

22nd Conference on Flavor Physics and CP Violation



# Charm results at Belle and Belle II



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On behalf of the Belle II Collaboration



# Outline

➤ Quick introduction to Belle (II)



➤ Belle results: Search for rare decays

✓  $D^0 \rightarrow hh'e^+e^-$  preliminary, intended to PRL

✓  $D \rightarrow p\ell$  PRD 109, L031101 (2024)

✓  $\Xi_c^0 \rightarrow \Xi^0 \ell^+ \ell^-$  PRD 109, 052003 (2024)



➤ Belle and Belle II analysis: Charmed Baryon

✓  $\Xi_c^0 \rightarrow \Xi^0 h^0$  preliminary, intended to JHEP

➤ Summary

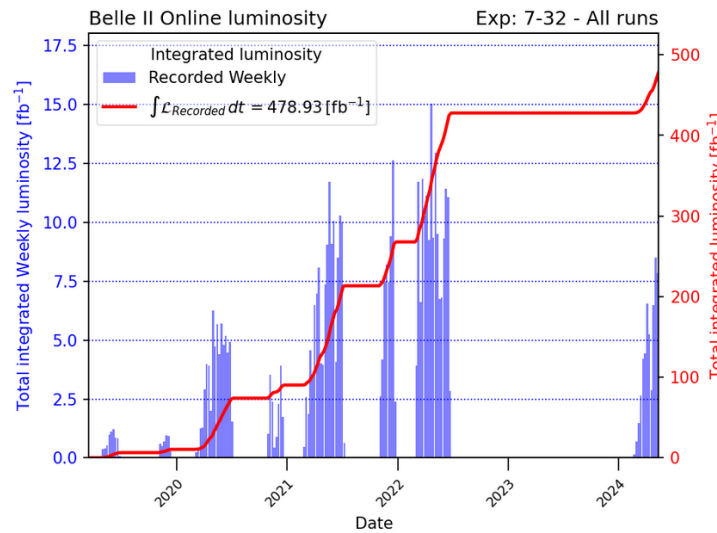
# Experiments

✓ Belle and Belle II operate at asymmetric  $e^+e^-$  colliders

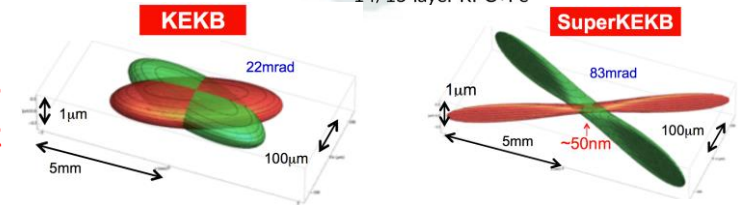
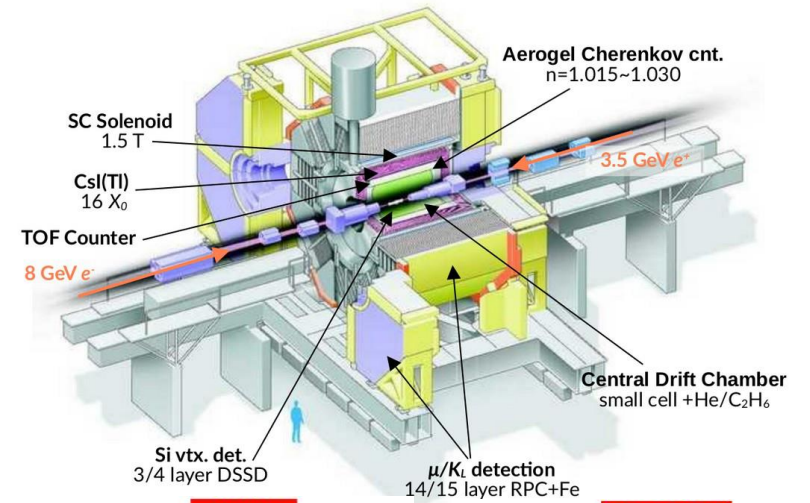
- Collisions at or near  $\Upsilon(4S)$ , B-factories
- KEKB (1999-2010), peak  $\mathcal{L} = 2 \times 10^{34} \text{ cm}^{-2} \text{ s}^{-1}$
- SuperKEKB, peak  $\mathcal{L} = 4.7 \times 10^{34} \text{ cm}^{-2} \text{ s}^{-1}$

just started Run2 (Feb. 2024)

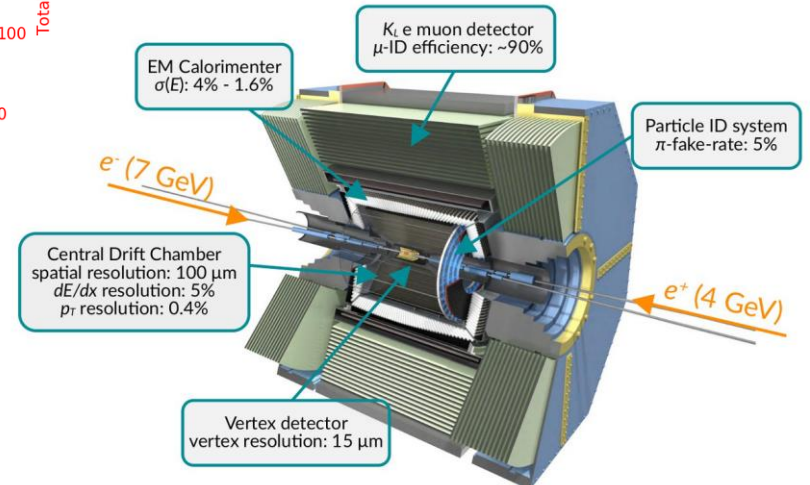
- Upgrade on collider and detector



Belle @KEKB, recorded  $\int L dt = 1 \text{ ab}^{-1}$



Belle II @SuperKEKB, Run1  
recorded  $\int L dt = 424 \text{ fb}^{-1}$  (2019-2022)



✓ Combined analyses at Belle & Belle II

- $\sim 1.4 \text{ ab}^{-1}$  in total
- Analyze Belle data with Belle II framework
- For charm analyses, large statistics to improve precision

