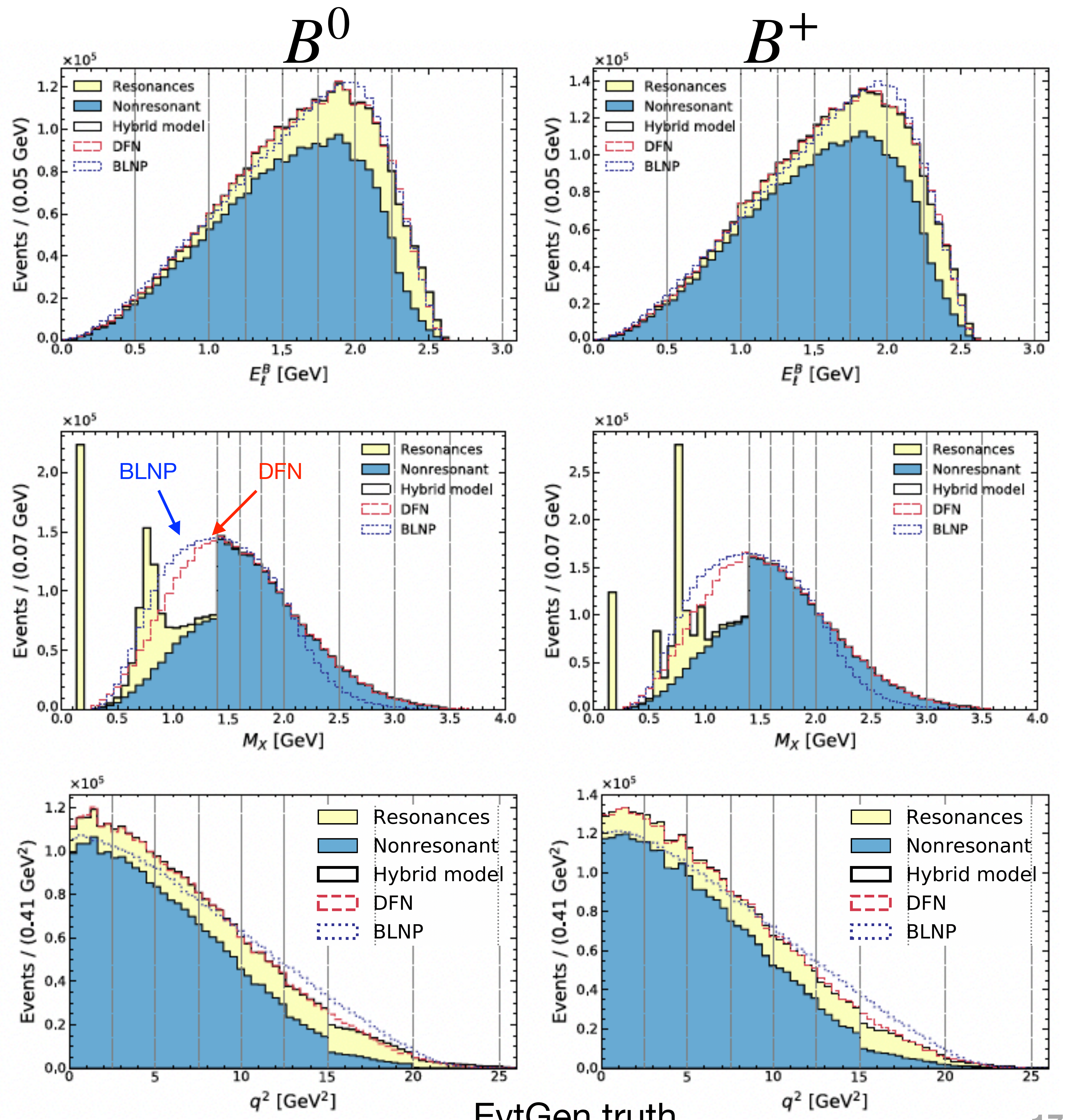
$\Delta\mathscr{B}(B \to X_u \ell \nu)$ and Inclusive |Vub|

Challenge 4: Seeking advanced signal modelling

DFN was used as nominal for inclusive contribution at Belle.

At **Belle II**, initially planned to switch to BLNP, but noticed two issues.

Hybrid recipe for combining excl. & incl.



EvtGen truth

$\Delta\mathcal{B}(B \to X_u \ell \nu)$ and Inclusive $|V_{ub}|$

Challenge 4: Seeking advanced signal modelling

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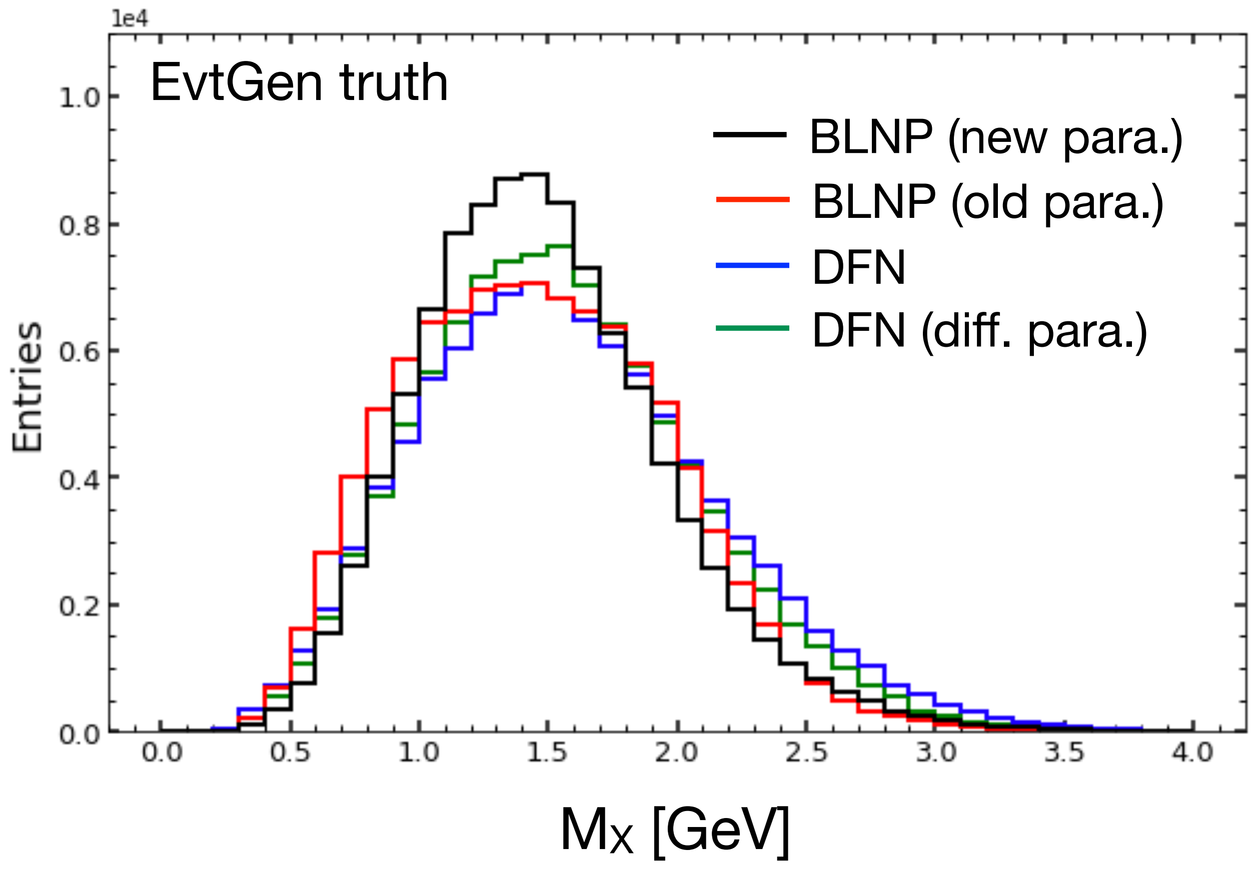
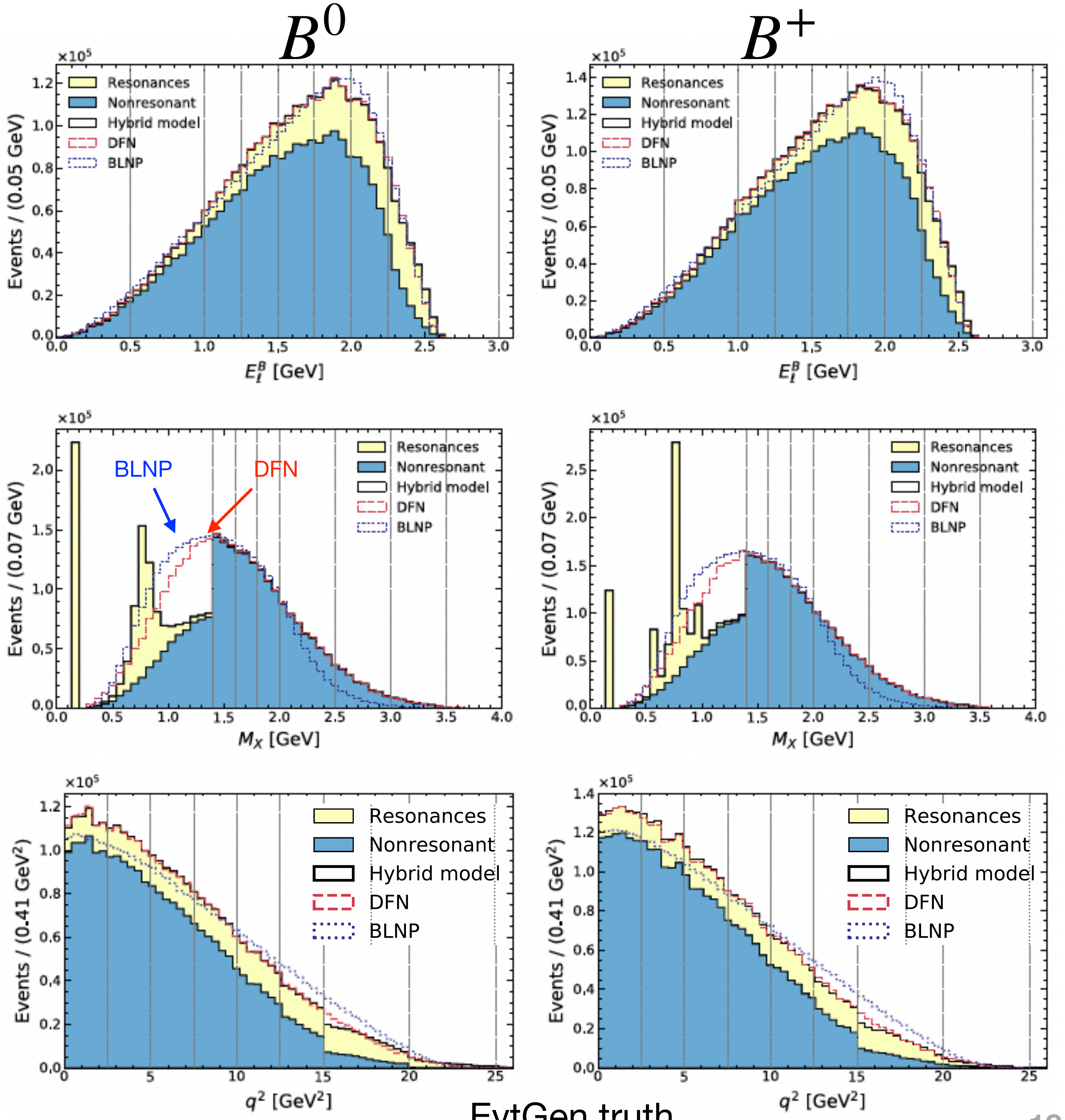
- More peaking M_X when take HQE para. from HFLAV21 as input



	m_b (SF)	mu2_pi (SF)
BLNP new	4.582	0.202
BLNP old	4.61	0.20
	Lambda (KN)	**lambda_1 (KN)**
DFN	0.621	-0.497
DFN diff.	0.588	-0.362

Hybrid recipe for combining excl. & incl.



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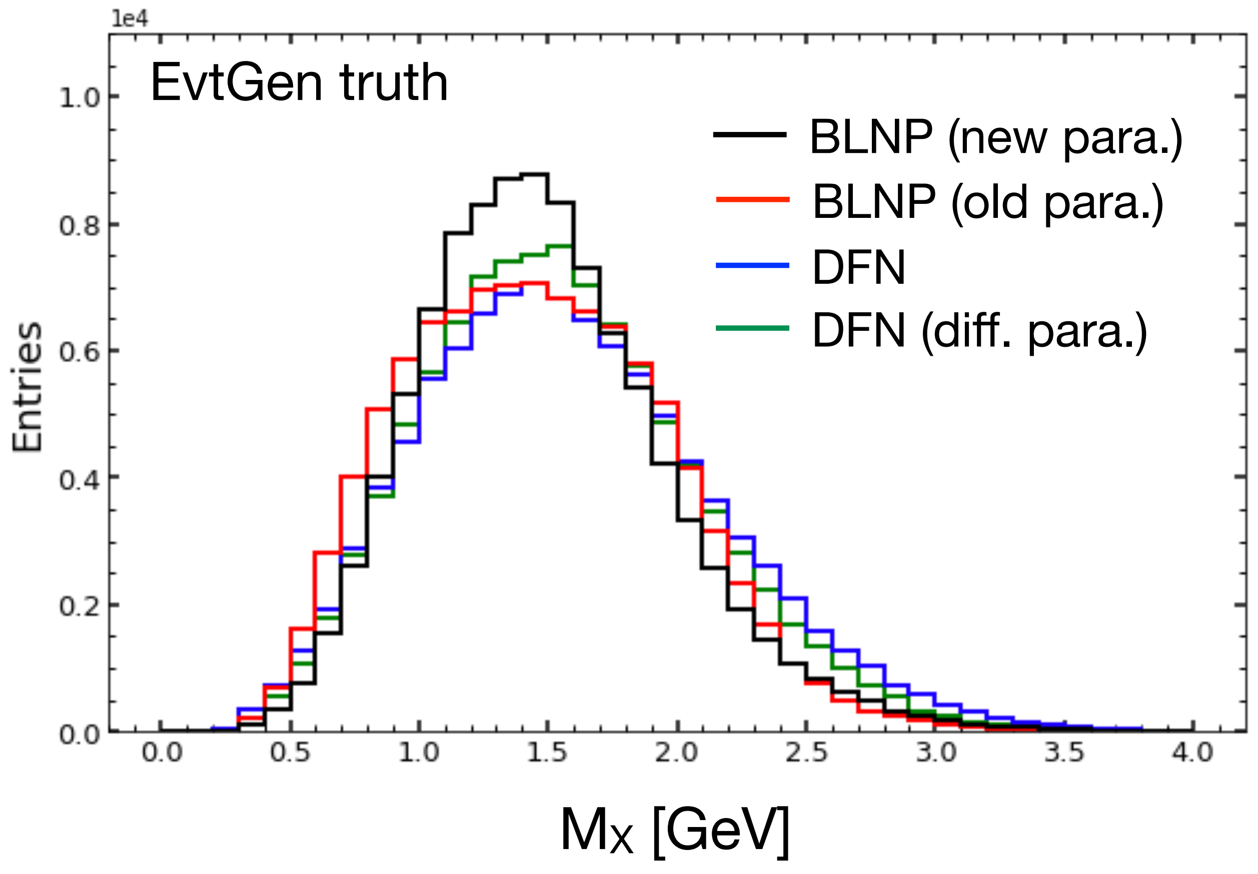
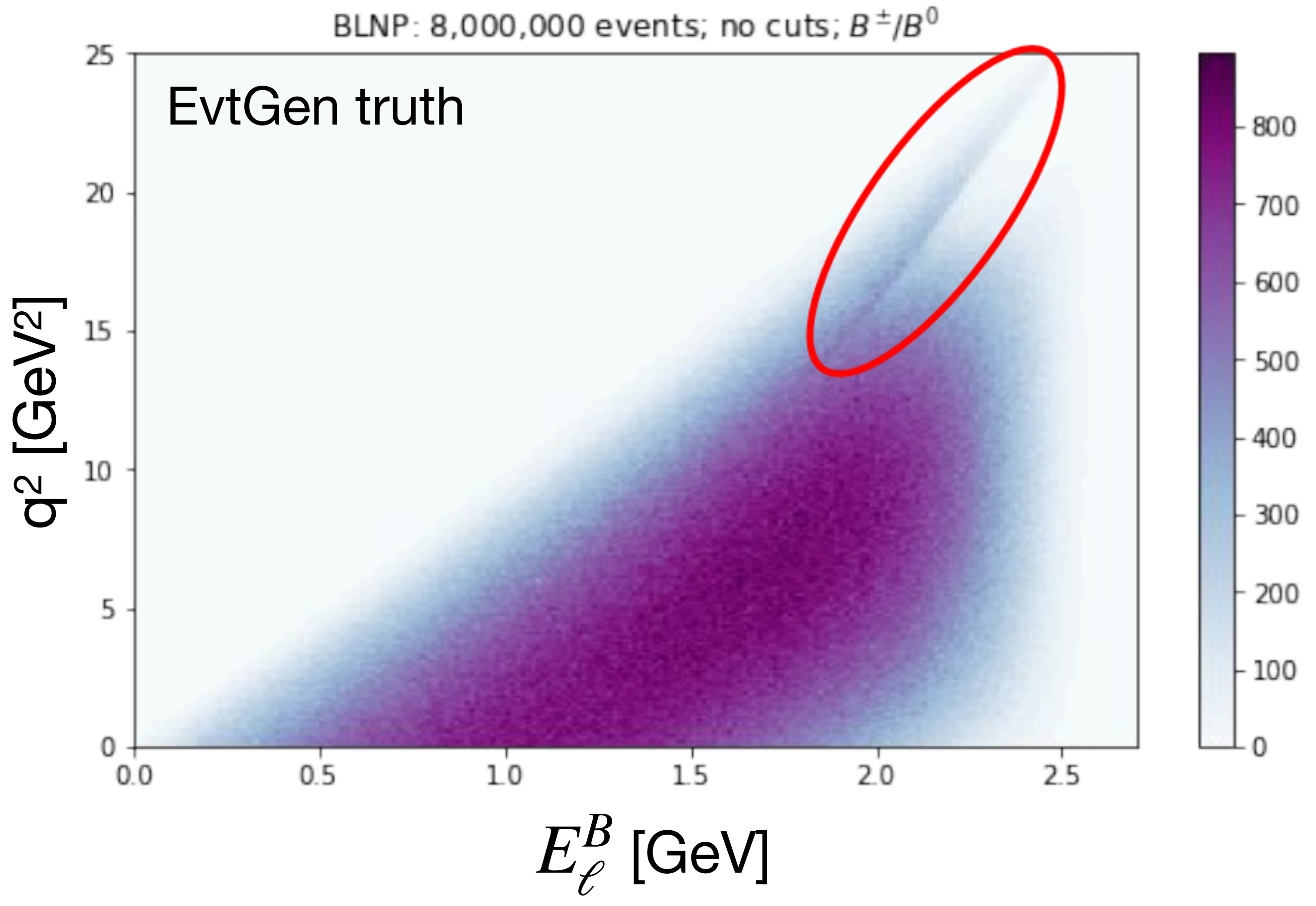
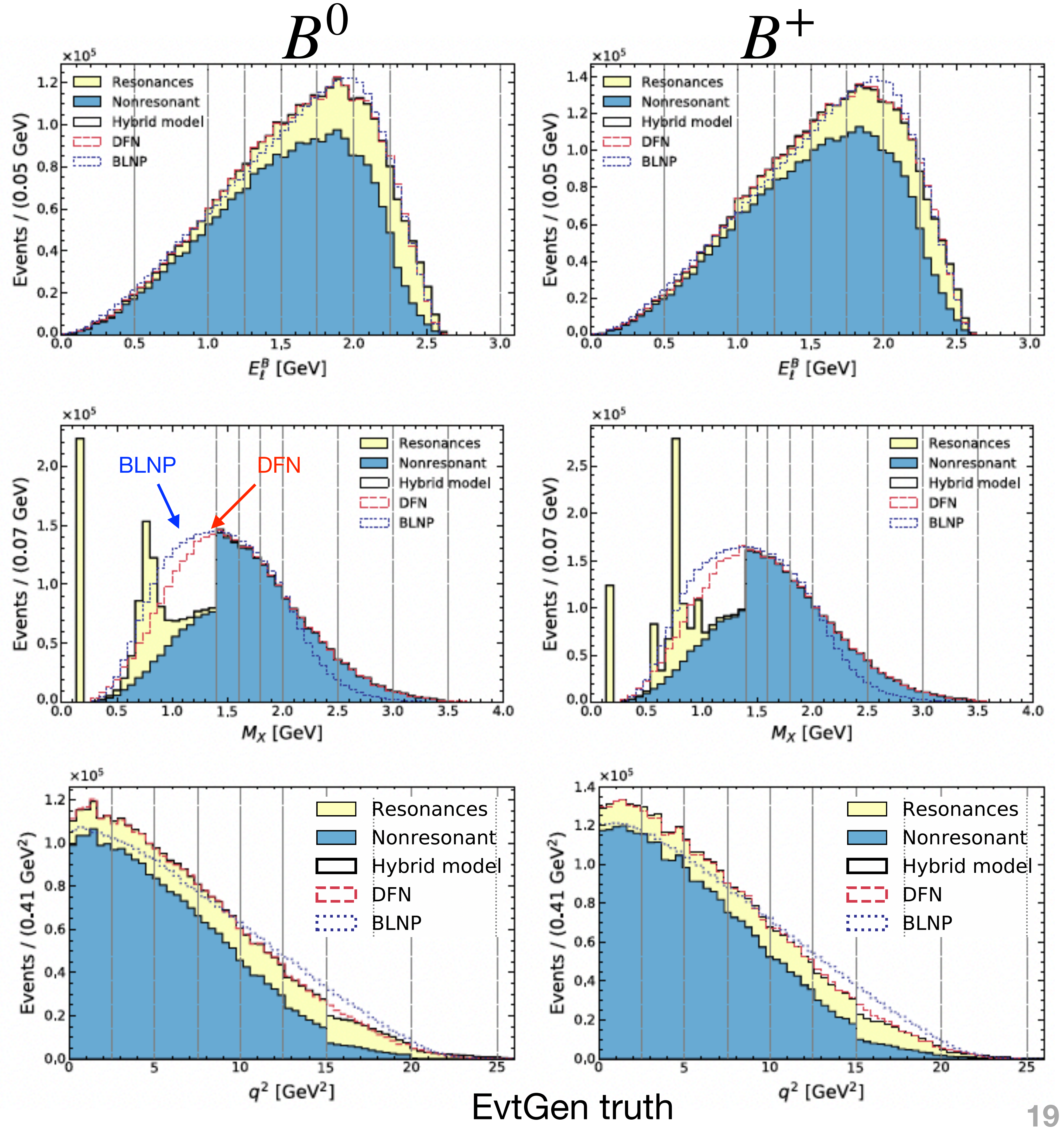


