Tau physics at Belle and Belle II

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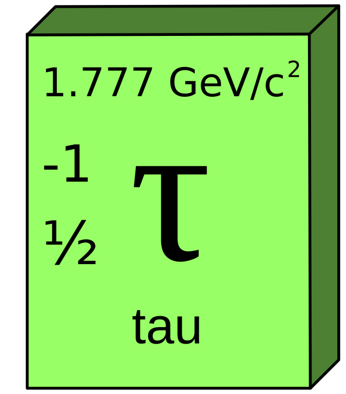
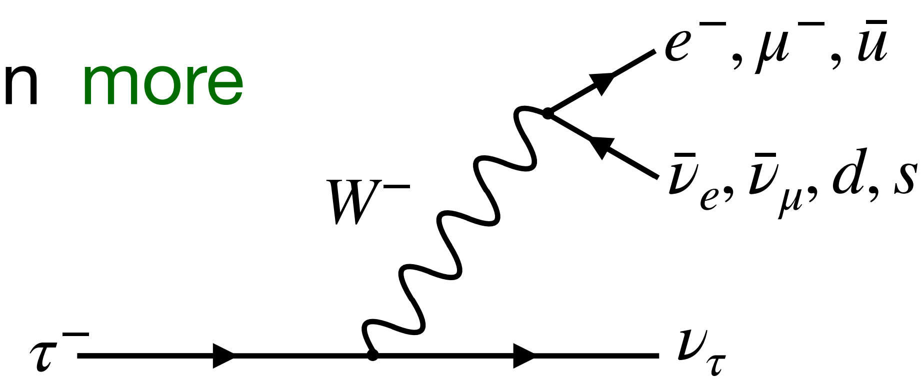
on behalf of the Belle II collaboration

The 2024 International Workshop on Future Tau Charm Facilities

Introduction: why τ lepton?



- τ lepton is the heaviest lepton in the Standard Model (SM) with both leptonic and hadronic decay modes
- Larger mass compared to muon makes τ lepton more sensitive to some models of New Physics (NP)



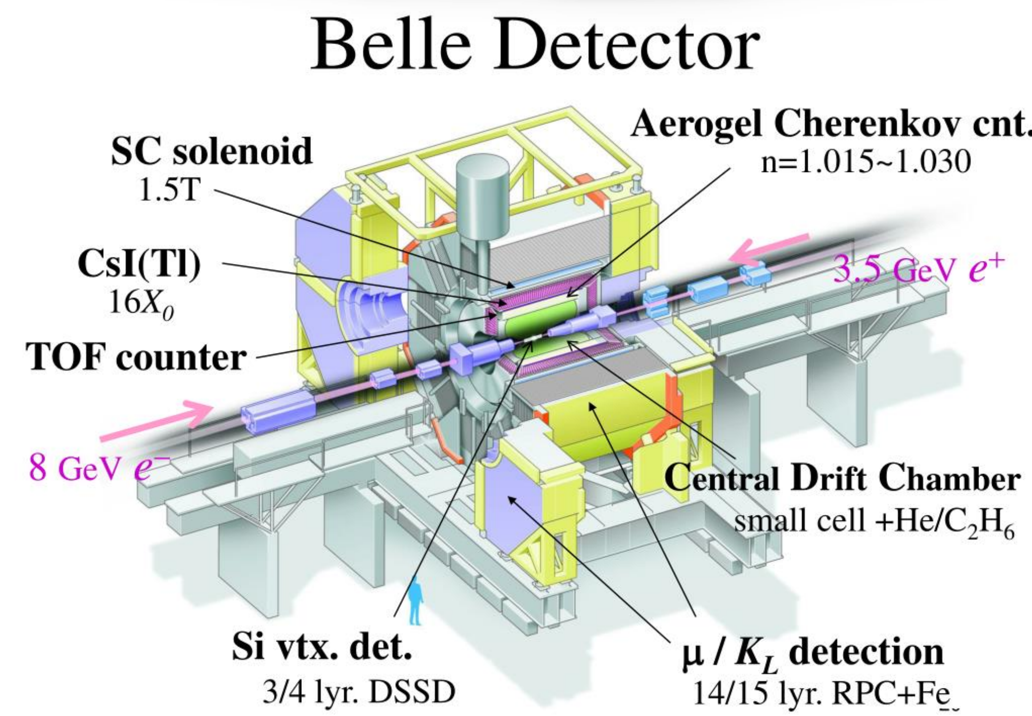
Broad range of available measurements :

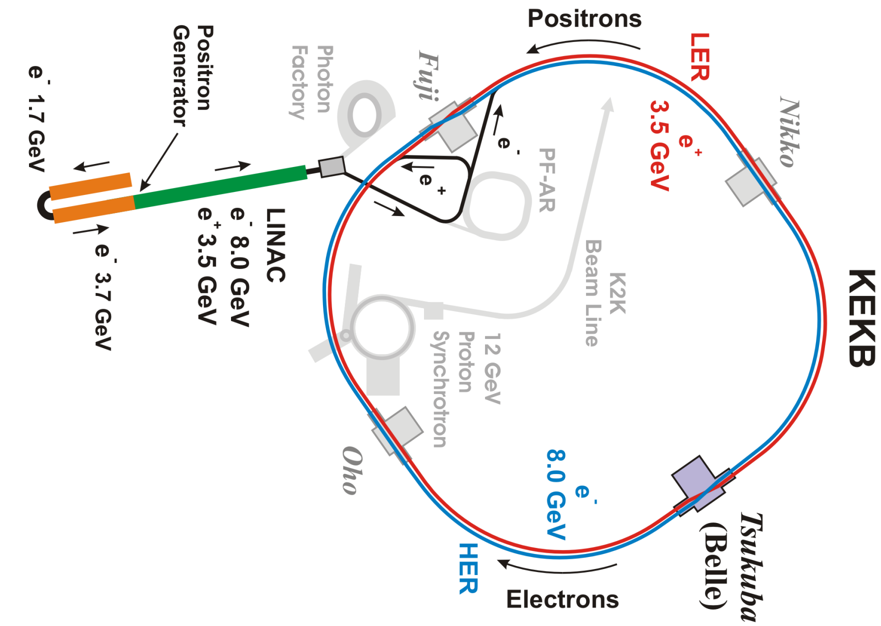
- Precise measurements of properties with possibility of CPT tests:
  - Mass
  - Lifetime
  - Electric and Magnetic DM
- Study of pure leptonic decays
  - Lepton flavor universality (LFU)
  - Michel parameters
- Study of hadronic decays
  - QCD at 1 GeV
  - LFU
  - CP violation (CPV)
- Direct search for New Physics
  - Lepton flavor violation (LFV)
  - Invisible particles

Belle as a τ factory

- e^+e^- colliders outperform hadron machines in τ physics because the $\tau^+\tau^-$ initial state is known, and the detectors are nearly hermetic
- Existing experiments:
    - BES III and KEDR (limited in statistics compared to Belle and Belle II)
    - *B*-factories Belle and BaBar (Belle II ancestors) are perfect for the τ lepton studies due to unprecedented $\tau^+\tau^-$ data samples
- Belle integrated luminosity of $\mathscr{L} = 1\,\text{ab}^{-1}$ provides 912×10^6 $\tau^+\tau^-$ pairs

The largest amount of $\tau^+\tau^-$ pairs





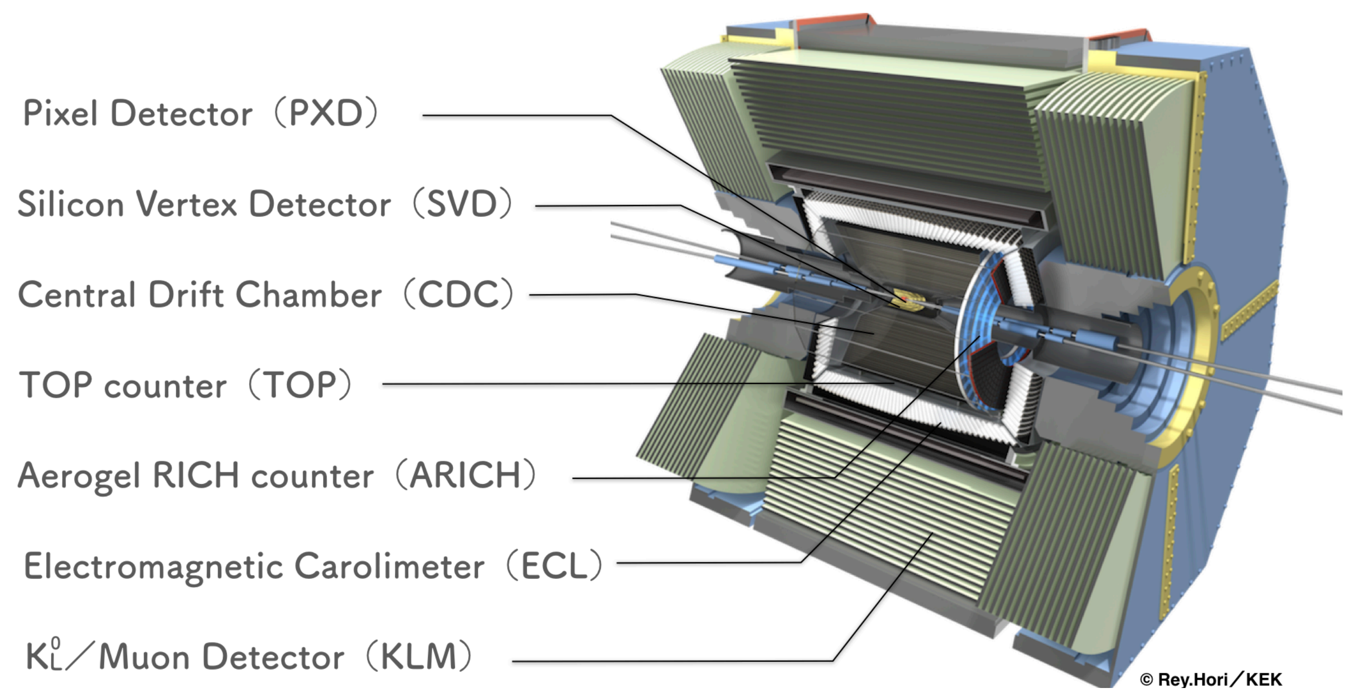
[PTEP 2012 (2012) 1, 04D001](#)

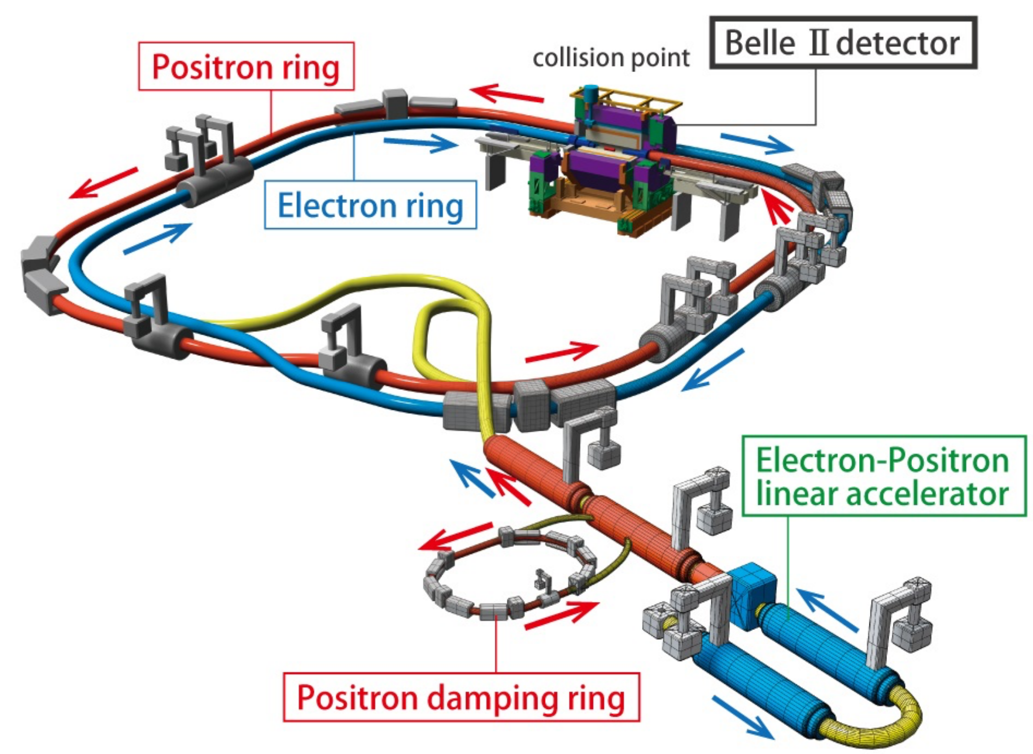
Belle II as a τ factory

- e^+e^- colliders outperform hadron machines in τ physics
- Existing experiments:
  - BES III and KEDR (limited in statistics compared to Belle II)
  - *B*-factories Belle and BaBar (Belle II ancestors) are perfect for the τ lepton studies due to unprecedented $\tau^+\tau^-$ data samples (for the time being, they surpass the Belle II statistics of $\mathscr{L} = 424\,\text{fb}^{-1}$)
- Belle II expects integrated luminosity of $\mathscr{L} = 50\,\text{ab}^{-1}$ (full luminosity or FL) providing 46×10^9 $\tau^+\tau^-$ pairs
- Significant improvements on the trigger for low-multiplicity events

The Future belongs to Belle II

PTEP 2019 (2019) 12, 123C01





Mass of the τ lepton

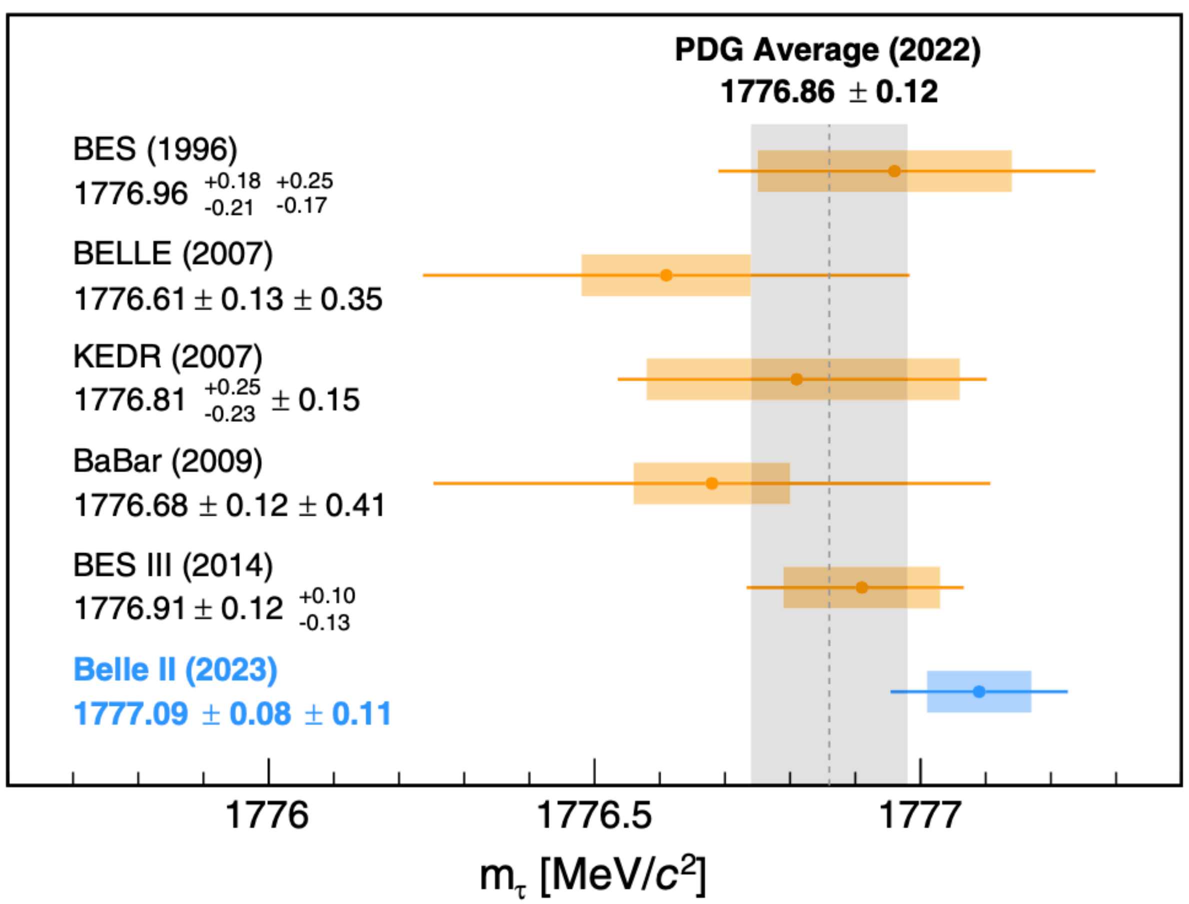
Precision is needed for LFU and $\alpha_s(m_\tau)$