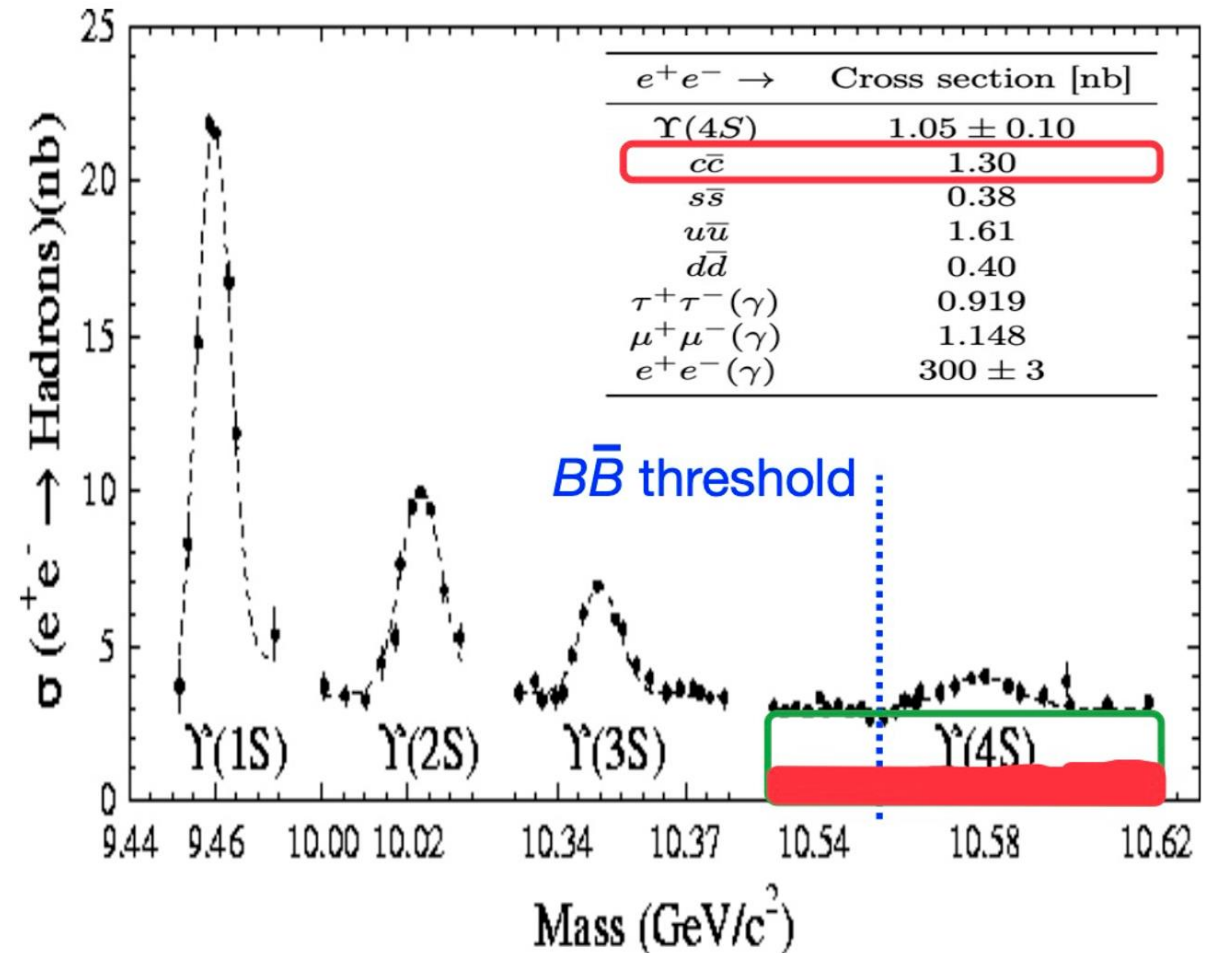
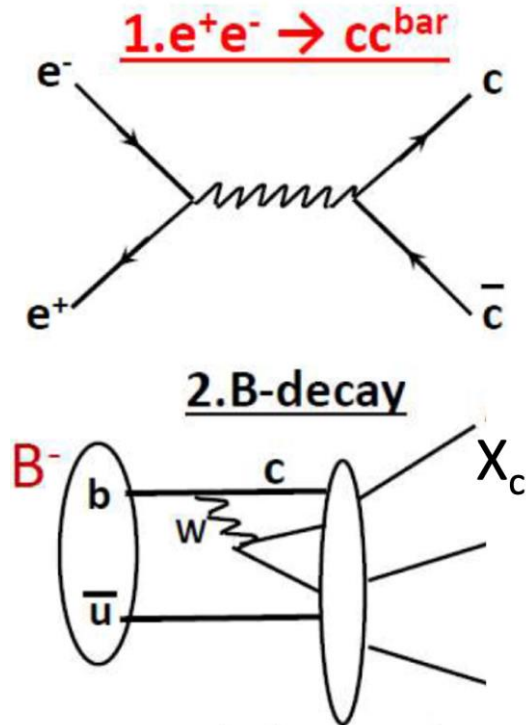
# Charm physics at Belle (II)

✓ Two ways to produce the charm sample at B-factories

- A large cross section for  $e^+e^- \rightarrow c\bar{c}$  continuum process
- B decays from  $\Upsilon(4S) \rightarrow B\bar{B}$

✓ Full topics for charm physics

- CP violation
- $D^0 - \bar{D}^0$  mixing
- Amplitude analysis
- Lifetime
- Rare decay
- Charmed baryon



# Search for $D^0 \rightarrow hh'e^+e^-$

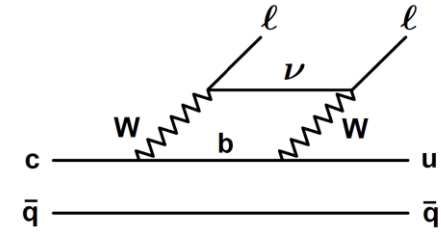
PRELIMINARY

Belle 942/fb

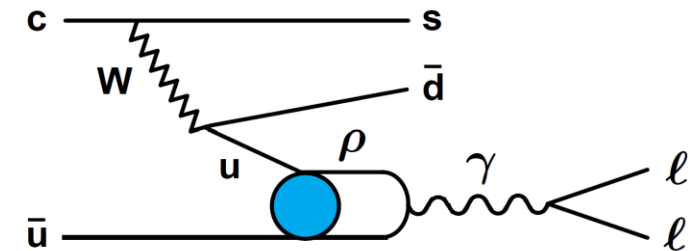
## ✓ Rare charmed meson decay

- Flavor changing neutral current (FCNC)  $c \rightarrow u\ell\ell$  process is suppressed in the SM, sensitive to BSM
- LD contributions from vector meson dominance mode
- Search for new physics and LFU (Lepton Flavor Universality) tests

Short Distance (SD)



Long Distance (LD)



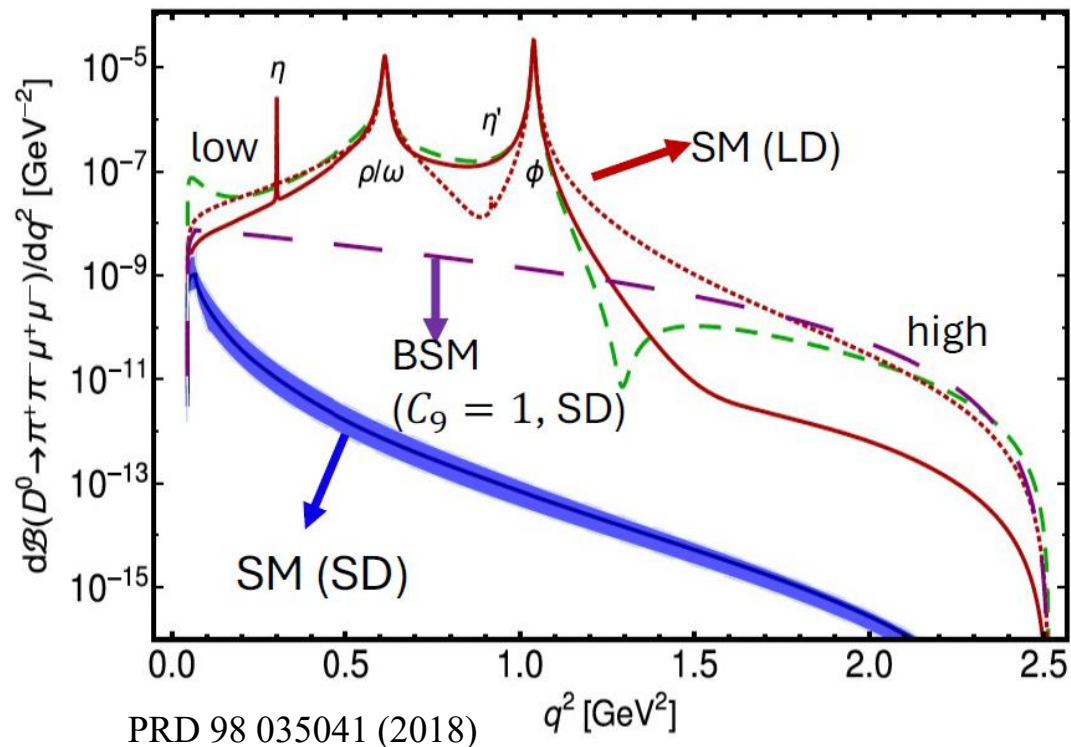
Previous measurements on BFs and ULs @90% [ $\times 10^{-7}$ ]