- Unphysical correlation pattern shown on kinematic variables (para. independent)



Compromise solution: theorists suggest to stay with DFN or glue DFN to BLNP for this problematic phase-space region

Hybrid recipe for combining excl. & incl.



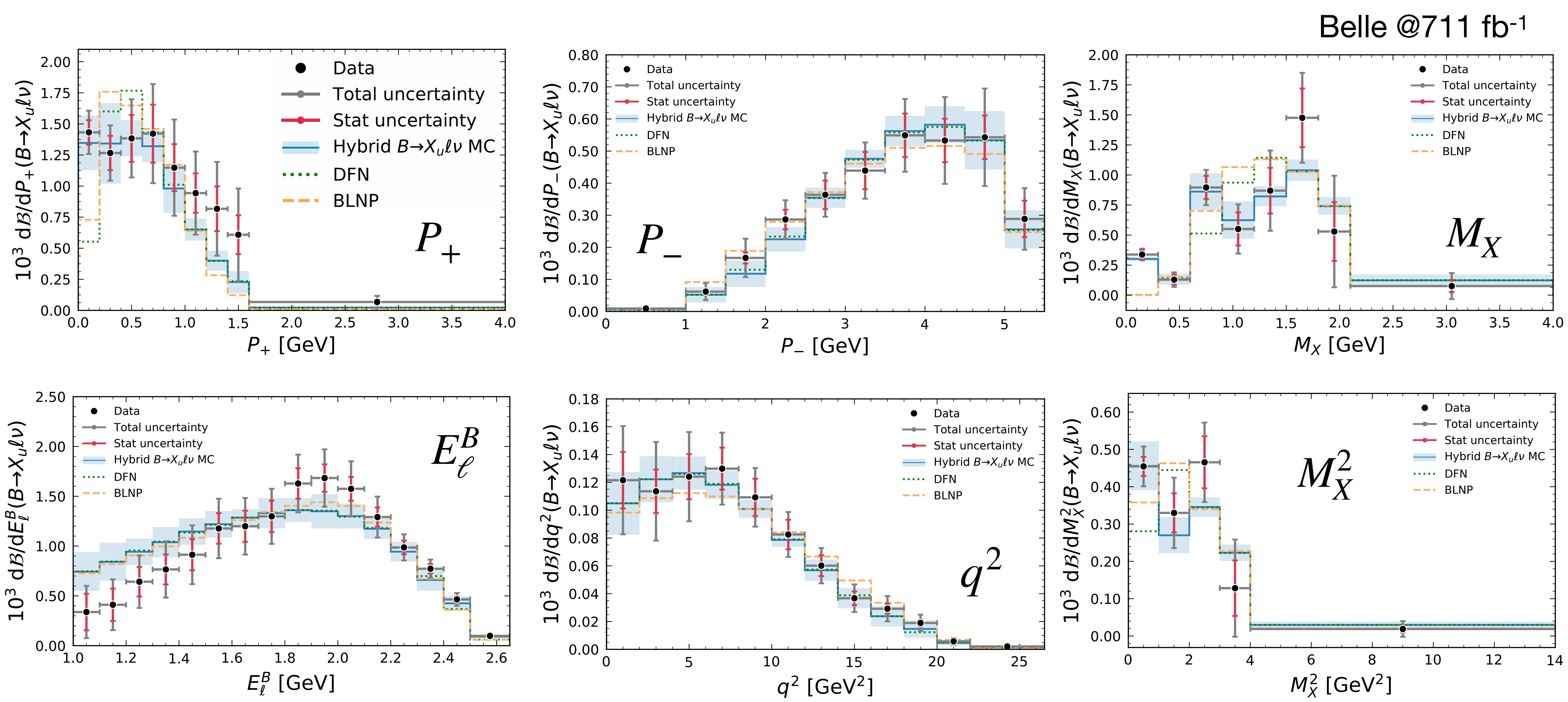
Differential Spectra of $B \to X_u \ell \nu$

Differential Spectra of $B \to X_u \ell \nu$

PRL 127, 261801 (2021)

https://www.hepdata.net/record/ins1895149

- Inherit **same analysis strategy** in the partial BF measurement [PRD 104 , 012008 (2021)]
- Additionally select |Emiss - Pmiss| < 0.1 GeV (not for E_ℓ^B) to **improve resolution**



Agreement between data and simulations:

χ^2	E_ℓ^B	M_X	M_X^2	q^2	P_+	P_-
n.d.f.	16	8	5	12	9	10
Hybrid	13.5	2.5	2.6	4.5	1.7	5.2
DFN	16.2	63.2	13.1	18.5	29.3	6.1
BLNP	16.5	61.0	6.3	20.6	23.6	13.7

Need theorists' great help to constrain/ improve the simulations with the observations **[long-term solution for Challenge 4]**

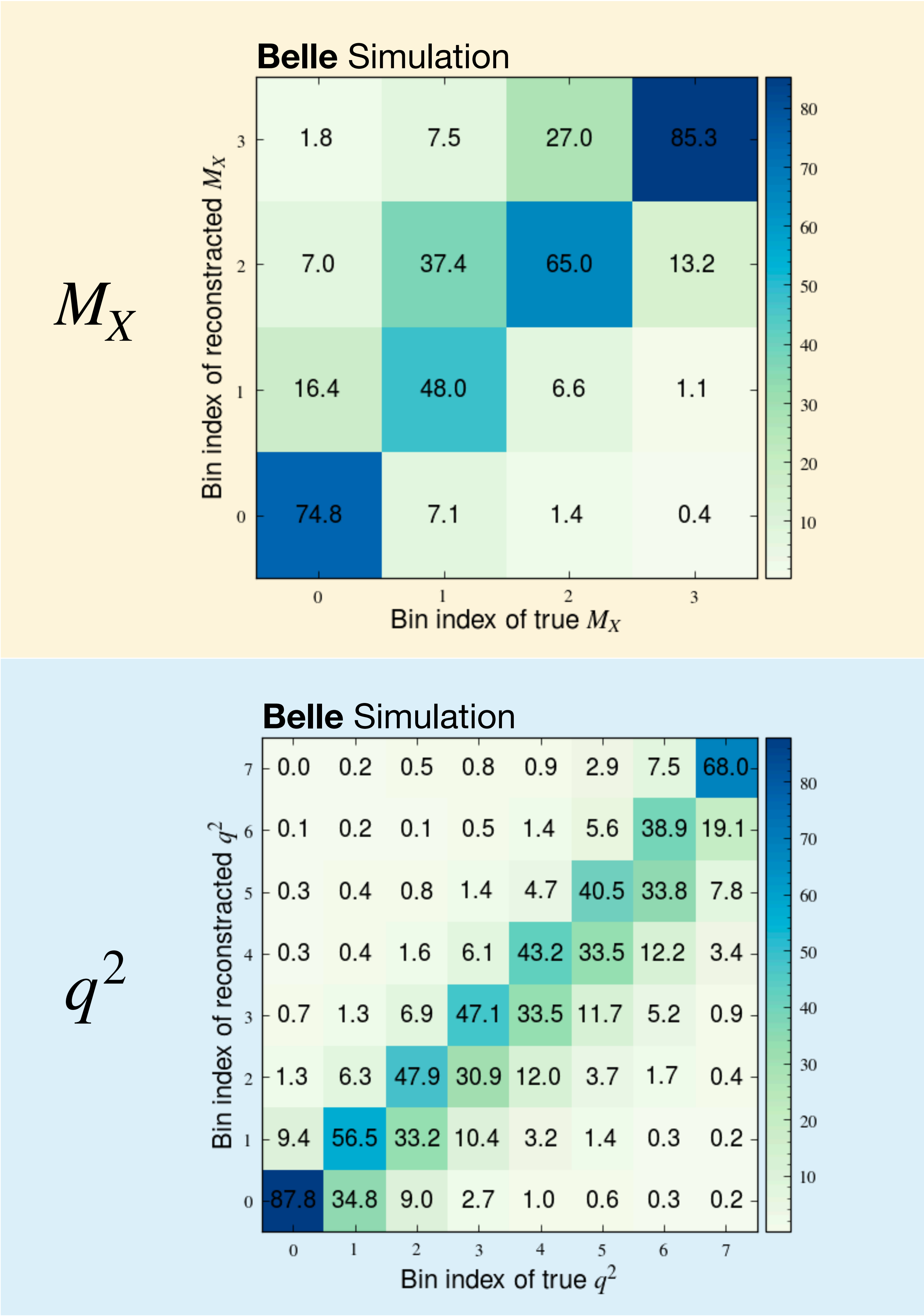
Differential Spectra of $B \to X_u \ell \nu$

Challenge 5:

- Resolution of kinematic variables

In the Belle study, applying **U_{miss} = |E_{miss} - P_{miss}| < 0.1 GeV** improves resolution by **21%~37%**, but looses 55% signals

All



Differential Spectra of $B \to X_u \ell \nu$

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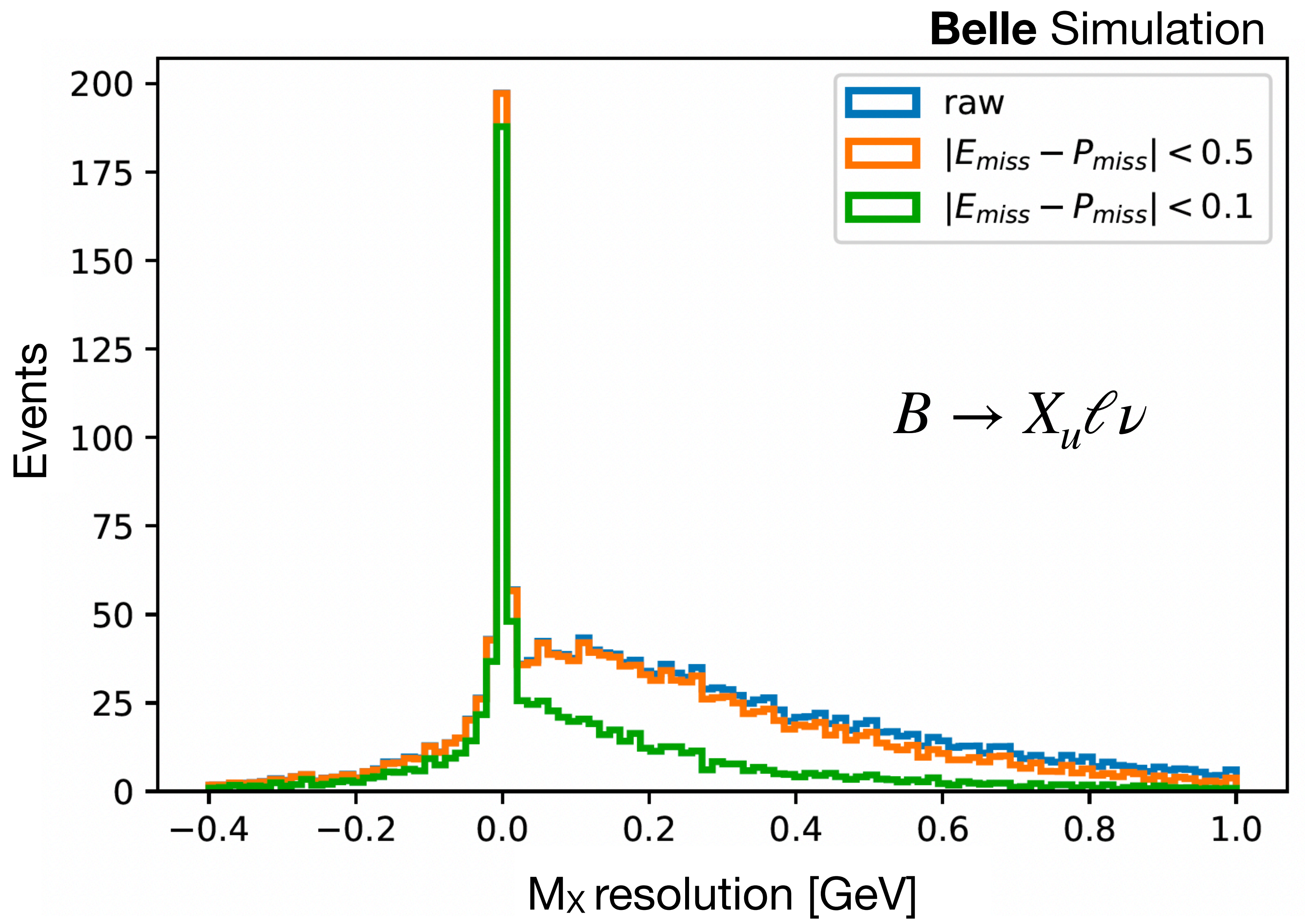
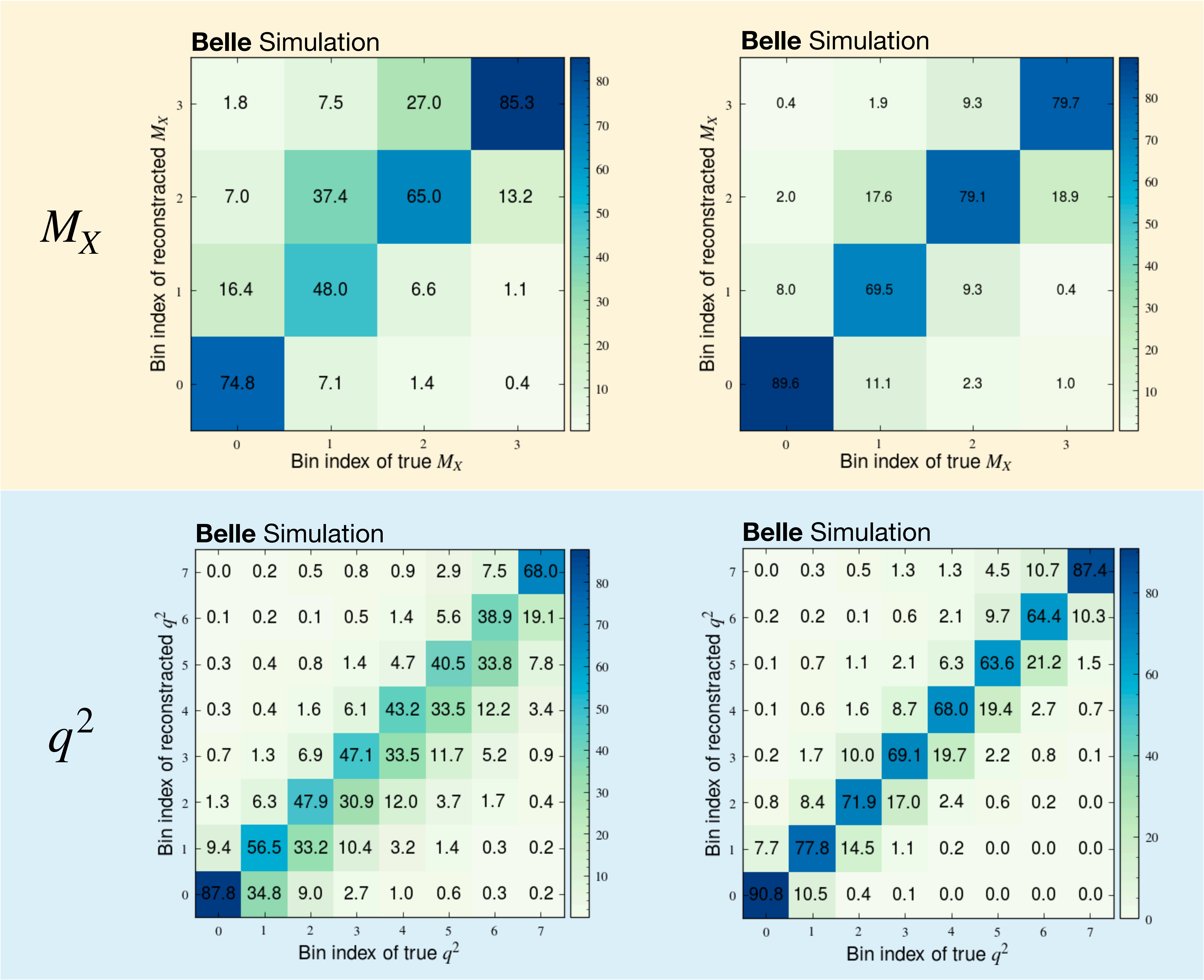