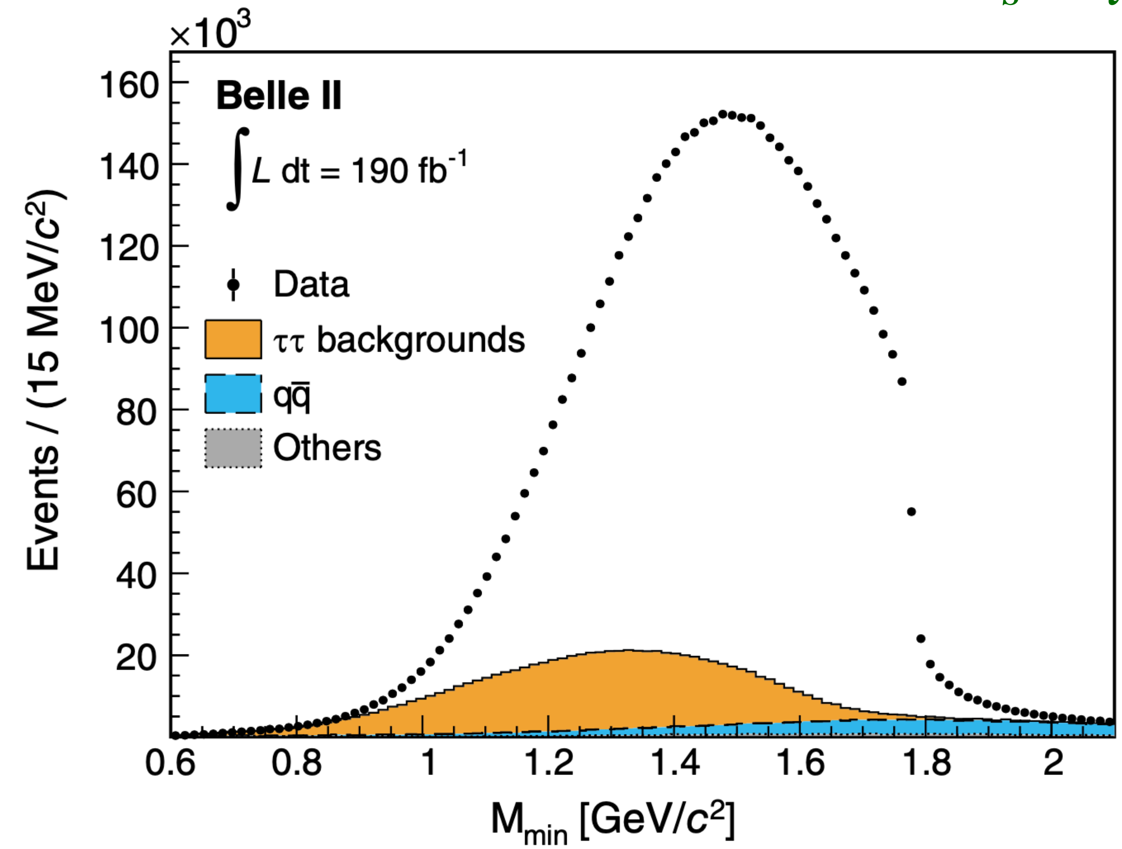
Phys.Rev.D 108 (2023) 032006

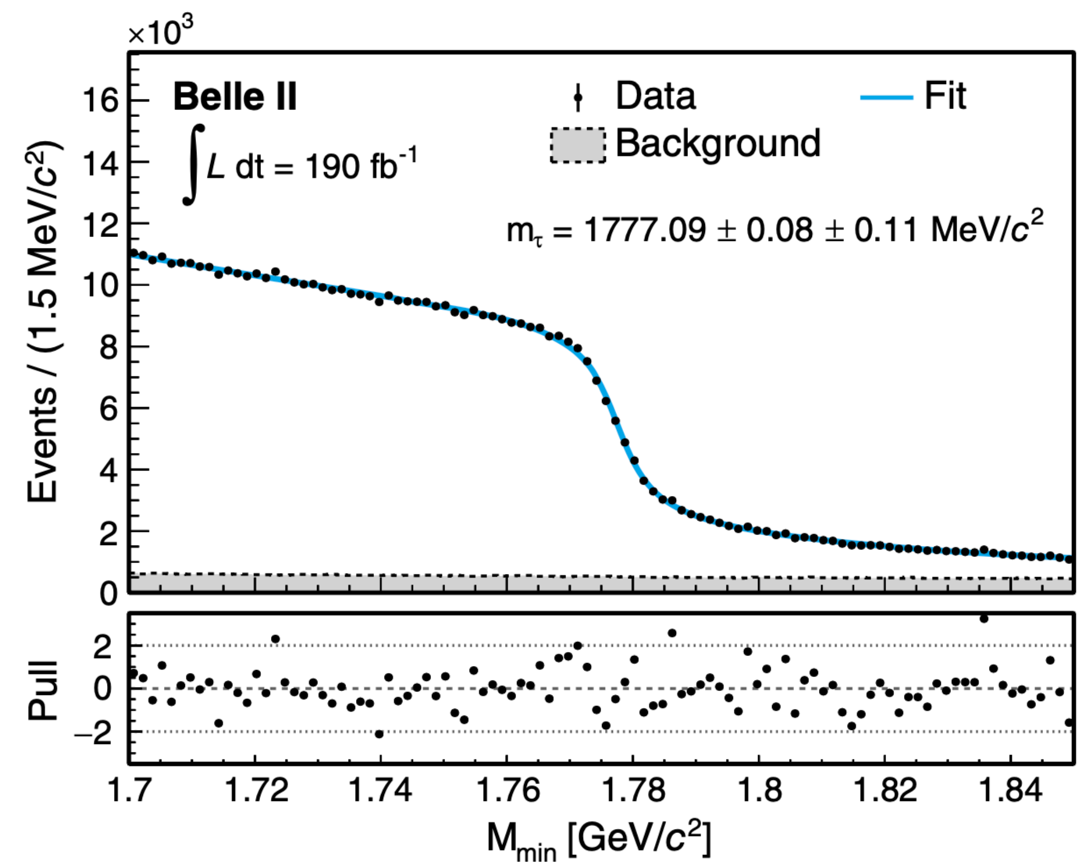
- Belle II in $\tau^- \to \pi^-\pi^+\pi^-\nu_\tau$ ($\mathscr{L} = 190\,\mathrm{fb}^{-1}$)
- Pseudomass method

$$M_{\min} = \sqrt{M_{3\pi}^2 + 2(\sqrt{s}/2 - E_{3\pi}^*)(E_{3\pi}^* - p_{3\pi}^*)} \leq m_\tau$$

Accuracy in $\sqrt{s}$ and p is the key to precision



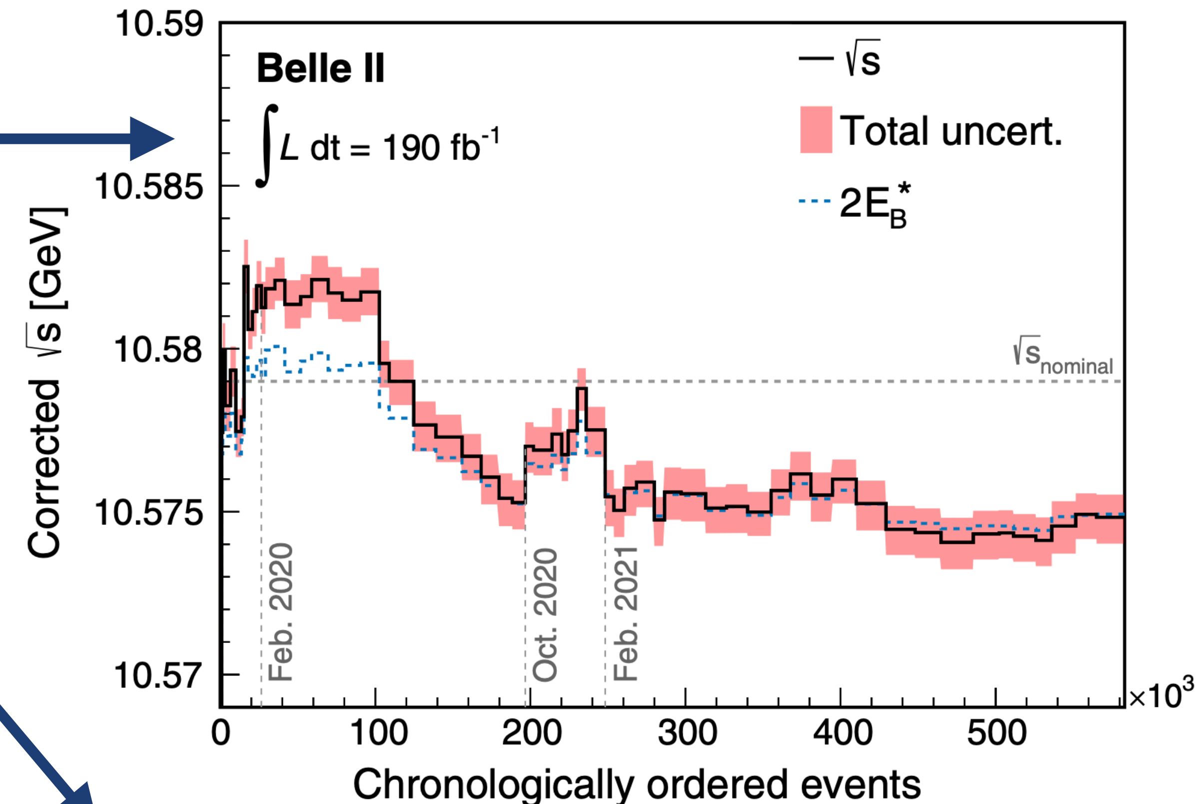




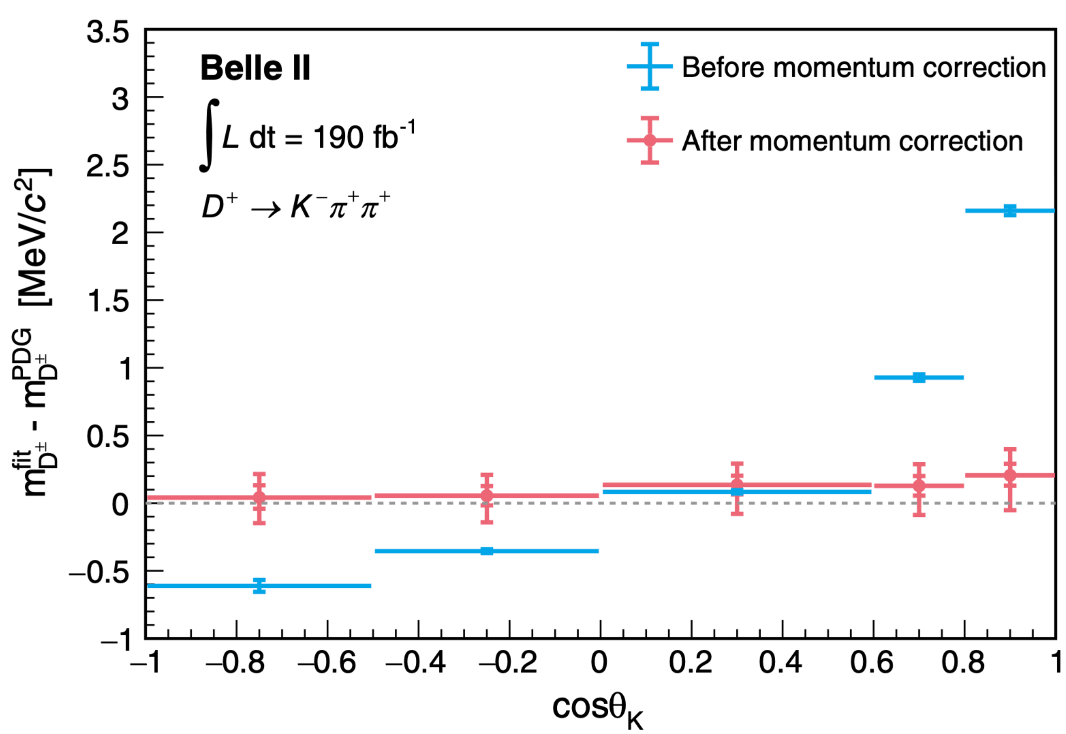
Belle II provides World's most precise result

Mass of the τ lepton (2)

- Systematics are crucial in this study
- Beam energy calibration using hadronic $B\bar{B}$-pair decays and $e^+e^- \to B\bar{B}$ cross section
- Charged-particle momentum correction using $D^0 \to K^-\pi^+$ sample with cross-checks in $D^+ \to K^-\pi^+\pi^+$, $D^0 \to K^-\pi^+\pi^-\pi^+$, and $J/\psi \to \mu^+\mu^-$

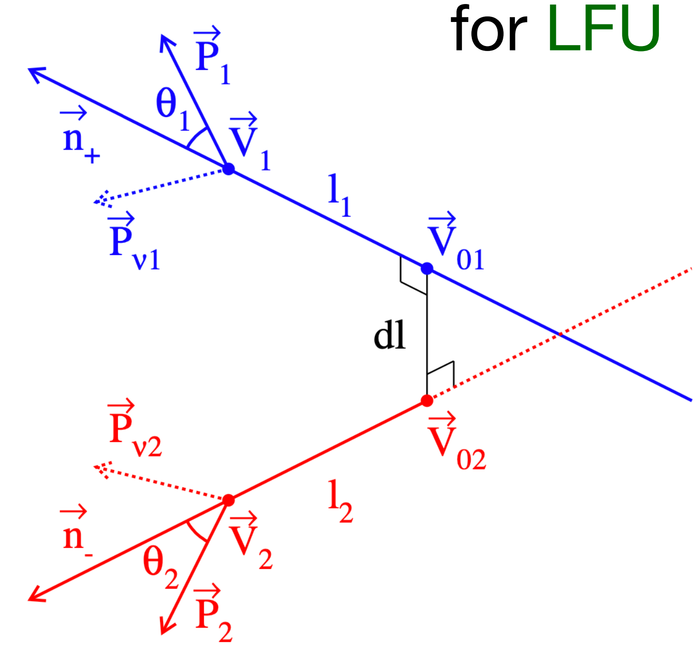


Source	Uncertainty (MeV/c^2)
Knowledge of the colliding beams:	
Beam-energy correction	0.07
Boost vector	< 0.01
Reconstruction of charged particles:	
Charged-particle momentum correction	0.06
Detector misalignment	0.03
Fit model:	
Estimator bias	0.03
Choice of the fit function	0.02
Mass dependence of the bias	< 0.01
Imperfections of the simulation:	
Detector material density	0.03
Modeling of ISR, FSR and τ decay	0.02
Neutral particle reconstruction efficiency	$\leq$ 0.01
Momentum resolution	< 0.01
Tracking efficiency correction	< 0.01
Trigger efficiency	< 0.01
Background processes	< 0.01
Total	0.11



Lifetime of the τ lepton

Precision is needed for LFU

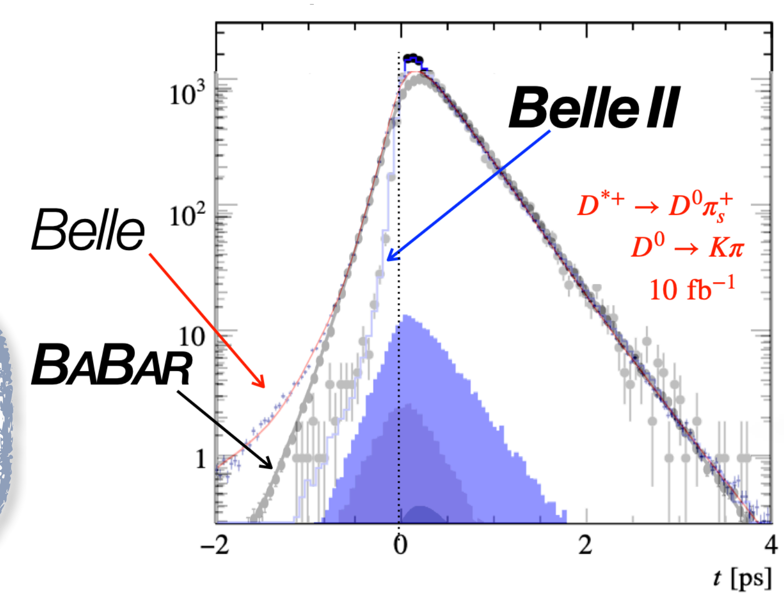


- Boost of the τ lepton in the Laboratory frame is required
- The most precise measurement is done by Belle using $\mathscr{L} = 711\ \text{fb}^{-1}$ in $e^+e^- \to \tau^+\tau^- \to (\pi^+\pi^-\pi^+\bar{\nu}_\tau, \pi^+\pi^-\pi^-\nu_\tau)$:
  $[290.17 \pm 0.53(\text{stat}) \pm 0.33(\text{syst})] \times 10^{-15}\ \text{s}$
- The CPT invariance was tested for the first time:
  $|\langle\tau_{\tau^+}\rangle - \langle\tau_{\tau^-}\rangle| / \langle\tau_\tau\rangle < 7.0 \times 10^{-3}$ (90 % CL)

