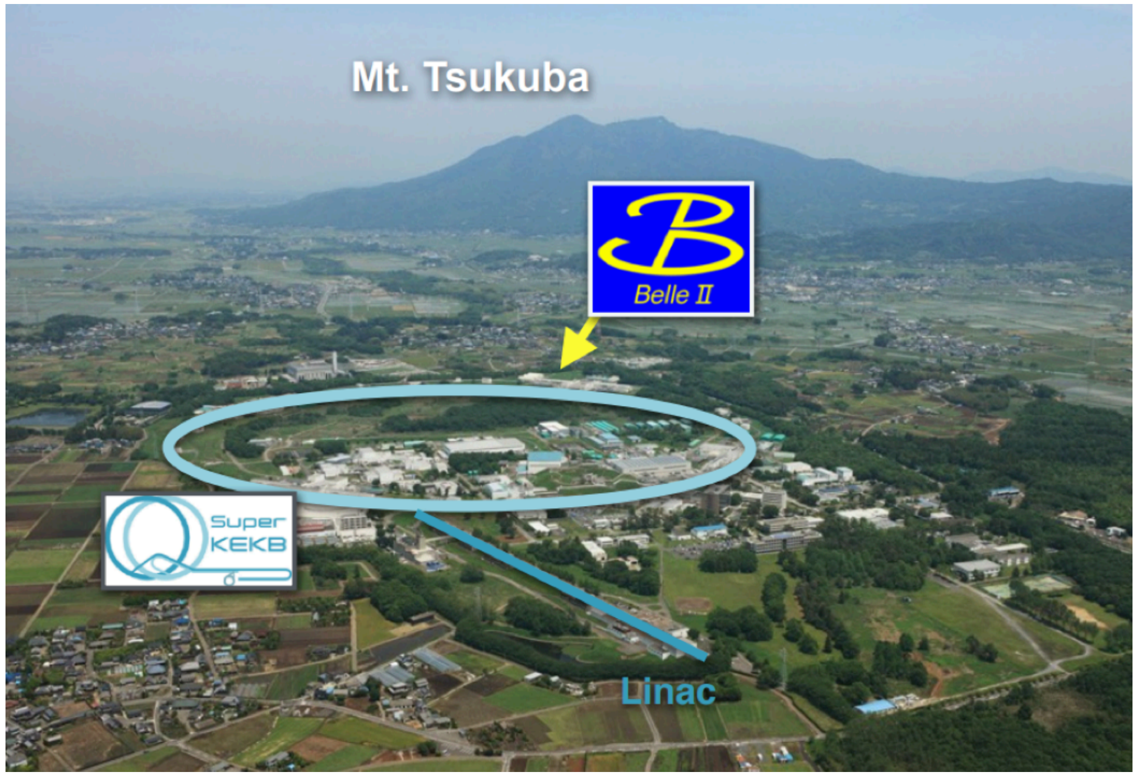
CPV in D meson decays at Belle I/II

Michel Bertemes on behalf of the Belle I/II collaboration
12th International Workshop on CKM Unitarity - 09/18/23

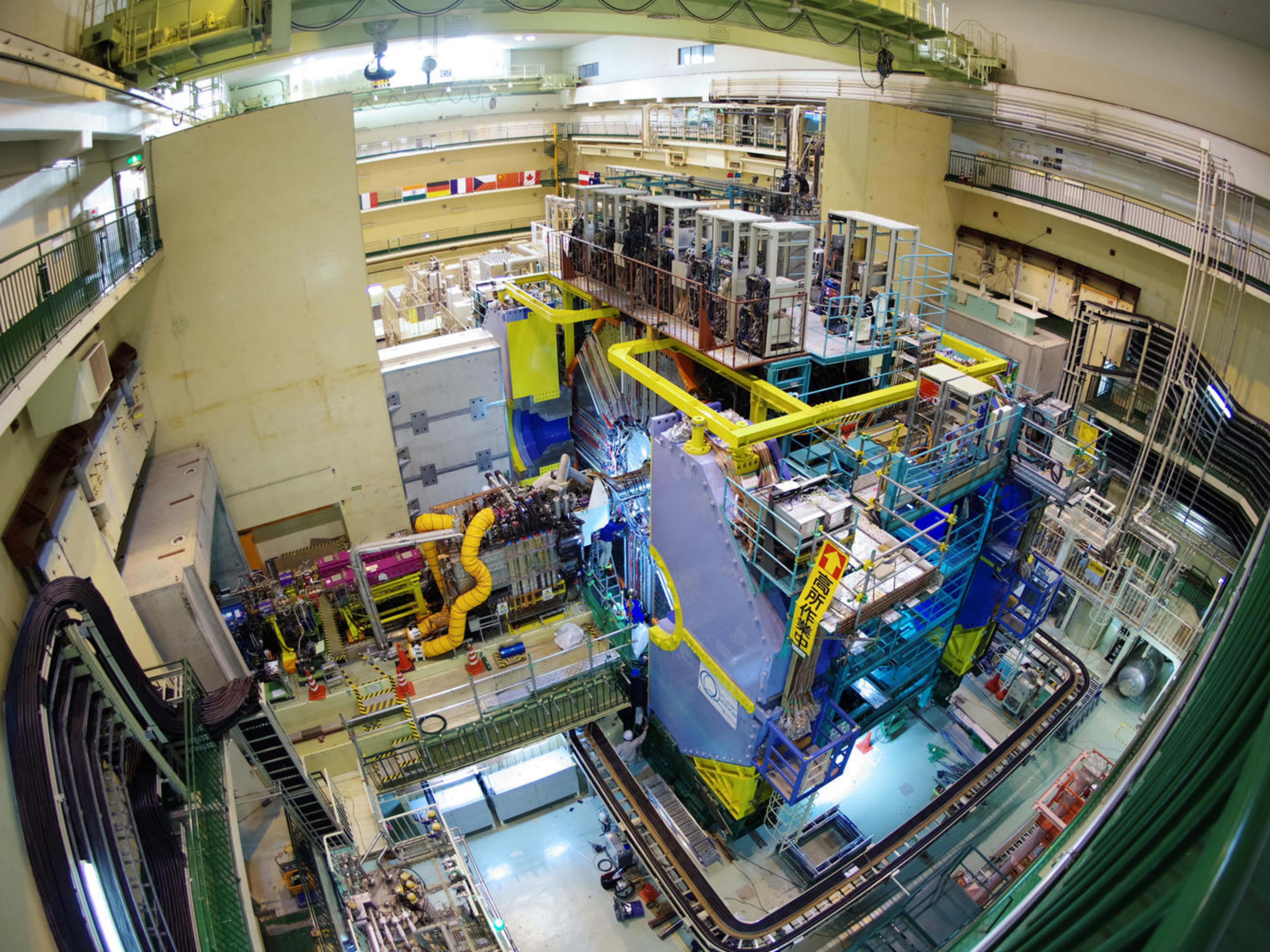
HEPHY INSTITUT FÜR HOCHENERGIEPHYSIK | Brookhaven National Laboratory | FWF Der Wissenschaftsfonds.

SuperKEKB and Belle II

- *SuperKEKB*
  - asymmetric e^+e^- collider in Tsukuba, Japan
  - nano-beam interaction point
  - $\mathscr{L}$=4.7 × 10^{34}cm^{-2}s^{-1} (record)
  - tunable E_{cm} around $\Upsilon(4S)$ mass
- *Belle II*
  - 4π spectrometer
  - successor to Belle with improved vertexing, tracking, PID and calorimetry capabilities
  - currently in LS1, resume in 2024

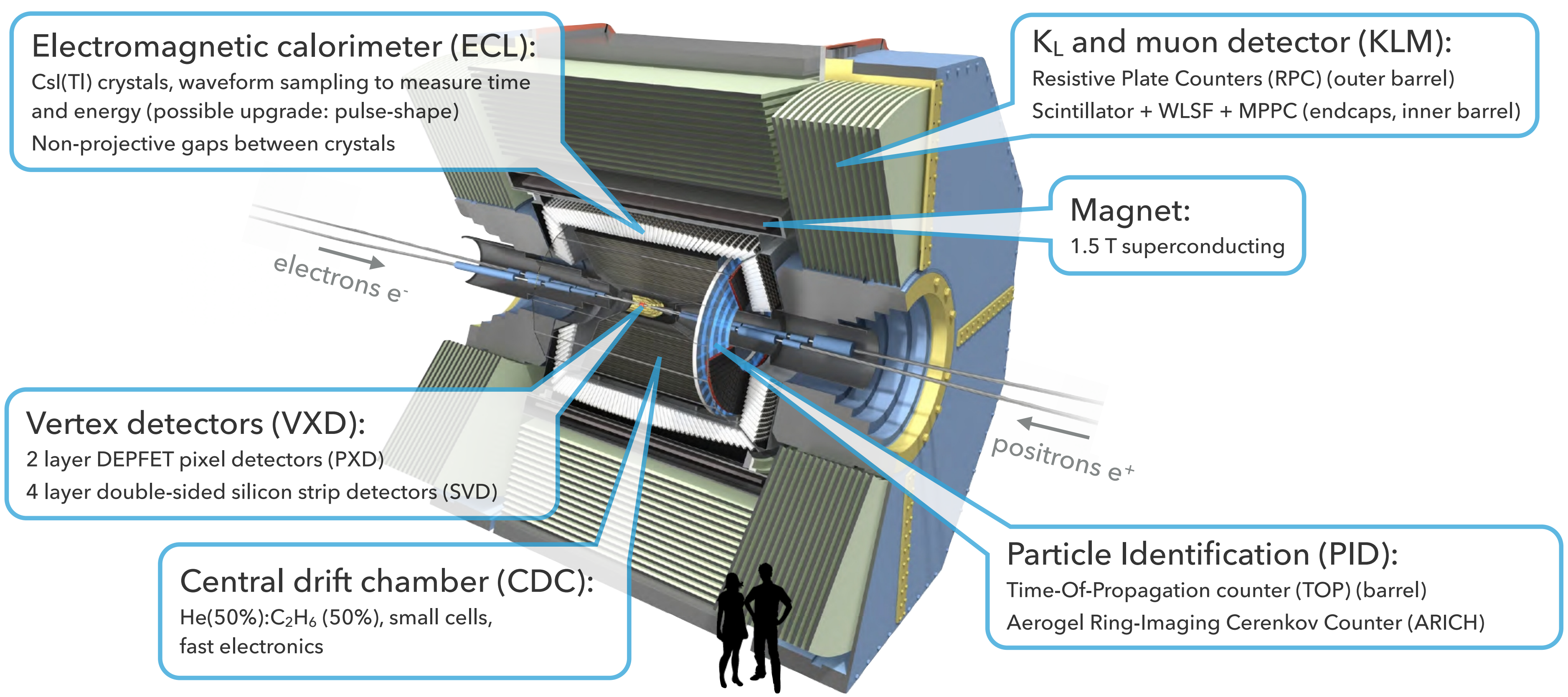


	Belle	Belle II
Years of operation	1999-2010	2019-
Beam energies	8 GeV (e^-) , 3.5 GeV (e^+)	7 GeV (e^-) , 4 GeV (e^+)
Data set	~1000fb^{-1}	424fb^{-1}