All | **|E_{miss} - P_{miss}| < 0.1**



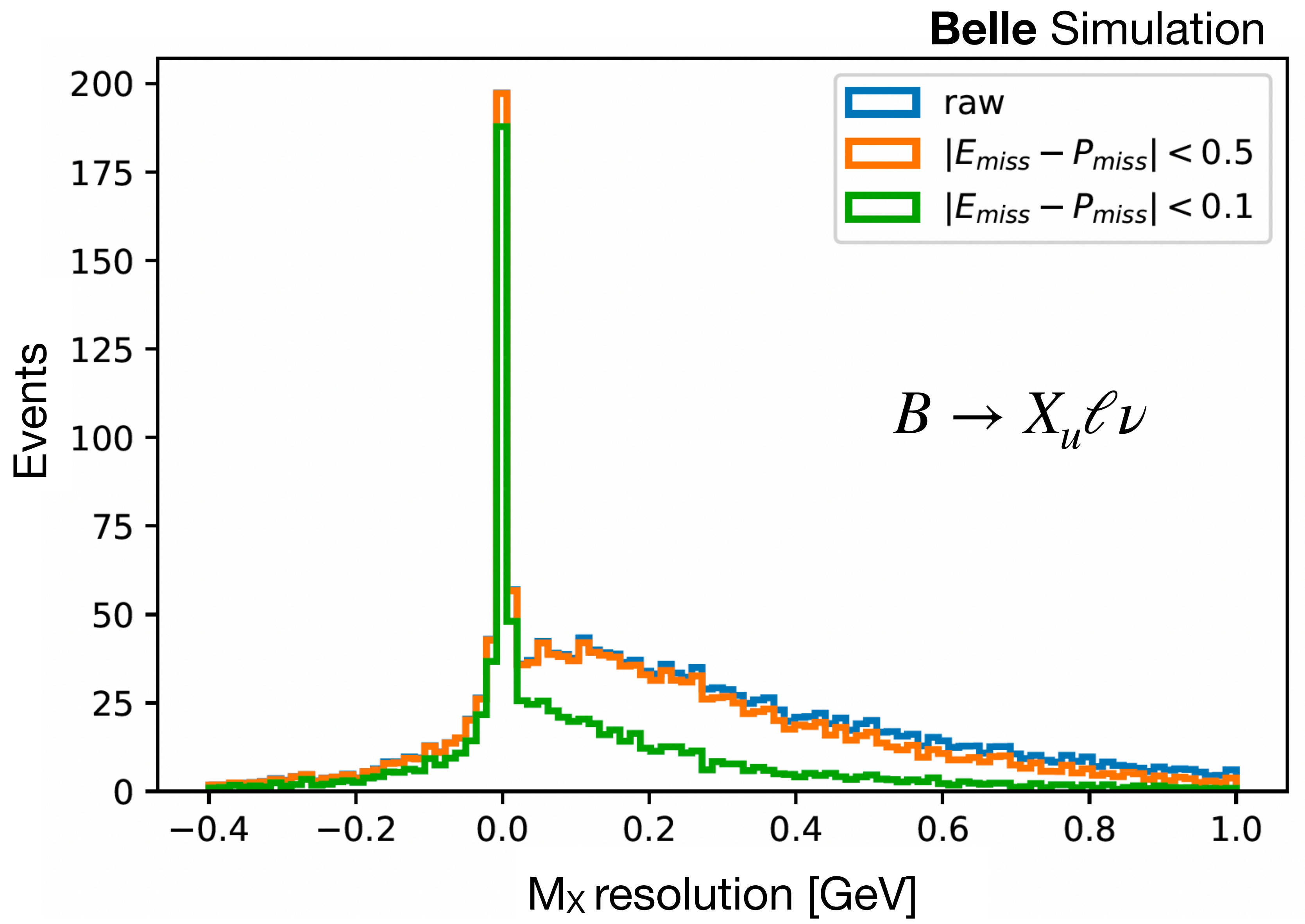
Differential Spectra of $B \to X_u \ell \nu$

Challenge 5:

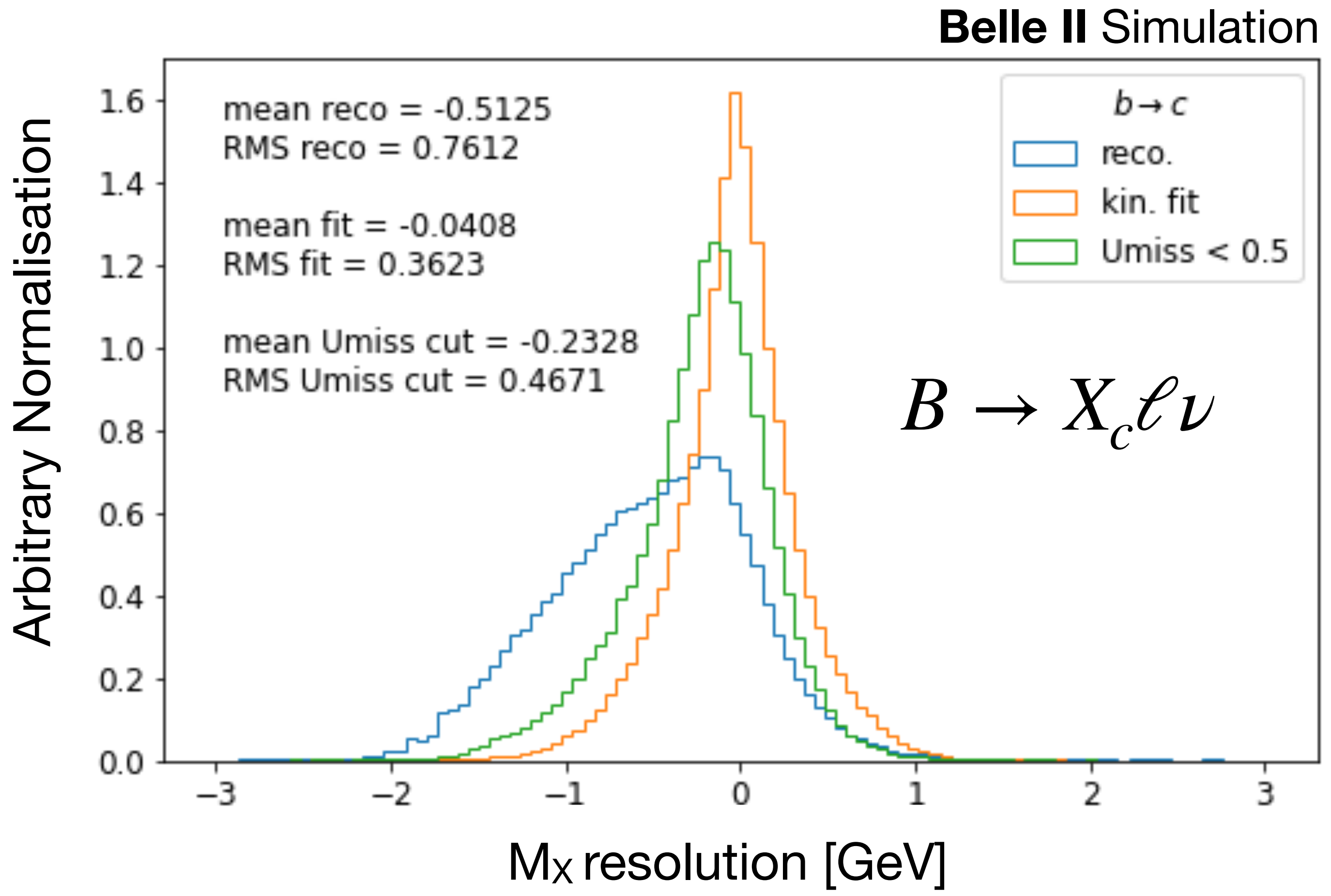
- Resolution of kinematic variables

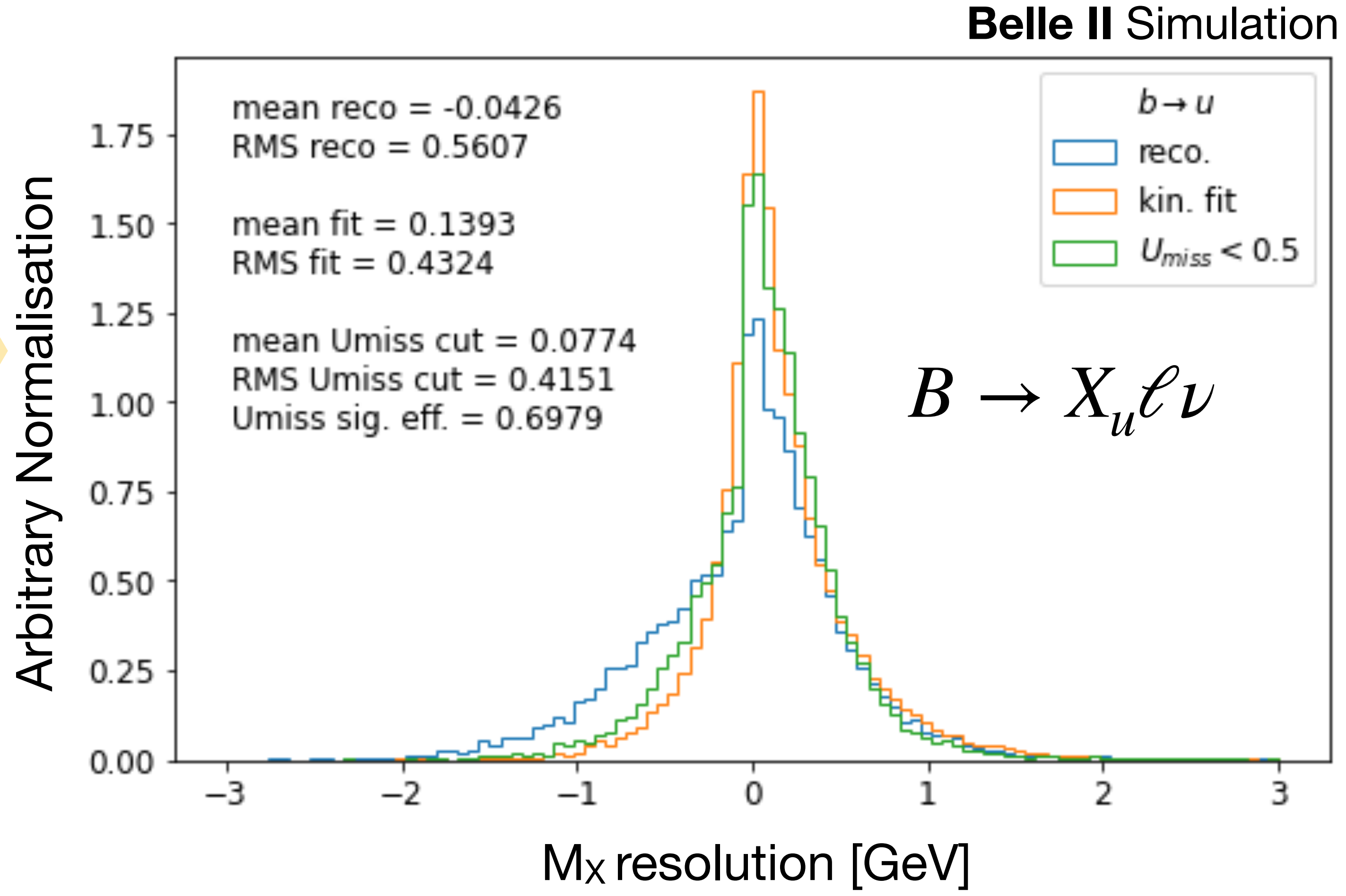
At **Belle II**, apply the **kinematic fit** strategy as developed for measuring the q² moments of $B \to X_c \ell \nu$ [PRD 107, 072002 (2023)] reject events of failed fitting ~100% efficiency.

In the Belle study, applying **U_{miss} = |E_{miss} - P_{miss}| < 0.1 GeV** improves resolution by **21%~37%**, but looses 55% signals



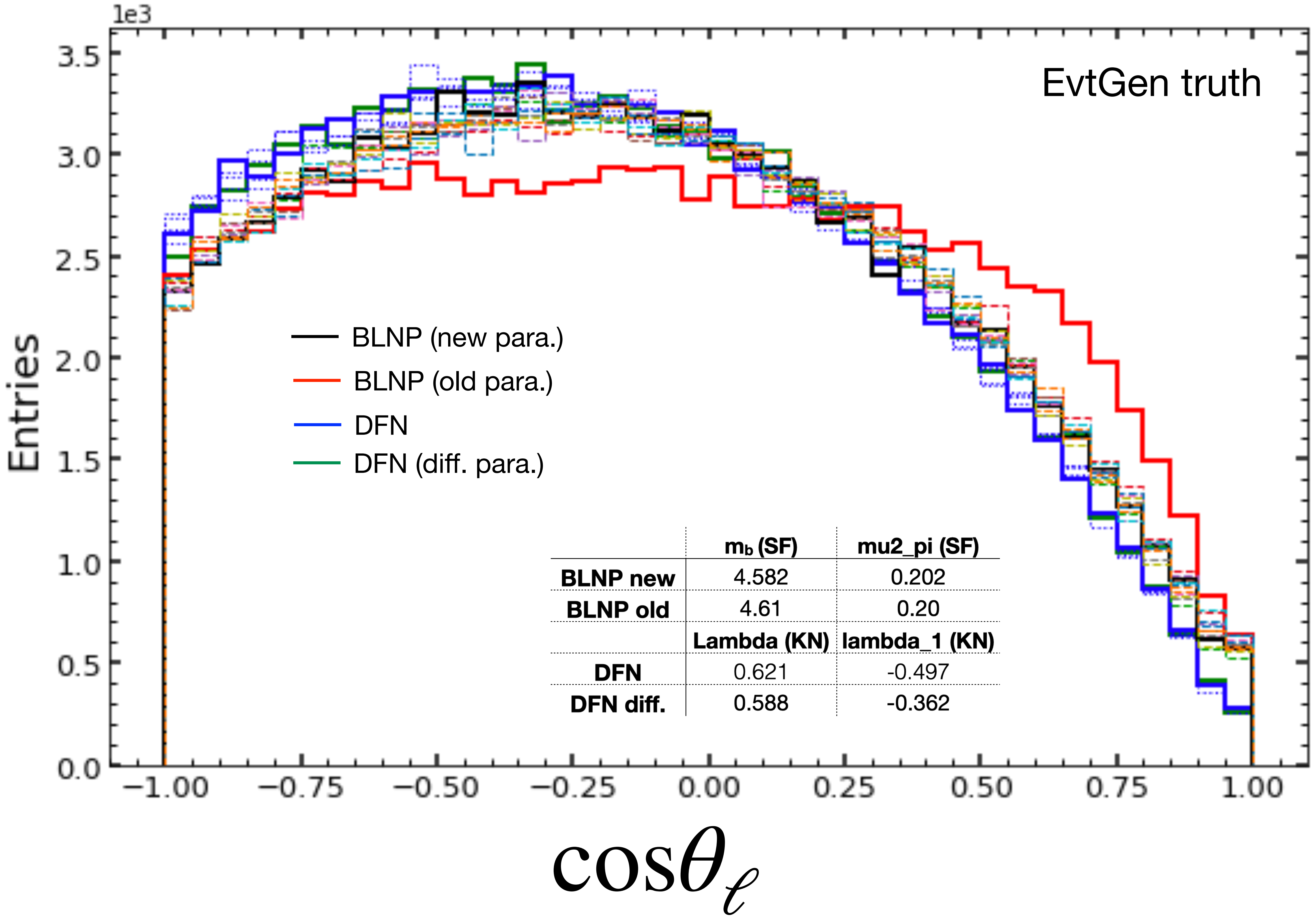
Improve resolution without dropping efficiency