Source	$\Delta\langle\tau\rangle$ (μm)
SVD alignment	0.090
Asymmetry fixing	0.030
Beam energy, ISR and FSR description	0.024
Fit range	0.020
Background contribution	0.010
τ-lepton mass	0.009
Total	0.101

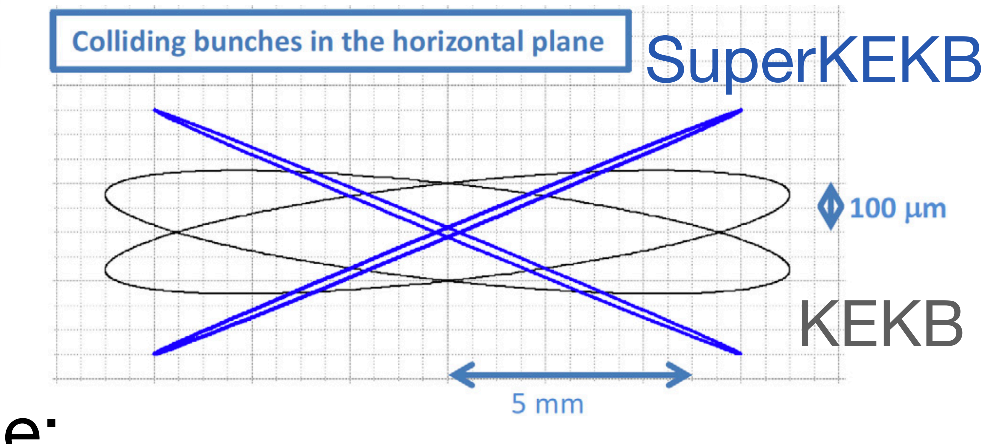
Phys.Rev.Lett. 112 (2014) 031801

- For lifetime, the statistical uncertainty is dominant, and the main systematics source is SVD alignment
- For CPT test, most systematics cancel
- The result can be improved by Belle II with more statistics and better vertex detector



x2 better time resolution (visible at $t < 0$)

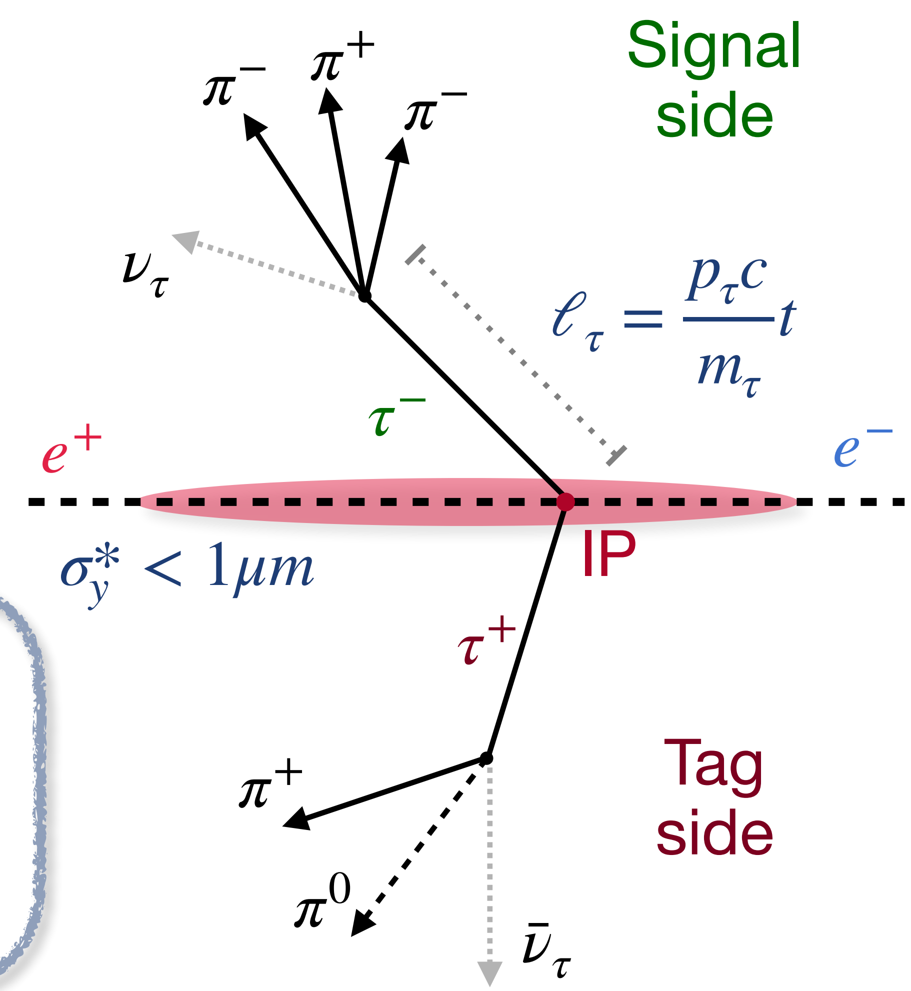
Lifetime of the τ lepton: new approach



- SuperKEKB uses nanobeam collision scheme: constraint to the beam-spot can be done

Nucl.Instrum.Meth.A 907 (2018) 188-199

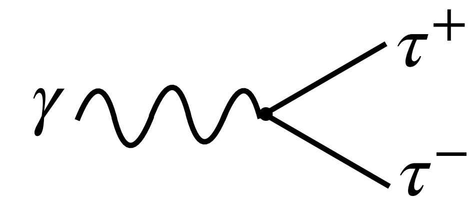
- Second τ lepton can be reconstructed in one-prong decay mode $\tau^+ \to \rho^+ \bar{\nu}_\tau$: increase in statistics compared to $\tau^+ \to \pi^+\pi^-\pi^+\bar{\nu}_\tau$ due to higher branching fraction
- One neutrino mode is still needed for the τ lepton momentum reconstruction
- Competitive results can be obtained with current statistics (check the talk by Stefano Moneta at the XXVII Cracow EPIPHANY Conference on Future of particle physics)



EDM and MDM

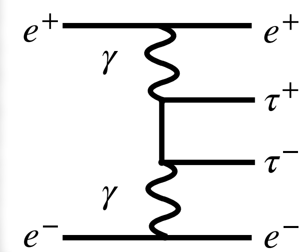
[1] JHEP 04 (2022) 110
[2] 2207.06307 [hep-ex]
[3] Eur.Phys.J.C 35 (2004) 159-170
[4] JHEP 10 (2019) 089

- General expression of the $\tau\tau\gamma$ vertex includes Electric and Magnetic Dipole Moments (EDM — d_τ and MDM — a_τ)



- In the SM, the first is forbidden by T-invariance, and the second is $a_\tau^{\text{SM}} = 117721(5) \times 10^{-8}$
- For EDM, matrix element can be written as $M^2 = M^2_{\text{SM}} + \Re(d_\tau)M^2_{\Re} + \Im(d_\tau)M^2_{\Im} + |d_\tau|^2 M^2_{d^2}$

MDM measurement by DELPHI [3]



Two photon approach is used

$-0.052 < a_\tau < 0.013$ (95 % CL)

EDM measurement by Belle ($\mathscr{L} = 833$ fb^{-1}) [1]

Triple momentum and spin correlation observables (so called optimal observables)

$$O_{\Re} = \frac{M^2_{\Re}}{M^2_{\text{SM}}}, \quad O_{\Im} = \frac{M^2_{\Im}}{M^2_{\text{SM}}} \quad \langle O_{\Re}\rangle = a_{\Re}\Re(d_\tau) + b_{\Re} \quad \langle O_{\Im}\rangle = a_{\Im}\Im(d_\tau) + b_{\Im}$$

$$-1.85 \cdot 10^{-17} < \Re(d_\tau) < 6.1 \cdot 10^{-18}\, e\text{cm (95 \% CL)}$$

$$-1.03 \cdot 10^{-17} < \Im(d_\tau) < 2.3 \cdot 10^{-18}\, e\text{cm (95 \% CL)}$$

Belle II (FL) expects $|a_\tau^{\text{NP}}| < 2 \times 10^{-5}$ [4]

Mode	$\text{Re}(d_\tau)(10^{-17}\,e\text{cm})$	$\text{Im}(d_\tau)(10^{-17}\,e\text{cm})$
$e\mu$	$-3.2 \pm 2.5 \pm 3.6$	$0.6 \pm 0.4 \pm 1.8$
$e\pi$	$0.7 \pm 2.3 \pm 4.8$	$2.4 \pm 0.5 \pm 2.2$
$\mu\pi$	$1.0 \pm 2.2 \pm 4.3$	$2.4 \pm 0.5 \pm 2.6$
$e\rho$	$-1.2 \pm 0.8 \pm 1.0$	$-1.1 \pm 0.3 \pm 0.6$
$\mu\rho$	$0.7 \pm 1.0 \pm 2.2$	$-0.5 \pm 0.3 \pm 0.8$
$\pi\rho$	$-0.6 \pm 0.7 \pm 1.0$	$0.4 \pm 0.3 \pm 1.2$
$\rho\rho$	$-0.4 \pm 0.5 \pm 0.9$	$-0.3 \pm 0.3 \pm 0.4$
$\pi\pi$	$-2.2 \pm 4.3 \pm 5.2$	$-0.9 \pm 0.9 \pm 1.2$

Belle II (FL) expects $|\Re, \Im(d_\tau)| < 10^{-18} - 10^{-19}$ [2]

Leptonic decays: LFU

Talk by Paul Feichtinger at TAU2023

- Precise test of $\mu - e$ universality:

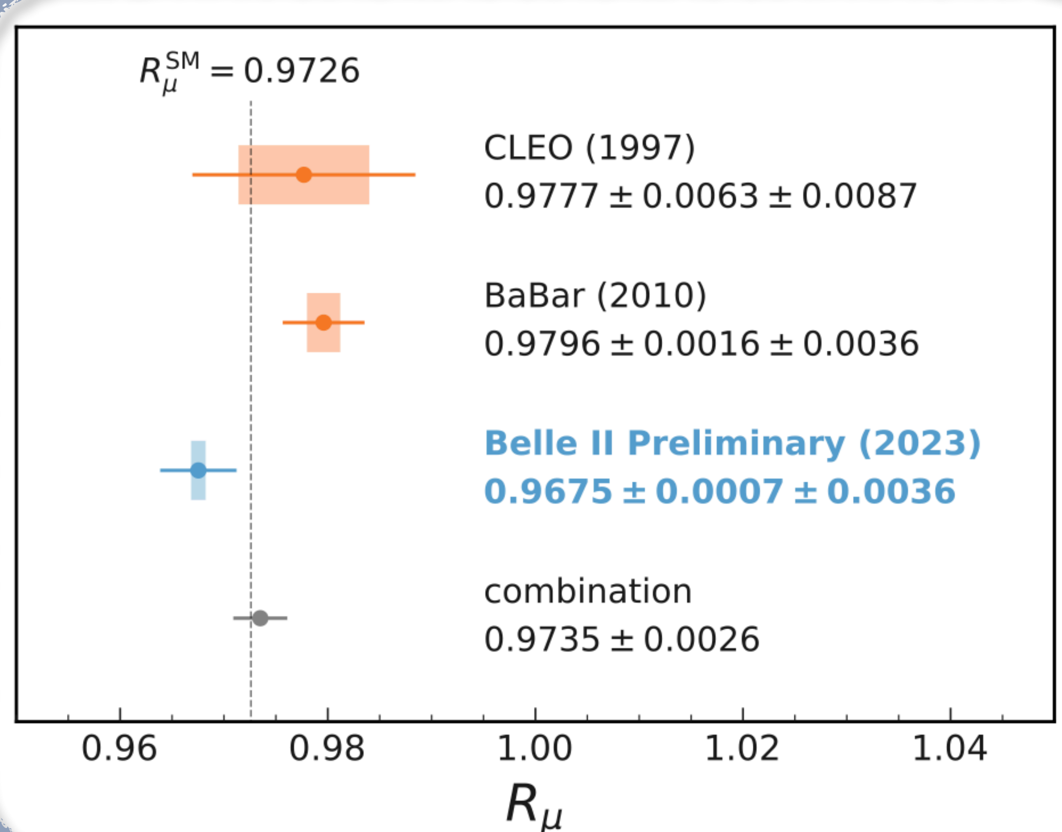
$$\left(\frac{g_\mu}{g_e}\right)_\tau = \sqrt{R_\mu \frac{f(m_e^2/m_\tau^2)}{f(m_\mu^2/m_\tau^2)}}$$

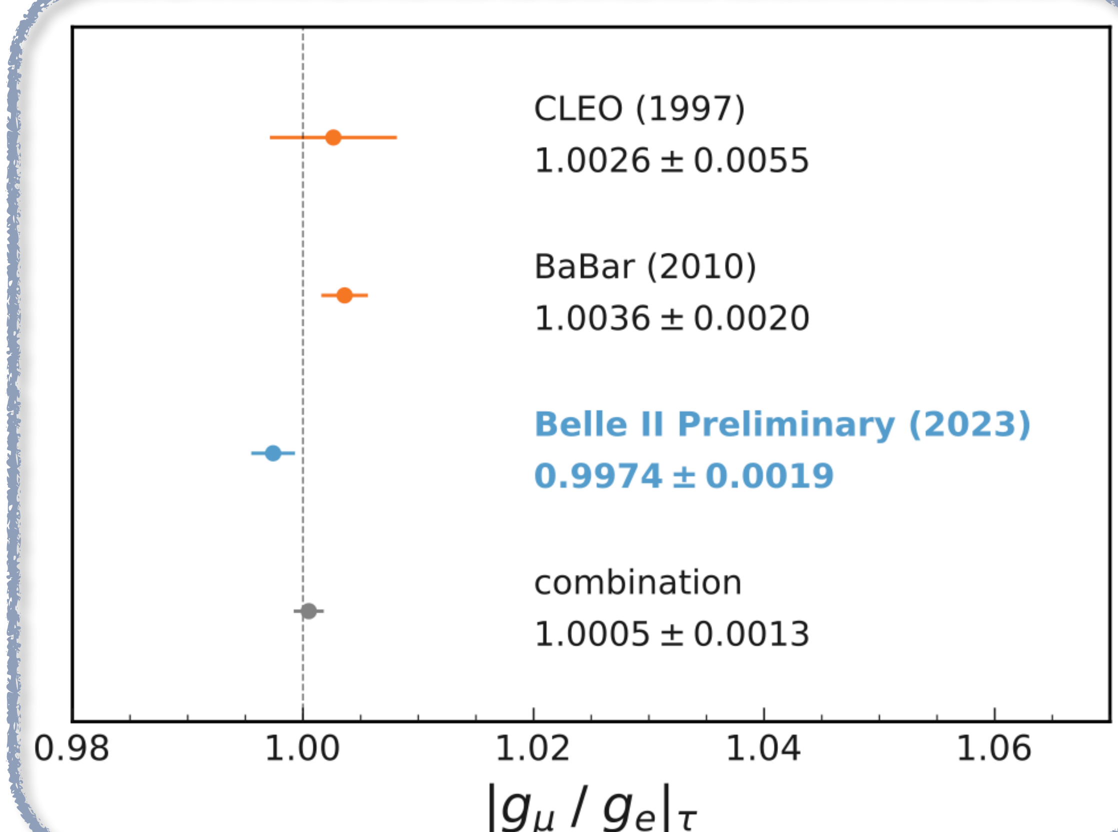
$$R_\mu = \frac{\mathscr{B}(\tau^- \to \mu^- \bar{\nu}_\mu \nu_\tau(\gamma))}{\mathscr{B}(\tau^- \to e^- \bar{\nu}_e \nu_\tau(\gamma))} \stackrel{\text{SM}}{=} 0.9726$$

$$f(x) = 1 - 8x + 8x^3 - x^4 - 12x^2 \ln x$$

- Belle II with $\tau^+ \to \rho^+ \bar{\nu}_\tau$ in tag side ($\mathscr{L} = 362\,\text{fb}^{-1}$)
- Selection based on rectangular criteria and neural network (NN) classifier
- Leading systematics from Particle IDentification (PID) ($0.32\,\%$) evaluated using control samples
- Sub-leading systematics from trigger ($0.10\,\%$)

Most precise test in single measurement





Leptonic decays: Michel parameters

- Michel parameters (MP) describe the Lorentz structure of the charged currents interaction in the theory of weak interaction and can be used to test the SM
- If daughter lepton polarization is not measured, only 4 MPs (ρ, η, ξ, and $\xi\delta$) are accessible
- Michel parameter ξ' describes longitudinal polarization of daughter lepton