Experiment	$K^-K^+e^+e^-$	$\pi^-\pi^+e^+e^-$	$K^-\pi^+e^+e^-$
Babar (2019)			$40.0 \pm 5.0 \pm 2.3$ ( $\rho^0/\omega$ ) stat syst
BESIII (2019)	$< 110$	$< 70$	$< 410$
	$K^-K^+\mu^+\mu^-$	$\pi^-\pi^+\mu^+\mu^-$	$K^-\pi^+\mu^+\mu^-$
LHCb (2016-2017)	$1.54 \pm 0.27 \pm 0.19$	$9.64 \pm 0.48 \pm 1.10$	$4.17 \pm 0.12 \pm 0.40$ ( $\rho^0/\omega$ )

BABAR: PRL 122, 081802

BESIII: PRD 97, 072015 (2019)

LHCb: PLB517, 558(2016); PRL 119, 181805 (2017)





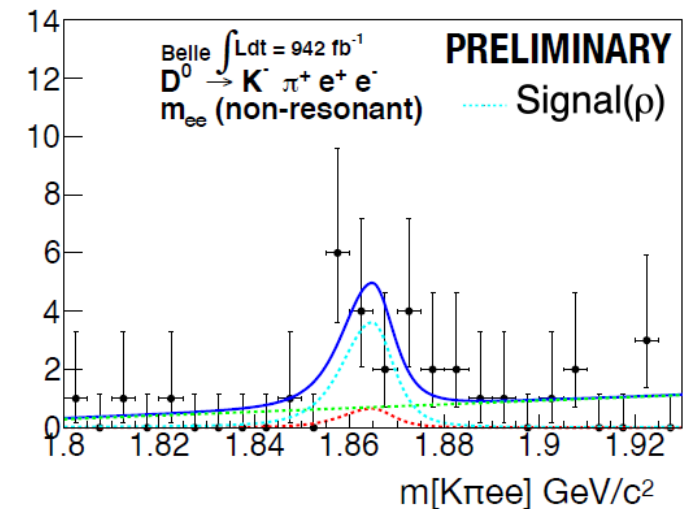
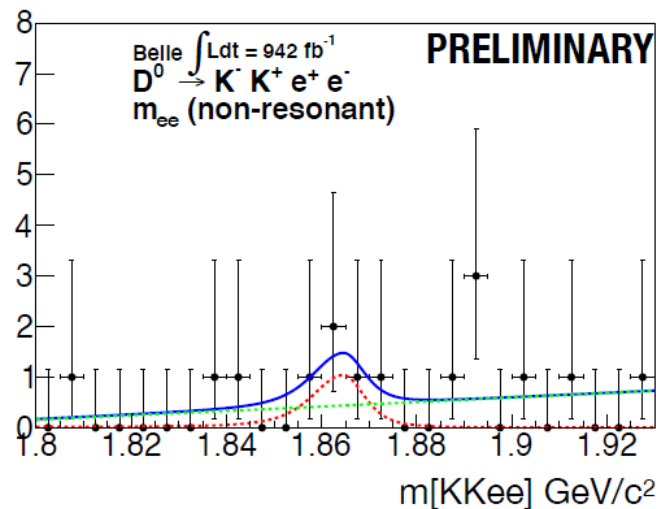
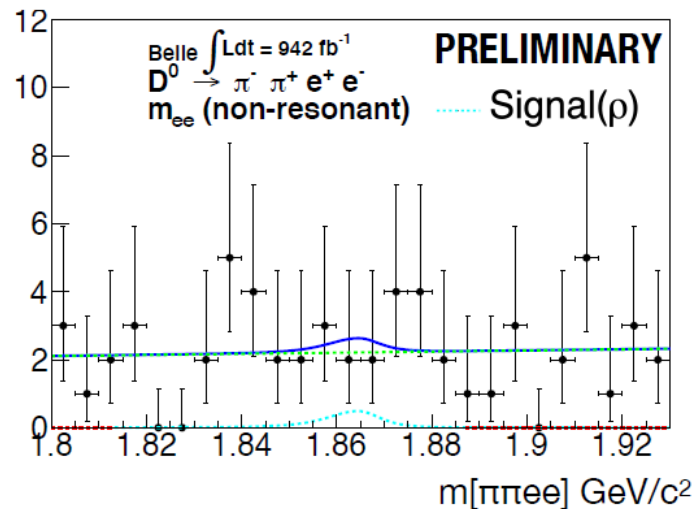
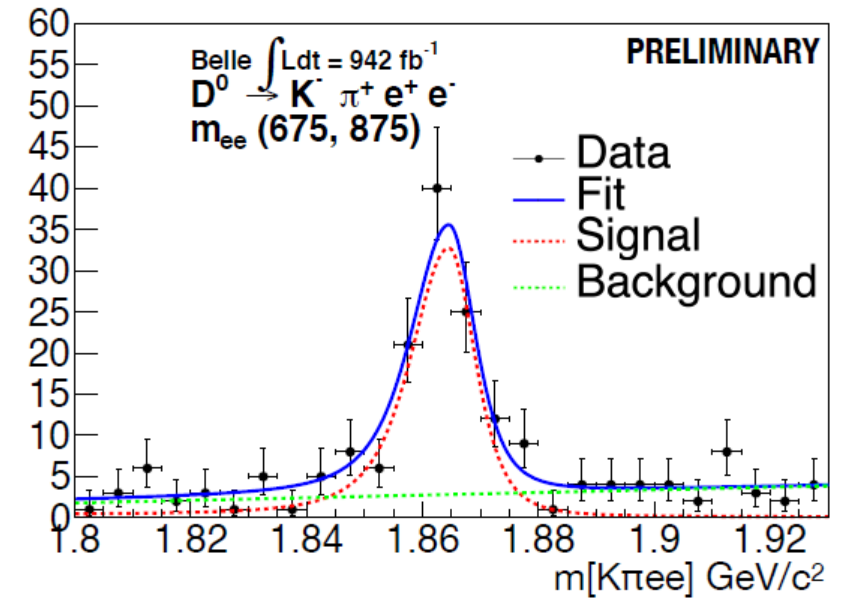
# $D^0 \rightarrow hh'e^+e^-$ results