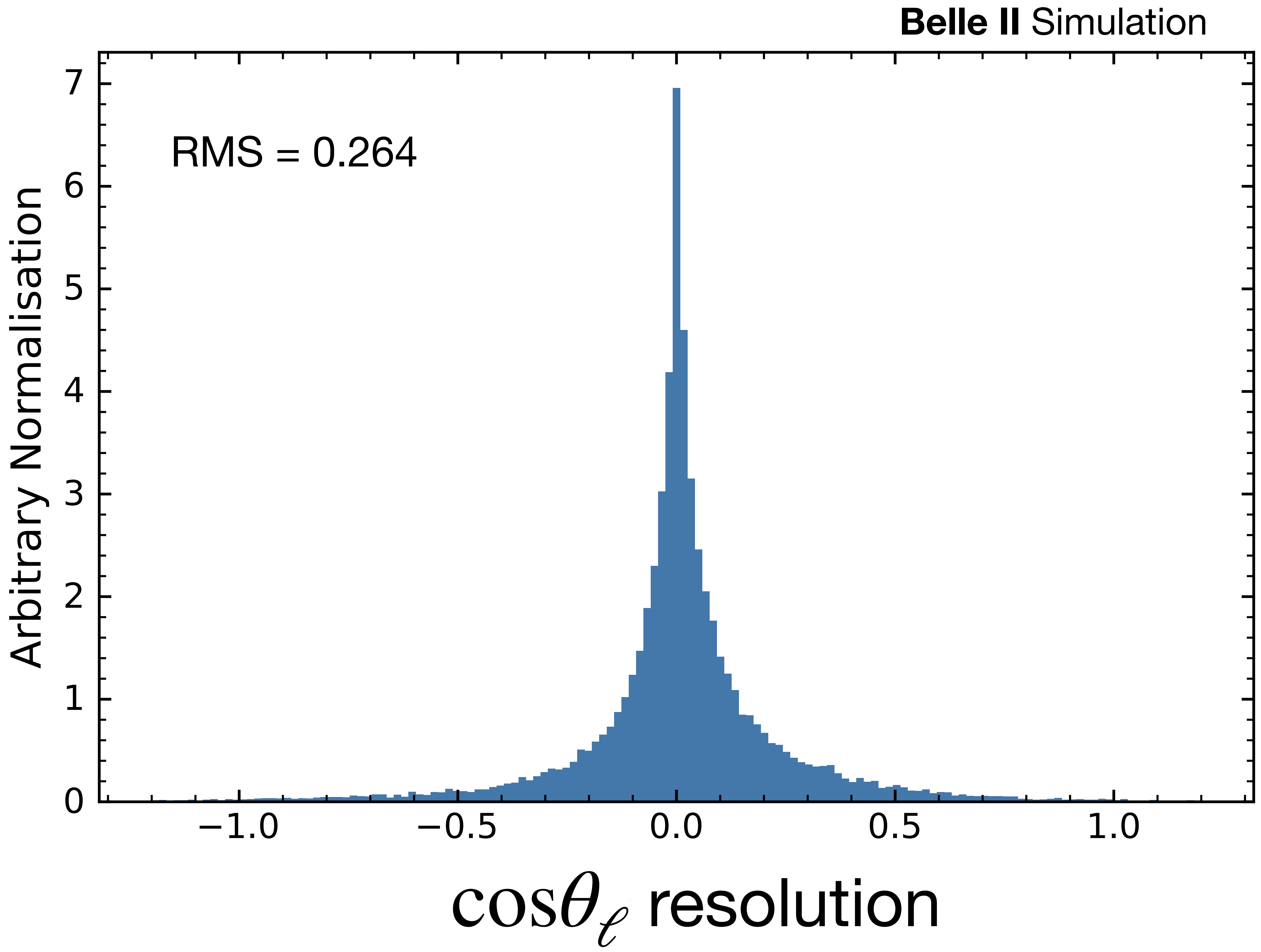




Differential Spectra of $B \to X_u \ell \nu$

In addition to Belle's $\mathrm{d}\mathscr{B}/\mathrm{d}x, x = M_X, M_X^2, q^2, P^\pm, E_l^B$, **angular variable cosθ_ℓ** (the angle between lepton and B in the rest frame of *W*) can be measured => extract forward-backward asymmetry



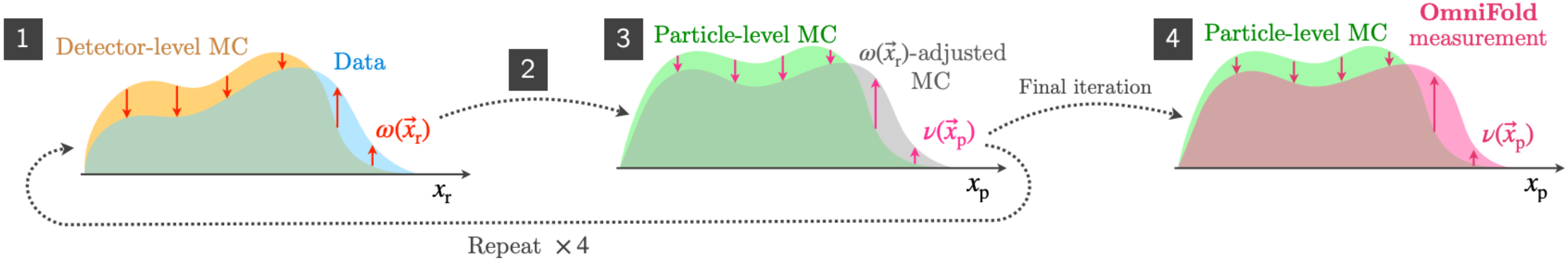


Differential Spectra of $B \to X_u \ell \nu$

Another option of unfolding:

OmniFold, unbinned & multi-dimensional unfolding based on ML [PRL 124, 182001 (2020)]

One of the recent applications is "A simultaneous unbinned differential cross section measurement of twenty-four Z+jets kinematic observables with the ATLAS detector"
[arXiv:2405.20041]



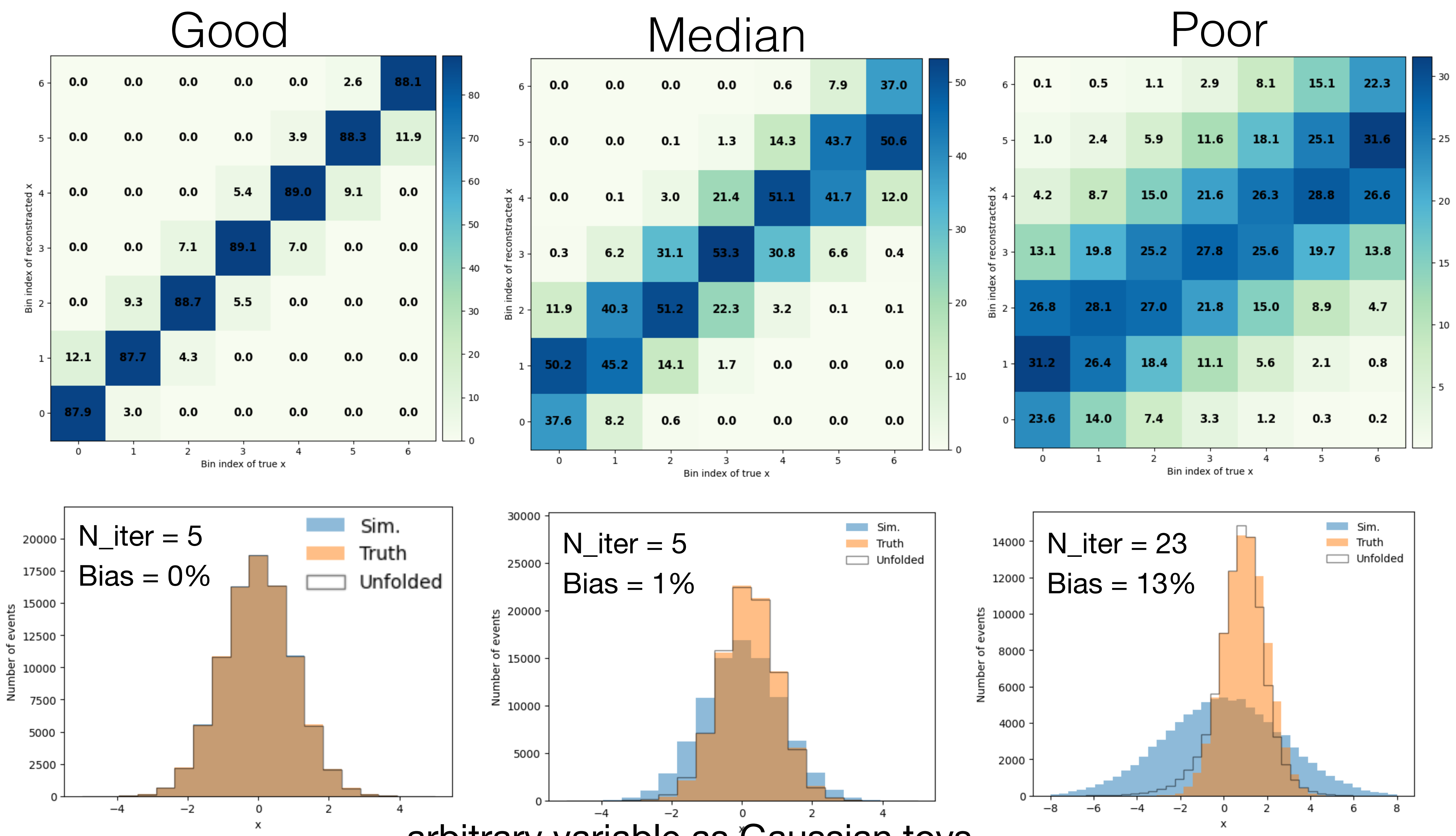
If this can be applied for $B \to X_u \ell \nu$, users will obtain **multi-variable differential spectra** with user-chosen bindings of user-chosen variables, and **all moments** with user-chosen thresholds of user-chosen variable **=> all you need at once**

Differential Spectra of $B \to X_u \ell \nu$

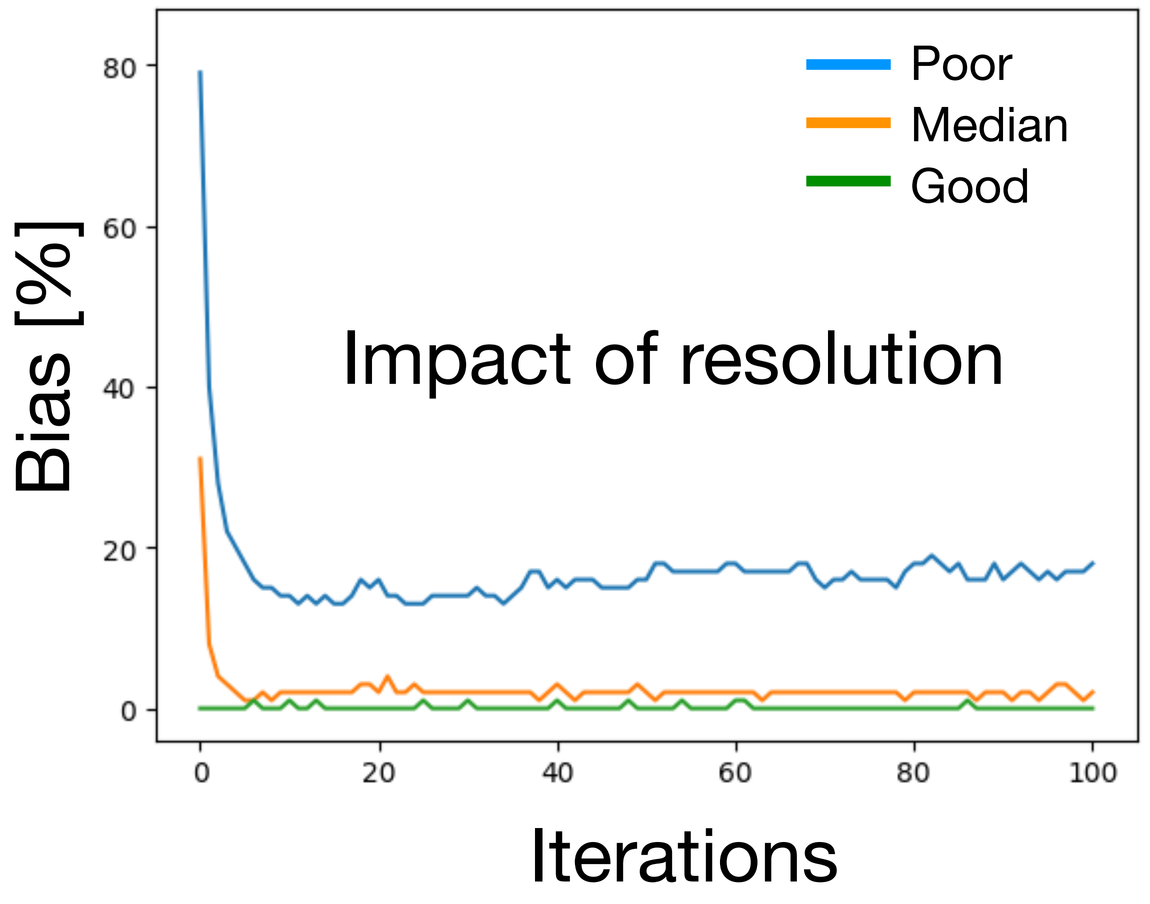
Another option of unfolding:

OmniFold, unbinned & multi-dimensional unfolding based on ML [PRL 124, 182001 (2020)]

1D test with 3 situations of resolution, compare bias: $\sum_i \left(\left| N^i_{true} - N^i_{unfold} \right| \right) / N_{true}$



arbitrary variable as Gaussian toys
(more sophisticated studies are planned)



Differential Spectra of $B \to X_u \ell \nu$

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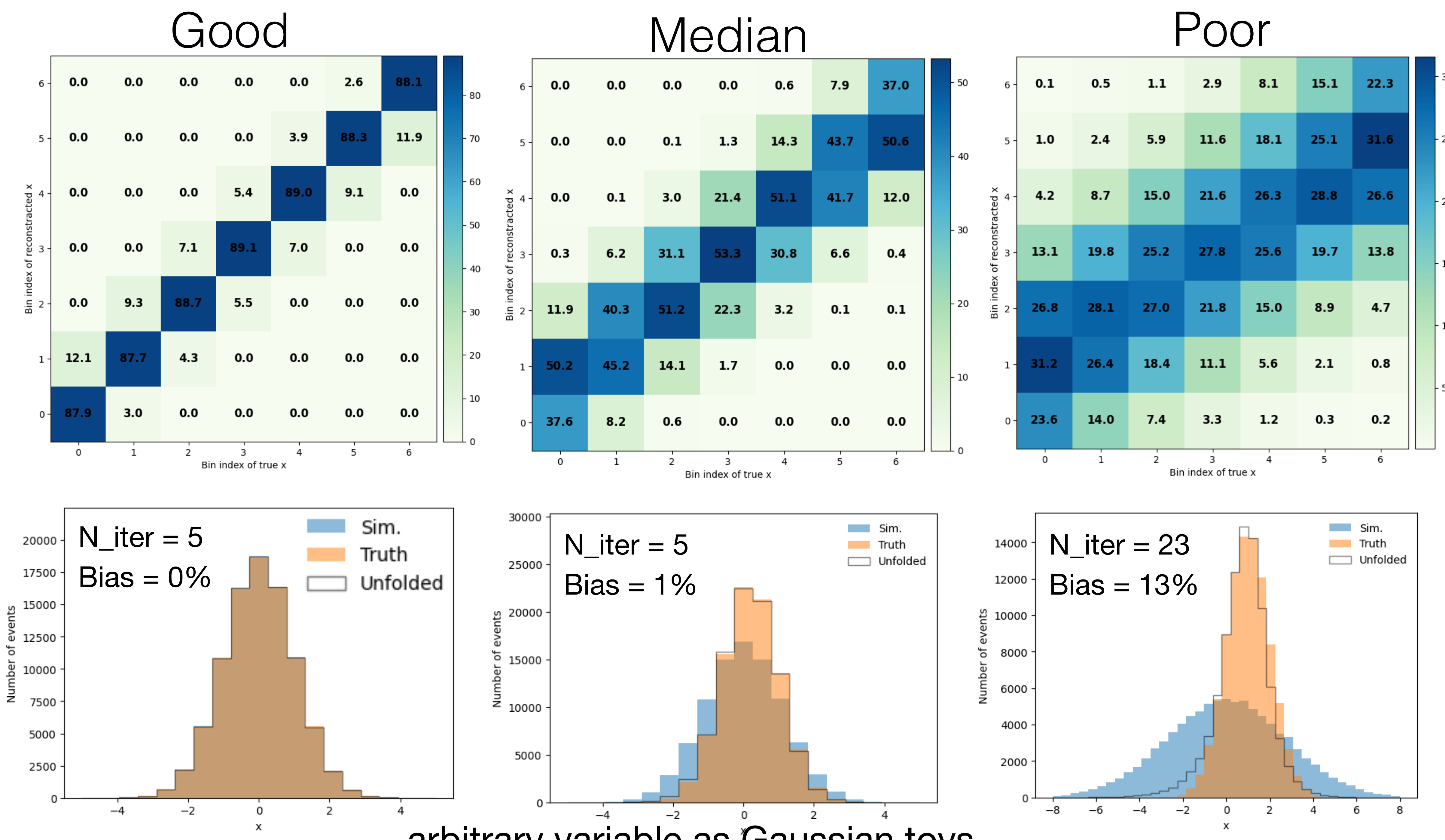