Current status of MP measurements

MP (SM)	$\tau \to e\nu_e\nu_\tau$	$\tau \to \mu\nu_\mu\nu_\tau$
$\rho\,(0.75)$	0.747 ± 0.010	0.763 ± 0.020
$\eta\,(0)$	0.013 ± 0.020	0.094 ± 0.073
$\xi\,(1)$	0.994 ± 0.040	1.030 ± 0.059
$\xi\delta\,(0.75)$	0.734 ± 0.028	0.778 ± 0.037
$\xi'\,(1)$	NM	0.22 ± 1.03

PTEP 2022 (2022) 083C01

Nucl.Part.Phys.Proc. 287-288 (2017)

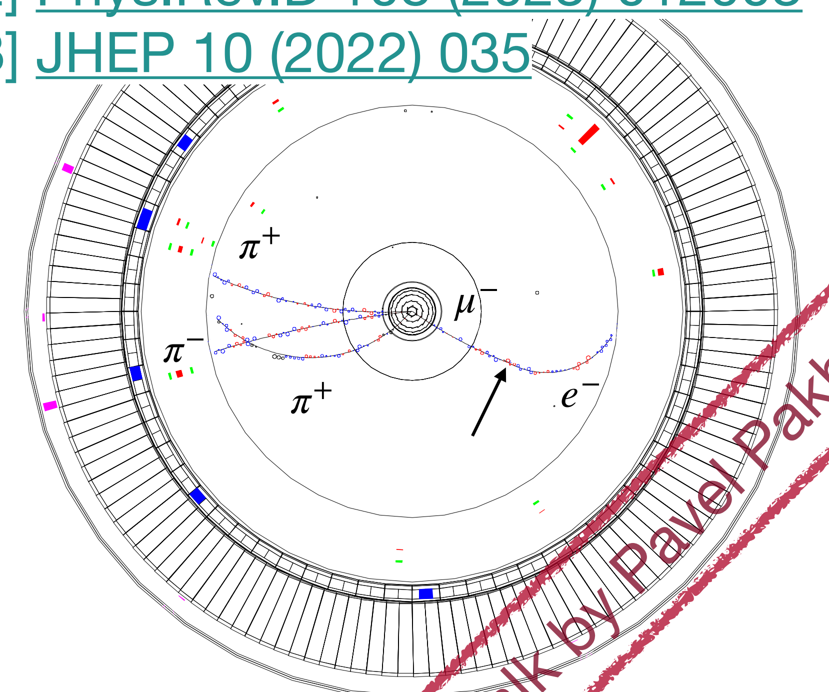
For MP ρ, η, ξ, and $\xi\delta$, Belle has already achieved statistical uncertainty of an order 10^{-3}, but systematics is around 10^{-2} (main contribution is from trigger efficiency)

At Belle II (FL), statistical uncertainties will be of the order 10^{-4}, and the systematic errors will be the dominant one

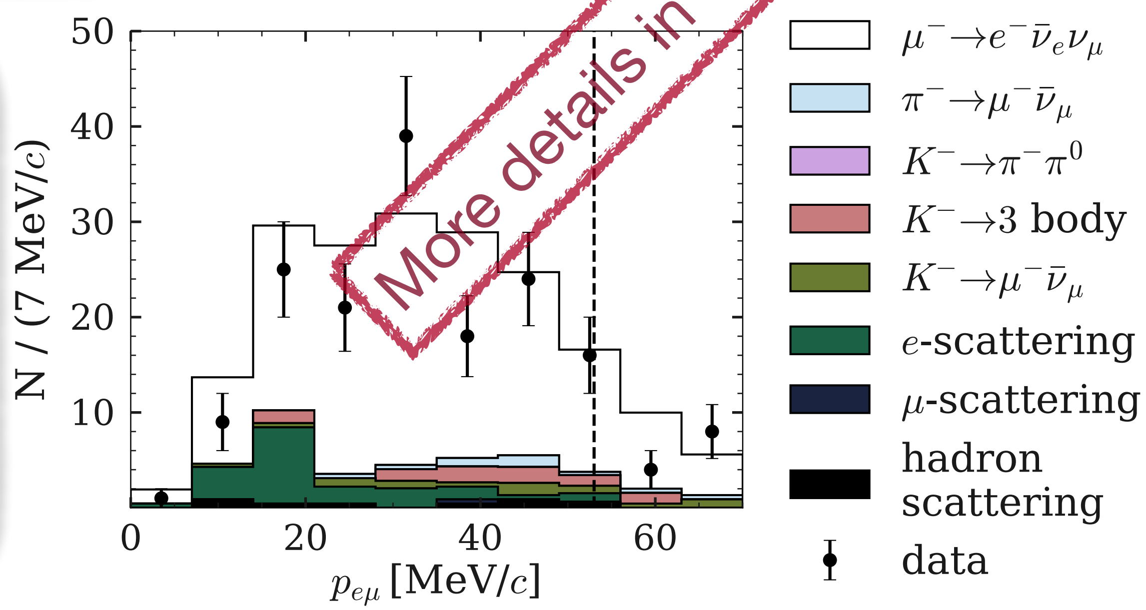
Measurement of the MP ξ' in the $\tau^- \to \mu^- \bar{\nu}_\mu \nu_\tau$ decay

[1] Phys.Rev.Lett. 131 (2023) 021801
[2] Phys.Rev.D 108 (2023) 012003
[3] JHEP 10 (2022) 035

- The method is based on the muon decay-in-flight reconstruction in the tracker as a kink
- The information about muon spin can be inferred from the daughter electron direction in the muon rest frame due to *P*-violation in the decay
- The first measurement was performed by the Belle collaboration ($\mathscr{L} = 988\,\mathrm{fb}^{-1}$) [1, 2]: $\xi' = 0.22 \pm 0.94(\mathrm{stat}) \pm 0.42(\mathrm{syst})$
- With enlarged CDC, special kink reconstruction algorithm (crucial), and record integrated luminosity, Belle II (FL) can improve the statistical uncertainty up to $\sigma_{\xi'} \approx 7 \times 10^{-3}$ [3]
- Systematics can be controlled at the same level with various data samples with kinks



More details in talk by Pavel Pakhlov



Radiative and five-body leptonic τ decays

- Radiative and five-body leptonic τ-decays provide information about Michel parameters that describe daughter lepton polarization in $\tau^- \to \ell^- \bar{\nu}_\ell \nu_\tau$
- Their understanding is also crucial for LFV studies as they are main background

Radiative leptonic τ-decay

Belle collaboration measured $\xi\kappa(e) = -0.4 \pm 1.2$, $\xi\kappa(\mu) = 0.8 \pm 0.6$, and $\bar{\eta}(\mu) = -1.3 \pm 1.7$ ($\mathscr{L} = 711\ \text{fb}^{-1}$)

PTEP 2018 (2018) 2, 023C01

$\xi\kappa = -1/4(\xi + \xi') + 2/3\xi\delta$ $\quad \bar{\eta} = 4/3\rho - 1/4\xi'' - 3/4$

Belle II can repeat with better precision!

Five-body leptonic τ-decay

Belle estimations for $\mathscr{L} = 700\ \text{fb}^{-1}$

Mode	SM Br	Measured	Expected N	Systematics
$\tau^- \to e^- e^+ e^- \bar{\nu}_e \nu_\tau$	$4.21(1) \times 10^{-5}$	$(1.8 \pm 1.5) \times 10^{-5}$	$1300\ (r_s = 47\%)$	$(6-12)\ \%$
$\tau^- \to \mu^- e^+ e^- \bar{\nu}_e \nu_\tau$	$1.984(4) \times 10^{-5}$	$< 3.2 \times 10^{-5}\ (90\%)$	$430\ (r_s = 50\%)$	$(8-13)\ \%$
$\tau^- \to e^- \mu^+ \mu^- \bar{\nu}_e \nu_\tau$	$1.247(1) \times 10^{-7}$	NM	$8\ (r_s = 37\%)$	$(36-72)\ \%$
$\tau^- \to \mu^- \mu^+ \mu^- \bar{\nu}_e \nu_\tau$	$1.183(1) \times 10^{-7}$	NM	$4\ (r_s = 16\%)$	$(36-72)\ \%$

JHEP 04 (2016) 185 J.Phys.Conf.Ser. 912 (2017) 1

CP violation

- No CPV is observed in the charged leptons sector (in the SM, it is predicted only in quarks sector)

- The most promising modes for the studies: $\tau^- \to K^-\pi^0\nu_\tau$, $\tau^- \to K_S^0\pi^-\nu_\tau$, $\tau^- \to K_S^0\pi^-\pi^0\nu_\tau$, $\tau^- \to (\rho\pi)^-\nu_\tau$, $\tau^- \to (\omega\pi)^-\nu_\tau$, and $\tau^- \to (a_1\pi)^-\nu_\tau$

The first measurement of the CP asymmetry was performed by BaBar in $\tau^- \to \pi^- K_S^0 \nu_\tau$:

Phys.Rev.D 85 (2012) 031102

$$A_\tau = \frac{\Gamma(\tau^+ \to \pi^+ K_S^0 \bar{\nu}_\tau) - \Gamma(\tau^- \to \pi^- K_S^0 \nu_\tau)}{\Gamma(\tau^+ \to \pi^+ K_S^0 \bar{\nu}_\tau) + \Gamma(\tau^- \to \pi^- K_S^0 \nu_\tau)}$$

$$A_\tau^{\text{SM}} = (0.36 \pm 0.01)\,\%$$

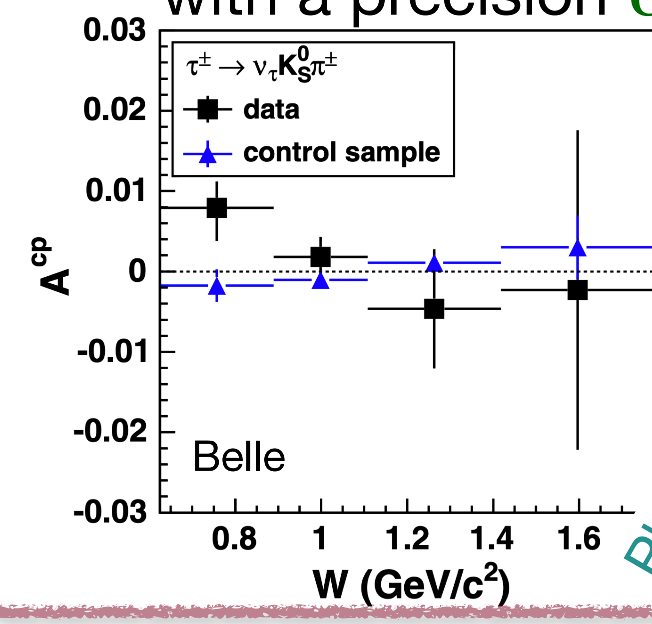
$$A_\tau = (-0.36 \pm 0.23 \pm 0.11)\,\%$$

- It is also possible to use a modified asymmetry with differential distributions integrated over a limited volume in the phase space with a specially selected kernel (done by Belle) →

- More complicated and most powerful method is to use unbinned maximum likelihood fit in the full phase space (not done at *B*-factories)

Belle II (FL) can approach the sensitivity level of 10^{-4}

A^{CP} is compatible with zero with a precision $O(10^{-3})$

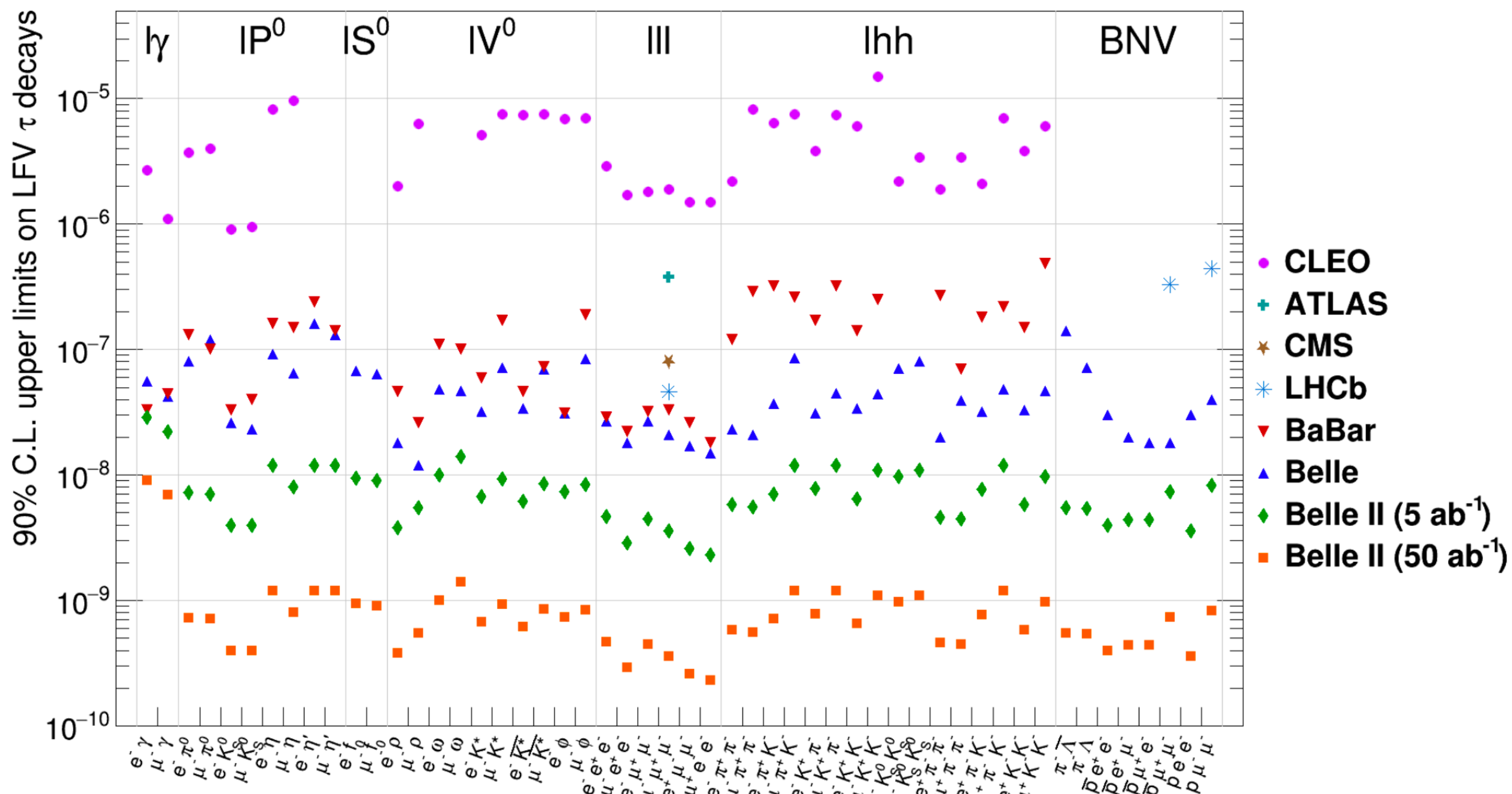


Phys.Rev.D 107 (2011) 131801

Charged Lepton Flavor Violation in τ decays

- Decays $\tau \to \ell\gamma$, $\tau \to \ell\ell\ell^{(\prime)}$, $\tau \to \ell h$ ($\ell, \ell' = e, \mu$ and h is a hadron system), and modes with baryons in the final state are sensitive to New Physics
- Different NP models predict branching fractions of such decays at the level 10^{-7}-10^{-10} (in the SM, $\sim 10^{-53}$ or even forbidden)
- The majority of World's leading results belong to Belle

2203.14919 [hep-ph]



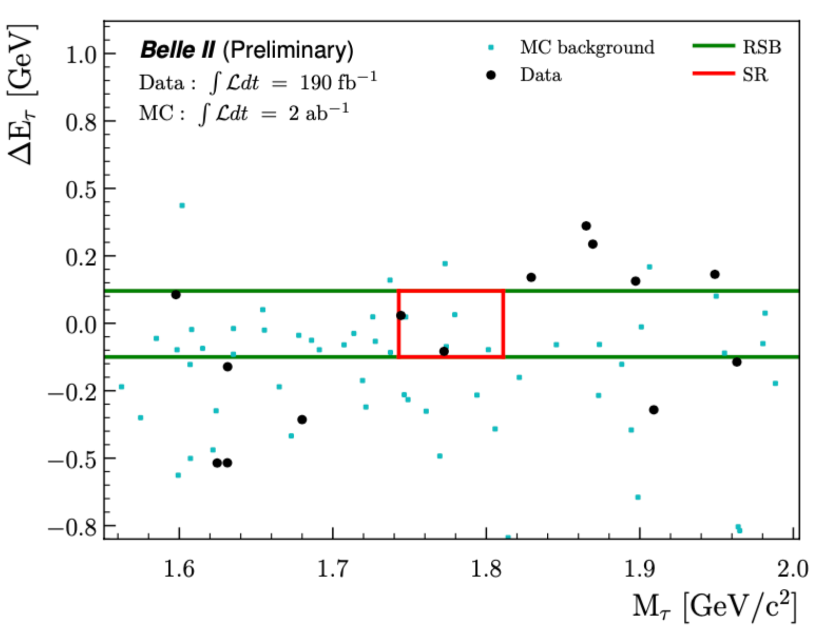
90% CL upper limits for measurements and extrapolation for Belle II from Belle results with respect to $5\,\text{ab}^{-1}$ and $50\,\text{ab}^{-1}$