✓ Signal observed for  $D^0 \rightarrow K^-\pi^+e^+e^-$  in  $\rho/\omega$  region ( $11.8\sigma$ )

- $\mathcal{B} = (39.6 \pm 4.5(\text{stat}) \pm 2.9(\text{syst})) \times 10^{-7}$
- Compatible with BABAR and with SM expectation

✓ No significant signal observed in other channels and regions

- $hh' = KK, K\pi, \pi\pi$ ;  $m_{ee}$  region:  $\eta, \rho/\omega, \phi$ , non-resonant
- Set upper limits in  $(2.3-7.7) \times 10^{-7}$  at 90% CL
- World's best limits to date



# Search for neutral $D \rightarrow p\ell$

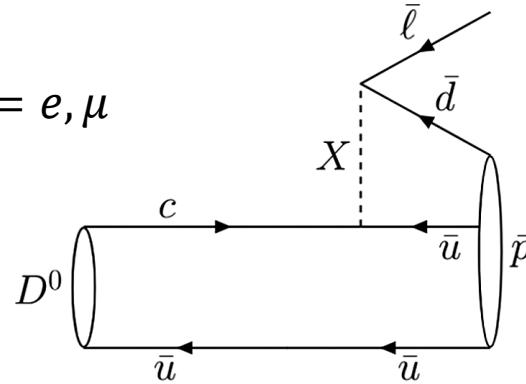
Belle 921/fb [PRD 109, L031101 (2024)]

✓ **Baryon Number Violation (BNV)** is one of the required conditions to explain matter-antimatter asymmetry

- Some models<sup>[1-5]</sup> allow violation of baryon (B) and lepton (L) numbers with the difference  $\Delta(B - L) = 0$  conserved.

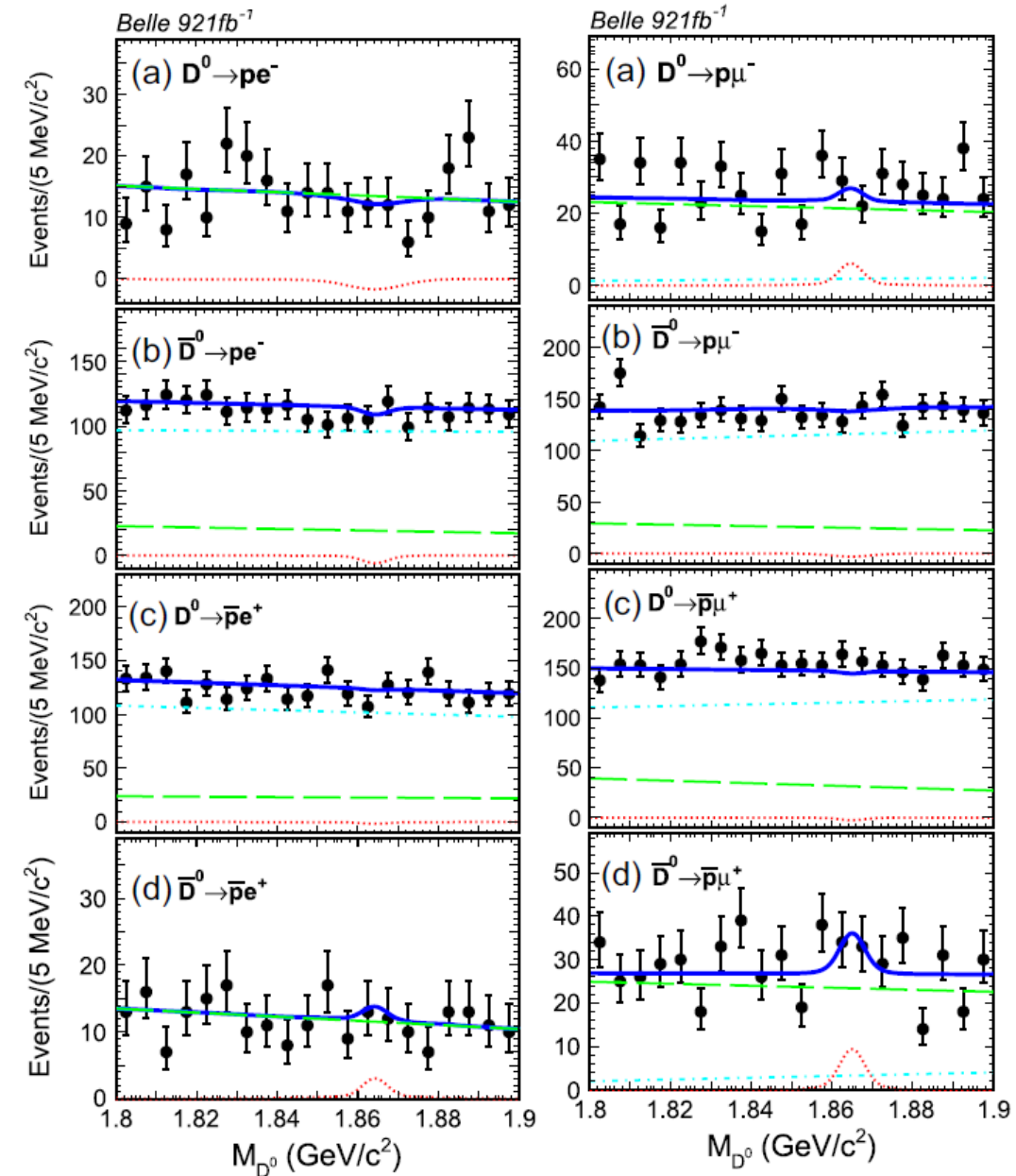
✓ Search in meson decays  $D \rightarrow p\ell$

- 8 channels in total for  $D^0/\bar{D}^0$ ,  $\ell = e, \mu$
- X: non-SM gauge boson



✓ **No significant signal observed**

- Set upper limits  $(5 - 8) \times 10^{-7}$  at 90% CL
- Most stringent limit to date
- First measurement for  $D \rightarrow p\mu$  modes



[1] PRD 8, 1240 (1973) [2] PRL 32, 438 (1974) [3] PRD 20,776 (1979) [4] PLB 91, 222 (1980) [5] PLB 314, 336 (1993)

# Search for $\Xi_c^0 \rightarrow \Xi^0 \ell^+ \ell^-$

Belle 980/fb [PRD 109, 052003 (2024)]

- [1] PRD 84, 072006(2011)
- [2] PRD 97, 091101(2018)
- [3] PRD 103, 013007 (2021)

✓ No neutrino-less semileptonic decays of charmed baryons observed yet.