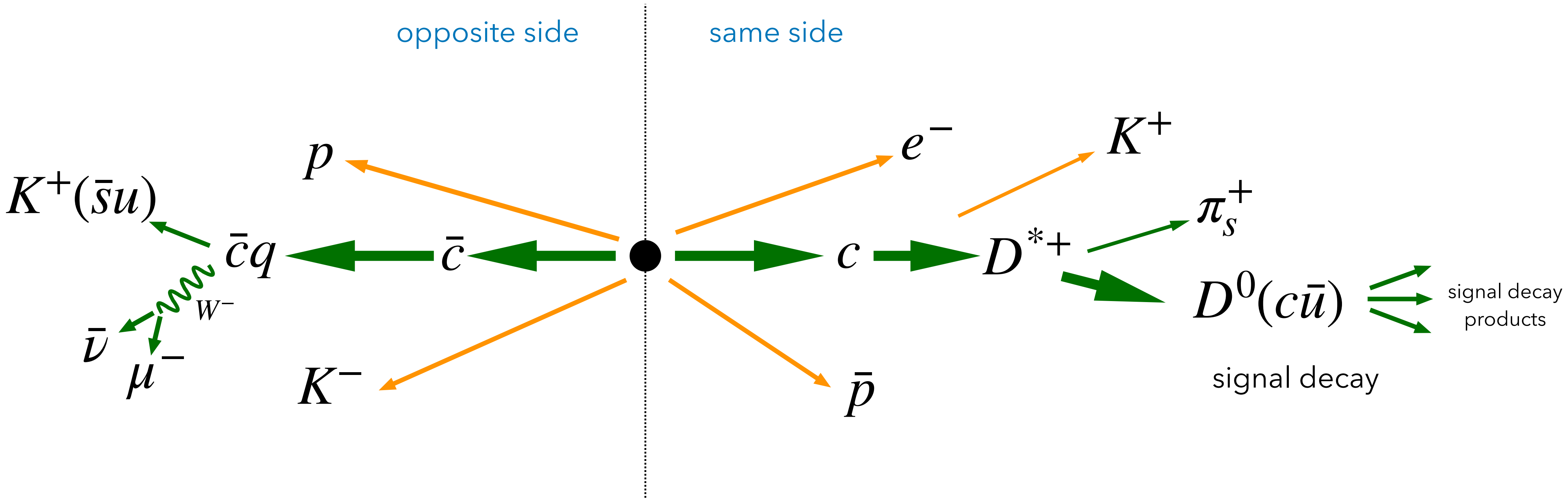

高所作業中

Charm physics at Belle II

- *"charm factory"*
  - **large** $e^+e^- \to c\bar{c}$ **cross-section** provides low-background event samples
  - 1.3M events per 1fb^{-1}
  - ~100% trigger efficiency uniform across decay time and kinematics
- *rich program*
  - excellent reconstruction of **final states with neutrals** e.g. $D^+ \to \pi^+\pi^0$, $D^0 \to \rho^0\gamma, \pi^0\pi^0, K_S^0K_S^0, K\pi\pi^0, \pi\pi\pi^0 \ldots$
  - unique access to final states with invisible particles: e.g. decay into neutrinos

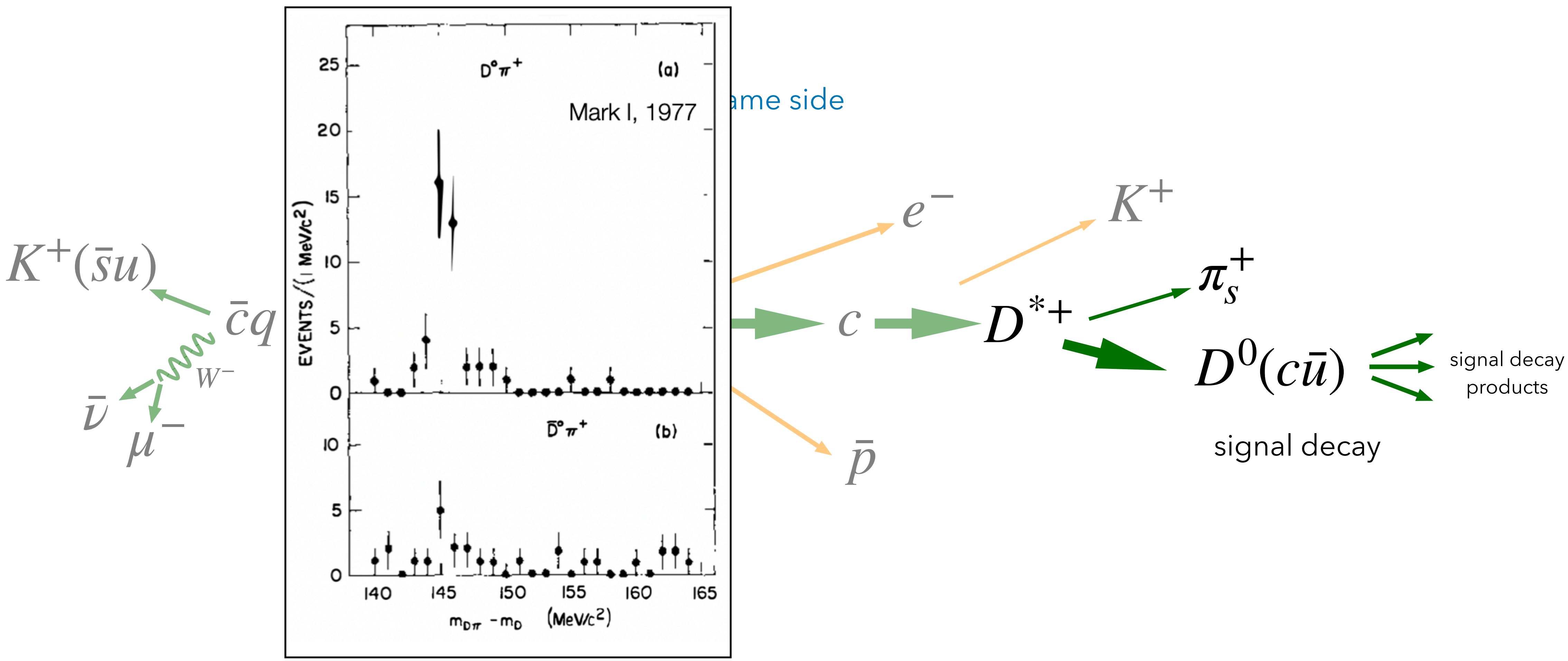


A beautiful charm event



- $e^+e^- \rightarrow$ two charm hadrons + fragmentation
  - no entanglement, inaccessible strong phase

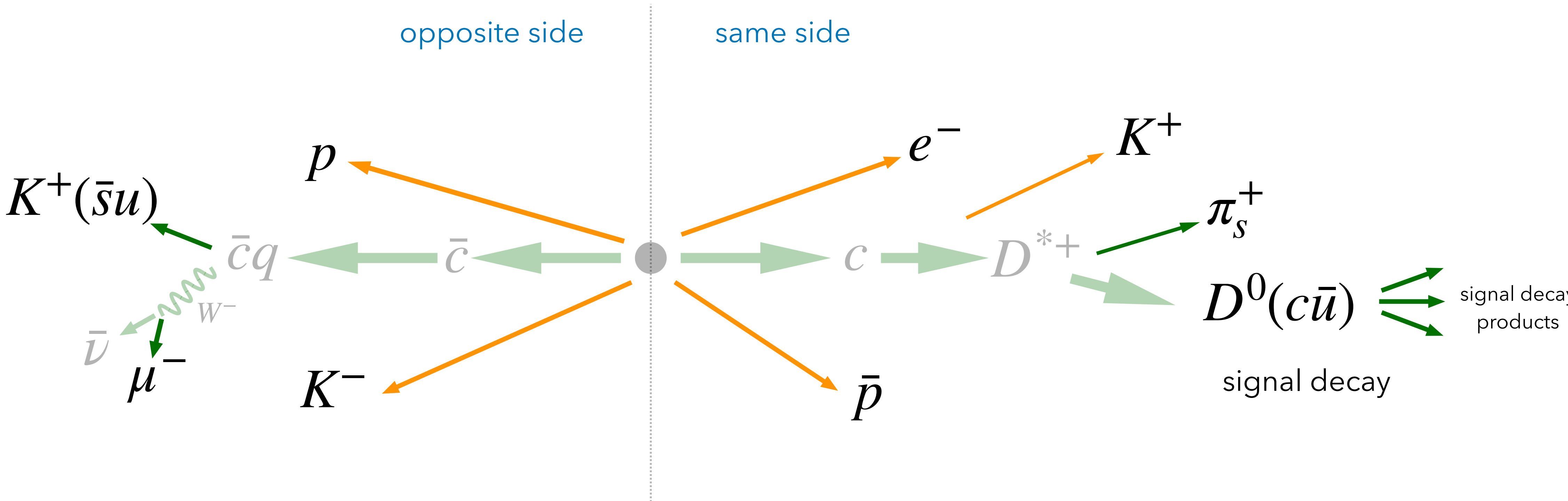
A beautiful charm event



- $e^+e^- \rightarrow$ two charm hadrons + fragmentation
  - no entanglement, inaccessible strong phase
- standard approach (since 1977): **exclusive reconstruction** of strong decay $D^{*+} \rightarrow D^0\pi_s^+$
  - inefficient reconstruction of slow=low momentum pion
  - loss in statistics (only ~25% of all charm quarks hadronize into D^*)

slow pion: $M(D^{*+}) - M(D^0) \approx 145\,\mathrm{MeV}/c^2$

A beautiful charm event



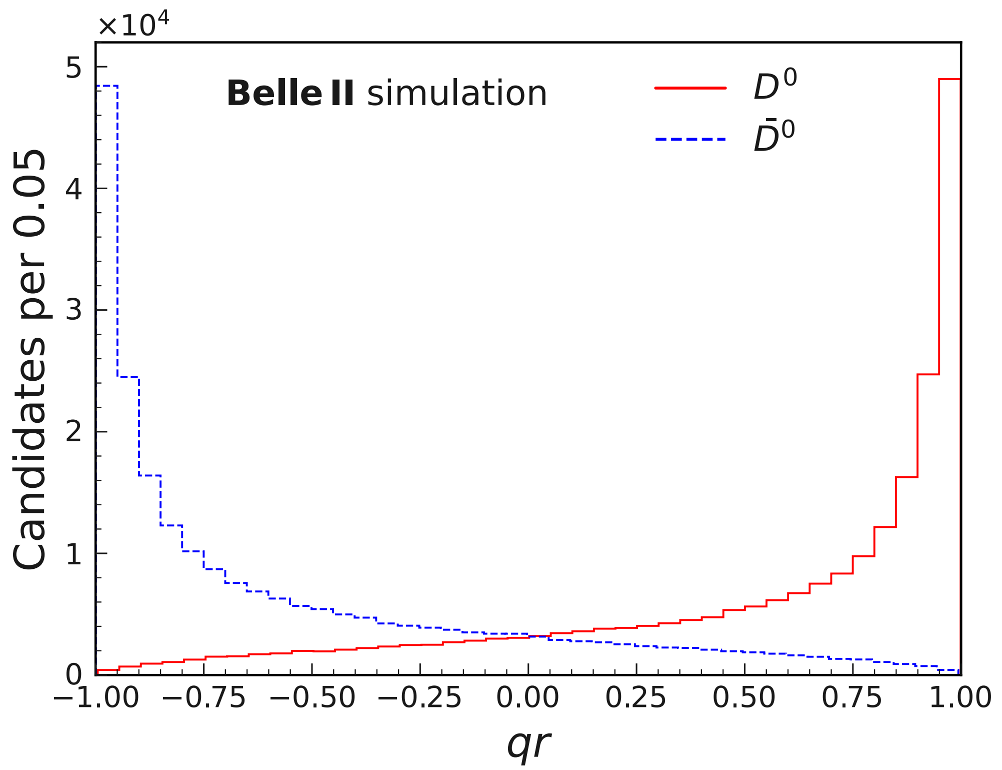
- $e^+e^- \to$ two charm hadrons + fragmentation
  - no entanglement, inaccessible strong phase
- standard approach (since 1977): **exclusive reconstruction** of strong decay $D^{*+} \to D^0\pi_s^+$
  - inefficient reconstruction of slow=low momentum pion
  - loss in statistics (only ~25% of all charm quarks hadronize into $D*$)
- a new **more inclusive** method is desirable to exploit correlation between signal flavor and charge of tagging particles