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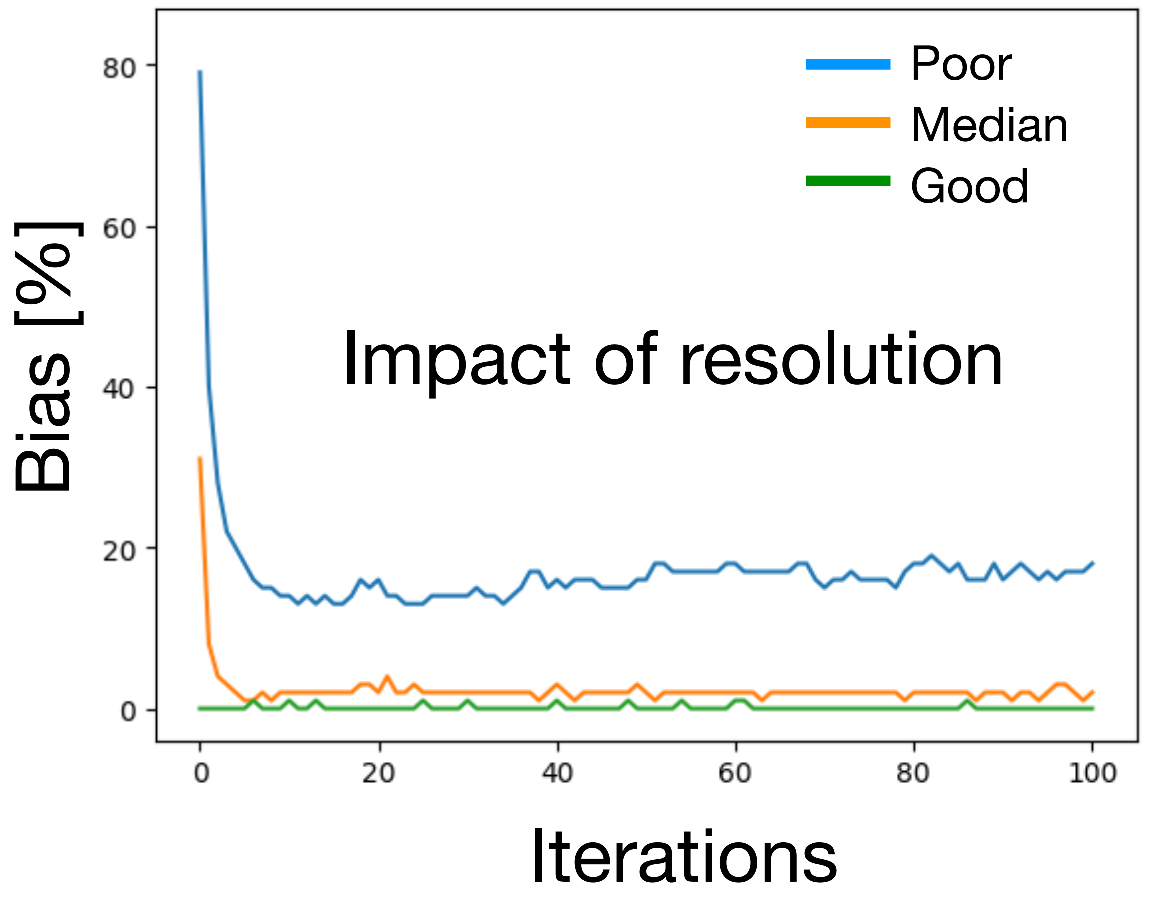
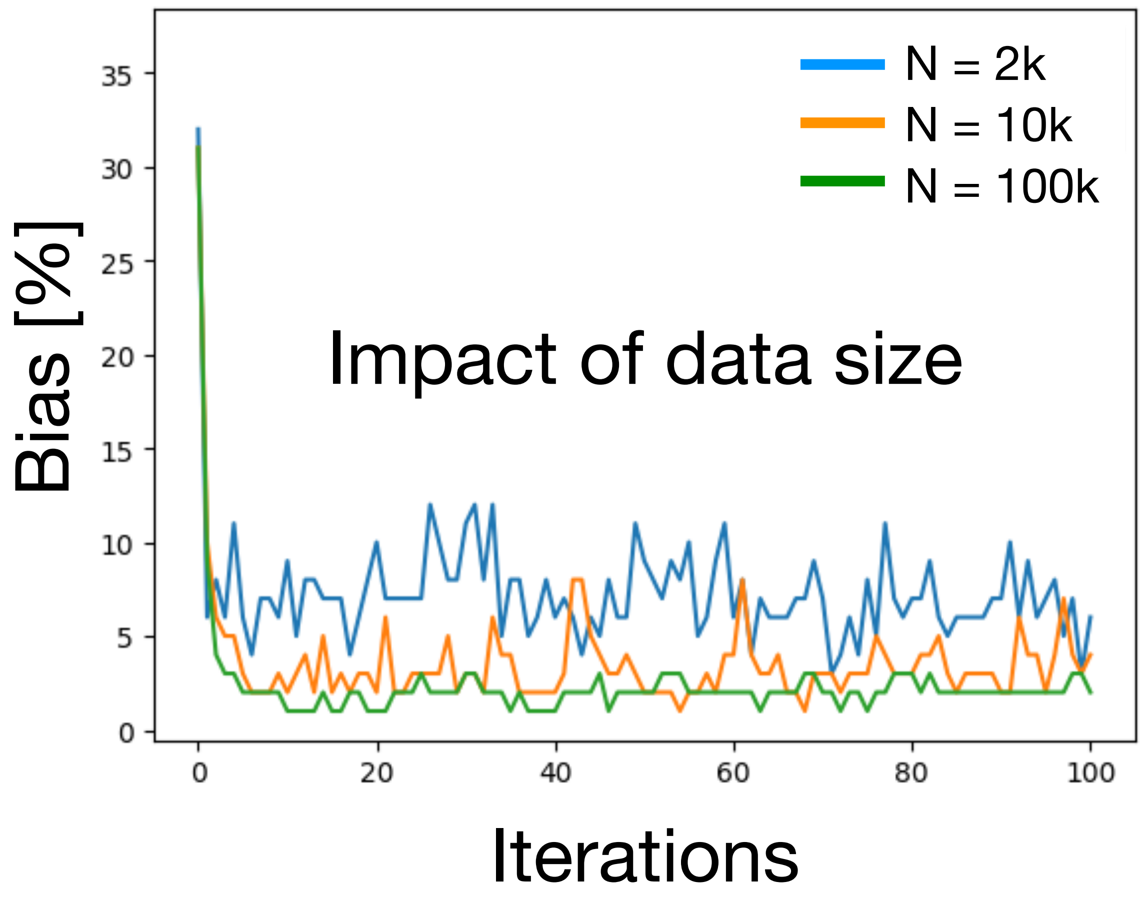




- Much <u>less sensitive to resolutions</u> than traditional binned unfolding
- Need large data set to achieve stable & reliable trainings, e.g. N $\gtrsim$ 10k
- Consider this new method for future 2 ab^{-1}@Belle II

Weak Annihilation Contribution in $B \to X_u \ell \nu$

Searching for Weak Annihilation in $B \to X_u \ell \nu$

- Last direct measurement is from CLEO@15.5 fb^{-1} [PRL 96, 121801 (2006)] with untagged: $\Gamma_{\mathbf{WA}}/\Gamma_{b \to u} < 7.4\%$ at 90% CL
- Aim for a new direct measurement and separate B$^+$ and B^0 from Belle II

Injected WA Model	
$M^2_{\ell\bar{\nu}}$ [GeV^2]	$\Gamma_{\ell\bar{\nu}}$ [GeV]
19.94	0.02
19.94	0.14
19.94	0.52
22.11	0.02
22.11	0.14
22.11	0.52
24.28	0.02
24.28	0.14
24.28	0.52

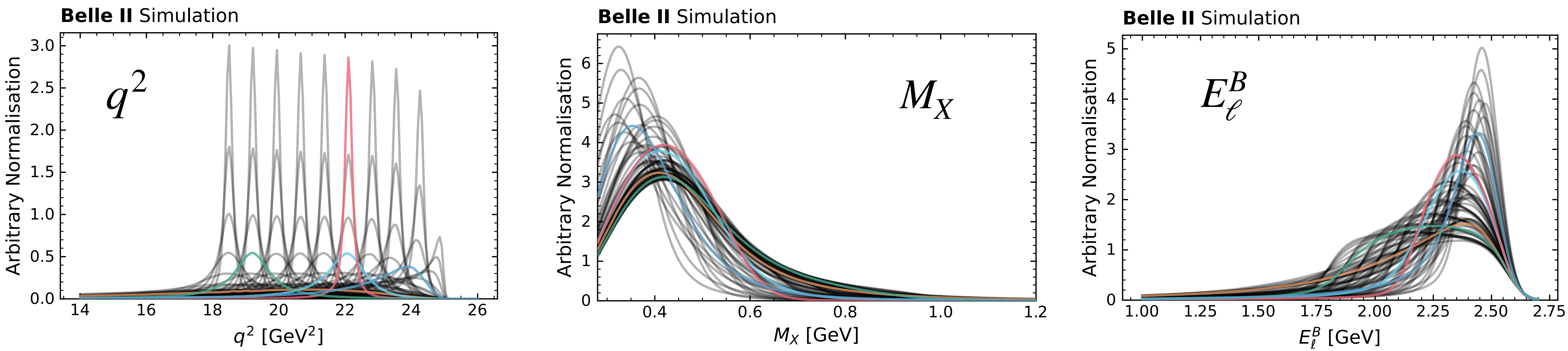
$B \to X_u W^*$ (PHSP)

$\quad | \quad \hookrightarrow \ell\nu$ (SLN)

$\quad \hookrightarrow \geq 2\pi$ (PYTHIA)

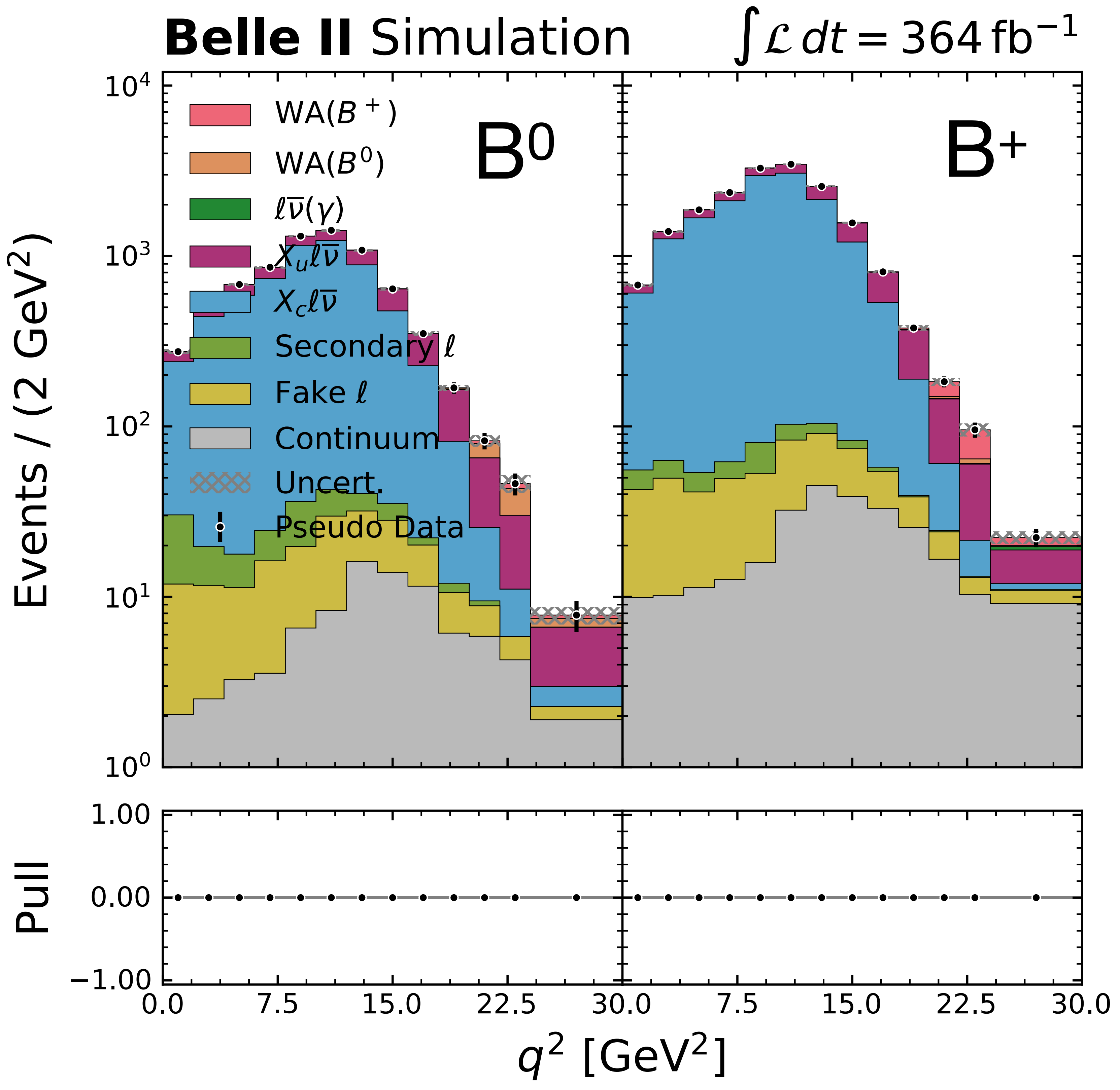
Soft hadronic Xu:
$M = 420^{+140}_{-130}$ MeV,
$\Gamma = 280 \pm 140$ MeV

Simulate WA models embodied $B \to X_u \ell \nu$ decays scanning various masses and widths of *W*



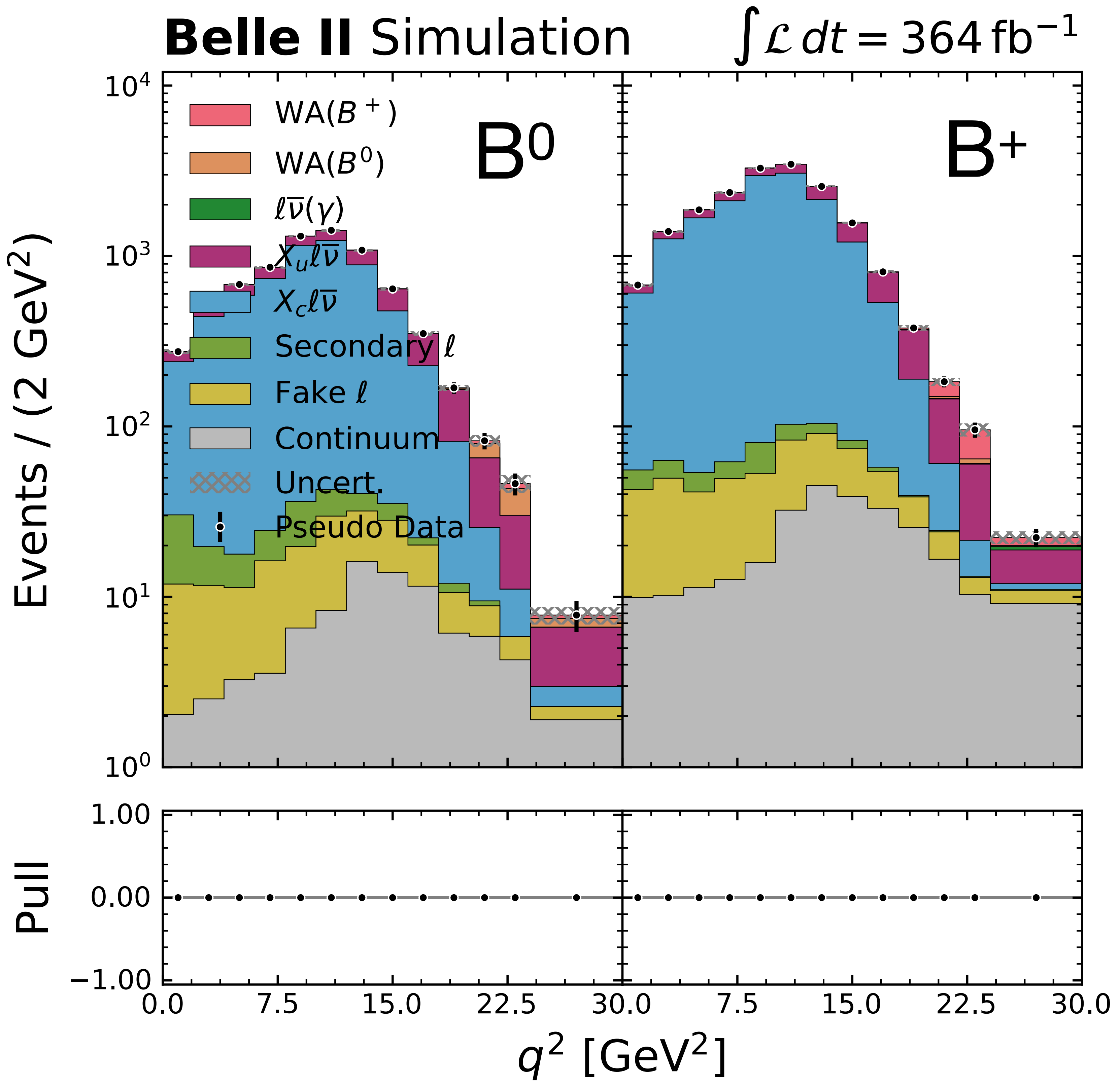
Searching for Weak Annihilation in $B \to X_u \ell \nu$

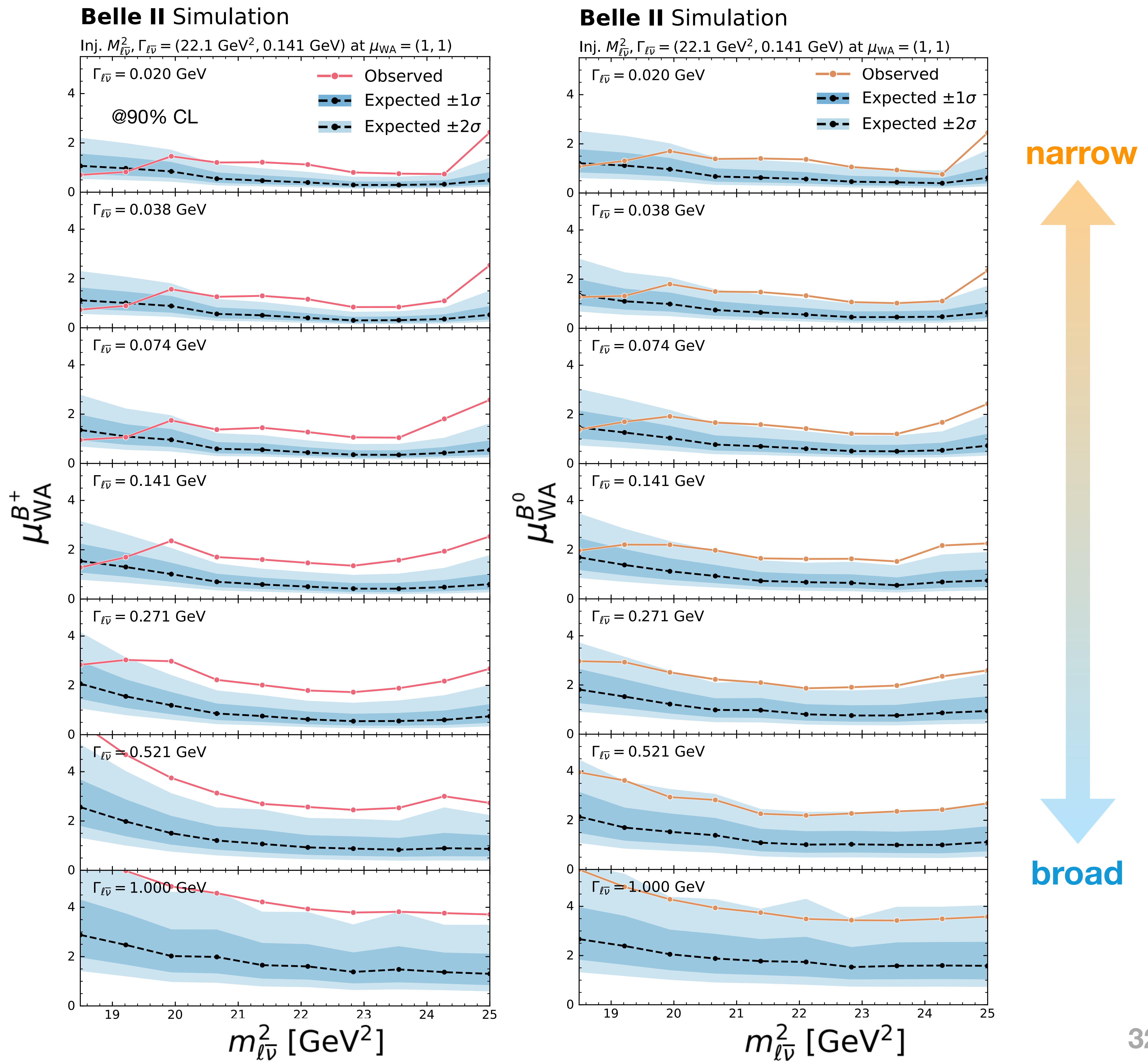
- Fit q² with various WA models embodied $B \to X_u \ell \nu$ decays to extract the normalised signal strength μ_{WA} (μ_{WA} = 1 for 1% of $\Gamma_{b \to u}$)



Searching for Weak Annihilation in $B \to X_u \ell \nu$

- Fit q² with various WA models embodied $B \to X_u \ell \nu$ decays to extract the normalised signal strength μ_{WA} (μ_{WA} = 1 for 1% of $\Gamma_{b \to u}$)
- Inject a WA model (e.g. m=4.7 GeV, Γ=0.141 GeV) with signal strength (1,1) to form a **pseudo data**
- **Fit** with **various WA configurations** (expected limit if no WA)





Simultaneous determination of excl. & incl. $|V_{ub}|$

First Simultaneous Determination of Incl. & Excl. |Vub|

PRL 131, 211801 (2023)