- Only upper limits of  $\Lambda_c \rightarrow p \ell^+ \ell^-$  decays were set for charmed baryons<sup>[1,2]</sup>, which receive both W-exchange and FCNC process contributions.
- Theoretically face difficulties from the Hamiltonian helicity structure and hadronic form factors
- To understand W-exchange contribution in  $\Lambda_c \rightarrow p \ell^+ \ell^-$
- If observed, the signal channels would allow to test LFU

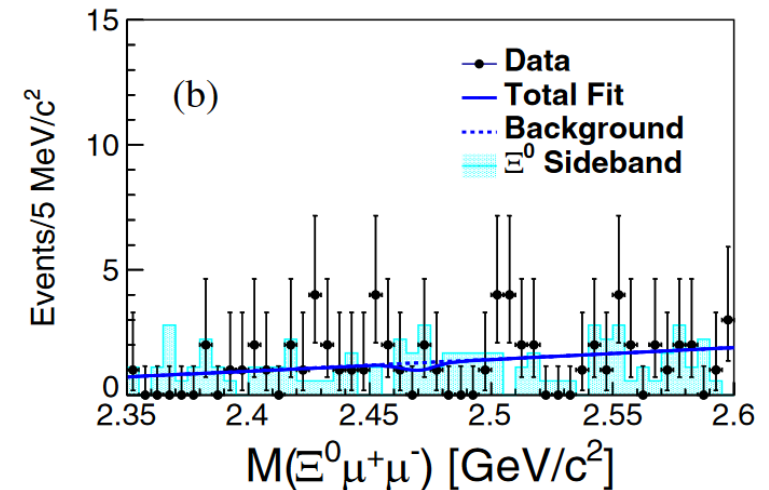
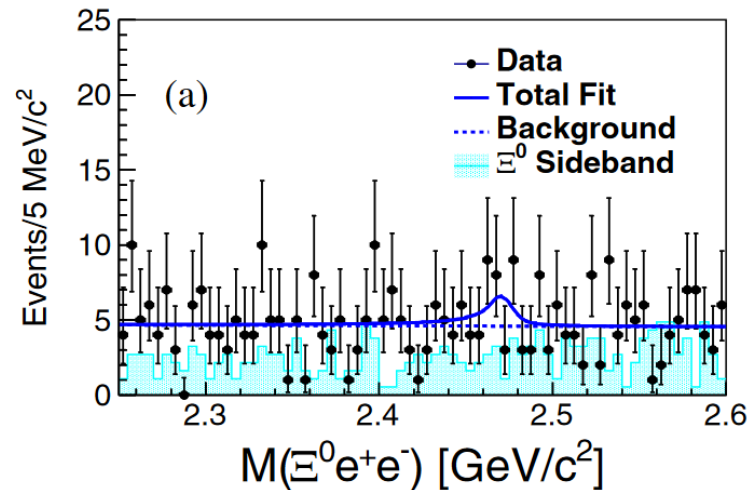
✓ First search for  $\Xi_c^0 \rightarrow \Xi^0 \ell^+ \ell^- (\ell = e, \mu)$

- No significant signals observed
- set upper limits at 90% CL

$$\mathcal{B}(\Xi_c^0 \rightarrow \Xi^0 e^+ e^-) < 9.9 \times 10^{-5}$$

$$\mathcal{B}(\Xi_c^0 \rightarrow \Xi^0 \mu^+ \mu^-) < 6.5 \times 10^{-5}$$

- Theoretical prediction gives upper limits at  $2.35 (2.25) \times 10^{-6}$  for electron (muon) mode<sup>[3]</sup>



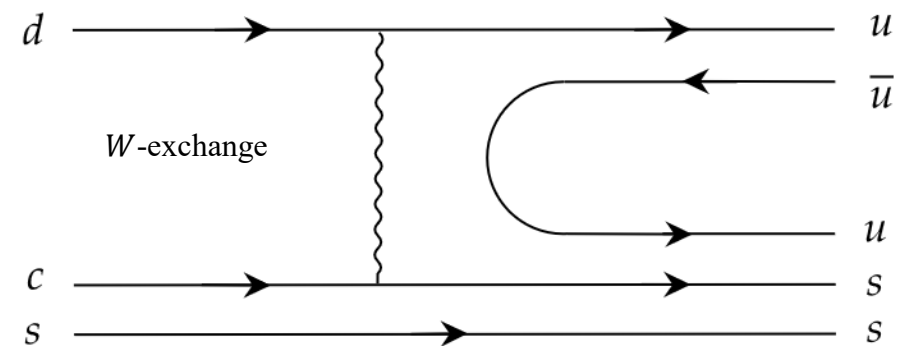
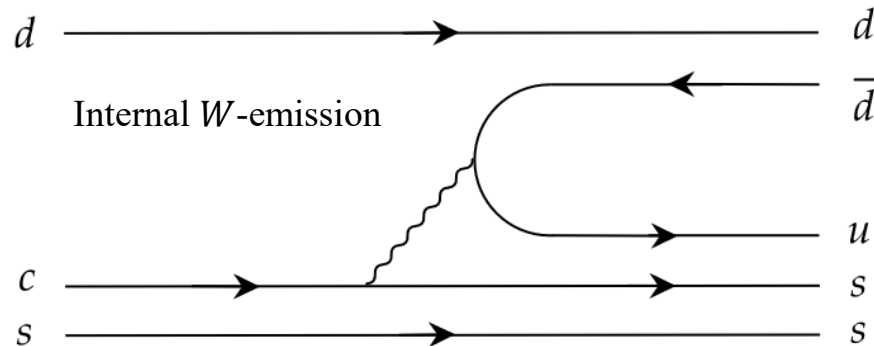


# Study of $\Xi_c^0 \rightarrow \Xi^0 h^0, h^0 = \pi^0, \eta, \eta'$

PRELIMINARY  
Belle + Belle II 1.4/ab

✓ Hadronic two-body decay of charmed baryons

- **Nonfactorizable amplitudes** from internal W-emission and W-exchange diagram lead to the difficulties for theoretical predictions
- Feynman diagrams [CJPH 78, 324 (2022)] for Cabibbo-favored signal modes  $\Xi_c^0 \rightarrow \Xi^0 h^0$ , **only nonfactorizable amplitudes contribute to.**



- Several theoretical approaches developed to deal with nonfactorizable contributions, give various predictions on branching fractions  $((0.5-26.7) \times 10^{-3})$  and decay asymmetry parameters [see backup].
- **Need experiment measurement to clarify the theoretical picture.**

# $\Xi_c^0 \rightarrow \Xi^0 h^0$ results

- ✓ First measurements of the branching fractions

$$\mathcal{B}(\Xi_c^0 \rightarrow \Xi^0 \pi^0) = (6.9 \pm 0.3(\text{stat.}) \pm 0.5(\text{syst.}) \pm 1.5(\text{norm.})) \times 10^{-3}$$

$$\mathcal{B}(\Xi_c^0 \rightarrow \Xi^0 \eta) = (1.6 \pm 0.2(\text{stat.}) \pm 0.2(\text{syst.}) \pm 0.4(\text{norm.})) \times 10^{-3}$$

$$\mathcal{B}(\Xi_c^0 \rightarrow \Xi^0 \eta') = (1.2 \pm 0.3(\text{stat.}) \pm 0.1(\text{syst.}) \pm 0.3(\text{norm.})) \times 10^{-3}$$

- taking  $\Xi_c^0 \rightarrow \Xi^- \pi^+$  as reference mode
- favoring predictions in SU(3) flavor symmetry [JHEP 02, 235 (2023)]

- ✓ First asymmetry parameter  $\alpha(\Xi_c^0 \rightarrow \Xi^0 \pi^0)$  measurement

$$\alpha(\Xi_c^0 \rightarrow \Xi^0 \pi^0) = -0.90 \pm 0.15(\text{stat.}) \pm 0.23(\text{syst.})$$