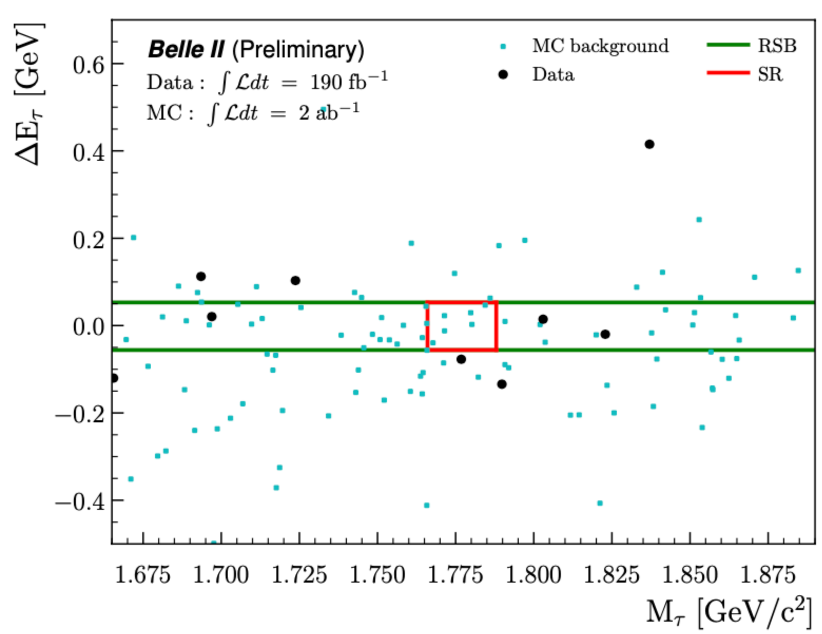
- In the zero-background scenarios, Belle II will improve Belle results linearly with the integrated-luminosity increase (assuming the same analysis efficiency)

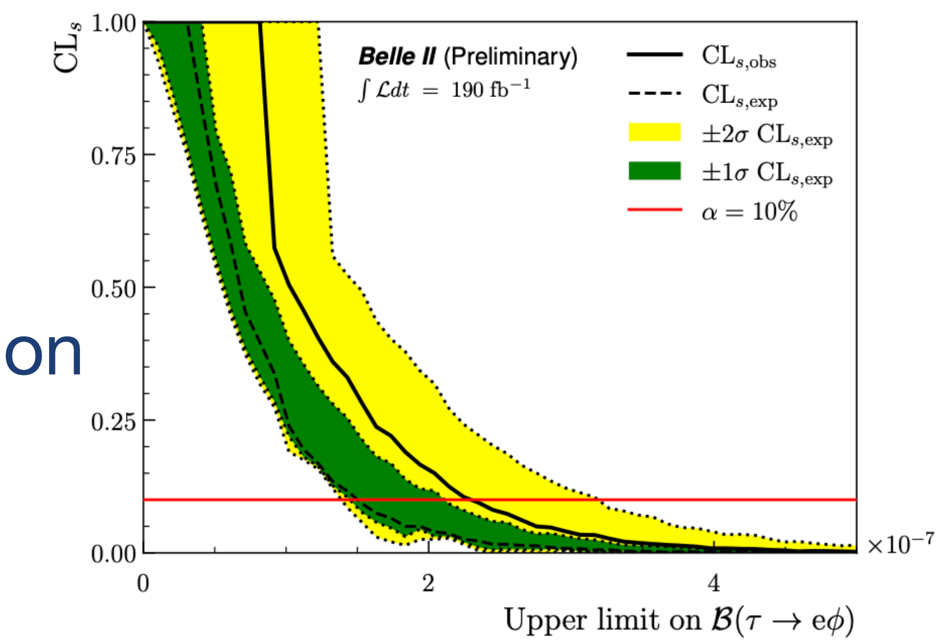
LFV: first result from Belle II

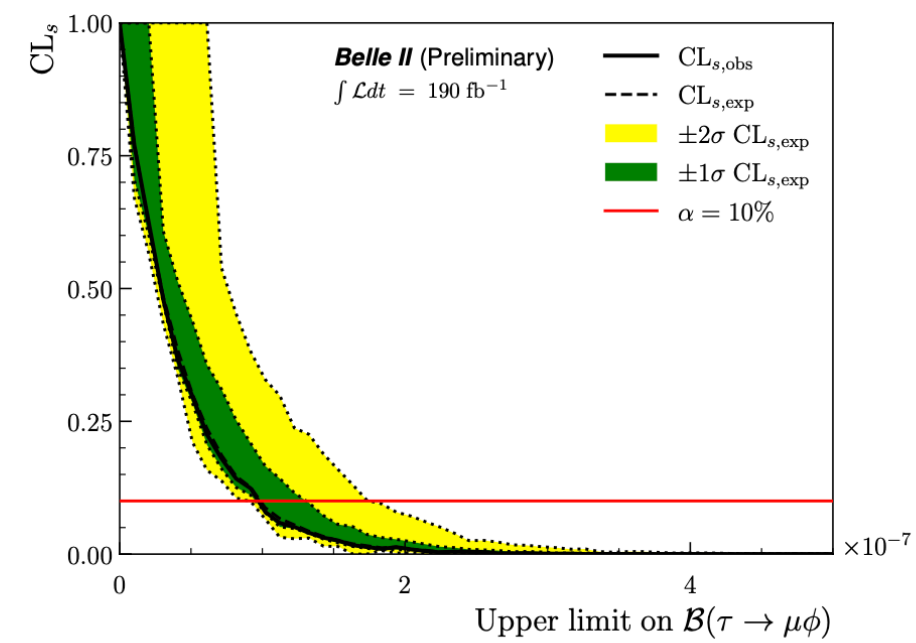
- Search for LFV $\tau^- \to \ell^- \phi$ decays ($\mathscr{L} = 190\,\text{fb}^{-1}$)
- For the first time, untagged approach is used
- Background is suppressed using BDT
- Twice the final signal efficiency improve for muon mode compared to previous studies
- Background is controlled by sidebands in data

2305.04759 [hep-ex]









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Belle

$\mathscr{B}(\tau^- \to e^-\phi) < 2.0 \times 10^{-8}\,(90\,\%\,\text{CL})$

$\mathscr{B}(\tau^- \to \mu^-\phi) < 2.3 \times 10^{-8}\,(90\,\%\,\text{CL})$

Belle II

$\mathscr{B}(\tau^- \to e^-\phi) < 23 \times 10^{-8}\,(90\,\%\,\text{CL})$

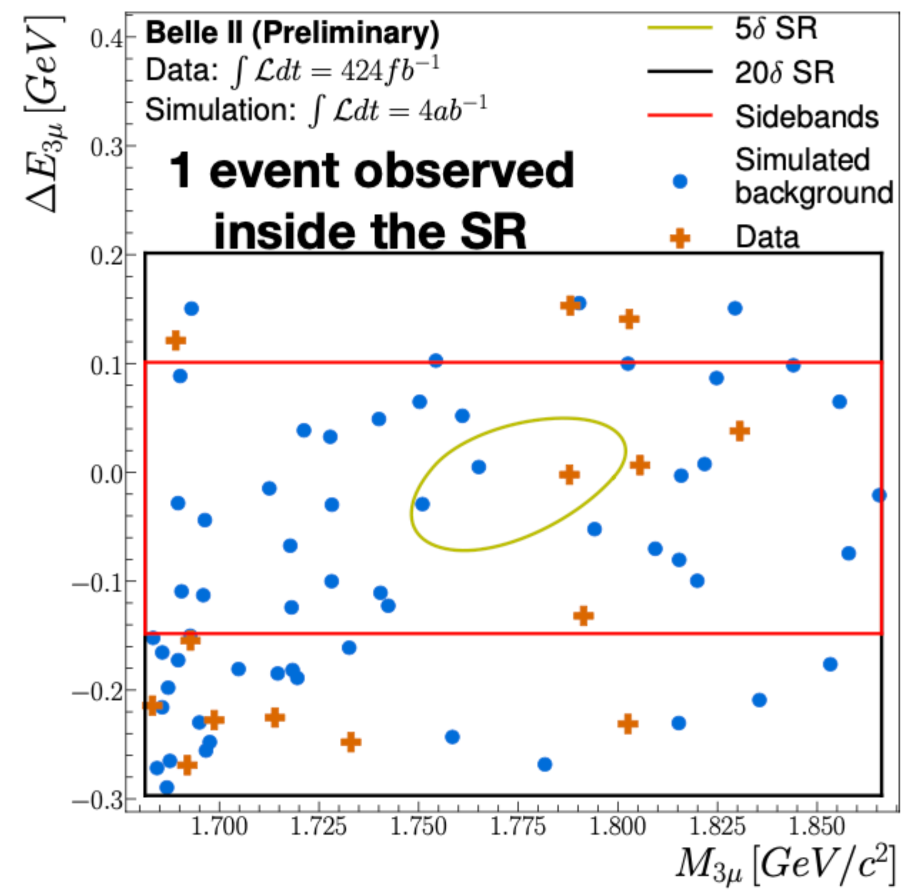
$\mathscr{B}(\tau^- \to \mu^-\phi) < 9.7 \times 10^{-8}\,(90\,\%\,\text{CL})$

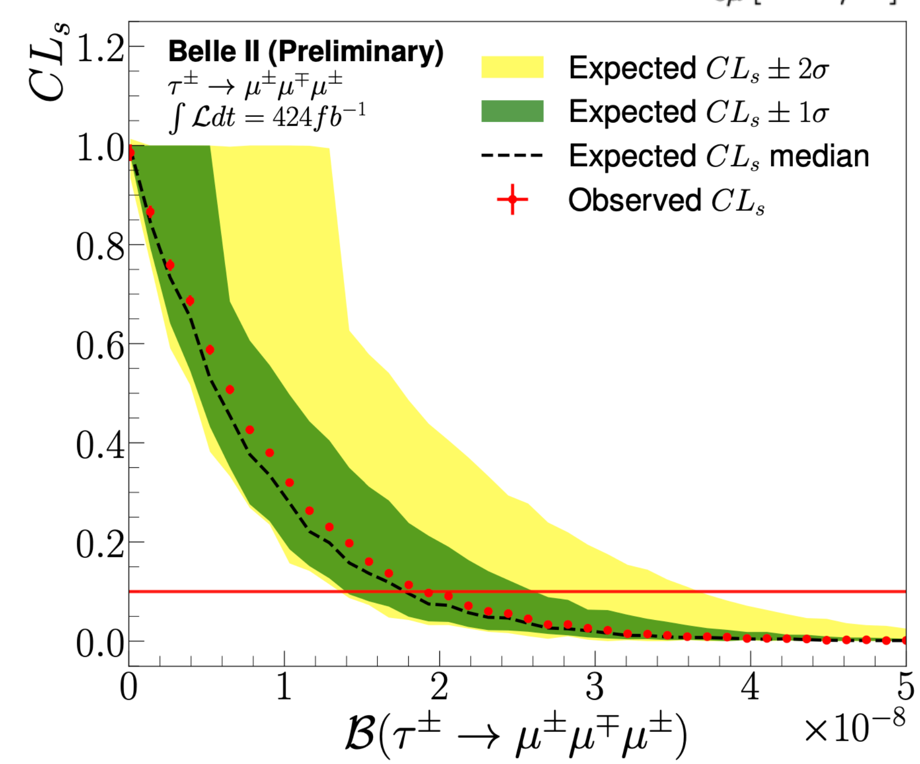
LFV: $\tau^- \to \mu^-\mu^+\mu^-$ at Belle II

- Search for LFV $\tau^- \to \mu^-\mu^+\mu^-$ decays (on-going)
- Belle II ($\mathscr{L} = 424\,\text{fb}^{-1}$) is already competitive with Belle ($\mathscr{L} = 782\,\text{fb}^{-1}$)
- Innovative inclusive untagged method with 1 and 3 tagging tracks and BDT selection
- Conventional method with muon ID and 1 tagging track is used as a cross-check
- $\varepsilon_{\text{sig}} = [20.42 \pm 0.06]\,\%$ (~ 3 times larger than Belle) with expected background $0.5^{+0.4}_{-1.5}$ events

Talk by Alberto Martini at TAU2023

Belle II $\mathscr{B}(\tau^- \to \mu^-\mu^+\mu^-) < 1.9 \times 10^{-8}\,(90\,\%\,\text{CL})$