Novel method for the identification of the production flavor of neutral charmed mesons

PRD 107, 112010 (2023)

The Charm Flavor Tagger (CFT)

- reconstruct particles most collinear with signal meson
- uses **kinematic features** (ΔR, recoiling mass) and **PID** of tagging particles
- based on BDT, **predicts** qr (tagging decision q and dilution r)
- trained using simulation and calibrated with Belle II data

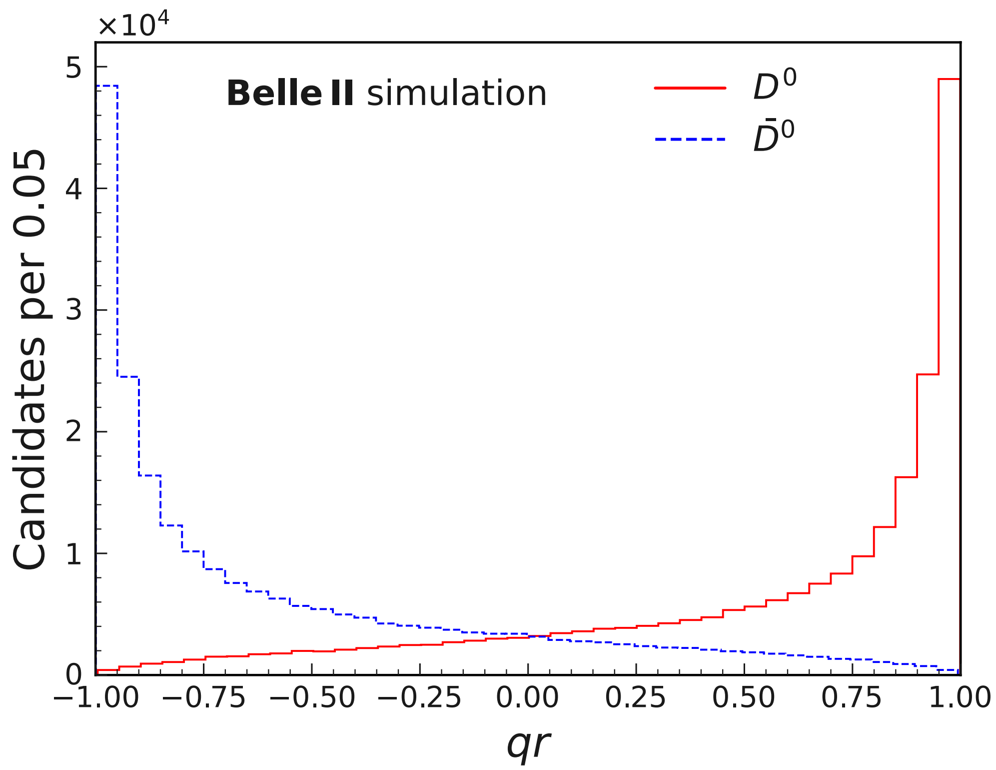


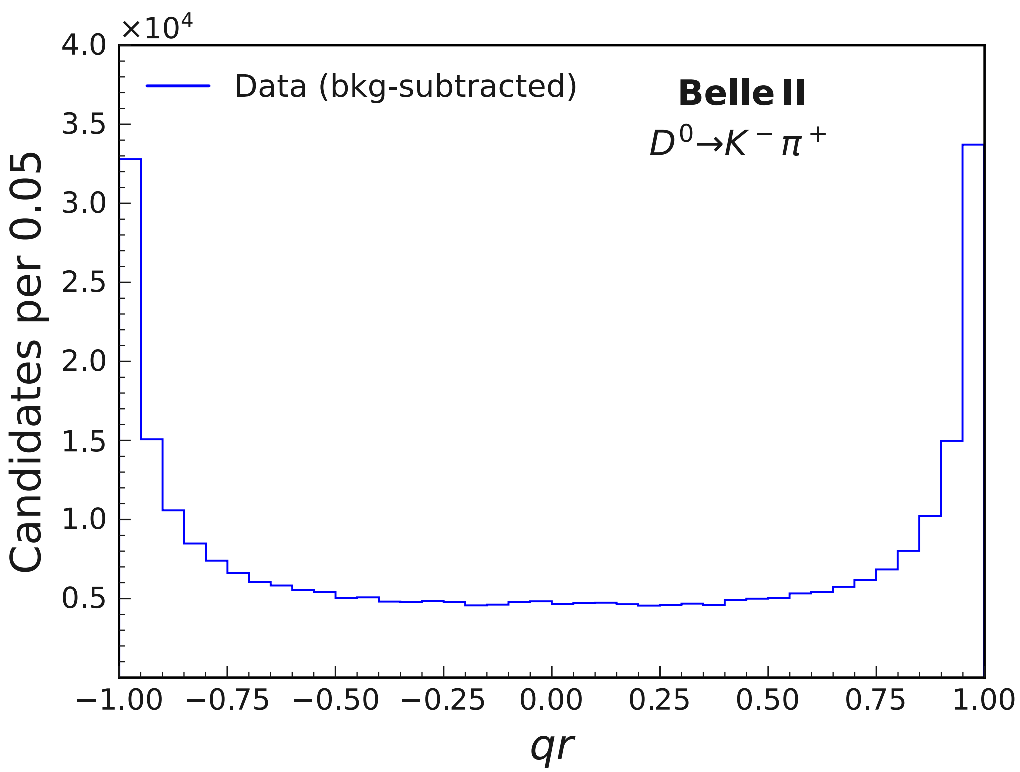
q=+1 for D^0 and -1 for $\bar{D}^0$

r=1 perfect prediction, r=0 random guessing

The Charm Flavor Tagger (CFT)

- reconstruct particles most collinear with signal meson
- uses **kinematic features** (ΔR, recoiling mass) and **PID** of tagging particles
- based on BDT, **predicts** qr (tagging decision q and dilution r)
- trained using simulation and calibrated with Belle II data





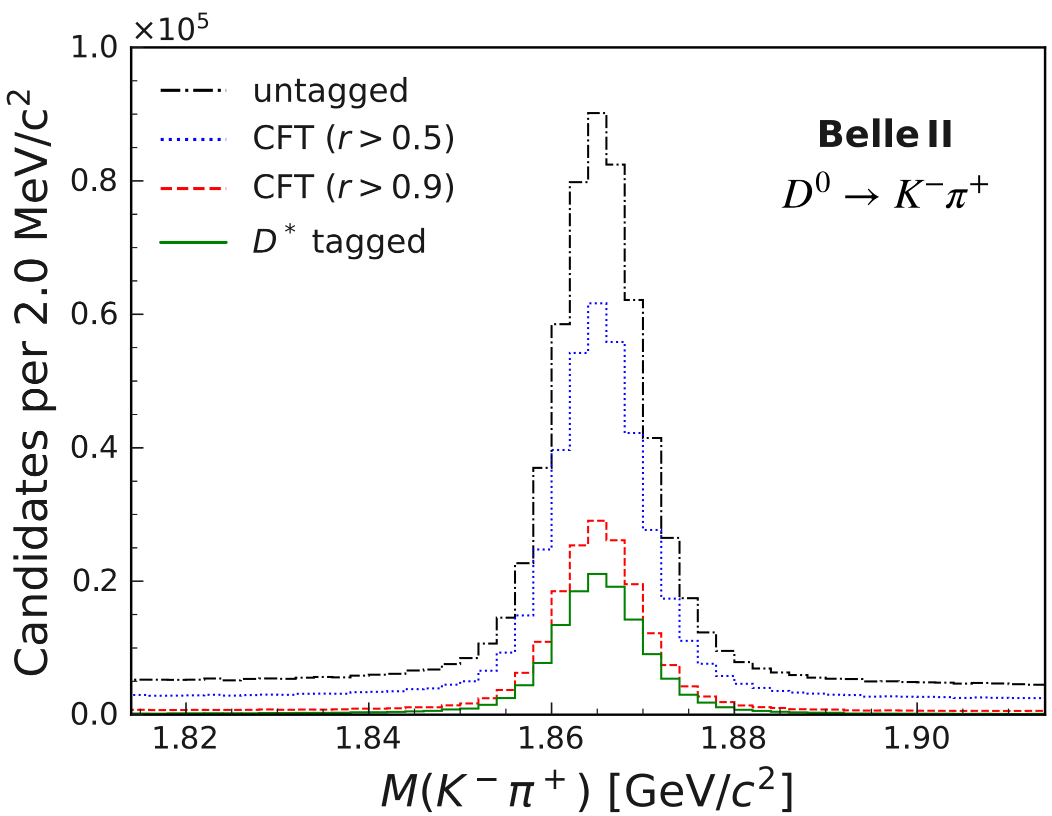
q=+1 for D^0 and -1 for $\bar{D}^0$
r=1 perfect prediction, r=0 random guessing