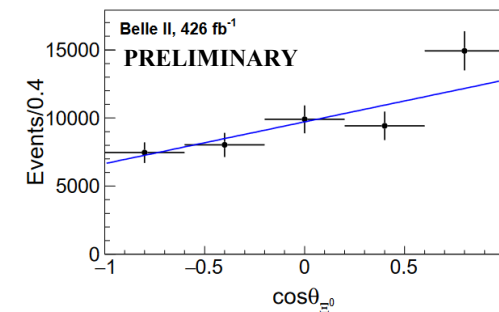
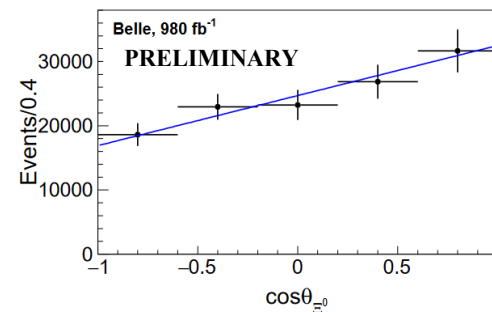
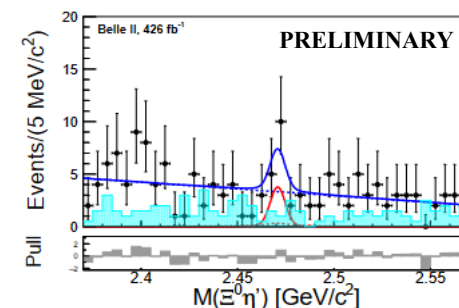
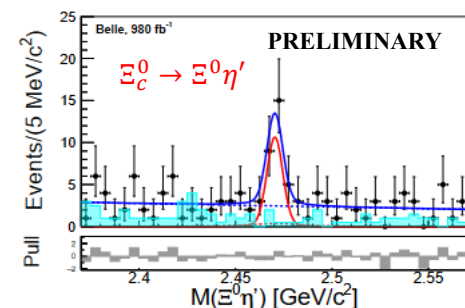
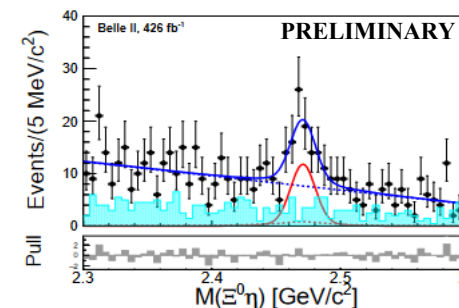
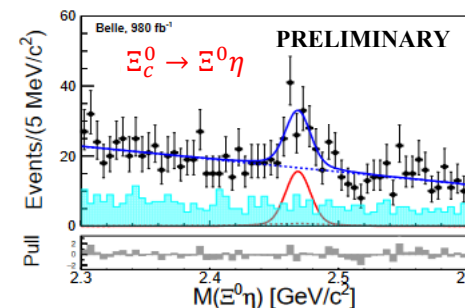
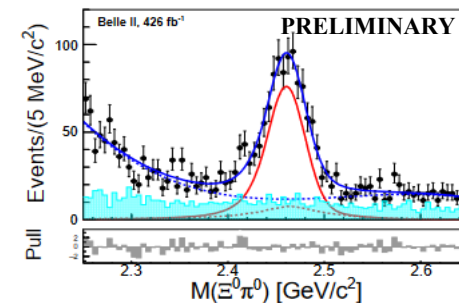
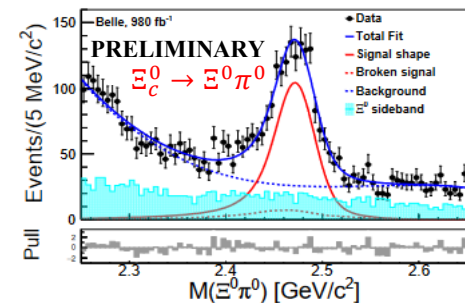
- through a simultaneous fit depending on differential decay rate

$$\frac{dN}{d\cos\theta_{\Xi^0}} \propto 1 + \alpha(\Xi_c^0 \rightarrow \Xi^0 h^0) \alpha(\Xi^0 \rightarrow \Lambda \pi^0) \cos\theta_{\Xi^0}$$

- consistent with predictions<sup>[1-4]</sup>

[1]PRD 48, 4188 (1993) [2] PRD 101, 014011 (2020) [3] EPJC 7, 217 (1999) [4]PLB 794, 19 (2019)

PRELIMINARY at Belle + Belle II 1.4/ab



# Summary

- Belle and Belle II provide a unique environment for charm physics both in meson and baryon decays
  - sensitive in SM measurements and search for BSM physics
- Belle is still producing important measurements for more than 10 years after the end of data taking
  - Search for rare decays: tightest to date/first measurement
  - FCNC  $D^0 \rightarrow hh'e^+e^-$
  - BNV  $D \rightarrow p\ell$
  - Semi-leptonic  $\Xi_c^0 \rightarrow \Xi^0\ell^+\ell^-$
- Belle + Belle II combined data sample provides the platform for further charm measurements
  - $\mathcal{B}$  and  $\alpha$  measurements for  $\Xi_c^0 \rightarrow \Xi^0 h^0$
- Belle II has started Run 2 data taking, expecting more physics results with a larger data sample

Thank you for your attention!

# Backups

# ULs ( $\times 10^{-7}$ ) for $D^0 \rightarrow hh'ee$

$m_{ee}$ region	[MeV/ $c^2$ ]	Yield	Significance	$\mathcal{B}$	<i>BELLE</i> UL @ 90% CL	Efficiency (%)	BESIII (UL @ 90% CL)	<i>BABAR</i>
$K^- K^+ e^+ e^-$								
$\eta$	520-560	-	$< 0.1\sigma$	-	$< 2.3$	$3.53 \pm 0.04$	$< 110$	-
$\rho^0/\omega$	$> 675$	$2.6 \pm 1.8$	$2.0\sigma$	$1.2 \pm 0.9 \pm 0.1$	$< 3.0$	$6.00 \pm 0.06$		
non-resonant	$> 200$ <sup>a</sup>	$3.5 \pm 3.3$	$1.5\sigma$	$3.1 \pm 3.0 \pm 0.4$	$< 7.7$	$3.19 \pm 0.04$		
$\pi^- \pi^+ e^+ e^-$								
$\eta$	520-560	$0.6 \pm 2.3$	$0.3\sigma$	$0.4 \pm 1.4 \pm 0.2$	$< 3.2$	$5.31 \pm 0.05$		
$\rho^0/\omega$	675-875	$3.7 \pm 4.1$	$0.9\sigma$	$2.0 \pm 2.2 \pm 0.8$	$< 6.1$	$5.69 \pm 0.05$	$< 70$	-
$\phi$	995-1035	$3.6 \pm 3.2$	$1.1\sigma$	$1.1 \pm 1.1 \pm 0.2$	$< 3.1$	$9.41 \pm 0.06$		
non-resonant	$> 200$	$-0.2 \pm 4.1$	$< 0.1\sigma$	$-0.2 \pm 3.4 \pm 0.9$	$< 7.2$	$3.69 \pm 0.04$		
$K^- \pi^+ e^+ e^-$								
$\eta$	520-560	$4.0 \pm 2.7$	$1.6\sigma$	$2.2 \pm 1.5 \pm 0.5$	$< 5.6$	$5.09 \pm 0.04$		
$\rho^0/\omega$	675-875	$110 \pm 13$	$11.8\sigma$	$39.6 \pm 4.5 \pm 2.9$	-	$8.01 \pm 0.06$	$< 410$	$< 31^*$
$\phi$	990-1034	$4.6 \pm 2.4$	$2.5\sigma$	$1.4 \pm 0.8 \pm 0.3$	$< 2.9$	$9.19 \pm 0.06$		
non-resonant	$> 560$	$2.2 \pm 4.2$	$0.4\sigma$	$1.3 \pm 2.4 \pm 0.6$	$< 6.5$	$4.89 \pm 0.09$		

<sup>a</sup> Excluding resonance regions, which is same for all three modes.