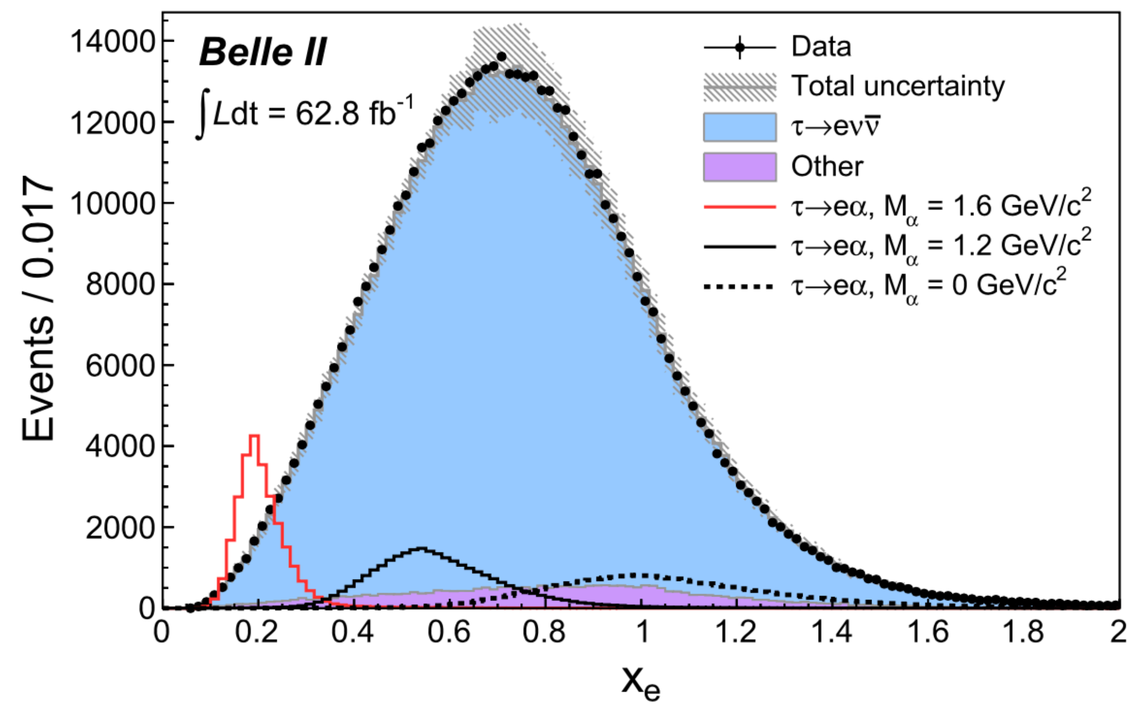


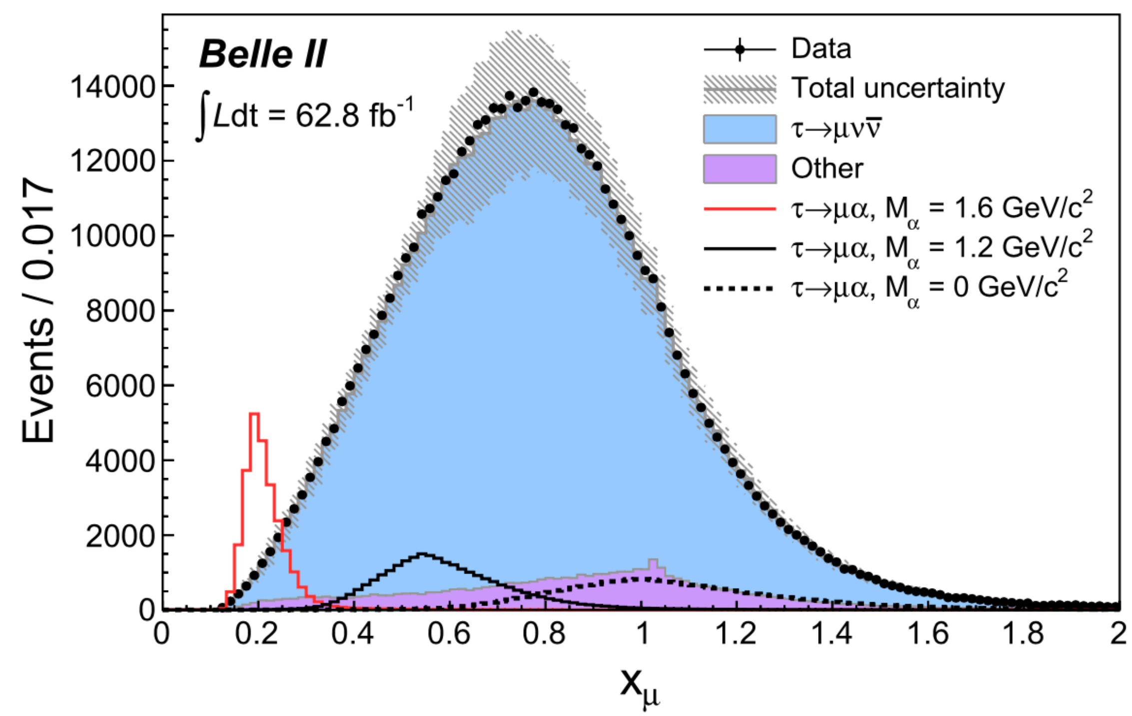


Search for LFV with Invisible boson

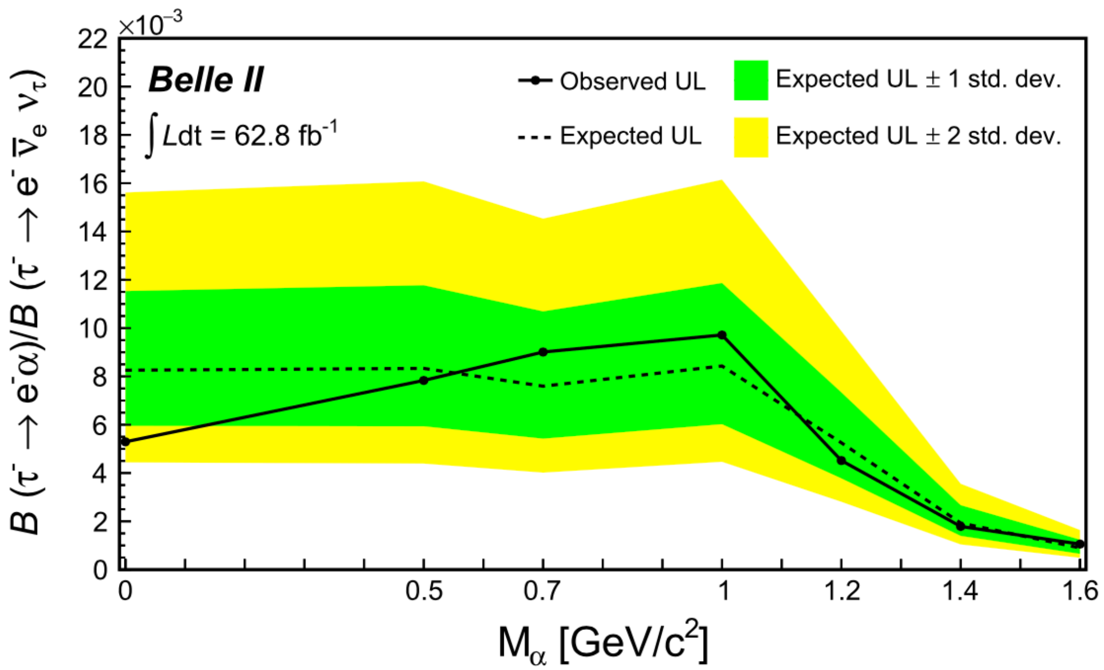
Phys.Rev.Lett. 130 (2023) 181803

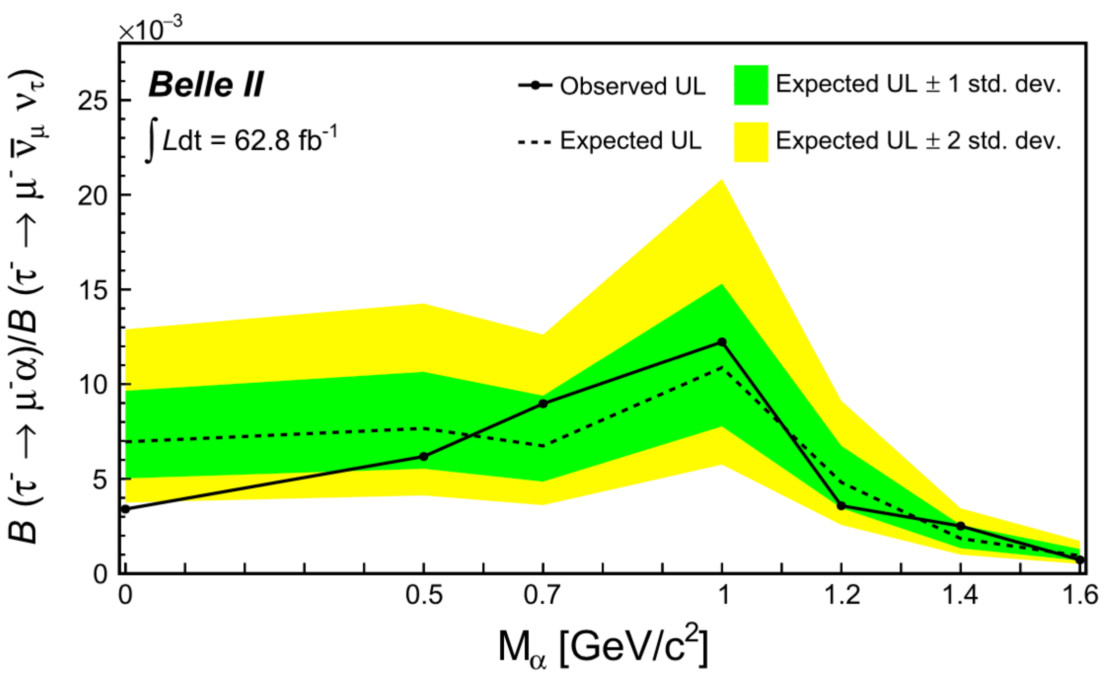
- Search for LFV $\tau^- \to \ell^- \alpha$ decays, where α is invisible spin-0 boson ($\mathscr{L} = 62.8\,\text{fb}^{-1}$)
- Predicted in models with axionlike particles
- Second τ lepton is reconstructed in $\tau^+ \to h^+ h^- h^+ \bar{\nu}_\tau$ decay mode ($h = \pi, K$)
- Pseudo τ rest frame is used ($\overrightarrow{p}_\tau \sim -\overrightarrow{p}_{3h}/|\overrightarrow{p}_{3h}|$)
- Looked for as an excess above $\tau^- \to \ell^- \bar{\nu}_\ell \nu_\tau$ spectrum
- $x_\ell = 2E_\ell/m_\tau$





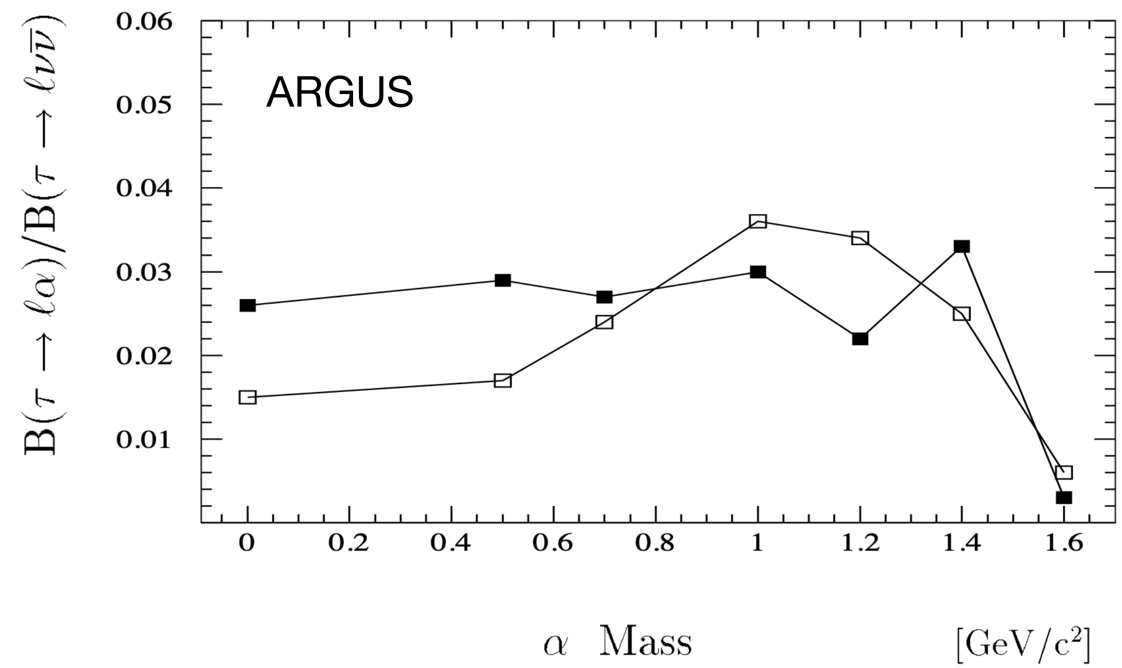
Search for LFV with Invisible boson (2)





2.2-14 times more stringent than the best previous bounds

Z.Phys.C 68 (1995) 25-28



For massless particle

Argus

$\mathscr{B}(\tau \to e\alpha) < 26 \cdot 10^{-3}$ (95 % CL)

$\mathscr{B}(\tau \to \mu\alpha) < 15 \cdot 10^{-3}$ (95 % CL)

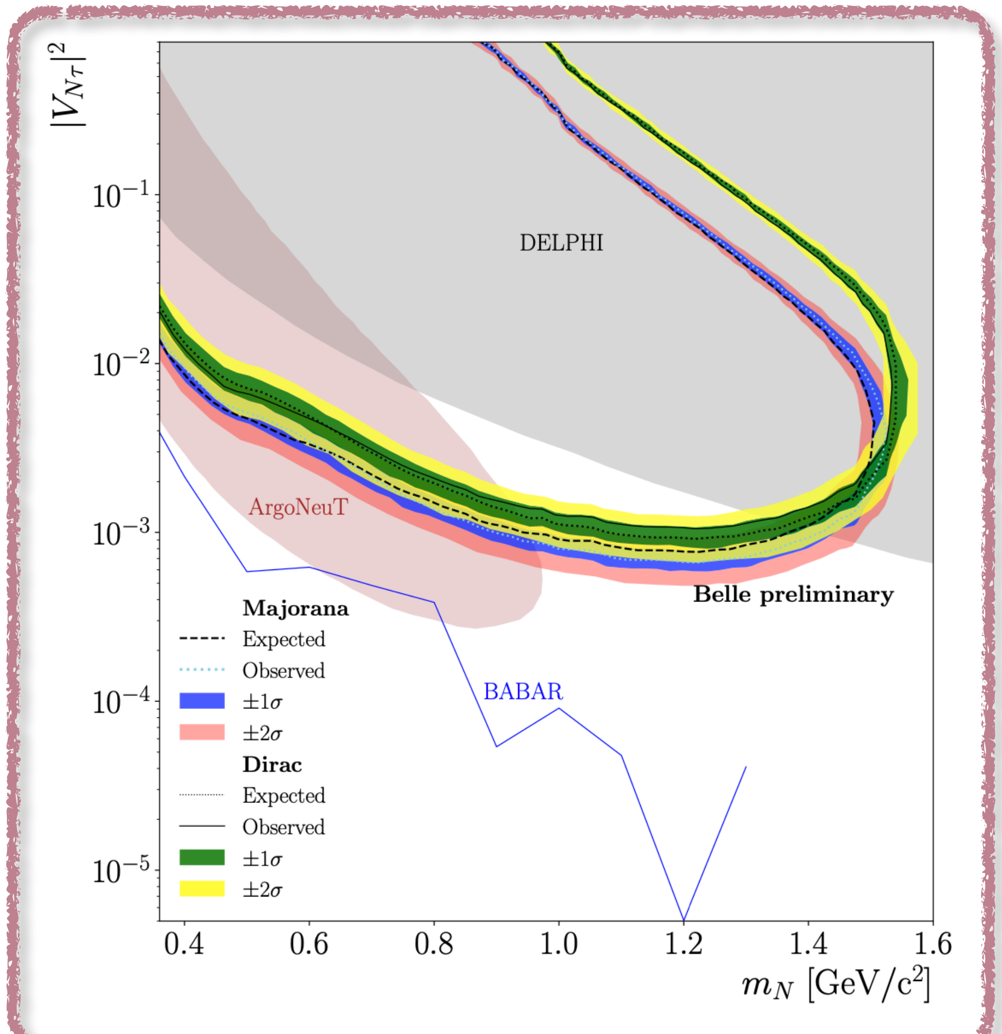
Belle II

$\mathscr{B}(\tau \to e\alpha) < 5.3 \cdot 10^{-3}$ (95 % CL)

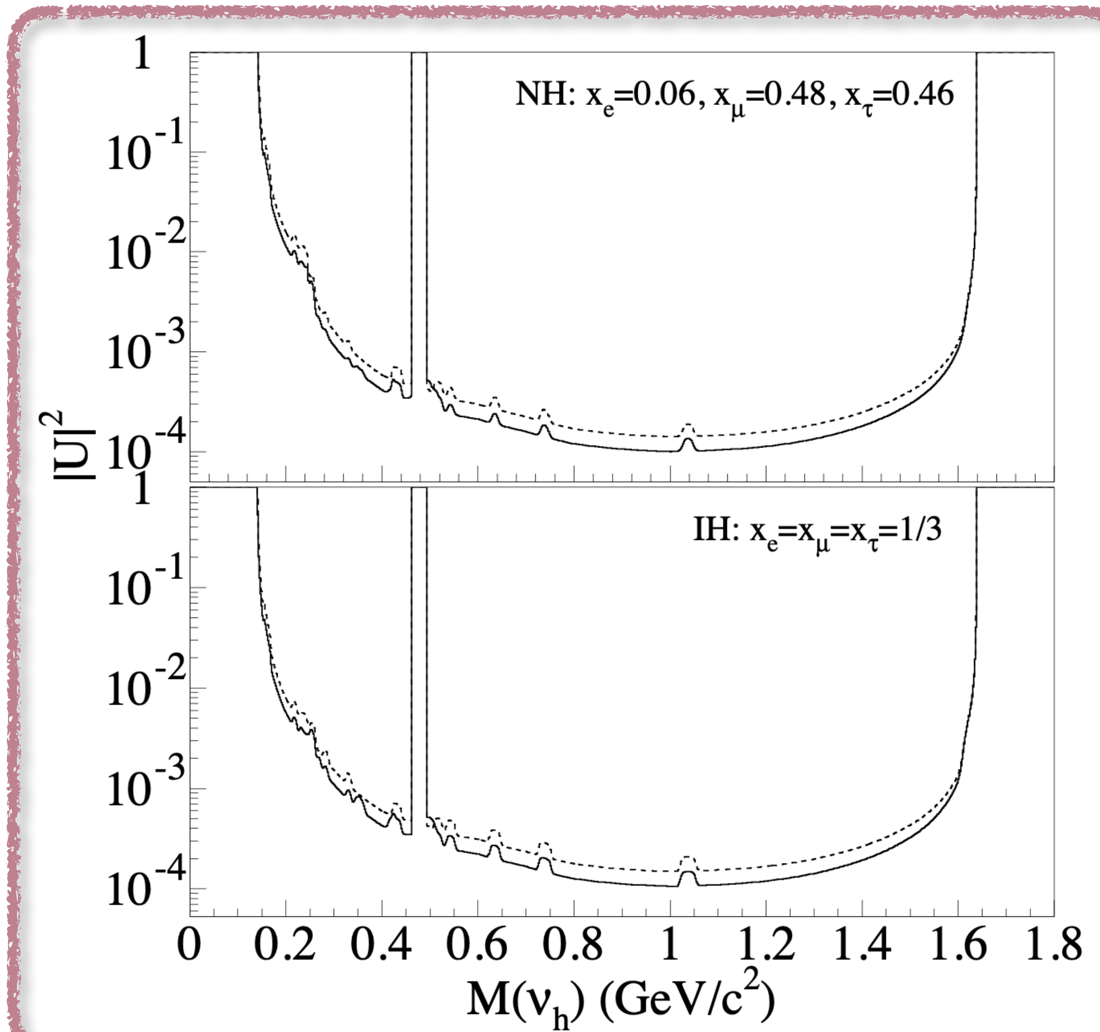
$\mathscr{B}(\tau \to \mu\alpha) < 3.4 \cdot 10^{-3}$ (95 % CL)

Heavy neutrinos searches at Belle

- Search for a sterile neutrino in $\tau^- \to \pi^- N(\to \mu^+\mu^-\nu_\tau)$ ($\mathscr{L} = 915\,\mathrm{fb}^{-1}$)
  - One prong tag side
  - $\mu^+\mu^-$ displaced vertex (> 15 cm from the beam axis)
  - $K_S^0 \to \pi^+\pi^-$ mass region is removed