- Extract signal in $\mathbf{q^2}$: $\mathbf{N_{\pi^\pm}}$ for $B \to \pi \ell \nu$ and $B \to X_u \ell \nu$ simultaneously
- Fitter corporates experimental observation of templates' **normalisations** η and $B \to \pi \ell \nu$ **form factor (FF) parameters** $\mathbf{a^{+,0}}$

$$-2\log\mathscr{L} = -2\log\prod_i \text{Poisson}\left(\eta_{\text{obs}}, \eta_{\text{pred}}\cdot(1+\epsilon\cdot\theta)\right) + \theta\rho_\theta^{-1}\theta^T + \chi^2_{\text{FF}}$$

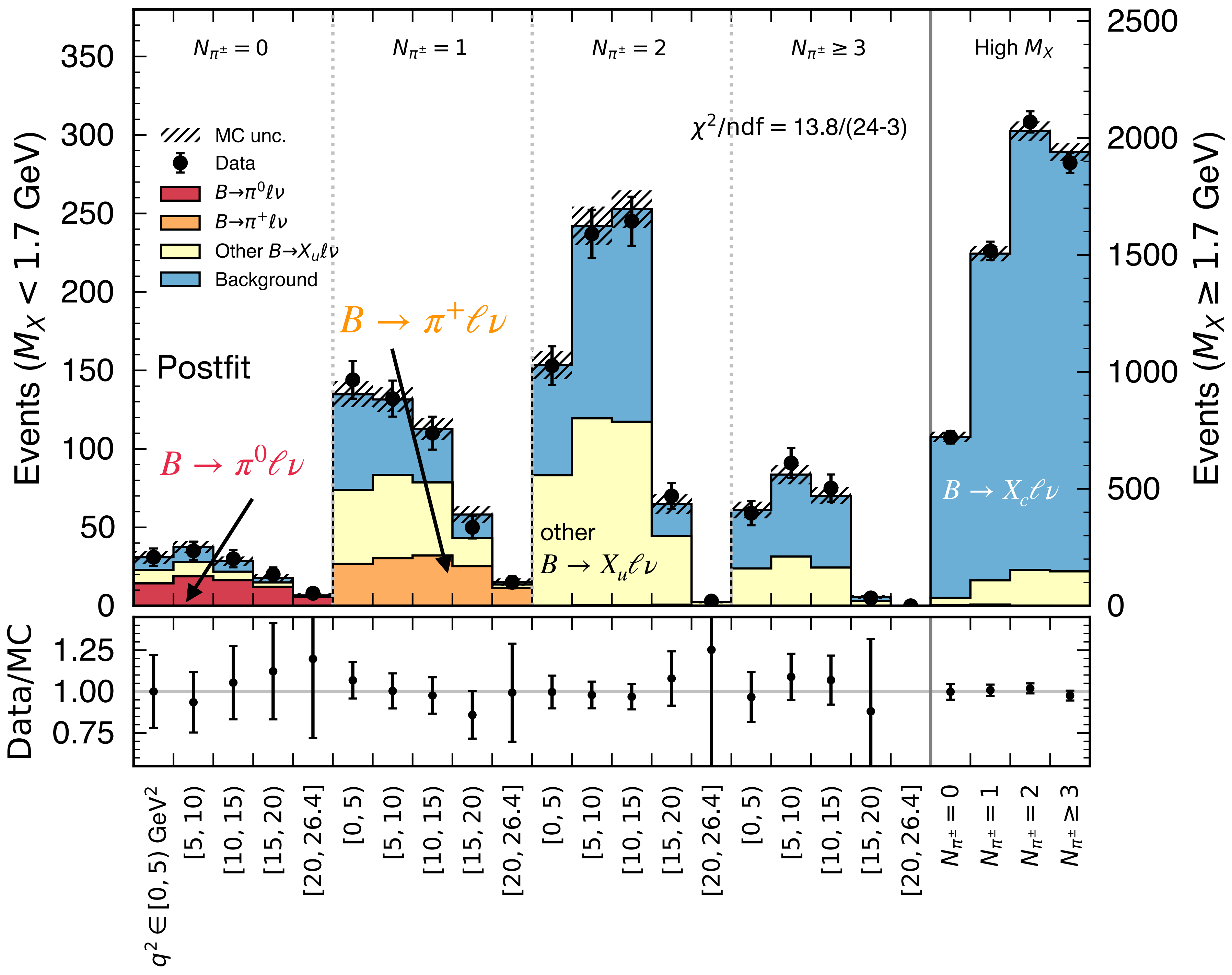
Constraints on BCL parameters , input taken from LQCD / LQCD+exp fits in FLAG 2021

Differential decay rates

Acceptance & reco. efficiency

Forward-folding q^2

PRL 131, 211801 (2023)



Normalisation can be linked with isospin relation, or floating separately **(nominal: linked)**

$B \to \pi^0\ell\nu$
$B \to \pi^+\ell\nu$
other $B \to X_u\ell\nu$
all background

Shape described by BCL para.

$$\mathscr{B}(B\to\pi^0\ell\nu) + \mathscr{B}(B\to\pi^+\ell\nu) + \mathscr{B}(B\to X_u^{\text{other}}\ell\nu) = \mathscr{B}(B\to X_u\ell\nu)$$

$$\Delta\mathscr{B}(B\to X_u\ell\nu) = \mathscr{B}(B\to X_u\ell\nu)\cdot\epsilon_{\Delta\text{PS:}E_\ell^B>1\text{GeV}}$$

$$\left|V_{ub}^{\text{excl.}}\right| = \sqrt{\frac{\mathscr{B}(B\to\pi\ell\nu)}{\tau_B\cdot\Gamma_{\text{FF}}}}$$

$$\left|V_{ub}^{\text{incl.}}\right| = \sqrt{\frac{\Delta\mathscr{B}(B\to X_u\ell\nu)}{\tau_B\cdot\Delta\Gamma_{\text{GGOU}}}}$$

First Simultaneous Determination of Incl. & Excl. $|V_{ub}|$

BELLE

PRL 131, 211801 (2023)

$$\left|V_{ub}^{\text{excl.}}\right| = (3.78 \pm 0.23_{\text{stat}} \pm 0.16_{\text{syst}} \pm 0.14_{\text{theo}}) \times 10^{-3} \quad \text{(LQCD + exp.)}$$

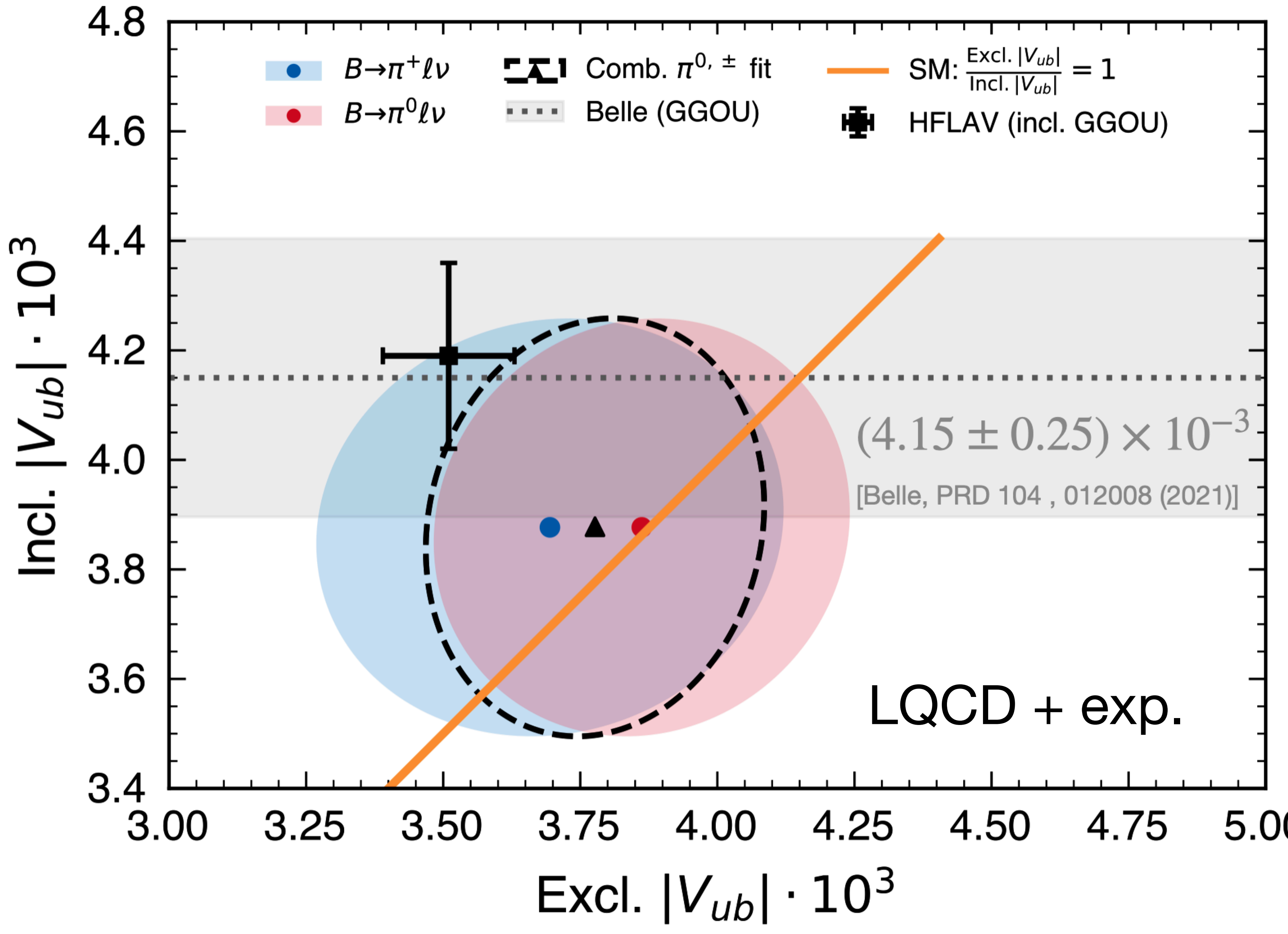
$$\left|V_{ub}^{\text{incl.}}\right| = (3.88 \pm 0.20_{\text{stat}} \pm 0.31_{\text{syst}} \pm 0.09_{\text{theo}}) \times 10^{-3} \quad \text{(LQCD + exp.)}$$

Ratio = $0.97 \pm 0.12 \quad (\rho = 0.11)$

$$\left|V_{ub}^{\text{excl.}}\right| = (4.05 \pm 0.30_{\text{stat}} \pm 0.16_{\text{syst}} \pm 0.16_{\text{theo}}) \times 10^{-3} \quad \text{(LQCD)}$$

$$\left|V_{ub}^{\text{incl.}}\right| = (3.87 \pm 0.20_{\text{stat}} \pm 0.31_{\text{syst}} \pm 0.09_{\text{theo}}) \times 10^{-3} \quad \text{(LQCD)}$$

Ratio = $1.05 \pm 0.14 \quad (\rho = 0.07)$



Weighted average of excl. & incl. :

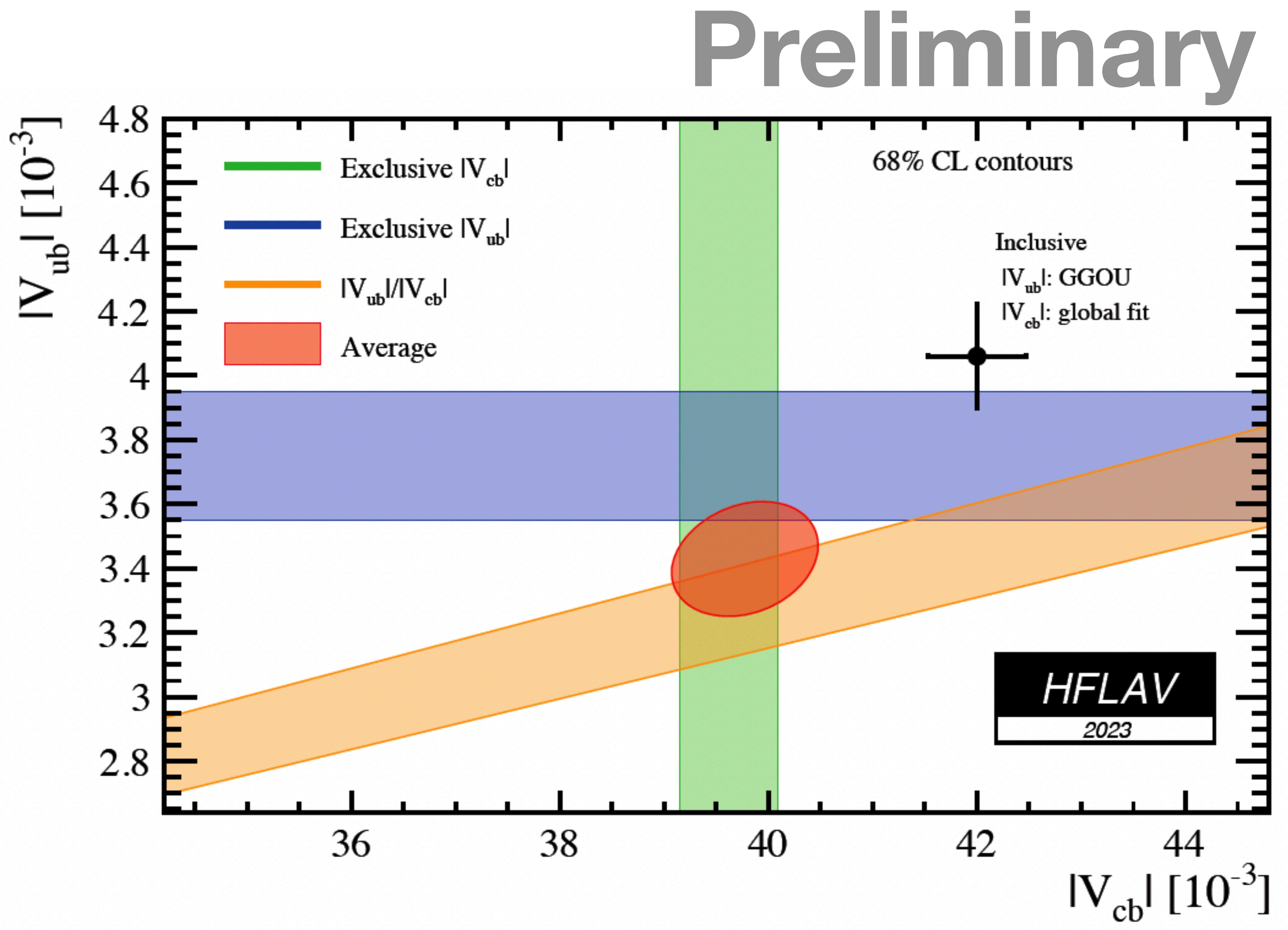
$$\left|V_{ub}\right| = (3.84 \pm 0.26) \times 10^{-3} \quad \text{(LQCD + exp.)}$$

$$\left|V_{ub}\right| = (3.96 \pm 0.27) \times 10^{-3} \quad \text{(LQCD)}$$

CKM global fit (w/o $|V_{ub}|$): $(3.64 \pm 0.07) \times 10^{-3}$, **compatible within 0.8σ and 1.2σ, respectively**

Summary & Outlook

- Beneficial experience from Belle has been carried over to ongoing Belle II studies
- Expected compatible precisions between Belle and Belle II Run1
- Resolving all challenges in $B \to X_u \ell \nu$ measurements require continuous efforts from **experiment** and **theory**
- Improvements on **inclusive modelling** are urgently needed!!

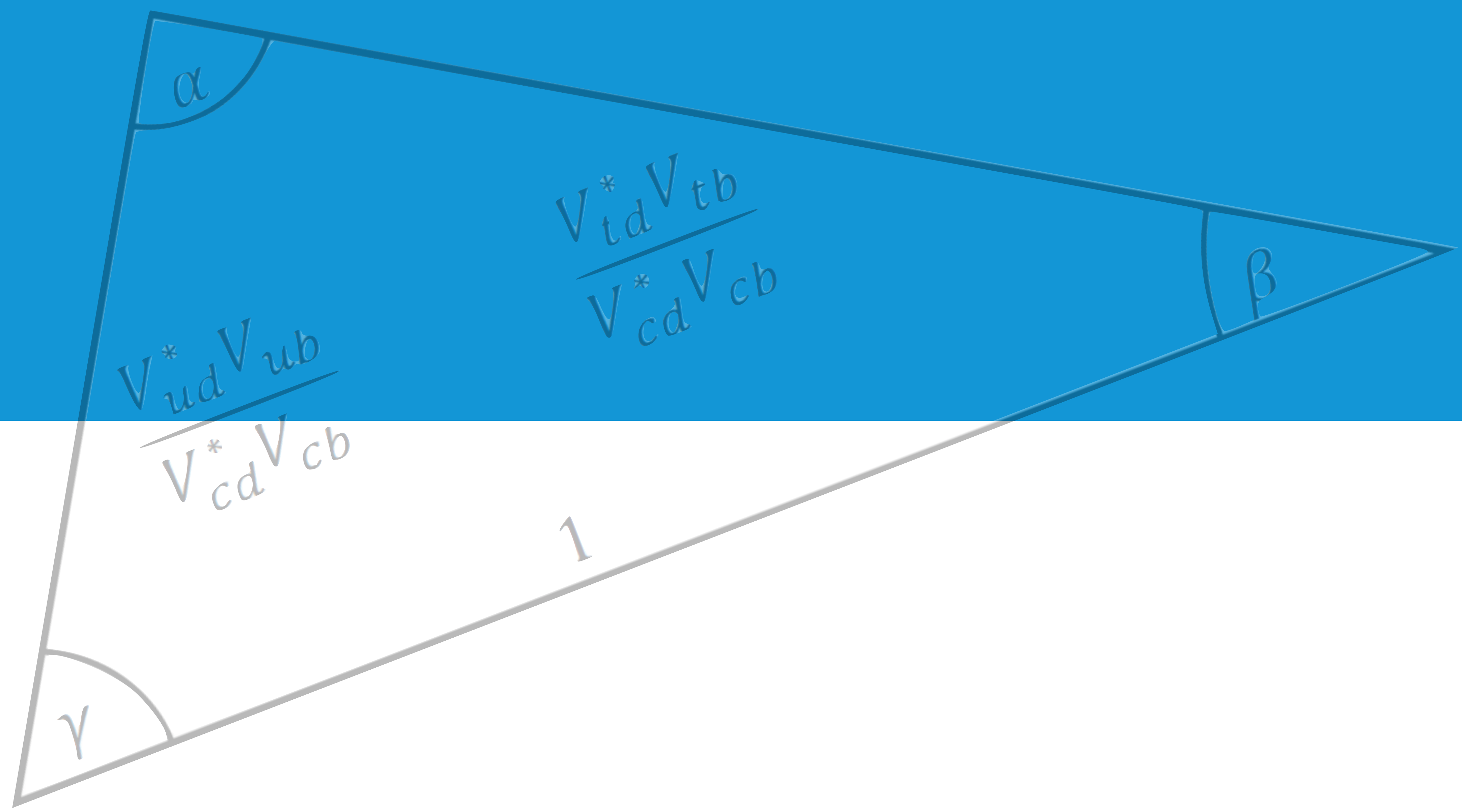


Can we say the 'Vub puzzle' has been resolved?