BESIII PRD97(2019)072015

BABAR PRL122(2019)081802



# ULs for $D \rightarrow p\ell$

TABLE I. Reconstruction efficiency ( $\epsilon$ ), signal yield ( $N_S$ ), signal significance ( $\mathcal{S}$ ), upper limit on the signal yield ( $N_{p\ell}^{UL}$ ), and branching fraction ( $\mathcal{B}$ ) at 90% confidence level for each decay mode.

Decay mode	$\epsilon$ (%)	$N_S$	$\mathcal{S}$ ( $\sigma$ )	$N_{p\ell}^{UL}$	$\mathcal{B} \times 10^{-7}$
$D^0 \rightarrow pe^-$	10.2	$-6.4 \pm 8.5$		17.5	$< 5.5$
$\bar{D}^0 \rightarrow pe^-$	10.2	$-18.4 \pm 23.0$		22.0	$< 6.9$
$D^0 \rightarrow \bar{p}e^+$	09.7	$-4.7 \pm 23.0$		22.0	$< 7.2$
$\bar{D}^0 \rightarrow \bar{p}e^+$	09.6	$7.1 \pm 9.0$	0.6	23.0	$< 7.6$
$D^0 \rightarrow p\mu^-$	10.7	$11.0 \pm 23.0$	0.9	17.1	$< 5.1$
$\bar{D}^0 \rightarrow p\mu^-$	10.7	$-10.8 \pm 27.0$		21.8	$< 6.5$
$D^0 \rightarrow \bar{p}\mu^+$	10.5	$-4.5 \pm 14.0$		21.1	$< 6.3$
$\bar{D}^0 \rightarrow \bar{p}\mu^+$	10.4	$16.7 \pm 8.8$	1.6	21.4	$< 6.5$

# Theoretical Predictions for $\Xi_c^0 \rightarrow \Xi^0 h^0$

**Table 1.** Theoretical predictions for the branching fractions and decay asymmetry parameters for  $\Xi_c^0 \rightarrow \Xi^0 h^0$  decays. Branching fractions are given in units of  $10^{-3}$ .

Reference	Model	$\mathcal{B}(\Xi_c^0 \rightarrow \Xi^0 \pi^0)$	$\mathcal{B}(\Xi_c^0 \rightarrow \Xi^0 \eta)$	$\mathcal{B}(\Xi_c^0 \rightarrow \Xi^0 \eta')$	$\alpha(\Xi_c^0 \rightarrow \Xi^0 \pi^0)$
Körner, Krämer [5]	quark	0.5	3.2	11.6	0.92
Ivanov <i>et al.</i> [6]	quark	0.5	3.7	4.1	0.94
Xu, Kamal [7]	pole	7.7	-	-	0.92
Cheng, Tseng [8]	pole	3.8	-	-	-0.78
Żenczykowski [9]	pole	6.9	1.0	9.0	0.21
Zou <i>et al.</i> [10]	pole	18.2	26.7	-	-0.77
Sharma, Verma [11]	CA	-	-	-	-0.8
Cheng, Tseng [8]	CA	17.1	-	-	0.54
Geng <i>et al.</i> [12]	SU(3) <sub>F</sub>	4.3±0.9	1.7 <sup>+1.0</sup> <sub>-1.7</sub>	8.6 <sup>+11.0</sup> <sub>-6.3</sub>	-
Geng <i>et al.</i> [13]	SU(3) <sub>F</sub>	7.6±1.0	10.3±2.0	9.1±4.1	-1.00 <sup>+0.07</sup> <sub>-0.00</sub>
Zhao <i>et al.</i> [14]	SU(3) <sub>F</sub>	4.7±0.9	8.3±2.3	7.2±1.9	-
Huang <i>et al.</i> [15]	SU(3) <sub>F</sub>	2.56±0.93	-	-	-0.23 ± 0.60
Hsiao <i>et al.</i> [16]	SU(3) <sub>F</sub>	6.0±1.2	4.2 <sup>+1.6</sup> <sub>-1.3</sub>	-	-
Hsiao <i>et al.</i> [16]	SU(3) <sub>F</sub> -breaking	3.6±1.2	7.3±3.2	-	-
Zhong <i>et al.</i> [17]	SU(3) <sub>F</sub>	1.13 <sup>+0.59</sup> <sub>-0.49</sub>	1.56±1.92	0.683 <sup>+3.272</sup> <sub>-3.268</sub>	0.50 <sup>+0.37</sup> <sub>-0.35</sub>
Zhong <i>et al.</i> [17]	SU(3) <sub>F</sub> -breaking	7.74 <sup>+2.52</sup> <sub>-2.32</sub>	2.43 <sup>+2.79</sup> <sub>-2.90</sub>	1.63 <sup>+5.09</sup> <sub>-5.14</sub>	-0.29 <sup>+0.20</sup> <sub>-0.17</sub>
Xing <i>et al.</i> [18]	SU(3) <sub>F</sub>	1.30±0.51	-	-	-0.28 ± 0.18