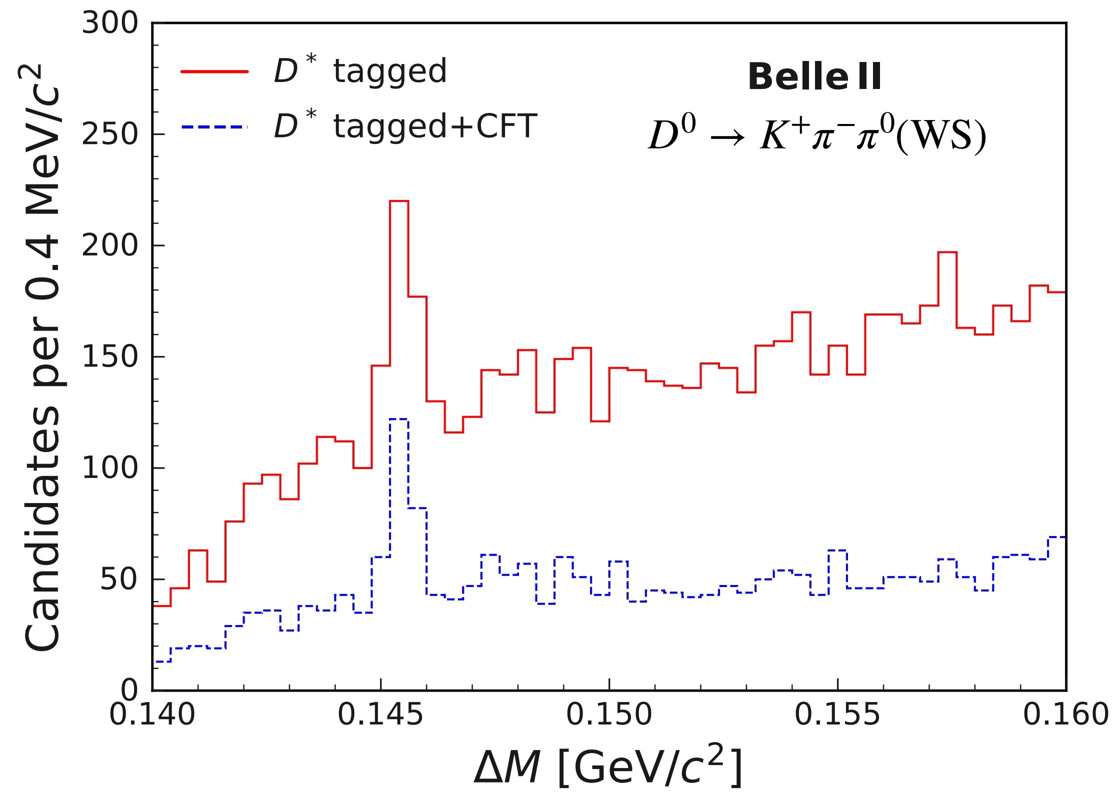
$$\epsilon_{\text{tag}}^{\text{eff}} = (47.91 \pm 0.07(\text{stat}) \pm 0.51(\text{syst}))\ \%$$

tagging power: $\epsilon_{\text{tag}}^{\text{eff}} = \epsilon_{\text{tag}}\langle r^2 \rangle$, $\epsilon_{\text{tag}}^{\text{eff}}(D^*) \sim 24\ \%$

The Charm Flavor Tagger (CFT)

- **double** the sample **size** w.r.t D^{*+}-tagged events
- provide discrimination between signal and background
- CFT will increase sensitivity for many charm decays:
  - $D^0 \to \pi^0\pi^0, K_S^0 K_S^0, K\pi\pi^0, \pi\pi\pi^0 \ldots$





Measurement of BR and search for CPV in $D^0 \to K_S^0 K_S^0 \pi^+ \pi^-$ decays

PRD 107, 052001 (2023)

Search for CPV in $D_{(s)}^+ \to K^+ K_S^0 h^+ h^-$ decays and observation of $D_s^+ \to K^+ K^- K_S^0 \pi^+$

arXiv:2305.11405

Search for CPV using T-odd correlations in $D_{(s)}^+ \to K^+ K^- \pi^+ \pi^0, K^+ \pi^- \pi^+ \pi^0$ and $D^+ \to K^- \pi^+ \pi^+ \pi^0$ decays

arXiv:2305.12806

Two approaches

$$A_{\text{raw}} = \frac{\Gamma(D \to f) - \Gamma(\bar{D} \to \bar{f})}{\Gamma(D \to f) + \Gamma(\bar{D} \to \bar{f})}$$

$$A_T = \frac{\Gamma(C_T > 0) - \Gamma(C_T < 0)}{\Gamma(C_T > 0) + \Gamma(C_T < 0)}$$

- obtain asymmetry from difference in partial widths
- A_{raw} includes asymmetries in production and reconstruction
  - A_{FB}: arising from $\gamma - Z^0$ interference
  - A_ϵ: reconstruction of final-state particles
  - need control channel to correct
- in charm: SCS two-body decays

- measure asymmetry in triple products $C_T = \vec{v_1} \cdot (\vec{v_2} \times \vec{v_3})$
- $A_T \neq 0$ can also arise from final-state interaction
  - isolate T- violation with $a_{CP}^{T-\text{odd}}$
  - $a_{CP}^{T-\text{odd}}$ is unaffected by production and reconstruction asymmetries
- in charm: four-body decays

Two approaches

$$A_{\rm raw} = \frac{\Gamma(D \to f) - \Gamma(\bar{D} \to \bar{f})}{\Gamma(D \to f) + \Gamma(\bar{D} \to \bar{f})}$$

- obtain asymmetry from difference in partial widths
- $A_{\rm raw}$ includes asymmetries in production and reconstruction
  - $A_{\rm FB}$: arising from $\gamma - Z^0$ interference
  - A_ϵ: reconstruction of final-state particles
  - need control channel to correct
- in charm: SCS two-body decays

$$A_T = \frac{\Gamma(C_T > 0) - \Gamma(C_T < 0)}{\Gamma(C_T > 0) + \Gamma(C_T < 0)} \qquad \bar{A}_T = \frac{\Gamma(-\bar{C}_T > 0) - \Gamma(-\bar{C}_T < 0)}{\Gamma(-\bar{C}_T > 0) + \Gamma(-\bar{C}_T < 0)}$$

$$a_{CP}^{T-\rm odd} = \frac{1}{2}(A_T - \bar{A}_T)$$

- measure asymmetry in triple products $C_T = \vec{v_1} \cdot (\vec{v_2} \times \vec{v_3})$
- $A_T \neq 0$ can also arise from final-state interaction
  - isolate T- violation with $a_{CP}^{T-\rm odd}$
  - $a_{CP}^{T-\rm odd}$ is unaffected by production and reconstruction asymmetries
- in charm: four-body decays

Two approaches

$$A_{\text{raw}} = \frac{\Gamma(D \to f) - \Gamma(\bar{D} \to \bar{f})}{\Gamma(D \to f) + \Gamma(\bar{D} \to \bar{f})}$$

- obtain asymmetry from difference in partial widths
- A_{raw} includes asymmetries in production and reconstruction
  - A_{FB}: arising from $\gamma - Z^0$ interference
  - A_{ϵ}: reconstruction of final-state particles
  - need control channel to correct
- in charm: SCS two-body decays

$$A_{CP} \propto \sin(\phi)\sin(\delta)$$

$$A_T = \frac{\Gamma(C_T > 0) - \Gamma(C_T < 0)}{\Gamma(C_T > 0) + \Gamma(C_T < 0)} \qquad \bar{A}_T = \frac{\Gamma(-\bar{C}_T > 0) - \Gamma(-\bar{C}_T < 0)}{\Gamma(-\bar{C}_T > 0) + \Gamma(-\bar{C}_T < 0)}$$

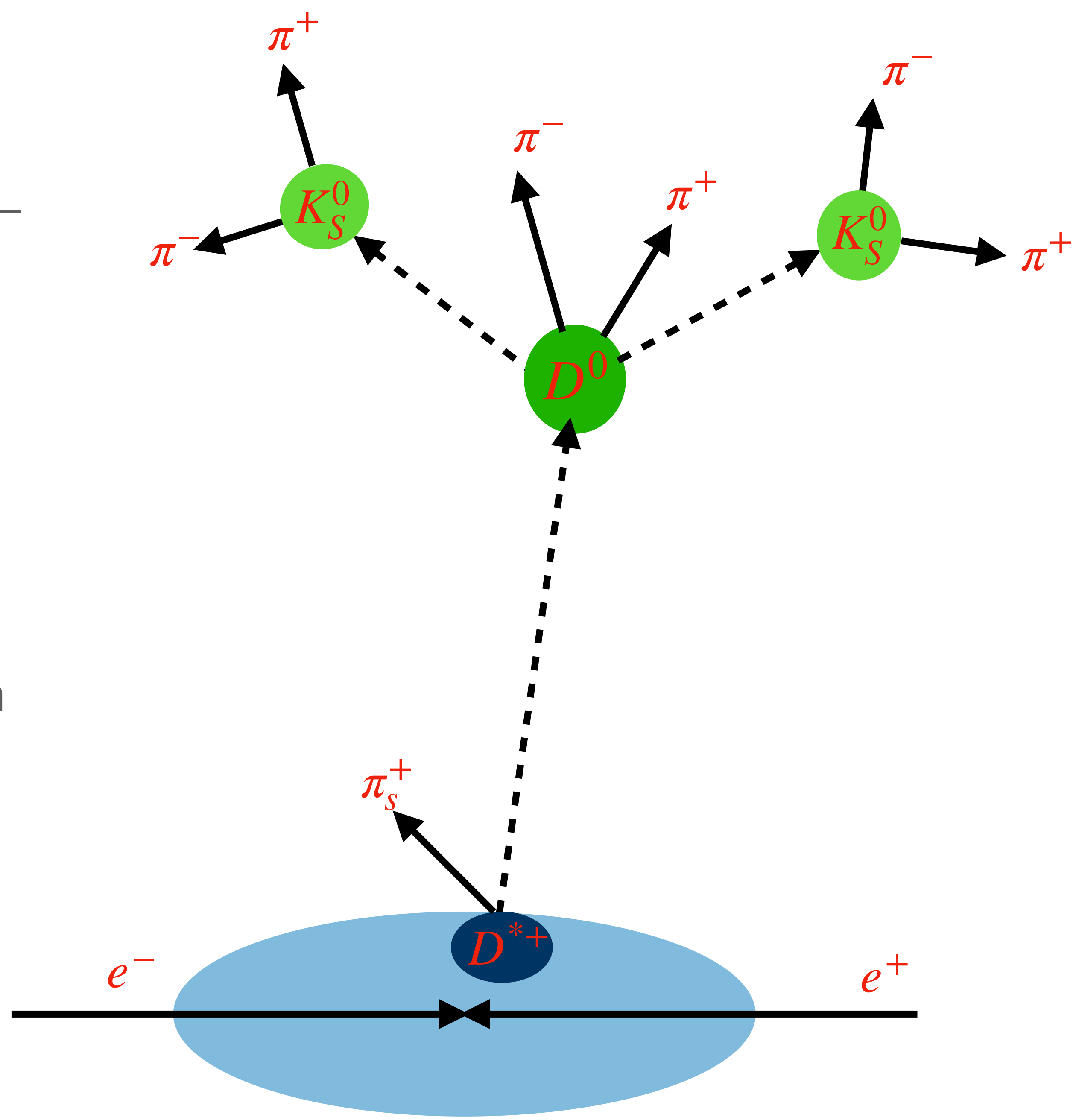
$$a_{CP}^{T\text{−odd}} = \frac{1}{2}(A_T - \bar{A}_T)$$