- Search for a heavy neutrino in $\tau^- \to \pi^- \nu_h(\to \pi^\pm \ell^\mp)$ ($\mathscr{L} = 988\,\mathrm{fb}^{-1}$)
  - No tag side requirements
  - $K_S^0 \to \pi^+\pi^-$ mass region is removed



2212.10095 [hep-ex]

Talk by Sourav Dey at TAU2023

Conclusions

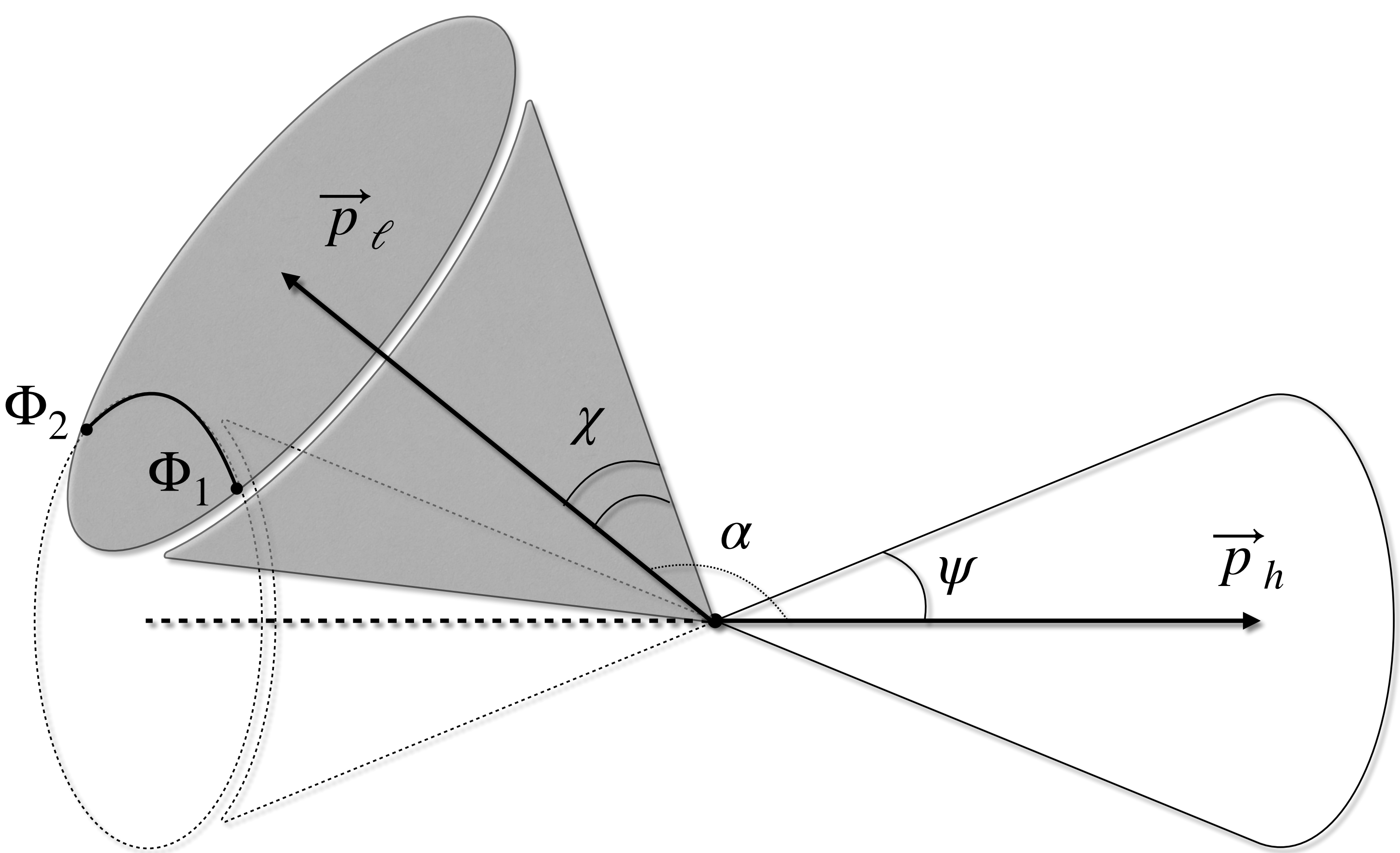
- Belle experiment is the World leading experiment in τ physics providing a big part of the World's best results
- Although Belle II is still in the beginning of its operation, it has already provided the community with competitive results and new methods applications (τ lepton mass measurement, search for LFV decays $\tau^- \to \ell^- \alpha$ and $\tau^- \to \ell^- \phi$), and more are upcoming ($\tau^- \to \mu^- \mu^+ \mu^-$)
- τ physics plays a significant role in the overall program of both Belle and Belle II experiment
- Belle II opens up an opportunity to repeat all the measurements done by Belle and BaBar with higher precision and to conduct new studies, not available for the previous generation
- Systematics become the dominant source of uncertainty in many analyses
- Belle II has a better sensitivity than STCF in measurements, depending on statistics, (most LFV searches) and in τ-lepton lifetime measurement
- By the end of operation, Belle II will accumulate unprecedented number of $\tau^+\tau^-$-pairs, which makes it, without any questions, the Super τ-factory

Thank you for attention!

Backup

τ lepton momentum reconstruction at Belle II

- The momentum of the τ lepton produced in $e^+e^- \to \tau^+\tau^-$ is impossible to reconstruct due to presence of undetectable neutrinos
- Precise knowledge of center-of-mass energy, back-to-back production of $\tau^+\tau^-$-pair, and zero mass (to a high extent) of neutrinos allows to restrict the possible directions of $\tau^+\tau^-$-pair (up to initial-state radiation)



$$\frac{2E_\tau E_\ell - M_\tau^2 - m_\ell^2}{2p_\tau p_\ell} \leq \cos\chi \leq \frac{E_\tau E_\ell - M_\tau m_\ell}{p_\tau p_\ell}$$

$$\cos\psi = \frac{2E_\tau E_h - M_\tau^2 - m_h^2}{2p_\tau p_h}$$

$$\Phi_1 = \pi + \arcsin\left(\frac{\cos\psi\cos\alpha + \cos\chi}{\sin\psi\sin\alpha}\right)$$

$$\Phi_2 = 2\pi - \arcsin\left(\frac{\cos\psi\cos\alpha + \cos\chi}{\sin\psi\sin\alpha}\right)$$

τ lepton polarization at Belle II

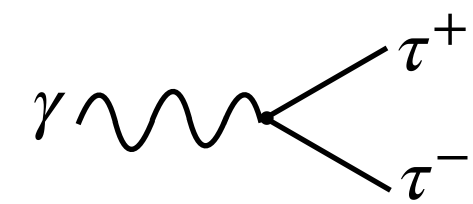
- The beams at Belle II are not polarized, so average τ lepton polarization is zero. Nevertheless, spins of τ leptons are correlated in $e^+e^- \to \tau^+\tau^-$:

$$\frac{d\sigma(e^+e^-(w^-) \to \tau_{\text{sig}}(\vec{s}_{\text{sig}})\tau_{\text{tag}}(\vec{s}_{\text{tag}}))}{d\Omega_\tau} = \frac{\alpha^2\beta}{64E^2}\left[A_0 + D_{ij}(\vec{s}_{\text{sig}})_i(\vec{s}_{\text{tag}})_j\right]$$

$$A_0 = 1 + \cos^2\theta_\tau + \frac{\sin^2\theta_\tau}{\gamma^2} \qquad D_{ij} = \begin{pmatrix} \left(1+\frac{1}{\gamma^2}\right)\sin^2\theta_\tau & 0 & \frac{1}{\gamma}\sin 2\theta_\tau \\ 0 & -\beta^2\sin^2\theta_\tau & 0 \\ \frac{1}{\gamma}\sin 2\theta_\tau & 0 & 1+\cos^2\theta_\tau - \frac{\sin^2\theta_\tau}{\gamma^2} \end{pmatrix}$$

- One can use tagging τ lepton as a spin analyzer with the decay mode $\tau^+ \to \pi^+\pi^0\bar{\nu}_\tau$. This mode has the largest branching fraction (around $25\,\%$), and it is also well-studied

EDM and MDM



- General expression of the $\tau\tau\gamma$ vertex can be parametrized as follows:

$$-ir\bar{u}(p')\left\{F_1(q^2)\gamma^\mu + iF_2(q^2)\sigma^{\mu\nu}\frac{q_\nu}{2m_\tau} + F_3(q^2)\gamma^5\sigma^{\mu\nu}\frac{q_\nu}{2m_\tau}\right\}u(p)\varepsilon_\mu(q)$$

$$F_1(0) = 1 \qquad F_2(0) = \frac{g_\tau - 2}{2} \equiv a_\tau \qquad F_3(0) = -\frac{2m_\tau d_\tau}{e_\tau}$$

- d_τ – EDM, a_τ – MDM

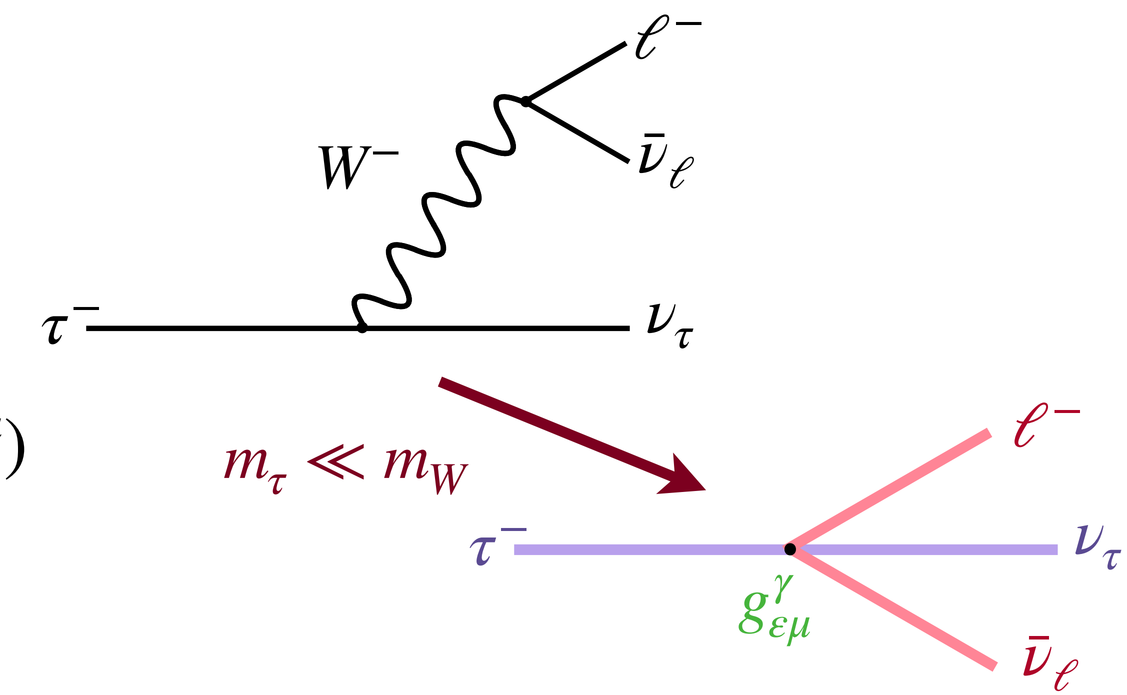
Michel parameters

- Michel parameters (MP) of a lepton decay are bilinear combinations of coupling constants arising in the most general expression for the decay matrix element

$$M = \frac{4G_F}{\sqrt{2}} \sum_{\substack{\gamma = S, V, T \\ \varepsilon, \mu = R, L}} g^{\gamma}_{\varepsilon\mu} \langle \bar{\ell}_{\varepsilon} | \Gamma^{\gamma} | ((\nu_{\ell})_{\alpha}) \rangle \langle (\bar{\nu}_{\tau})_{\beta} | \Gamma_{\gamma} | \tau_{\mu} \rangle$$

$$\Gamma^S = 1,\ \Gamma^V = \gamma^{\mu},\ \Gamma^T = \frac{1}{\sqrt{2}}\sigma^{\mu\nu} = \frac{i}{2\sqrt{2}}(\gamma^{\mu}\gamma^{\nu} - \gamma^{\nu}\gamma^{\mu})$$

Scalar Vector Tensor



- Michel parameters describe the Lorentz structure of the charged currents interaction in the theory of weak interaction and can be used to test the SM

- Deviations can be caused by anomalous coupling with the W-boson, new gauge or charged Higgs bosons, presence of massive neutrinos

- The only nonzero term in the SM theory of weak interaction: $g^{V}_{LL} = 1$

Leptonic decays: Michel parameters (2)

- Differential decay width of τ lepton integrated over neutrino momenta:

$$\frac{d^2\Gamma}{dx\, d\cos\theta} = \frac{m_\tau}{4\pi^3} W_{\ell\tau}^4 G_F^2 \sqrt{x^2 - x_0^2} \left(F_{IS}(x) \pm F_{AS}(x) P_\tau \cos\theta + F_{T_1}(x) P_\tau \sin\theta \zeta_1 \right.$$
$$\left. + F_{T_2}(x) P_\tau \sin\theta \zeta_2 + (\pm F_{IP}(x) + F_{AP}(x) P_\tau \cos\theta) \zeta_3 \right)$$

$$W_{\ell\tau} = \max E_\ell = \frac{m_\tau^2 + m_\ell^2}{2m_\tau},\ x = \frac{E_\ell}{\max E_\ell},\ x_0 = \frac{m_\ell}{\max E_\ell},\ P_\tau = |\boldsymbol{P}_\tau|$$

Functions parameters:

$F_{IS}(x) : \rho, \eta$; $F_{AS}(x) : \xi, \xi\delta$; $F_{IP}(x) : \xi', \xi, \xi\delta$; $F_{AP}(x) : \xi'', \rho, \eta''$;
$F_{T_1}(x) : \xi'', \rho, \eta, \eta''$; $F_{T_2}(x) : \alpha'/A, \beta'/A$

- Differential decay width of radiative leptonic τ decay ($\vec{S}_\tau - \tau$ polarization):

$$\frac{d\Gamma(\tau^- \to \ell^- \bar{\nu}_\ell \nu_\tau \gamma)}{dE_\ell\, d\Omega_\ell\, dE_\gamma\, d\Omega_\gamma} = \left(A_0 + \bar{\eta} A_1\right) + \left(\vec{B}_0 + \xi\kappa \vec{B}_1\right) \cdot \vec{S}_\tau$$

Leptonic differential decay width parametric functions definition

$$F_{IS}(x) = x(1-x) + \frac{2}{9}\rho(4x^2 - 3x - x_0^2) + \eta x_0(1-x)$$

$$F_{AS}(x) = \frac{1}{3}\xi\sqrt{x^2 - x_0^2}\left[1 - x + \frac{2}{3}\delta\left(4x - 3 - \frac{x_0^2}{2}\right)\right]$$

$$F_{IP}(x) = \frac{1}{54}\sqrt{x^2 - x_0^2}\left[-9\xi'\left(2x - 3 + \frac{x_0^2}{2}\right) + 4\xi\left(\delta - \frac{3}{4}\right)\left(4x - 3 - \frac{x_0^2}{2}\right)\right]$$

$$F_{AP}(x) = \frac{1}{6}\left[\xi''\left(2x^2 - x - x_0^2\right) + 4\left(\rho - \frac{3}{4}\right)\left(4x^2 - 3x - x_0^2\right) + 2\eta'' x_0(1-x)\right]$$

$$F_{T_1}(x) = -\frac{1}{12}\left[2\left(\xi'' + 12\left(\rho - \frac{3}{4}\right)\right)(1-x)x_0 + 3\eta(x^2 - x_0^2) + \eta''(3x^2 - 4x + x_0^2)\right]$$

$$F_{T_2}(x) = \frac{1}{3}\sqrt{x^2 - x_0^2}\left(3\frac{\alpha'}{A}(1-x) + \frac{\beta'}{A}(2 - x_0^2)\right)$$

MP parameters through coupling constants

$$\rho = \frac{3}{4} - \frac{3}{4}\left[\left(|g^V_{RL}|^2 + |g^V_{LR}|^2\right) + 2\left(|g^T_{LR}|^2 + |g^T_{RL}|^2\right) + \Re\left\{g^S_{RL}g^{T*}_{RL} + g^S_{LR}g^{T*}_{LR}\right\}\right]$$

$$\eta = \frac{1}{2}\Re\left\{g^V_{RL}\left(g^{S*}_{LR} + 6g^{T*}_{LR}\right) + g^V_{LR}\left(g^{S*}_{RL} + 6g^{T*}_{RL}\right) + \left(g^V_{RR}g^{S*}_{LL} + g^V_{LL}g^{S*}_{RR}\right)\right\}$$

$$\xi = 4\Re\left\{g^S_{LR}g^{T*}_{LR} - g^S_{RL}g^{T*}_{RL}\right\} + \left(|g^V_{LL}|^2 - |g^V_{RR}|^2\right) + 3\left(|g^V_{LR}|^2 - |g^V_{RL}|^2\right)$$
$$+5\left(|g^T_{LR}|^2 - |g^T_{RL}|^2\right) + \frac{1}{4}\left(|g^S_{LL}|^2 - |g^S_{RR}|^2 + |g^S_{RL}|^2 - |g^S_{LR}|^2\right)$$

$$\xi\delta = \frac{3}{16}\left(|g^S_{LL}|^2 - |g^S_{RR}|^2 + |g^S_{RL}|^2 - |g^S_{LR}|^2\right) + \frac{3}{4}\left(|g^V_{LL}|^2 - |g^V_{RR}|^2 - |g^T_{LR}|^2\right.$$
$$\left. + |g^T_{RL}|^2 + \Re\left\{g^S_{LR}g^{T*}_{LR} - g^S_{RL}g^{T*}_{RL}\right\}\right)$$