- Ref. [17] with breaking scenario suits best for  $\mathcal{B}$  measurements

- [5] J. G. Körner and M. Krämer, *Exclusive non-leptonic charm baryon decays*, Z. Phys. C **55** (1992) 659.
- [6] M. A. Ivanov, J. G. Korner, V. E. Lyubovitskij, and A. G. Rusetsky, *Exclusive nonleptonic decays of bottom and charm baryons in a relativistic three-quark model: Evaluation of nonfactorizing diagrams*, Phys. Rev. D **57** (1998) 5632.
- [7] Q. P. Xu and A. N. Kamal, *Cabibbo-favored nonleptonic decays of charmed baryons*, Phys. Rev. D **46** (1992) 270.
- [8] H. Y. Cheng and B. Tseng, *Cabibbo-allowed nonleptonic weak decays of charmed baryons*, Phys. Rev. D **48** (1993) 4188.
- [9] P. Żenczykowski, *Nonleptonic charmed-baryon decays: Symmetry properties of parity-violating amplitudes*, Phys. Rev. D **50** (1994) 5787.
- [10] J. Q. Zou, F. R. Xu, G. B. Meng, and H. Y. Cheng, *Two-body hadronic weak decays of antitriplet charmed baryons*, Phys. Rev. D **101** (2020) 014011.
- [11] K. K. Sharma and R. C. Verma, *A study of weak mesonic decays of  $\Lambda_c$  and  $\Xi_c$  baryons on the basis of HQET results*, Eur. Phys. J. C **7** (1999) 217.
- [12] C. Q. Geng, Y. K. Hsiao, C. W. Liu, and T. H. Tsai, *Antitriplet charmed baryon decays with SU(3) flavor symmetry*, Phys. Rev. D **97** (2018) 073006.
- [13] C. Q. Geng, C. W. Liu, and T. H. Tsai, *Asymmetries of anti-triplet charmed baryon decays*, Phys. Lett. B **794** (2019) 19.
- [14] H. J. Zhao, Y. L. Wang, Y. K. Hsiao, and Y. Yu, *A Diagrammatic Analysis of Two-Body Charmed Baryon Decays with Flavor Symmetry*, JHEP **02** (2020) 165.
- [15] F. Huang, Z. P. Xing, and X. Z. He, *A global analysis of charmless two body hadronic decays for anti-triplet charmed baryons*, JHEP **03** (2022) 143.
- [16] Y. K. Hsiao, Y. L. Wang, and H. J. Zhao, *Equivalent SU(3)<sub>f</sub> approaches for two-body anti-triplet charmed baryon decays*, JHEP **09** (2022) 35.
- [17] H. Zhong, F. Xu, Q. Wen and Y. Gu, *Weak decays of antitriplet charmed baryons from the perspective of flavor symmetry*, JHEP **02** (2023) 235.
- [18] Z. P. Xing, *et al.*, *Global analysis of measured and unmeasured hadronic two-body weak decays of antitriplet charmed baryons*, Phys. Rev. D **108** (2023) 053004.

# Systematic uncertainties for $\mathcal{B}(\Xi_c^0 \rightarrow \Xi^0 h^0)$

**Table 5.** Relative systematic uncertainties (%) for branching fraction ratio measurements. The uncertainties in last two rows are correlated systematic uncertainties from intermediate branching fractions and background shape, and others are uncorrelated ones.

Source	$\frac{\mathcal{B}(\Xi_c^0 \rightarrow \Xi^0 \pi^0)}{\mathcal{B}(\Xi_c^0 \rightarrow \Xi^- \pi^+)}$		$\frac{\mathcal{B}(\Xi_c^0 \rightarrow \Xi^0 \eta)}{\mathcal{B}(\Xi_c^0 \rightarrow \Xi^- \pi^+)}$		$\frac{\mathcal{B}(\Xi_c^0 \rightarrow \Xi^0 \eta')}{\mathcal{B}(\Xi_c^0 \rightarrow \Xi^- \pi^+)}$	
	Belle	Belle II	Belle	Belle II	Belle	Belle II
Tracking	0.7	0.8	0.7	0.7	1.0	1.5
$\pi^\pm$ PID	0.4	0.2	0.4	0.2	1.4	0.2
$\pi^0$ reconstruction	4.4	8.8	2.3	4.3	2.3	4.2
Photon reconstruction	-	-	4.0	2.0	4.0	1.9
MC statistics	0.8	0.7	0.9	0.9	1.2	1.0
$\alpha$ uncertainty	1.1	1.2	3.0	3.4	1.0	3.5
$\Xi^0$ signal mass window	0.5	2.0	0.5	2.0	0.5	2.0
Normalization mode statistics	1.0	1.3	1.0	1.3	1.0	1.3
Broken-signal ratio ( $n_{\text{broken}}/n_{\text{sig}}$ )	2.1	1.5	3.5	3.6	3.6	5.7
Broken-signal PDF	0.2	0.1	7.3	7.5	2.0	1.1
Mass Resolution	-	-	7.2	7.0	2.4	1.4
Intermediate states $\mathcal{B}$	-	-	0.5	0.5	1.3	1.3
Background shape	4.9	4.9	9.2	9.2	6.8	6.8
Total	7.2	10.6	15.3	15.6	9.9	11.2

# Values for $\mathcal{B}(\Xi_c^0 \rightarrow \Xi^0 h^0)$

Mode	$N_{\text{Belle}}^{\text{obs}}$	$\varepsilon_{\text{Belle}} (\%)$	$N_{\text{Belle II}}^{\text{obs}}$	$\varepsilon_{\text{Belle II}} (\%)$
$\Xi_c^0 \rightarrow \Xi^- \pi^+$	$36340 \pm 348$	$13.92 \pm 0.05$	$13719 \pm 184$	$13.38 \pm 0.03$
$\Xi_c^0 \rightarrow \Xi^0 \pi^0$	$1315 \pm 66$	$1.09 \pm 0.01$	$869 \pm 46$	$1.71 \pm 0.01$
$\Xi_c^0 \rightarrow \Xi^0 \eta$	$81 \pm 15$	$0.80 \pm 0.01$	$60 \pm 11$	$1.12 \pm 0.01$
$\Xi_c^0 \rightarrow \Xi^0 \eta'$	$23 \pm 6$	$0.46 \pm 0.01$	$8 \pm 4$	$0.81 \pm 0.01$

Results	Belle	Belle II	Combined
$\mathcal{B}(\Xi_c^0 \rightarrow \Xi^0 \pi^0) / \mathcal{B}(\Xi_c^0 \rightarrow \Xi^- \pi^+)$	$0.47 \pm 0.02 \pm 0.03$	$0.51 \pm 0.03 \pm 0.05$	$0.48 \pm 0.02 \pm 0.03$
$\mathcal{B}(\Xi_c^0 \rightarrow \Xi^0 \eta) / \mathcal{B}(\Xi_c^0 \rightarrow \Xi^- \pi^+)$	$0.10 \pm 0.02 \pm 0.01$	$0.14 \pm 0.02 \pm 0.02$	$0.11 \pm 0.01 \pm 0.01$
$\mathcal{B}(\Xi_c^0 \rightarrow \Xi^0 \eta') / \mathcal{B}(\Xi_c^0 \rightarrow \Xi^- \pi^+)$	$0.12 \pm 0.03 \pm 0.01$	$0.06 \pm 0.03 \pm 0.01$	$0.08 \pm 0.02 \pm 0.01$

are taken from ref. [39]. We combine the Belle and Belle II branching fraction ratios and uncertainties using formulas in ref. [44]:

$$\begin{aligned}
 r &= \frac{r_1 \sigma_2^2 + r_2 \sigma_1^2}{\sigma_1^2 + \sigma_2^2 + (r_1 - r_2)^2 \epsilon_r^2}, \\
 \sigma &= \sqrt{\frac{\sigma_1^2 \sigma_2^2 + (r_1^2 \sigma_2^2 + r_2^2 \sigma_1^2) \epsilon_r^2}{\sigma_1^2 + \sigma_2^2 + (r_1 - r_2)^2 \epsilon_r^2}},
 \end{aligned}
 \tag{5.3}$$

where  $r_i$ ,  $\sigma_i$  and  $\epsilon_r$  are the branching fraction ratio, uncorrelated uncertainty, and relative correlated systematic uncertainty from each data sample, respectively. The branching

[44] G. D'Agostini, *On the use of the covariance matrix to fit correlated data*, Nucl. Instrum. Methods Phys. Res., Sect. A **346** (1994) 306.