- measure asymmetry in triple products $C_T = \vec{v_1} \cdot (\vec{v_2} \times \vec{v_3})$
- $A_T \neq 0$ can also arise from final-state interaction
  - isolate T- violation with $a_{CP}^{T\text{−odd}}$
  - $a_{CP}^{T\text{−odd}}$ is unaffected by production and reconstruction asymmetries
- in charm: four-body decays

$$a_{CP}^{T\text{−odd}} \propto \sin(\phi)\cos(\delta)$$

Analysis details

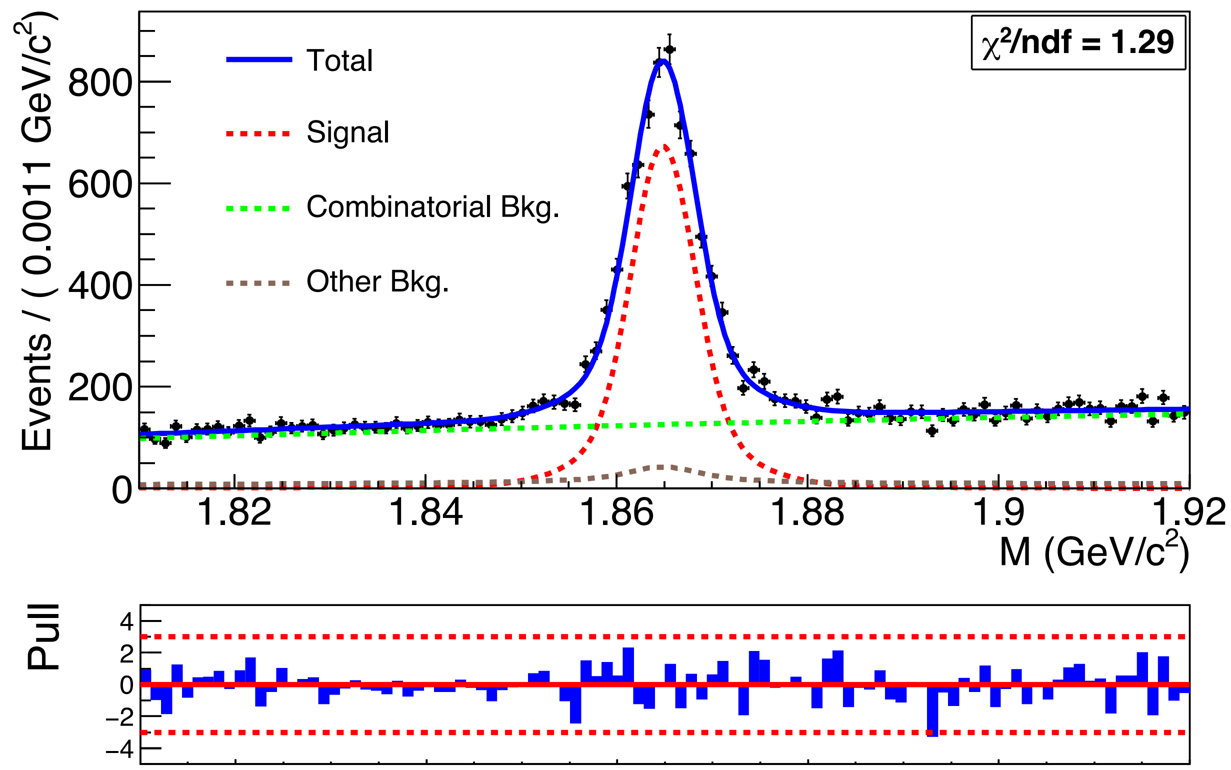
- focus on
  - A_{CP} in $D^0 \to K_S^0 K_S^0 \pi^+ \pi^-$ (D^*-tagged)
  - T-odd correlation in $D^+ \to K^+ K_S^0 \pi^+ \pi^-$
- common features
  - based on full Belle data set collected on or near $\Upsilon(nS)$, ~1ab^{-1}
  - identification of K_S^0 candidates with NN based on kinematic features
  - further background suppression with sum of vertex fit qualities, flight-length significance



A_{CP} in $D^0 \to K^0_S K^0_S \pi^+ \pi^-$

$$A_{\rm raw} = \frac{N(D^0 \to f) - N(\bar{D}^0 \to \bar{f})}{N(D^0 \to f) + N(\bar{D}^0 \to \bar{f})} \longrightarrow A_{\rm raw} = A_{CP} + A_{\rm FB} + A_{\epsilon}^{\pi_s}$$

- obtain asymmetry from different D^0 and $\bar{D}^0$ yields



A_{CP} in $D^0 \to K_S^0 K_S^0 \pi^+ \pi^-$

$$A_{\rm raw} = A_{CP} + A_{\rm FB} + A_\epsilon^{\pi_s}$$

- obtain asymmetry from different D^0 and $\bar{D}^0$ yields
- slow pion asymmetry $A_\epsilon^{\pi_s}$
  - use (un)-tagged $D^0 \to K^- \pi^+$ decays
  - measured in bins of transverse momentum and polar angle

$$w_{D^0} = 1 - A_\epsilon^{\pi_s}(p_T, \cos\theta_{\pi_s})$$

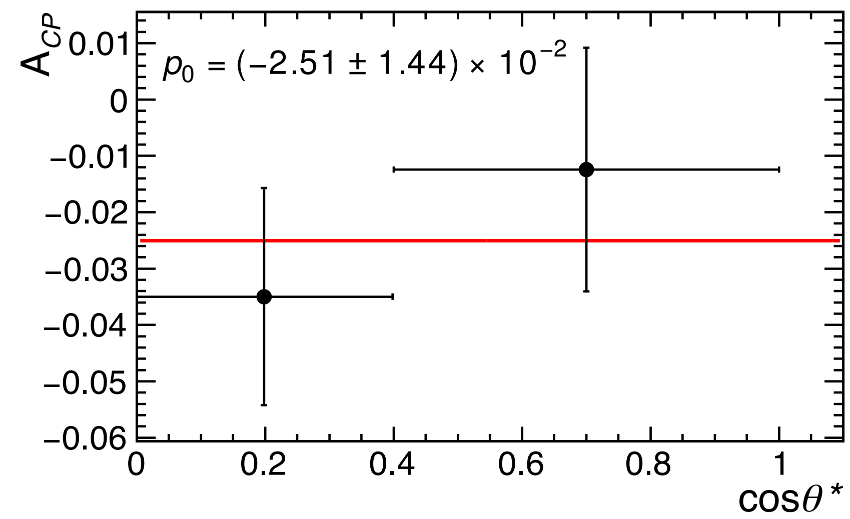
$$w_{\bar{D}^0} = 1 + A_\epsilon^{\pi_s}(p_T, \cos\theta_{\pi_s})$$

correct for $A_\epsilon^{\pi_s}$ by separately weighting D^0 and $\bar{D}^0$ yields

A_{CP} in $D^0 \to K_S^0 K_S^0 \pi^+ \pi^-$

$$A_{\rm raw}^{\rm corr} = A_{CP} + A_{\rm FB}$$

- obtain asymmetry from different D^0 and $\bar{D}^0$ yields
- slow pion asymmetry $A_{\epsilon}^{\pi_s}$
  - use (un)-tagged $D^0 \to K^- \pi^+$ decays
  - measured in bins of transverse momentum and polar angle
- forward-backward asymmetry $A_{\rm FB}$
  - odd function of polar angle of D^* momentum



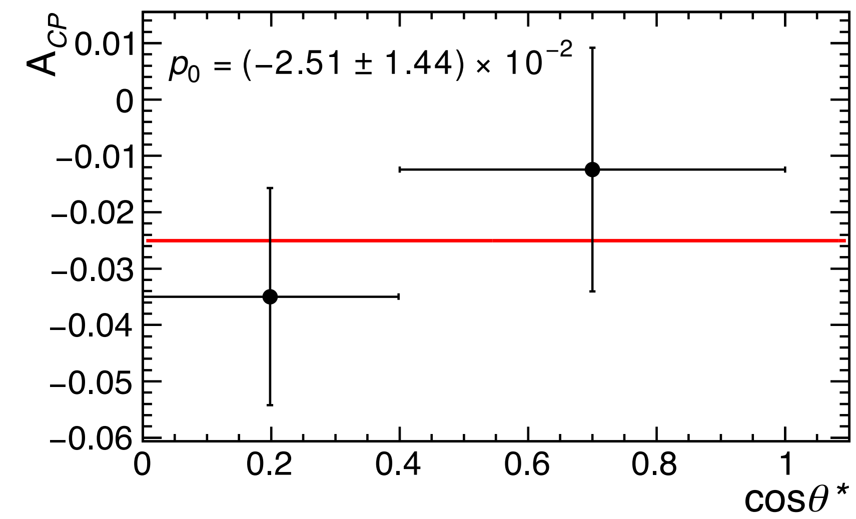
$$A_{CP} = \frac{A_{\rm raw}^{\rm corr}(\cos\theta^*) + A_{\rm raw}^{\rm corr}(-\cos\theta^*)}{2}$$

obtain A_{CP} by averaging over bins of polar angle

A_{CP} in $D^0 \to K_S^0 K_S^0 \pi^+ \pi^-$

$$A_{\rm raw}^{\rm corr} = A_{CP} + A_{\rm FB}$$

- obtain asymmetry from different D^0 and $\bar{D}^0$ yields
- slow pion asymmetry $A_\epsilon^{\pi_s}$
  - use (un)-tagged $D^0 \to K^- \pi^+$ decays
  - measured in bins of transverse momentum and polar angle
- forward-backward asymmetry $A_{\rm FB}$
  - odd function of polar angle of D^* momentum
- first such measurement for this decay



$$A_{CP} = \frac{A_{\rm raw}^{\rm corr}(\cos\theta^*) + A_{\rm raw}^{\rm corr}(-\cos\theta^*)}{2}$$

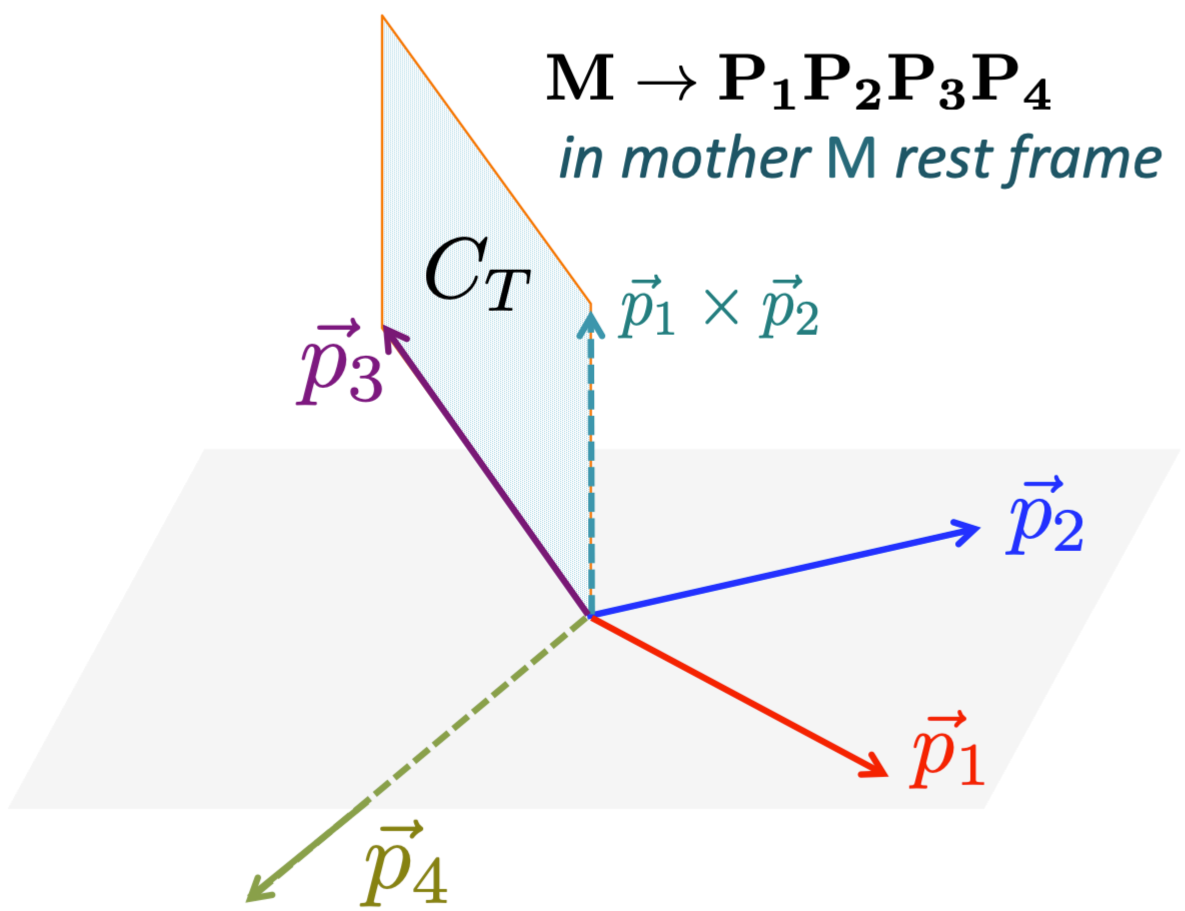
obtain A_{CP} by averaging over bins of polar angle

$$A_{CP}(D^0 \to K_S^0 K_S^0 \pi^+ \pi^-) = (-2.51 \pm 1.44({\rm stat})^{+0.35}_{-0.52}({\rm syst}))\ \%$$