MP parameters through coupling constants (2)

$$\xi' = -\left[3\left(|g^T_{RL}|^2 - |g^T_{LR}|^2\right) + \left(|g^V_{RR}|^2 + |g^V_{RL}|^2 - |g^V_{LR}|^2 - |g^V_{LL}|^2\right)\right.$$
$$\left. + \frac{1}{4}\left(|g^S_{RR}|^2 + |g^S_{RL}|^2 - |g^S_{LR}|^2 - |g^S_{LL}|^2\right)\right]$$

$$\xi'' = 1 - \frac{1}{2}\left(|g^S_{RL}|^2 + |g^S_{LR}|^2\right) + 2\left(|g^V_{RL}|^2 + |g^V_{LR}|^2 + |g^T_{RL}|^2 + |g^T_{LR}|^2\right)$$
$$+4\Re\left\{g^S_{RL}g^{T*}_{RL} + g^S_{LR}g^{T*}_{LR}\right\}$$

$$\eta'' = \frac{1}{2}\Re\left\{3g^V_{RL}\left(g^{S*}_{LR} + 6g^{T*}_{LR}\right) + 3g^V_{LR}\left(g^{S*}_{RL} + 6g^{T*}_{RL}\right) - \left(g^V_{RR}g^{S*}_{LL} + g^V_{LL}g^{S*}_{RR}\right)\right\}$$

$$\frac{\alpha'}{A} = \frac{1}{2}\Im\left\{g^V_{LR}\left(g^{S*}_{RL} + 6g^{T*}_{RL}\right) - g^V_{RL}\left(g^{S*}_{LR} + 6g^{T*}_{LR}\right)\right\}$$

$$\frac{\beta'}{A} = \frac{1}{4}\Im\left\{g^V_{RR}g^{S*}_{LL} - g^V_{LL}g^{S*}_{RR}\right\}$$

Five-body leptonic τ-decays branching fractions

J.Phys.Conf.Ser. 912 (2017) 1

$$BR_{\text{exp}}^{\tau^-\to e^-e^+e^-\bar\nu_e\nu_\tau} = BR_{\text{SM}}^{\tau^-\to e^-e^+e^-\bar\nu_e\nu_\tau}\{[Q_{LL} + (1.051\pm0.036)Q_{LR} + (-0.2053\pm0.1431)B_{LR} + L\leftrightarrow R] + (0.2416\pm0.0002)I_\alpha + (0.8606\pm0.0001)I_\beta\}.$$

$$BR_{\text{exp}}^{\tau^-\to \mu^-e^+e^-\bar\nu_\mu\nu_\tau} = BR_{\text{SM}}^{\tau^-\to \mu^-e^+e^-\bar\nu_\mu\nu_\tau}\{[Q_{LL} + (1.220\pm0.049)Q_{LR} + (-0.8717\pm0.1957)B_{LR} + L\leftrightarrow R] + \underline{(181.3\pm0.1)I_\alpha + (104.4\pm0.1)I_\beta}\}.$$

$$BR_{\text{exp}}^{\tau^-\to e^-\mu^+\mu^-\bar\nu_e\nu_\tau} = BR_{\text{SM}}^{\tau^-\to e^-\mu^+\mu^-\bar\nu_e\nu_\tau}\{[Q_{LL} + (1.226\pm0.001)Q_{LR} + (-0.8456\pm0.0001)B_{LR} + L\leftrightarrow R] + (0.2253\pm0.0001)I_\alpha + (0.5231\pm0.0001)I_\beta\}.$$

$$BR_{\text{exp}}^{\tau^-\to \mu^-\mu^+\mu^-\bar\nu_\mu\nu_\tau} = BR_{\text{SM}}^{\tau^-\to \mu^-\mu^+\mu^-\bar\nu_\mu\nu_\tau}\{[Q_{LL} + (1.216\pm0.005)Q_{LR} + (-0.8459\pm0.0005)B_{LR} + L\leftrightarrow R] - (18.00\pm0.01)I_\alpha + (197.3\pm0.1)I_\beta\}.$$

- Underlined part is the most sensitive to Michel parameters: $I_\alpha = 2(\alpha + i\alpha')/A$ and $I_\beta = -2(\beta + i\beta')/A$. Here $\eta = (\alpha - 2\beta)/A$ and $\eta'' = (3\alpha + 2\beta)/A$
- Here an alternative Michel-like parametrization from Phys.Lett.B 173 (1986) 102-106 is used

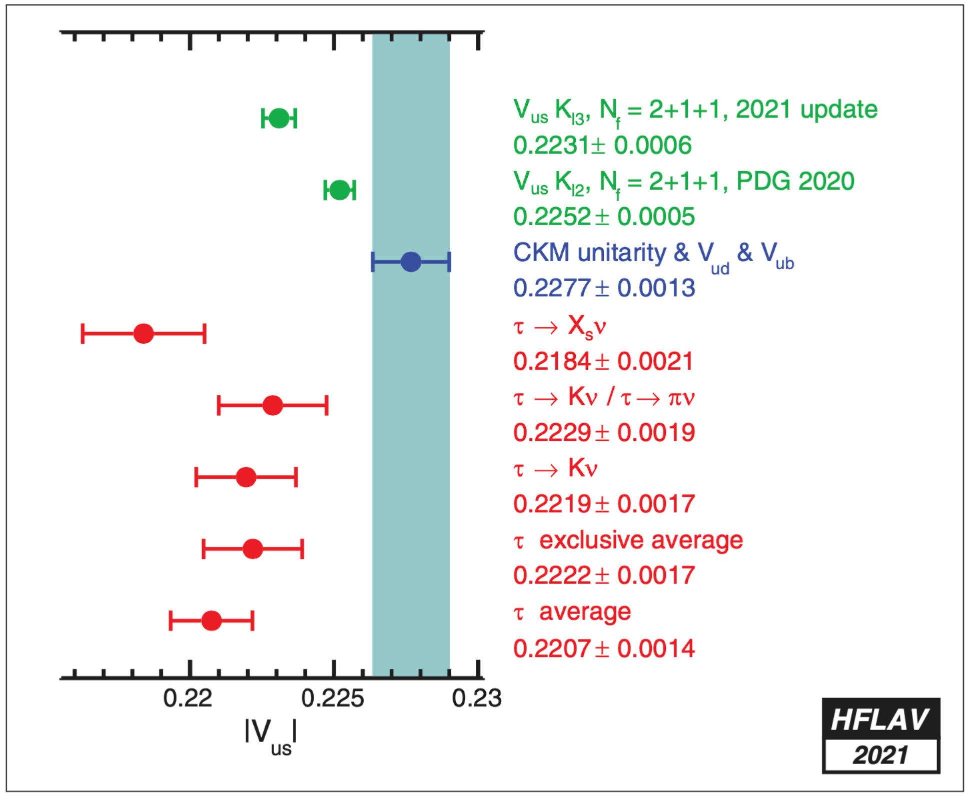
Hadronic decays

- Hadronic decays of τ lepton are unique laboratory to determine $\alpha_s(m_\tau)$, m_s, and V_{us}
- They also can be used for the lepton universality tests: $\tau^- \to \pi^- \nu_\tau$ and $\tau^- \to K^- \nu_\tau$ decays are analogous to $\pi^- \to \mu^- \bar{\nu}_\mu$ and $K^- \to \mu^- \bar{\nu}_\mu$

$$R_{\tau/P} = \frac{\Gamma(\tau^- \to P^- \nu_\tau)}{\Gamma(P^- \to \mu^- \bar{\nu}_\tau)} = \left|\frac{g_\tau}{g_\mu}\right|^2 \frac{m_\tau^3}{2 m_P m_\mu^2} \frac{(1 - m_P^2/m_\tau^2)^2}{(1 - m_\mu^2/m_P^2)^2} (1 + \delta R_{\tau/P})$$

$$|g_\tau/g_\mu|_\pi = 0.9959 \pm 0.0038 \quad |g_\tau/g_\mu|_K = 0.9855 \pm 0.0075$$

Phys.Rev.D 107 (2023) 052008



- Determination of $|V_{us}|$

$$\frac{|V_{us}| f_K}{|V_{ud}| f_\pi} = \frac{m_\tau^2 - m_\pi^2}{m_\tau^2 - m_K^2} \sqrt{\frac{\mathcal{B}(\tau^- \to K^- \nu_\tau)}{\mathcal{B}(\tau^- \to \pi^- \nu_\tau)} \frac{1 + \delta R_{\tau/\pi}}{1 + \delta R_{\tau/K}} \frac{1}{1 + \delta R_{K/\pi}}}$$

$$|V_{us}| = 0.2229 \pm 0.0019 \qquad |V_{us}|_{\text{unitarity}} = 0.2277 \pm 0.0013$$

Belle II can measure $\Gamma(\tau^- \to \pi^- \nu_\tau)$ and $\Gamma(\tau^- \to K^- \nu_\tau)$ that has not been done at *B*-factories before

Hadronic decays (2)

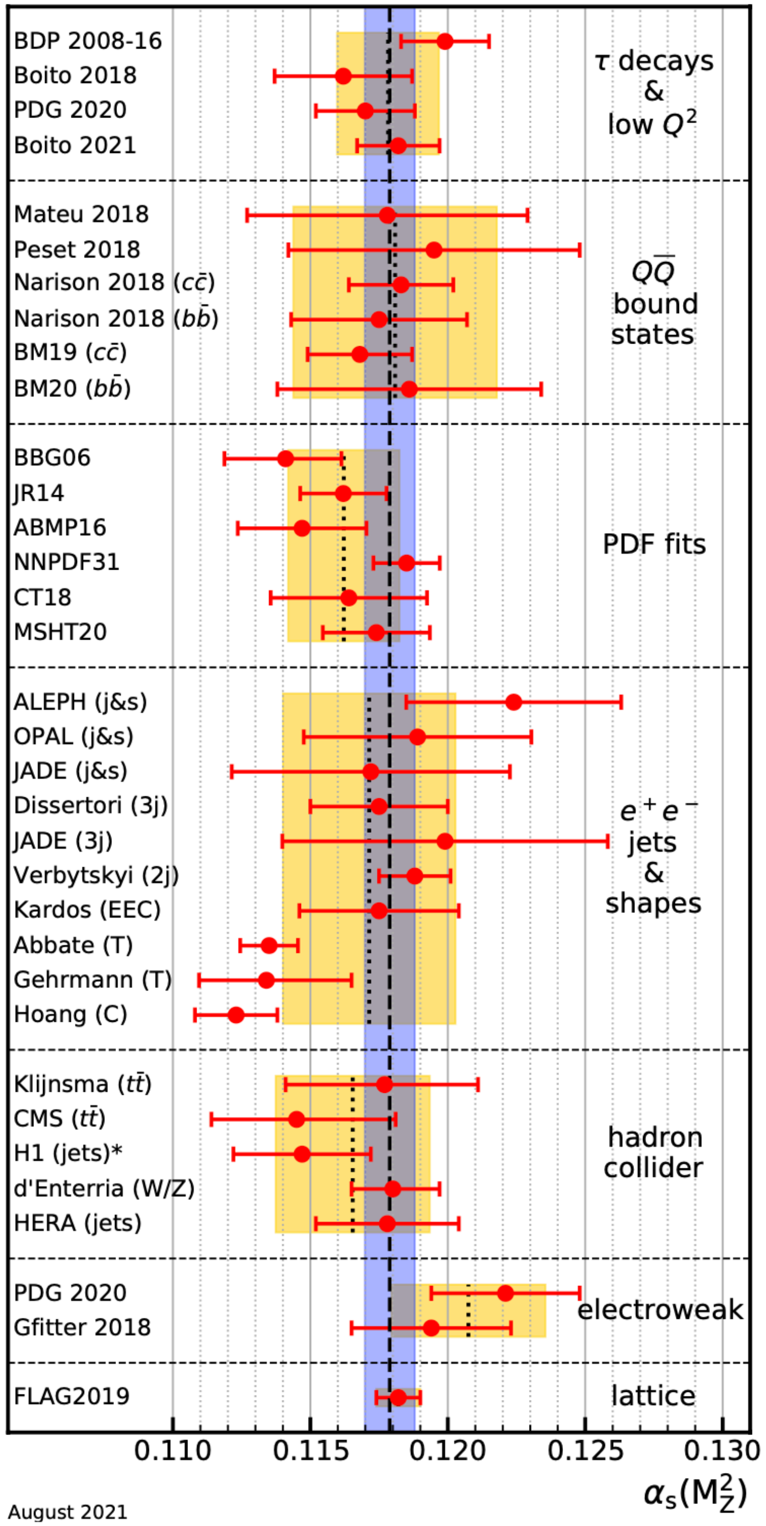
- More precise knowledge of already measured hadron modes is desirable for more accurate determination of α_s and for other studies, where these modes play the role of background (e.g. the partial-wave analysis of $\tau^- \to \pi^-\pi^+\pi^-\nu_\tau$ being done by Belle [1] before done only by CLEO II [2])

- Higher statistics of Belle II will also allow for observation of various hadron modes not accessible in the previous-generation *B*-factories

- Studies of hadronic modes of τ lepton can be used in the theoretical calculation of the hadronic contribution in the $a_\mu \equiv (g_\mu - 2)/2$

- Belle II can confirm or resolve current deviation of $a_\mu^{\text{had}}(\tau) = (703.0 \pm 4.4) \cdot 10^{-10}$ from $a_\mu^{\text{had}}(e^+e^-) = (692.3 \pm 4.2) \cdot 10^{-10}$ [3]

PTEP 2022 (2022) 8, 083C01



[1] Talk by Andrei Rabusov at TAU2023
[2] Phys.Rev.D 61 (1999) 012002
[3] Talk by Pablo Roig Garcés at TAU2023

Search for LFV in decays with τ lepton in final state at Belle

- Search for $B_s^0 \to \ell^\pm \tau^\mp$ with the semileptonic tagging method [1] ($\mathscr{L} = 121\ \text{fb}^{-1}$)
- Search for LFV decays $B^+ \to K^+ \tau^\pm \ell^\mp$ [2] ($\mathscr{L} = 711\ \text{fb}^{-1}$)
  - Full reconstruction of the tag side in the hadronic mode
  - Recoil mass of the $K^+ \ell^\mp$ system
  - BDT for background suppression
- Search for LFV decays of $\Upsilon(1S)$ [3] ($\mathscr{L} = 24.9\ \text{fb}^{-1}$)
  - Di-pion tagging from $\Upsilon(2S) \to \Upsilon(1S)\pi^+\pi^-$
  - Recoil mass of the $\pi^+\pi^-\ell^\mp(\gamma)$ system

[1] JHEP 08 (2023) 178
[2] Phys.Rev.Lett. 130 (2023) 261802
[3] JHEP 05 (2022) 095

Process	Upper limit (90% C.L.)
$\mathscr{B}(B_s^0 \to e^\mp \tau^\pm)$	$< 14 \times 10^{-4}$
$\mathscr{B}(B_s^0 \to \mu^\mp \tau^\pm)$	$< 7.3 \times 10^{-4}$
$\mathscr{B}(B^+ \to K^+ \tau^+ \mu^-)$	$< 0.6 \times 10^{-5}$
$\mathscr{B}(B^+ \to K^+ \tau^- \mu^+)$	$< 2.5 \times 10^{-5}$
$\mathscr{B}(B^+ \to K^+ \tau^+ e^-)$	$< 1.5 \times 10^{-5}$
$\mathscr{B}(B^+ \to K^+ \tau^- e^+)$	$< 1.5 \times 10^{-5}$
$\mathscr{B}(\Upsilon(1S) \to \mu^\pm \tau^\mp)$	$< 2.7 \times 10^{-6}$
$\mathscr{B}(\Upsilon(1S) \to e^\pm \tau^\mp)$	$< 2.7 \times 10^{-6}$
$\mathscr{B}(\Upsilon(1S) \to \gamma\mu^\pm \tau^\mp)$	$< 6.1 \times 10^{-6}$
$\mathscr{B}(\Upsilon(1S) \to \gamma e^\pm \tau^\mp)$	$< 6.5 \times 10^{-6}$

Best and first results by Belle