If not, what key elements are still missing?

THANK YOU

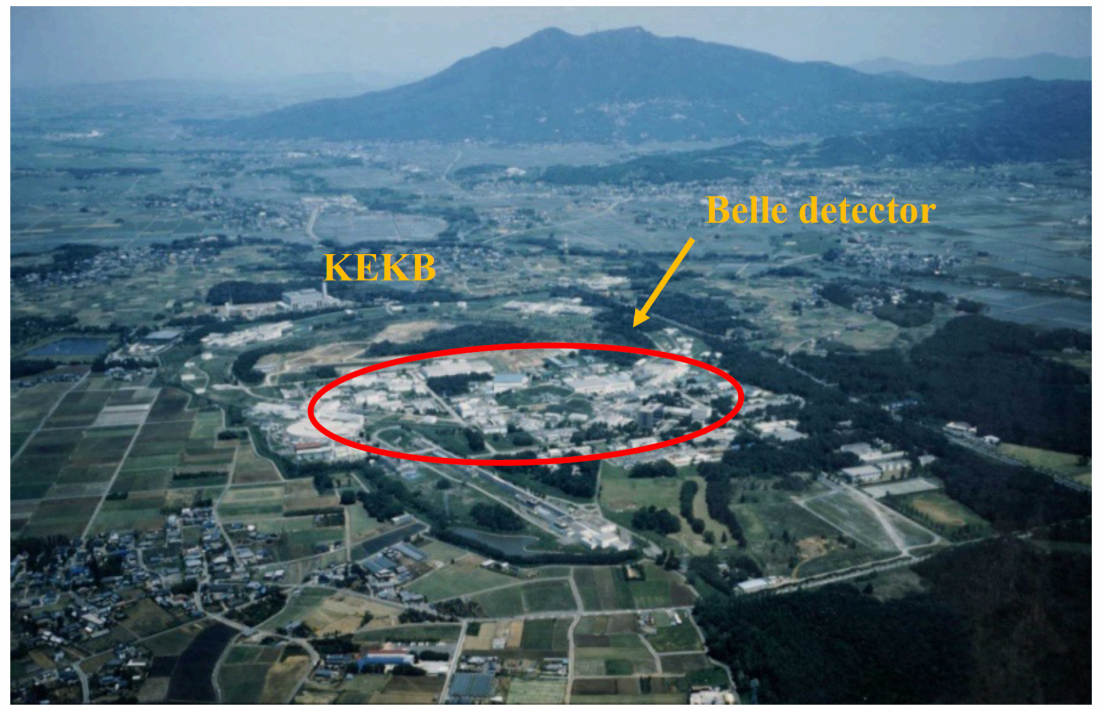
THANK YOU

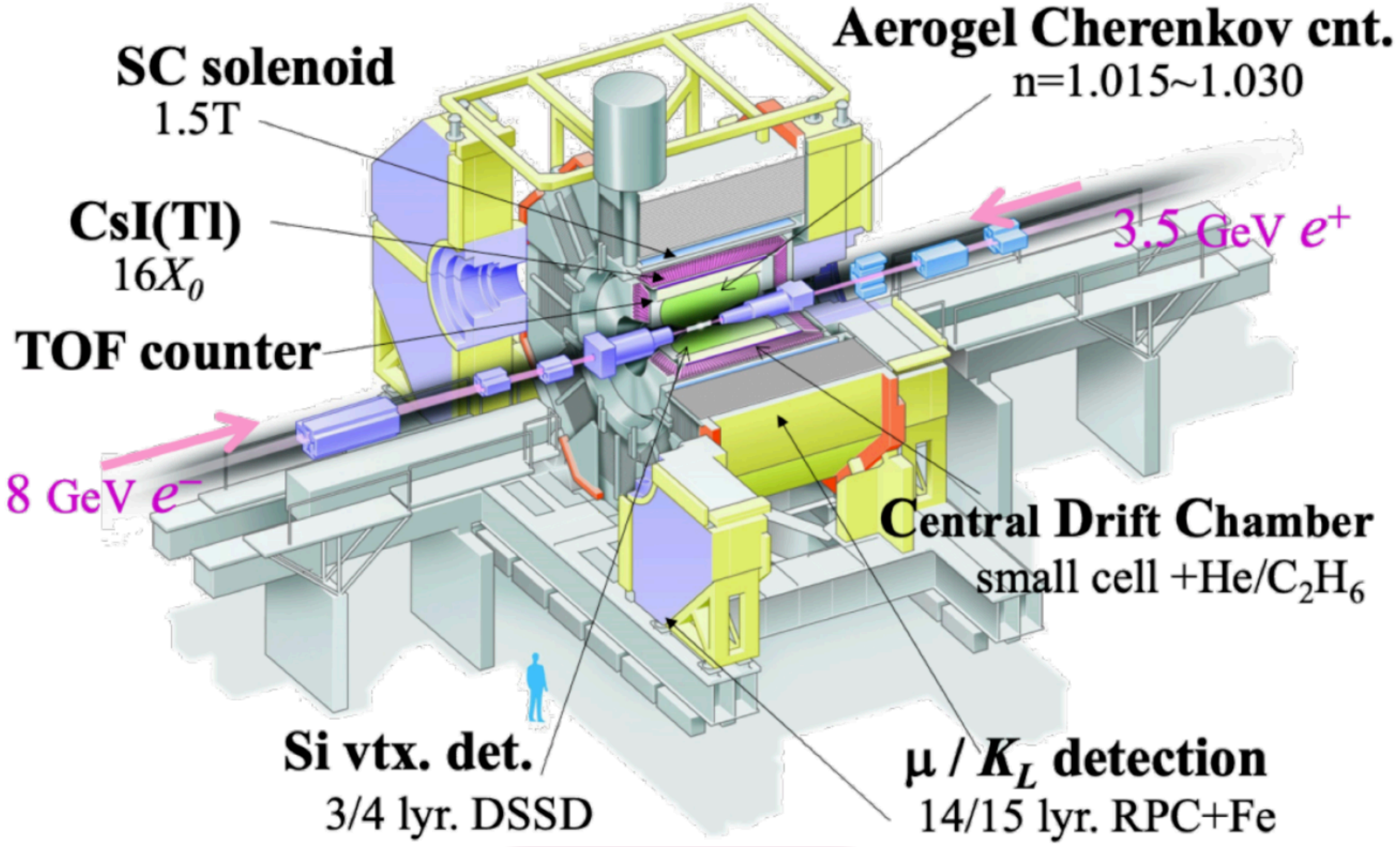


Belle Experiment

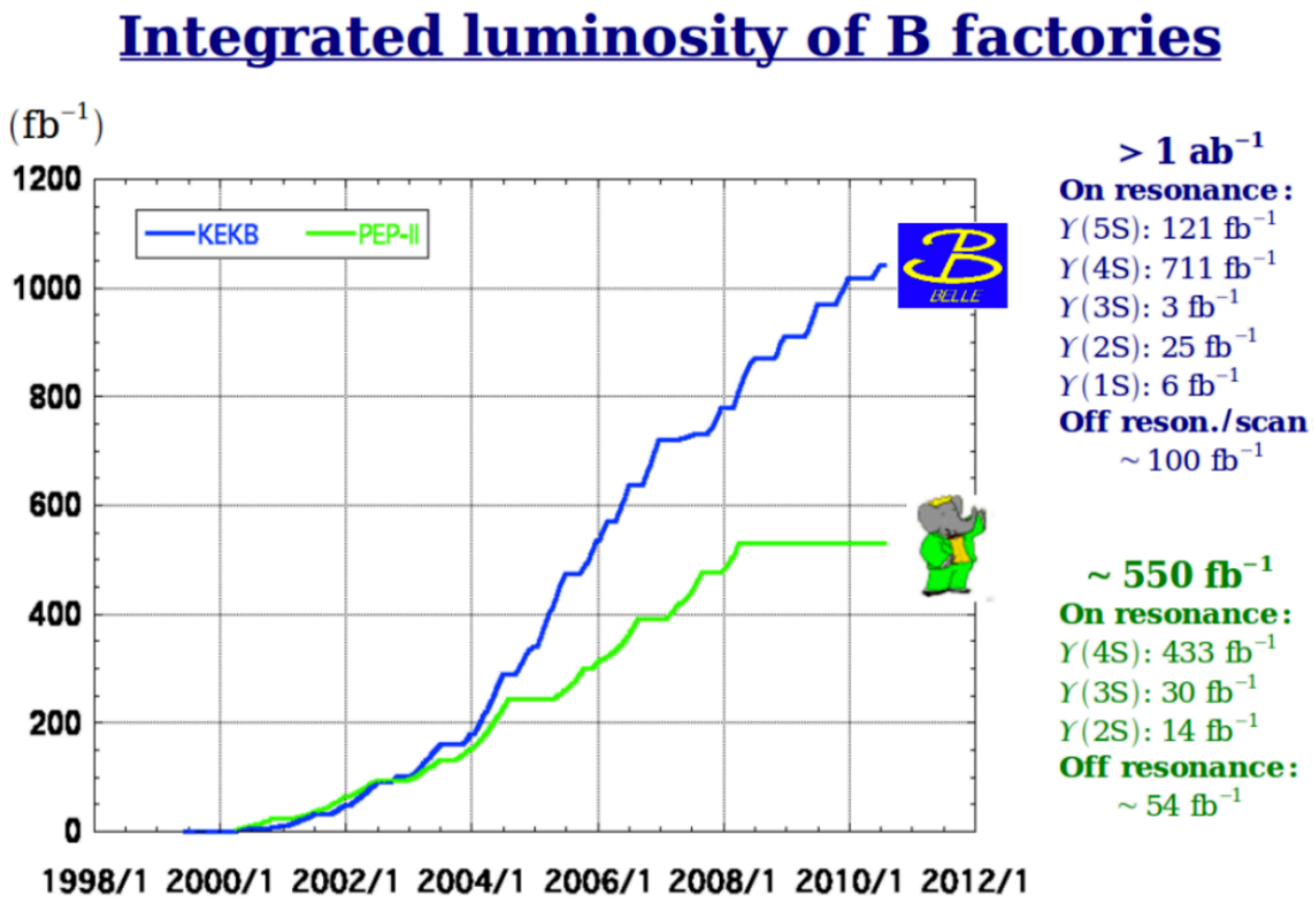
- KEKB is an asymmetric-energy e^+e^- collider operating near $\Upsilon(4S)$ mass peak
- Belle detector: nearly 4π coverage, good performances on momentum/vertex resolution, particle identification
- Unique advantages for analysing inclusive decays and process involving multiple neutrals

KEK
Tsukuba, Japan



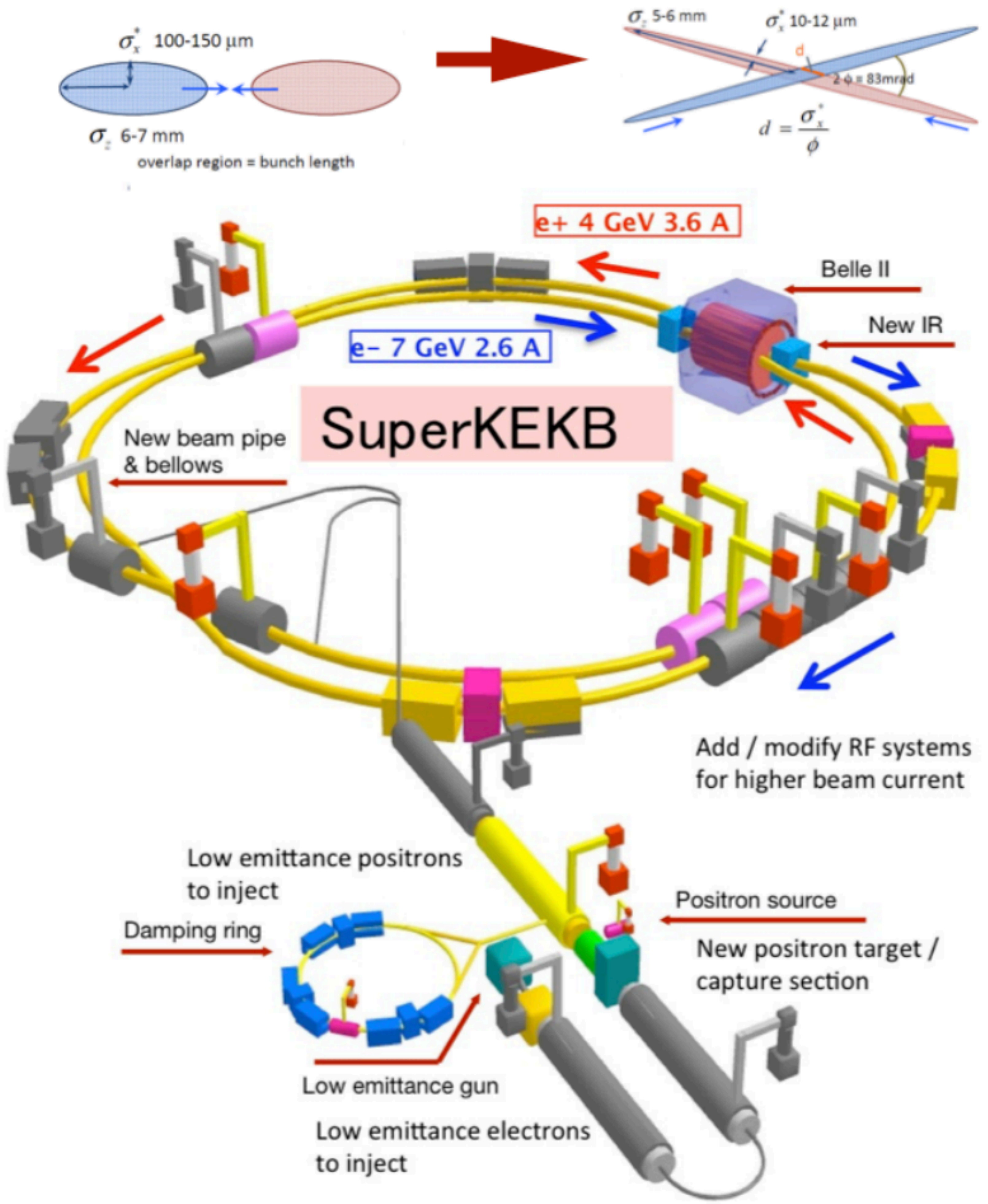


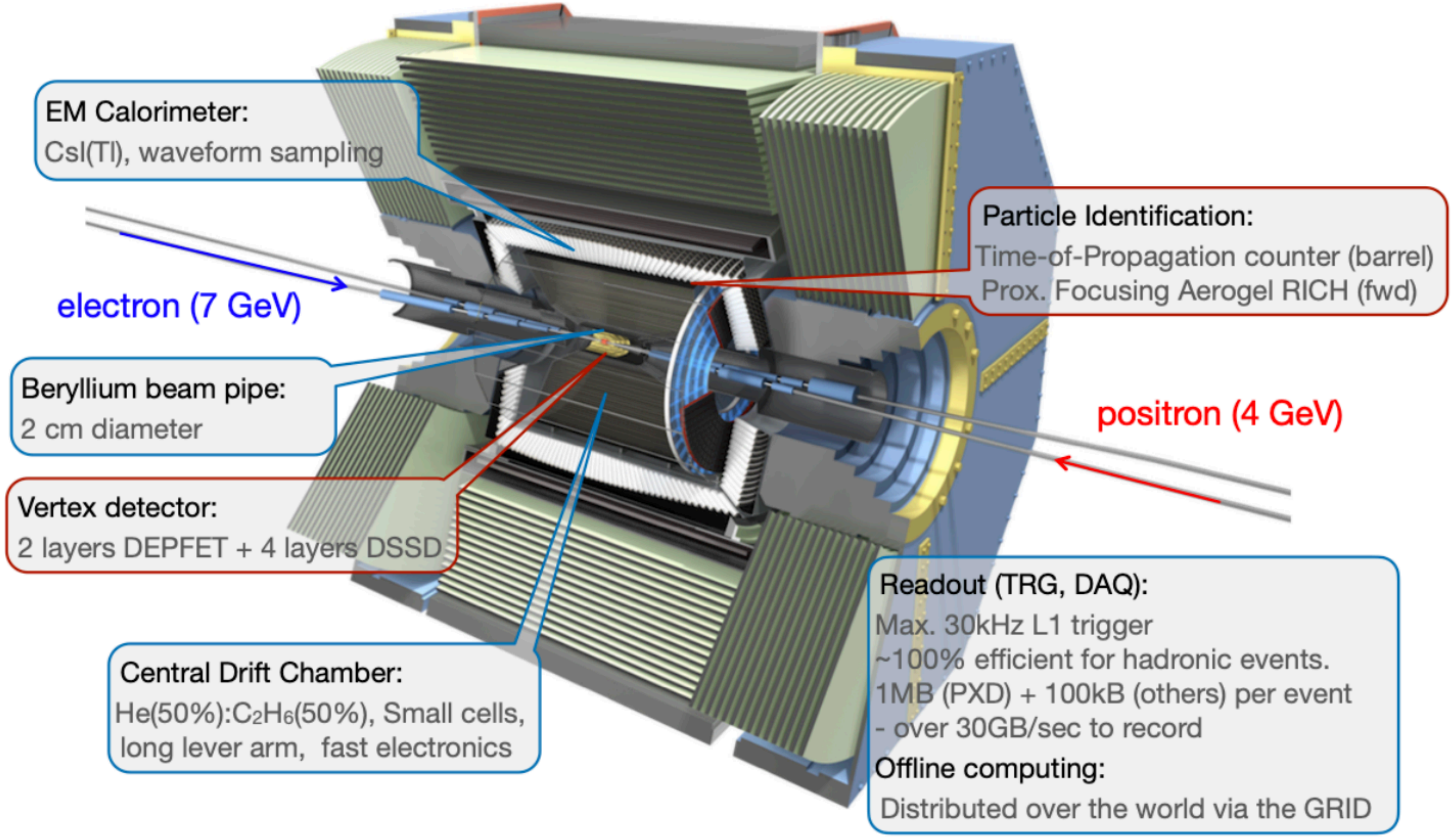
PEAK
L = 2.1 x $10^{34}/cm^2/sec$



Belle II Experiment

Upgraded detector and accelerator





arXiv:1011.0352 [physics.ins-det]

Hybrid Model of $B \to X_u \ell \nu$

Hybrid MC is a **combination** of **resonances** (exclusive decays) and **non-resonant** contribution in the inclusive $B \to X_u \ell \nu$ decays

- EvtGen simulation:

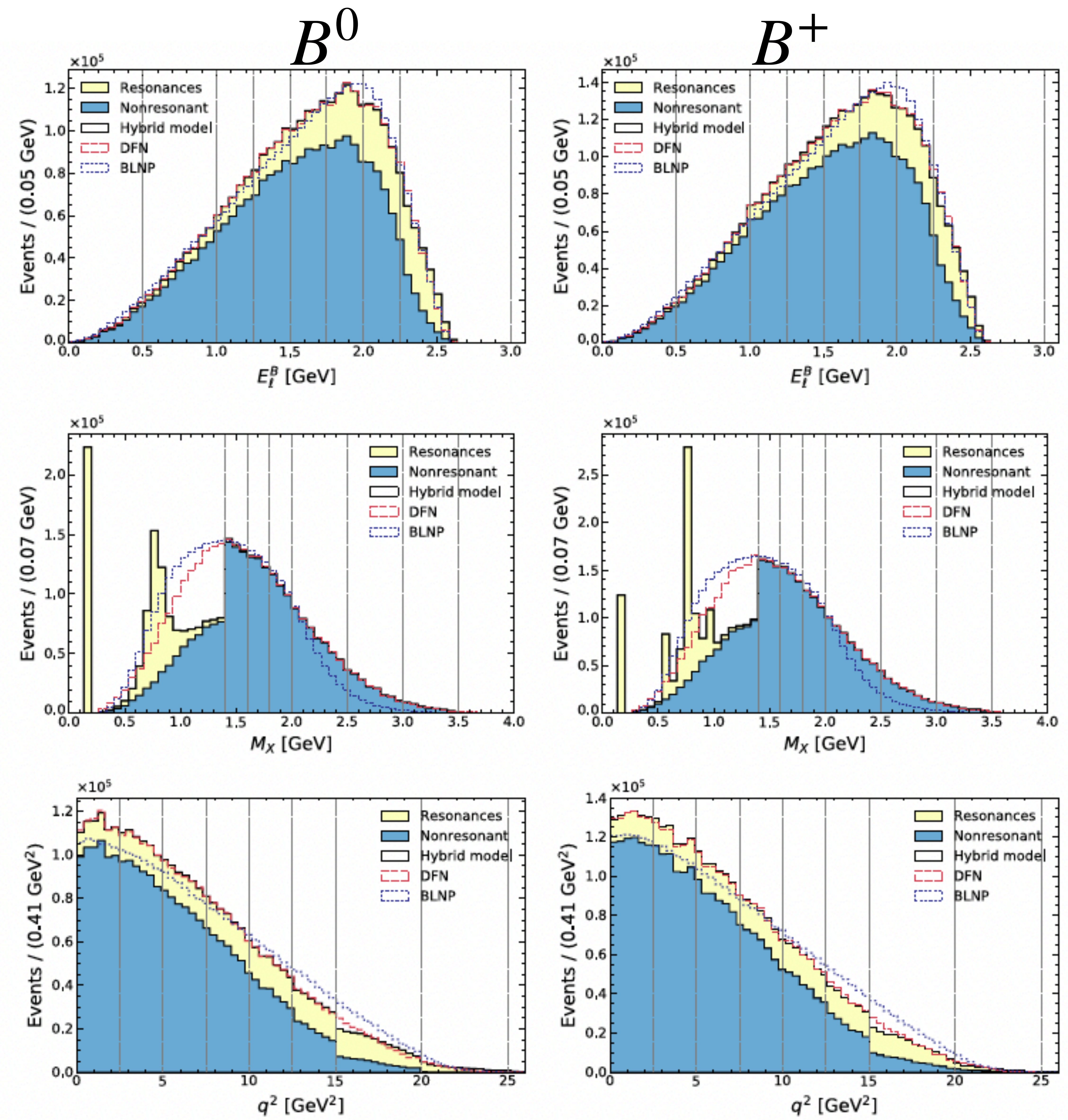
(1) exclusive modes $B \to (\pi, \rho, \omega, \eta^{(\prime)}) \ell \nu$ with latest WA form factors & branching fractions

(2) fully inclusive $B \to X_u \ell \nu$ (only non-resonant shapes, e.g. BLNP, GGOU)

- Calculate hybrid weights to mix resonance & non-res. in **3D binning of** (q^2, E_ℓ^B, M_X) to recover total $\mathscr{B}(B \to X_u \ell \nu)$ in each bin

$$H_i = R_i + \omega_i N_i$$

- Systematic uncertainties include the impact from exclusive FFs & BRs, total $\mathscr{B}(B \to X_u \ell \nu)$, inclusive models



EvtGen truth

First Simultaneous Determination of Incl. & Excl. $|V_{ub}|$

PRL 131, 211801 (2023)

Leading Systematic Uncertainties

Sources	Relative Syst. Uncertainty
Exclusive mode	$\mathscr{B}(B \to \pi \ell \nu)$
Tagging efficiency	**4.1%**
$B \to X_u \ell \nu$ modelling	**3.5%**
$B \to X_c \ell \nu$ modelling	**1.2%**
Inclusive mode	$\Delta\mathscr{B}(B \to X_u \ell \nu)$
$B \to X_u \ell \nu$ modelling	**10.9%**
Fragmentation	**5.3%**
Tagging efficiency	**3.4%**
$B \to X_c \ell \nu$ modelling	**2.8%**

Values from HFLAV 2021

[arXiv:2206.07501]

- Endpoint of electron spectra from BaBar (2016)

$m_b^{\mathrm{SF}} = 4.561 \pm 0.023$, $\mu_\pi^{2(\mathrm{SF})} = 0.149 \pm 0.04$, $\mu_i = 2.0$, $\mu_h = 4.25$

7.4.1 BLNP

Bosch, Lange, Neubert and Paz (BLNP) [594–597] provide theoretical expressions for the triple differential decay rate for $B \to X_u \ell^+ \nu_\ell$ events, incorporating all known contributions, whilst smoothly interpolating between the "shape-function region" of large hadronic energy and small invariant mass, and the "OPE region" in which all hadronic kinematical variables scale with the b-quark mass. BLNP assign uncertainties to the b-quark mass, which enters through the leading shape function, to sub-leading shape function forms, to possible weak annihilation contribution, and to matching scales. The BLNP calculation uses the shape function renormalization scheme; the heavy quark parameters determined from the global fit in the kinetic scheme, described in 7.2.2, were therefore translated into the shape function scheme by using a prescription by Neubert [598, 599]. The resulting parameters are $m_b(\mathrm{SF}) = (4.582 \pm 0.023 \pm 0.018)$ GeV, $\mu_\pi^2(\mathrm{SF}) = (0.202 \pm 0.089^{+0.020}_{-0.040})$ GeV/c^2, where the second uncertainty is due to the scheme translation. The extracted values of $|V_{ub}|$ for each measurement along with their average are given in Table 93 and illustrated in Fig. 63(a). The total uncertainty is $^{+5.6}_{-5.7}\%$ and is due to: statistics ($^{+1.5}_{-1.6}\%$), detector effects ($^{+1.7}_{-1.7}\%$), $B \to X_c \ell^+ \nu_\ell$ model ($^{+0.9}_{-1.0}\%$), $B \to X_u \ell^+ \nu_\ell$ model ($^{+1.8}_{-1.8}\%$), heavy quark parameters ($^{+2.7}_{-2.8}\%$), SF functional form ($^{+0.1}_{-0.3}\%$), sub-leading shape functions ($^{+0.8}_{-0.8}\%$), matching scales in BLNP μ, μ_i, μ_h ($^{+3.8}_{-3.8}\%$), and weak annihilation ($^{+0.0}_{-0.7}\%$). The error assigned to the matching scales is the source of the largest uncertainty, while the uncertainty due to HQE parameters (b-quark mass and $\mu_\pi^2(\mathrm{SF})$) is second. The uncertainty due to weak annihilation is assumed to be asymmetric, *i.e.* it only tends to decrease $|V_{ub}|$.

Evaluation of Leading Uncertainties

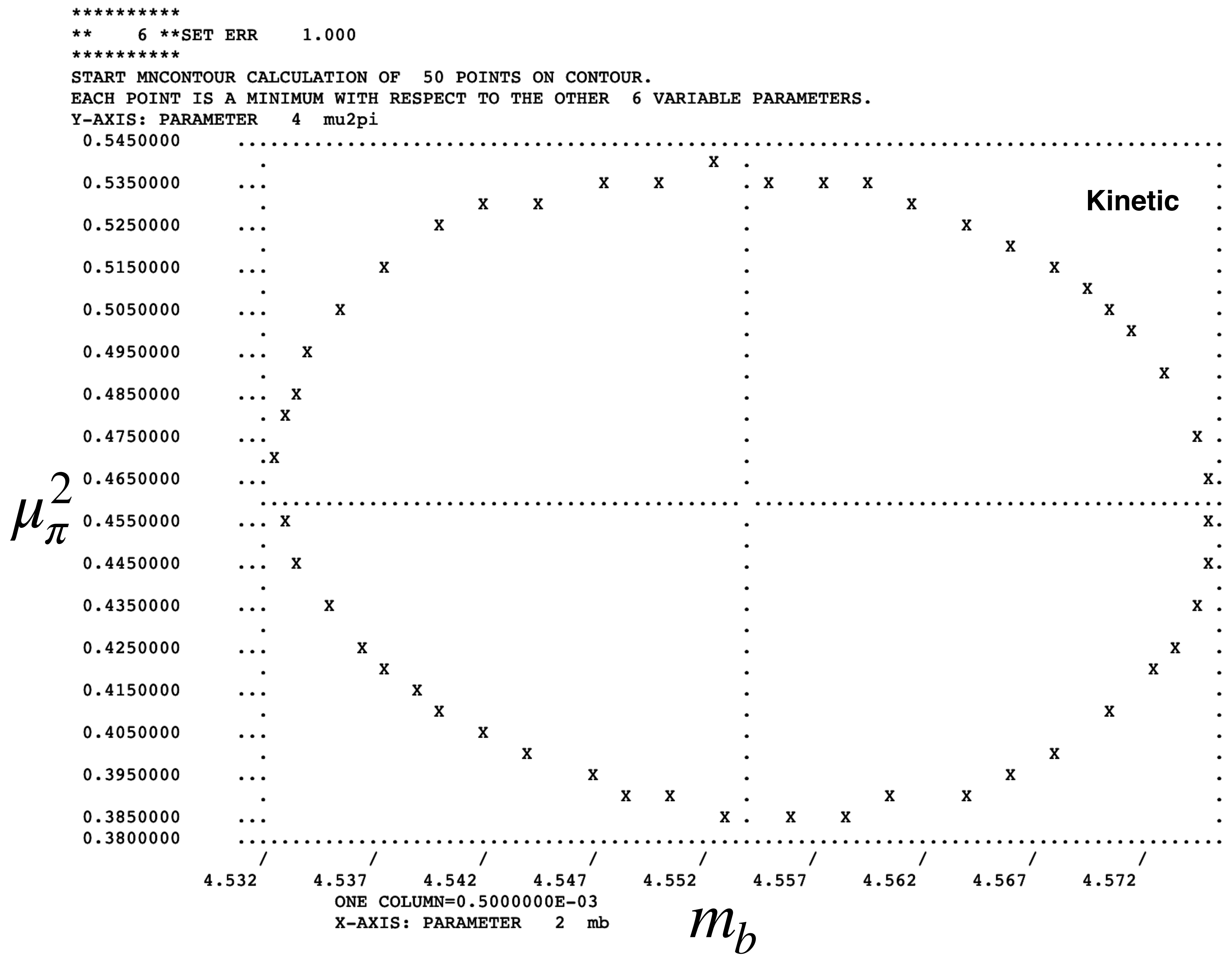
Leading errors for $|V_{ub}|_{BLNP}$:

- **HQE para (m_b, μ_π^2)**

1. taken from $B \to X_c \ell \nu$ global fit in kinetic scheme [details]
2. convert to shape function scheme (two steps: kinetic -> pole -> SF)
3. compute with BLNP framework (50 points of mb-mupi ellipse)
4. take max +/- differences on $|V_{ub}|$

$$m_b(\mathrm{SF}) = (4.582 \pm 0.023 \pm 0.018_{\mathrm{scheme}})$$

$$\mu_\pi^2(\mathrm{SF}) = (0.202 \pm 0.089\ ^{+0.020_{\mathrm{sche.}}}_{-0.040_{\mathrm{sche.}}})$$

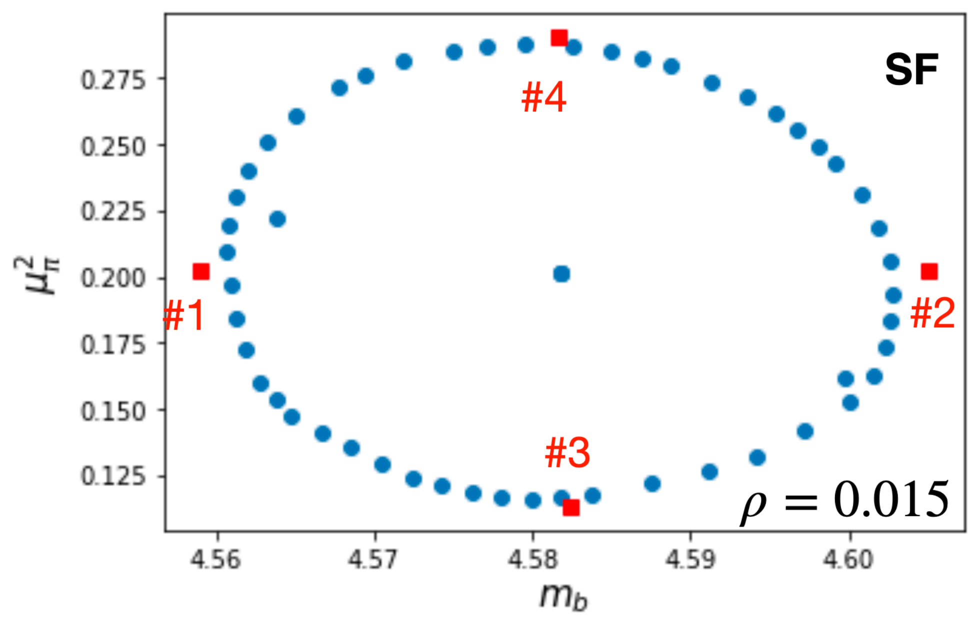


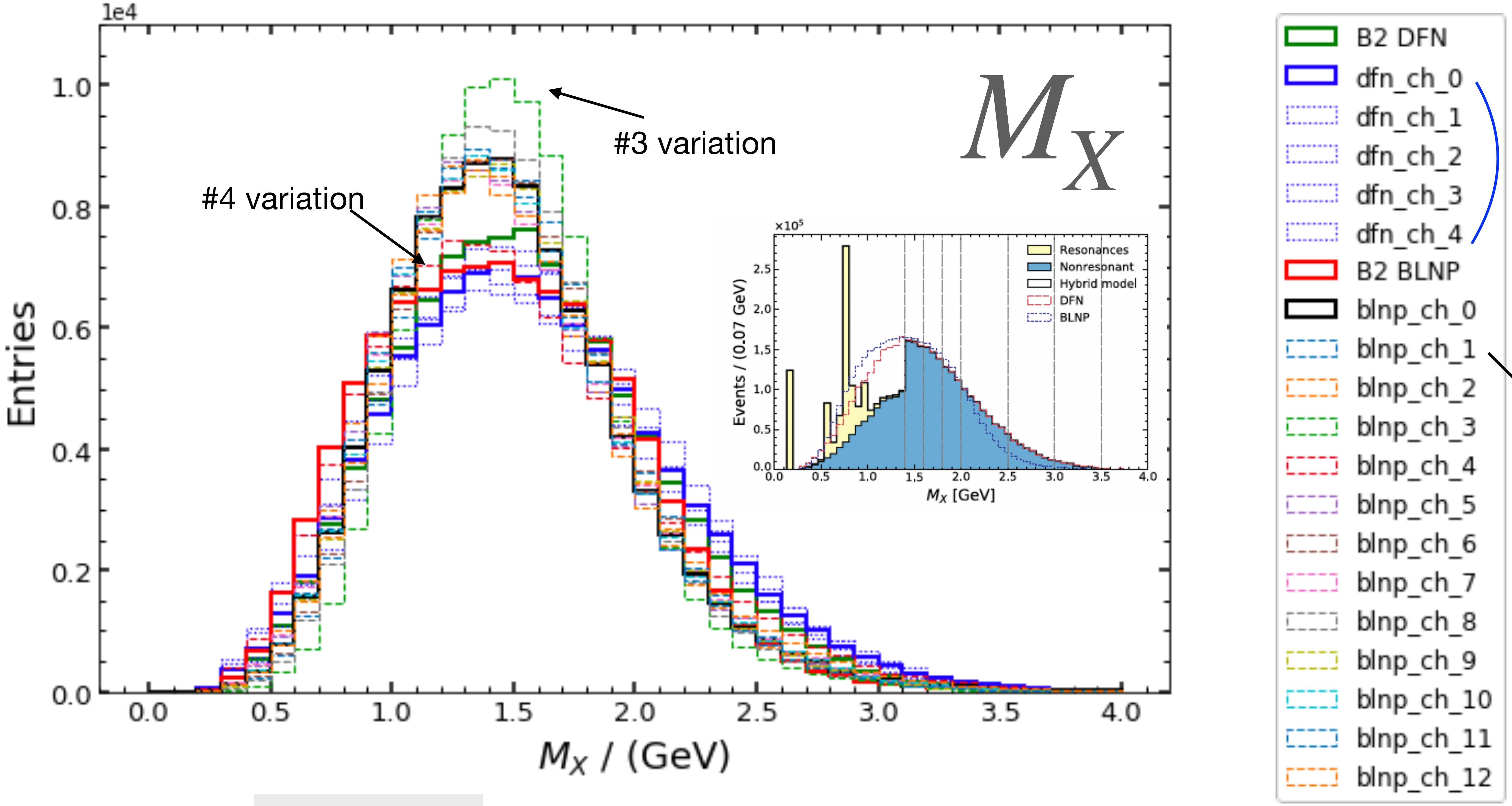
- **Matching scales (μ_h, μ_i, $\bar{\mu}$)**

1. No correlations assigned
2. Central values: μ_i=1.5GeV, μ_h=m$_b$/2, $\bar{\mu}=\mu_i$
3. Vary μ_h, $\bar{\mu}$ independently by sqrt(2)^{±1}; the intermediate scale μ_i fixed at 1.5 GeV

Generator-level Truth

- Very large deviation between the old BLNP (red) and current one (black).
- The old one is close to the current variation #4, while #3 pulls to the other side.





Belle para.(correct scheme conversion)

Old para. used in BaBar/Belle

New para. for BLNP

#	
0	Nominal
1	HQE eigen vari(1up)
2	HQE eigen vari(1down)
3	HQE eigen vari(2up)
4	HQE eigen vari(2down)
5	mb scheme. up
6	mb schem. dwon
7	mu_pi^2 shem. up
8	mu_pi^2 shem. down
9	mu_h up
10	mu_h down
11	mu_bar up
12	mu_bar down

Central BLNP: **red** -> **black**