T-odd correlation in $D^+ \to K^+ K_S^0 \pi^+ \pi^-$

- define C_T as triple product of momenta of charged final-state particles

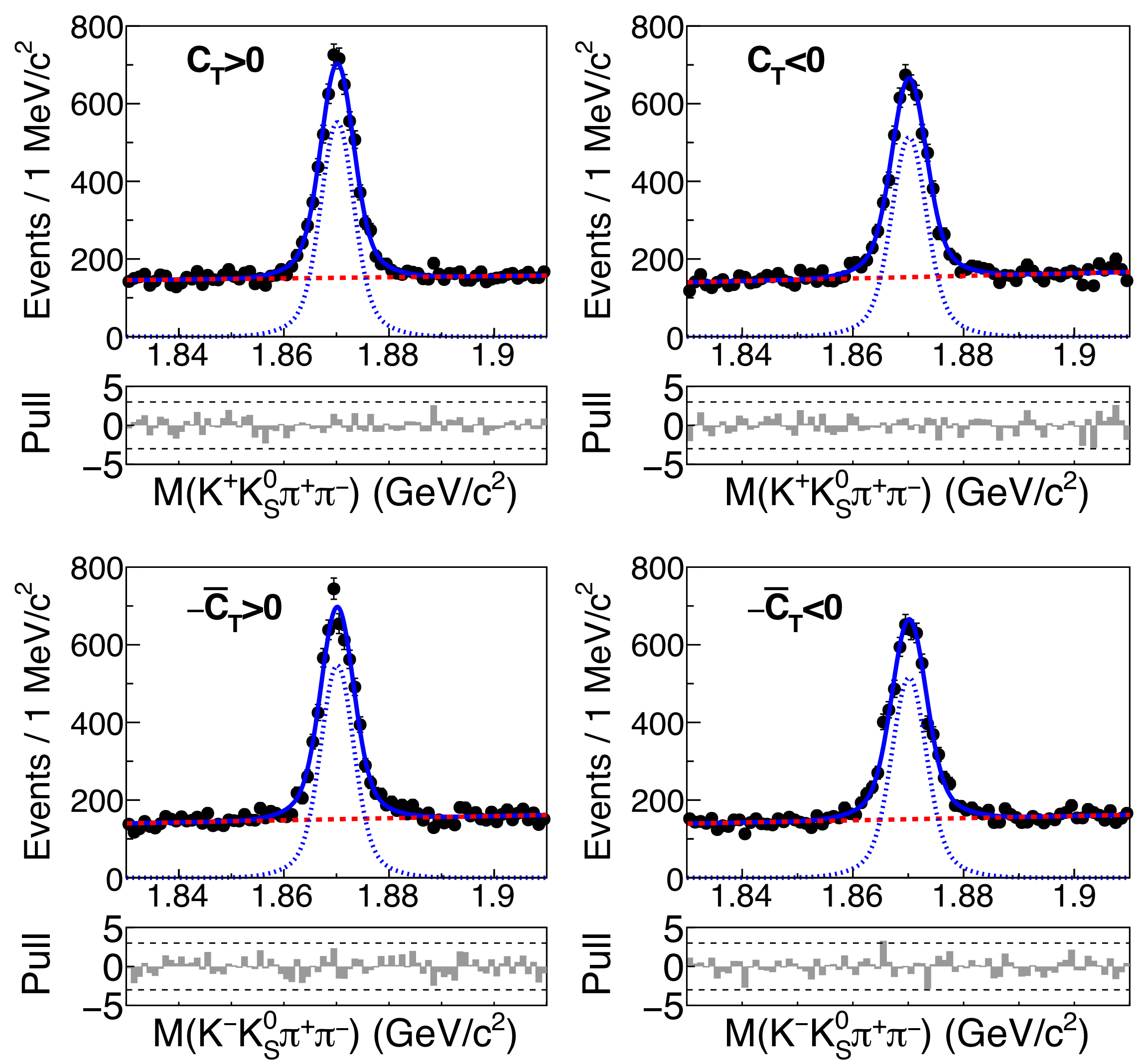
$$C_T = \vec{p}_{K^+} \cdot (\vec{p}_{\pi^+} \times \vec{p}_{\pi^-})$$



T-odd correlation in $D^+ \to K^+ K^0_S \pi^+ \pi^-$

- define C_T as triple product of momenta of charged final-state particles
- divide D candidates into four subsamples based on D flavor/charge and sign of C_T
- obtain N_D, $N_{\bar{D}}$, A_T and $a_{CP}^{T-\mathrm{odd}}$ from simultaneous fit to subsamples

total signal yield: 18632±214



$$N(C_T > 0) = \frac{N(D^+_{(s)})}{2}(1 + A_T),$$

$$N(C_T < 0) = \frac{N(D^+_{(s)})}{2}(1 - A_T),$$

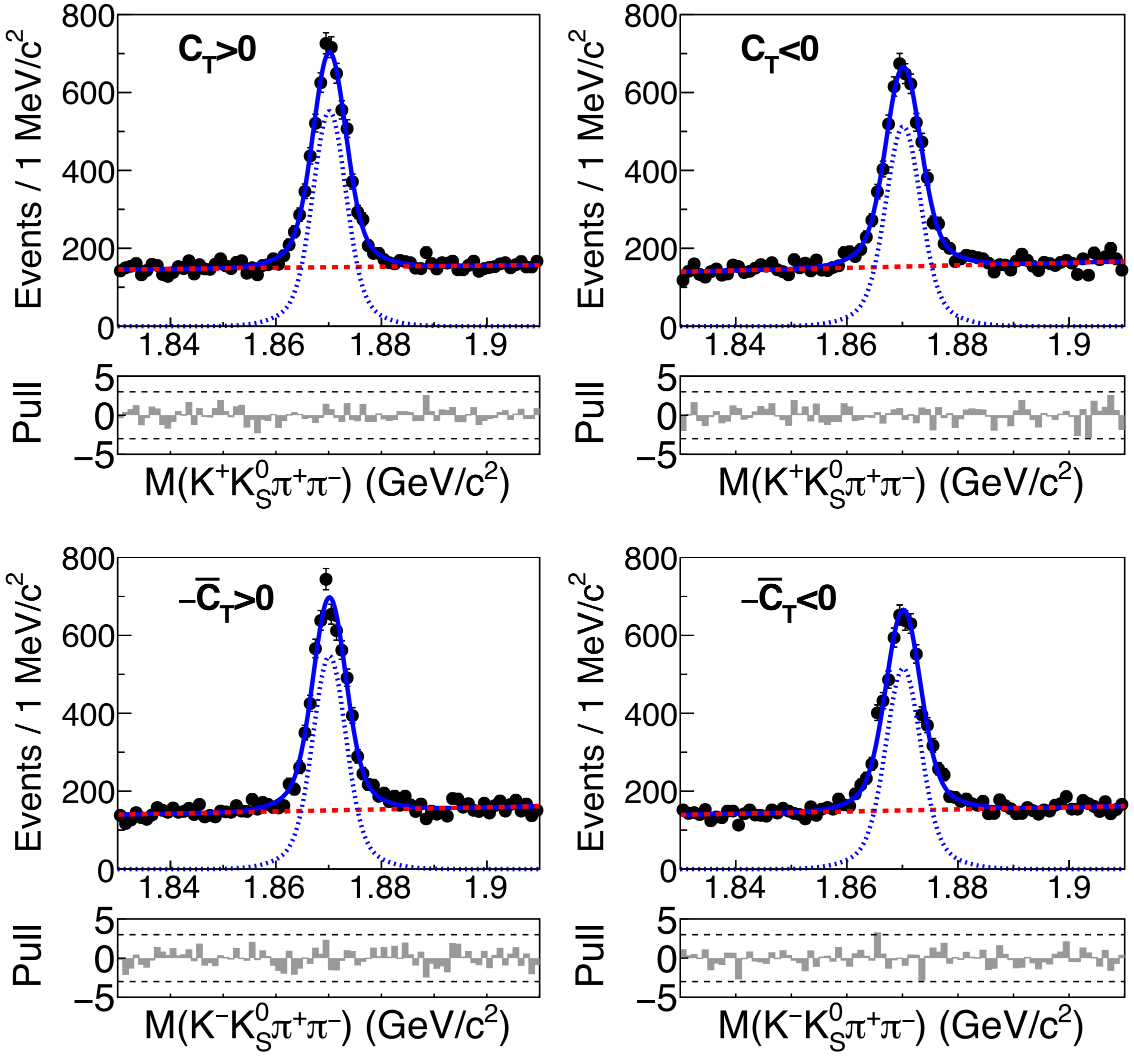
$$N(-\overline{C}_T > 0) = \frac{N(D^-_{(s)})}{2}(1 + A_T - 2a_{CP}^{T\text{-odd}}),$$

$$N(-\overline{C}_T < 0) = \frac{N(D^-_{(s)})}{2}(1 - A_T + 2a_{CP}^{T\text{-odd}}).$$

T-odd correlation in $D^+ \to K^+ K_S^0 \pi^+ \pi^-$

- define C_T as triple product of momenta of charged final-state particles
- divide D candidates into four subsamples based on D flavor/charge and sign of C_T
- obtain N_D, $N_{\bar{D}}$, A_T and $a_{CP}^{T\text{−odd}}$ from simultaneous fit to subsamples
- systematic effects related to efficiency variation of C_T

total signal yield: 18632±214

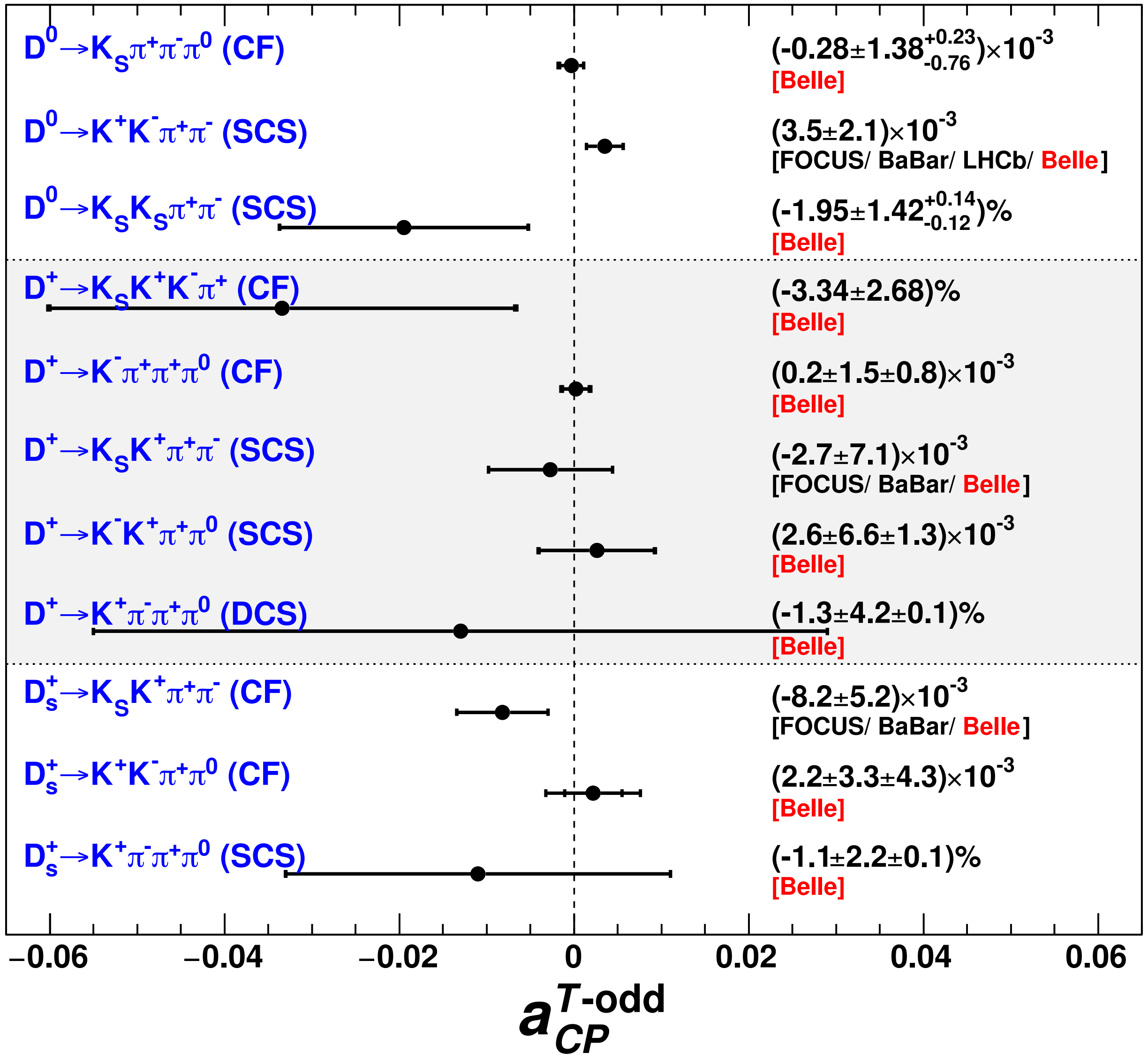


$$a_{CP}^{T\text{−odd}} = (0.34 \pm 0.87(\text{stat.}) \pm 0.32(\text{syst.}))\ \%$$

T-odd correlation

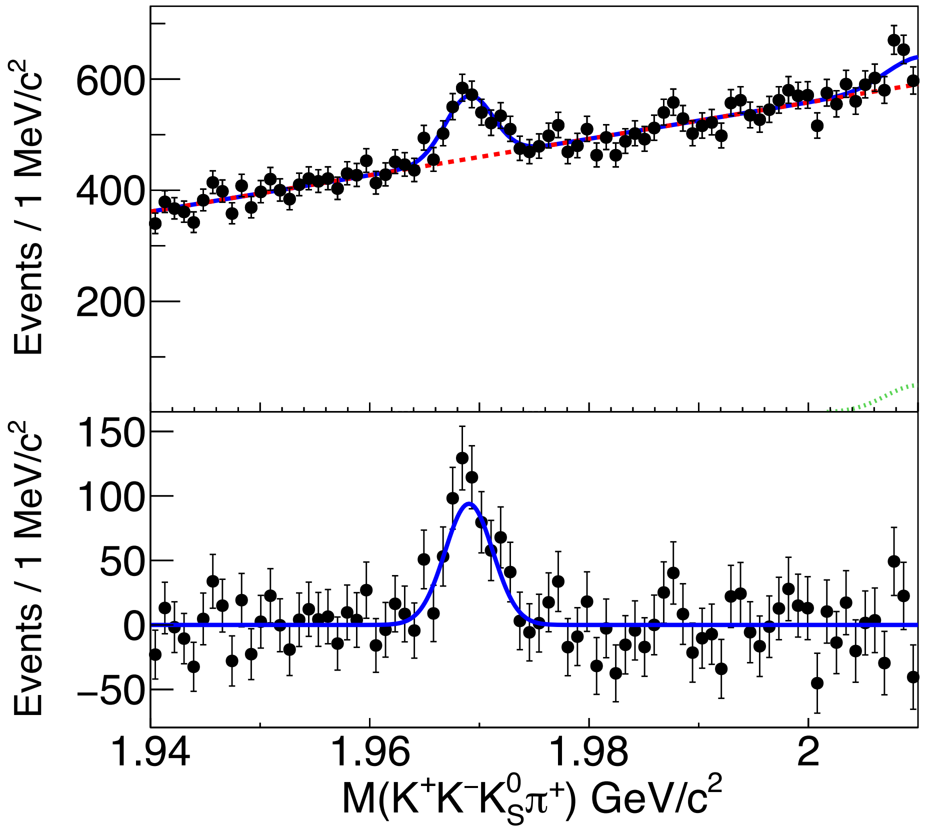
- define C_T as triple product of momenta of charged final-state particles
- divide D candidates into four subsamples based on D flavor/ charge and sign of C_T
- obtain N_D, $N_{\bar{D}}$, A_T and $a_{CP}^{T-\mathrm{odd}}$ from simultaneous fit to subsamples
- systematic effects related to efficiency variation of C_T
- results are among world's most precise measurements, no evidence of CPV

made by Longke Li



Noteworthy

- additional branching fractions measurements:
  - $B(D^0 \to K_S^0 K_S^0 \pi^+ \pi^-) = (4.82 \pm 0.08(\text{stat})^{+0.10}_{+0.11}(\text{syst}) \pm 0.31(\text{norm})) \times 10^{-4}$
    - most precise measurement to date
    - norm. channel: $D^0 \to K_S^0 \pi^+ \pi^-$
  - $B(D_s^+ \to K^+ K^- K_S^0 \pi^+) = (1.29 \pm 0.14(\text{stat}) \pm 0.04(\text{syst}) \pm 0.11(\text{norm})) \times 10^{-4}$
    - first observation of this decay
    - norm. channel: $D_s^+ \to K^+ K_S^0 \pi^+ \pi^-$
- measurement of $a_{CP}^{T\text{-odd}}$ in subregions of phase space:
  - largest asymmetry found in $D_s^+ \to K^{*0} \rho^+$
  - $a_{CP}^{T\text{-odd}} = (6.2 \pm 3.0(\text{stat}) \pm 0.4(\text{syst}))\ \%$



Conclusion

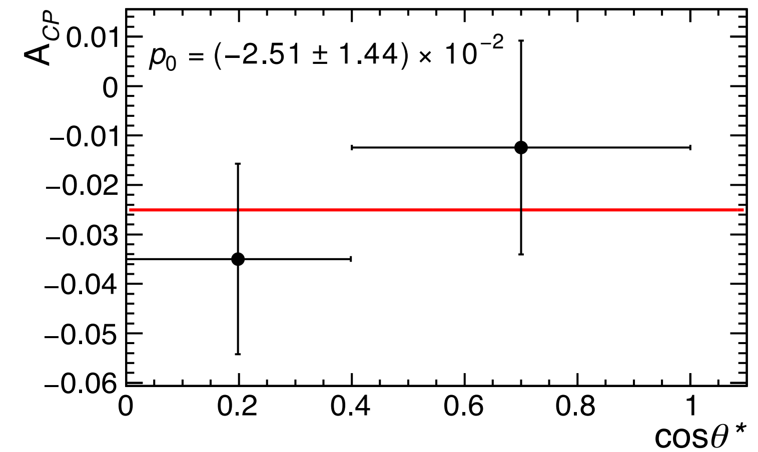
- Charm Flavor Tagger
  - new inclusive algorithm that exploits correlation between signal flavor and charge of tagging particles
  - significantly enlarge the available sample size
- A_{CP} and $a_{CP}^{T\text{-odd}}$ measurements
  - two complementary approaches to measure CPV
  - use four-body charm decays, efficient reconstruction at Belle
  - world's most precise results

Backup

$D^0 \to K_S^0 K_S^0 \pi^+ \pi^-$

TABLE I. Systematic uncertainties (fractional) for the branching fraction measurement.

Source	$K_S^0 K_S^0 \pi^+\pi^-$ (%)	$K_S^0 \pi^+\pi^-$ (%)
Fixed PDF parameters	0.14	0.09
$D^0 \to K_S^0 K_S^0 K_S^0$ background	0.11	–
Broken charm background	0.98	–
MC statistics	0.26	0.17
PID efficiency correction	0.80	0.74
K_S^0 reconstruction efficiency	0.83	0.36
Tracking Efficiency	0.70	
$M(\pi^+\pi^-)$ veto efficiency	$^{+0.42}_{-0.93}$	–
Fraction of mis-reconst. signal	$^{+0.02}_{-0.03}$	–
$D^0 \to K_S^0 K_S^0 \pi^+\pi^-$ decay model	0.73	
$\mathcal{B}(K_S^0 \to \pi^+\pi^-)$	0.07	–
Total for $\mathcal{B}_{K_S^0 K_S^0 \pi^+\pi^-}/\mathcal{B}_{K_S^0\pi^+\pi^-}$	$^{+2.07}_{-2.23}$	



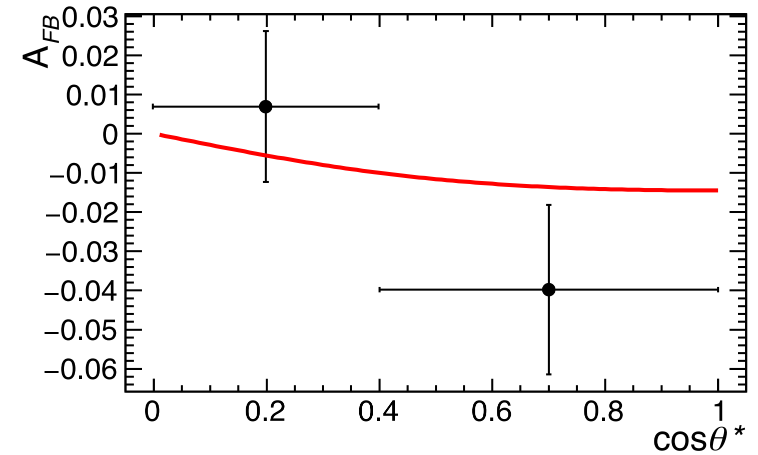


TABLE II. Systematic uncertainties (absolute) for A_{CP}.

Sources	(%)
Fixed PDF parameters	± 0.01
$D^0 \to K_S^0 K_S^0 K_S^0$ background	$^{+0.02}_{-0.03}$
Broken charm background	$^{+0.09}_{-0.07}$
Binning in $\cos\theta^*$	$^{+0.33}_{-0.51}$
Reconstruction asymmetry $A_\epsilon^{\pi_s}$	± 0.01
Fixed background fractions	± 0.04
Total	$^{+0.35}_{-0.52}$

TABLE III. Systematic uncertainties (absolute) for the a_{CP}^T measurement.

Source	(%)
Fixed PDF parameters	0.010
$D^0 \to K_S^0 K_S^0 K_S^0$ background	$^{+0.000}_{-0.013}$
Broken charm background	$^{+0.014}_{-0.040}$
Efficiency variation with C_T, $\overline{C}_T$	$^{+0.14}_{-0.11}$
Total	$^{+0.14}_{-0.12}$

Search for CPV in $D_{(s)}^{+} \to K^{+}K_{S}^{0}h^{+}h^{-}$ decays and observation of $D_{s}^{+} \to K^{+}K^{-}K_{S}^{0}\pi^{+}$

TABLE I. Results of A_T and $a_{CP}^{T\text{-odd}}$ measurements. The uncertainties listed are statistical.

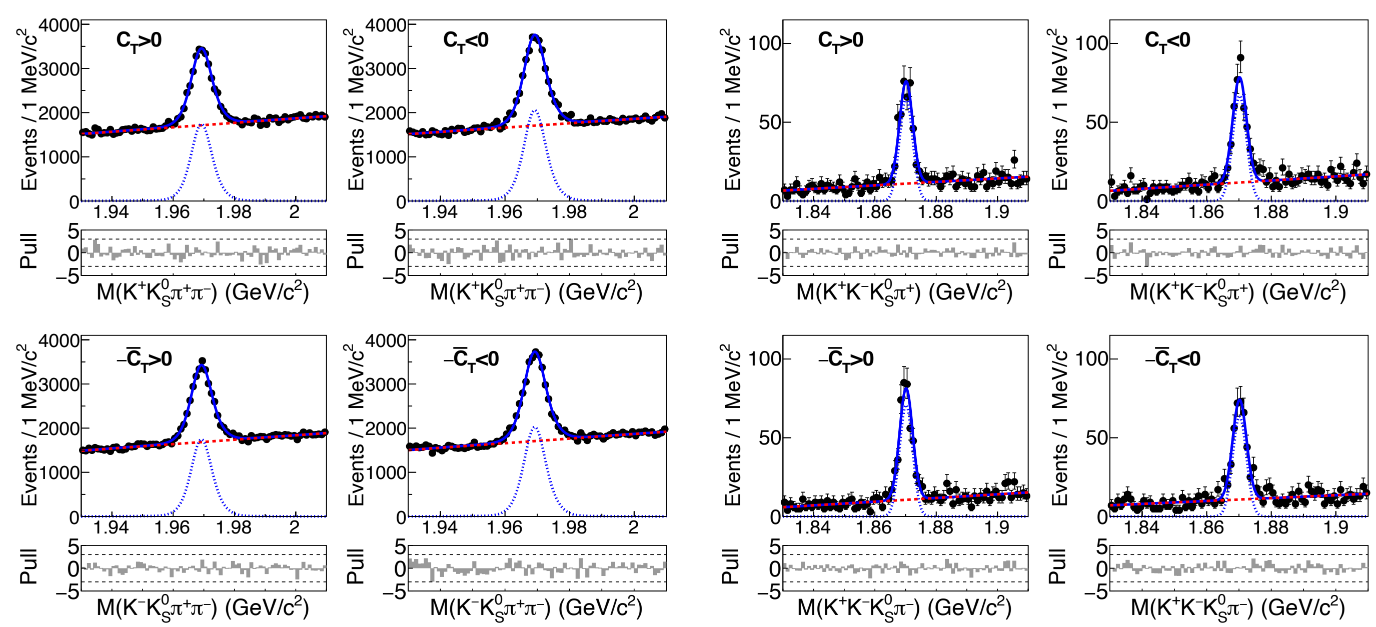
Mode	A_T (%)	$a_{CP}^{T\text{-odd}}$ (%)
$D^+ \to K^+K_S^0\pi^+\pi^-$	(3.67 ± 1.23)	(0.34 ± 0.87)
$D_s^+ \to K^+K_S^0\pi^+\pi^-$	(-8.31 ± 8.89)	(-0.46 ± 0.63)
$D^+ \to K^+K^-K_S^0\pi^+$	(-1.40 ± 4.23)	(-3.34 ± 2.66)

TABLE II. Contributions to the absolute systematic uncertainty for $a_{CP}^{T\text{-odd}}$ in units of % for each mode.

Sources	D^+(CS)	D_s^+(CF)	D^+(CF)
Fit model	0.01	0.02	0.12
Detector bias	0.32	0.32	0.32
Efficiency variation with C_T, $\overline{C}_T$	0.03	0.20	0.06
Total	0.32	0.38	0.35

TABLE III. Fitted signal yields and $a_{CP}^{T\text{-odd}}$ values. The first uncertainties are statistical and the second are systematic.

Mode	$N(D_{(s)}^+)$	$a_{CP}^{T\text{-odd}}$ (%)
$D^+ \to K^+K_S^0\pi^+\pi^-$	18632 ± 214	$(0.34 \pm 0.87 \pm 0.32)$
$D_s^+ \to K^+K_S^0\pi^+\pi^-$	70080 ± 676	$(-0.46 \pm 0.63 \pm 0.38)$
$D^+ \to K^+K^-K_S^0\pi^+$	1425 ± 44	$(-3.34 \pm 2.66 \pm 0.35)$



Search for CPV using T-odd correlations in $D^+_{(s)} \to K^+K^-\pi^+\pi^0, K^+\pi^-\pi^+\pi^0$ and $D^+ \to K^-\pi^+\pi^+\pi^0$ decays

TABLE I. Fitted $D^+_{(s)}$ and $D^-_{(s)}$ signal yields (N_D and $N_{\overline{D}}$, respectively), T-odd asymmetry A_T, and T-odd CP-violating asymmetry $a_{CP}^{T\text{-odd}}$, obtained from a simultaneous fit to the four subsamples of each decay mode. The uncertainties listed are statistical only.

Decay	$D^+ \to f$			$D^+_s \to f$	
Final state (f)	$K^+K^-\pi^+\pi^0$	$K^+\pi^-\pi^+\pi^0$	$K^-\pi^+\pi^+\pi^0$	$K^+\pi^-\pi^+\pi^0$	$K^+K^-\pi^+\pi^0$
N_D	27284 ± 254	2062 ± 127	438432 ± 947	15197 ± 484	167357 ± 786
$N_{\overline{D}}$	27177 ± 255	2044 ± 125	450667 ± 961	14945 ± 479	167064 ± 788
A_T (%)	$+3.63 \pm 0.93$	-0.4 ± 6.0	-0.76 ± 0.22	$+1.4 \pm 3.2$	$+2.96 \pm 0.47$
$a_{CP}^{T\text{-odd}}$ (%)	$+0.26 \pm 0.66$	-1.3 ± 4.2	$+0.02 \pm 0.15$	-1.1 ± 2.2	$+0.22 \pm 0.33$

TABLE II. T-odd CP-violating asymmetries ($a_{CP}^{T\text{-odd}}$) in seven subregions of phase space (see text) corresponding to the intermediate $D^+_{(s)} \to VV$ processes listed. The uncertainties listed are statistical and systematic, respectively.

Subregion	$D^+_{(s)} \to VV$	Signal region (SR)	$a_{CP}^{T\text{-odd}}$ ($\times 10^{-2}$)
(1) SCS	$D^+ \to \phi\rho^+$	ϕ-SR, ρ^+-SR	$0.85 \pm 0.95 \pm 0.25$
(2) SCS	$D^+ \to \overline{K}^{*0}K^{*+}$	$K^{*(0,+)}$-SR, veto ϕ-SR	$0.17 \pm 1.26 \pm 0.13$
(3) CF	$D^+ \to \overline{K}^{*0}\rho^+$	K^{*0}-SR, ρ^+-SR	$0.25 \pm 0.25 \pm 0.13$
(4) SCS	$D^+_s \to K^{*0}\rho^+$	K^{*0}-SR, ρ^+-SR	$6.2 \pm 3.0 \pm 0.4$
(5) SCS	$D^+_s \to K^{*+}\rho^0$	K^{*+}-SR, ρ^0-SR	$1.7 \pm 6.1 \pm 1.5$
(6) CF	$D^+_s \to \phi\rho^+$	ϕ-SR, ρ^+-SR	$0.31 \pm 0.40 \pm 0.43$
(7) CF	$D^+_s \to \overline{K}^{*0}K^{*+}$	$K^{*(0,+)}$-SR, veto ϕ-SR	$0.26 \pm 0.76 \pm 0.37$

TABLE III. Systematic uncertainties for $a_{CP}^{T\text{-odd}}$ in % for five $D^+_{(s)}$ decay channels: (a) $D^+ \to K^-K^+\pi^+\pi^0$; (b) $D^+ \to K^+\pi^-\pi^+\pi^0$; (c) $D^+ \to K^-\pi^+\pi^+\pi^0$; (d) $D^+_s \to K^+\pi^-\pi^+\pi^0$; and (e) $D^+_s \to K^-K^+\pi^+\pi^0$.

Decay channel	(a)	(b)	(c)	(d)	(e)
C_T-dependent efficiency	0.13	0.02	0.08	0.02	0.41
C_T resolution	0.01	0.06	0.01	0.07	0.02
PDF parameters	0.01	0.07	0.01	0.07	0.04
Mass resolution	0.03	0.01	...	0.02	0.11
Fit bias	0.01	0.07	0.00	0.06	0.02
Total syst.	0.13	0.12	0.08	0.12	0.43

Variables used for K_S^0 reconstruction by Belle neural network based method

- K_S^0 momentum in lab frame.
- Distance along the z axis between two track helices at their closest approach.
- Flight length in x-y plane.
- Angle between K_S^0 momentum and the vector joining IP to K_S^0 decay vertex.
- Angle between π momentum and laboratory frame direction in K_S^0 rest frame.
- Distance of closest approach in the x-y plane between the IP and the two pion helices.
- Total number of hits in SVD (silicon vertex detector) and CDC (central drift chamber) for two pion tracks.

$A_\varepsilon^{\pi_s}$ weights for A_{CP} :

$$A_\varepsilon^{\pi_s} = \frac{\varepsilon_f^+ - \varepsilon_f^-}{\varepsilon_f^+ + \varepsilon_f^-}$$

$\varepsilon_f^\pm$: reconstruction efficiency for $\pi_{slow}^\pm$

$$w_{D^0} = 1 + A_\varepsilon^{\pi_s} = \frac{2\varepsilon_f^+}{\varepsilon_f^+ + \varepsilon_f^-}$$

$$w_{\bar{D}^0} = 1 - A_\varepsilon^{\pi_s} = \frac{2\varepsilon_f^-}{\varepsilon_f^+ + \varepsilon_f^-}$$

Efficiency for D^0 and $\bar{D}^0$ become same after applying these weights