Recent Semileptonic Results

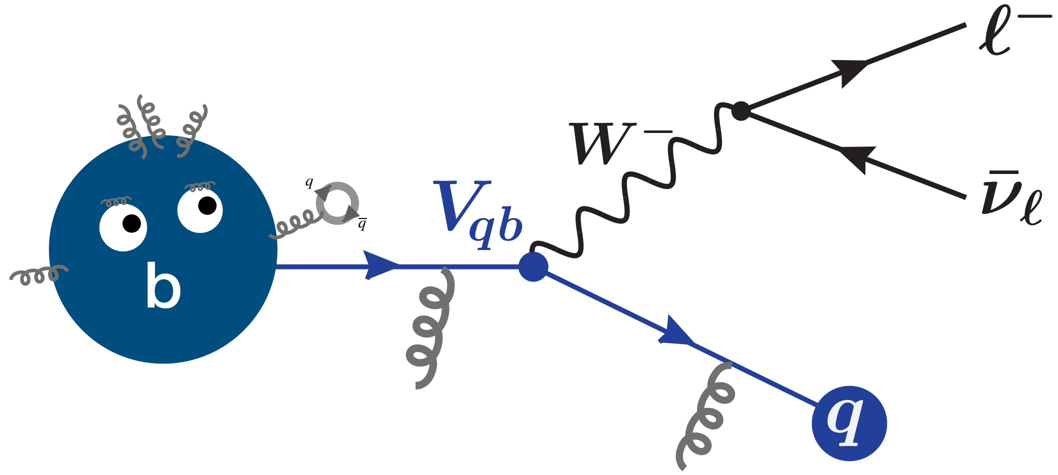
from Belle and Belle II

Flavor@TH Workshop at CERN

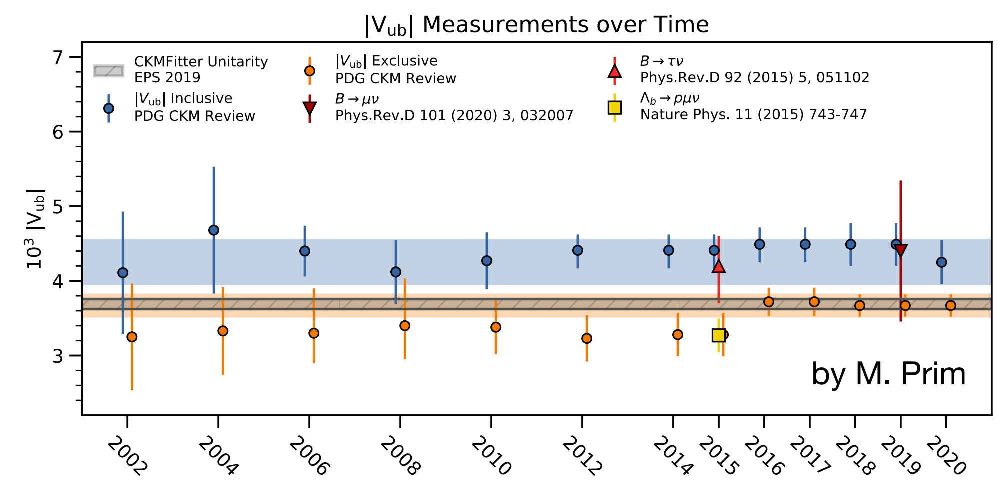
Florian Bernlochner (florian.bernlochner@uni-bonn.de)

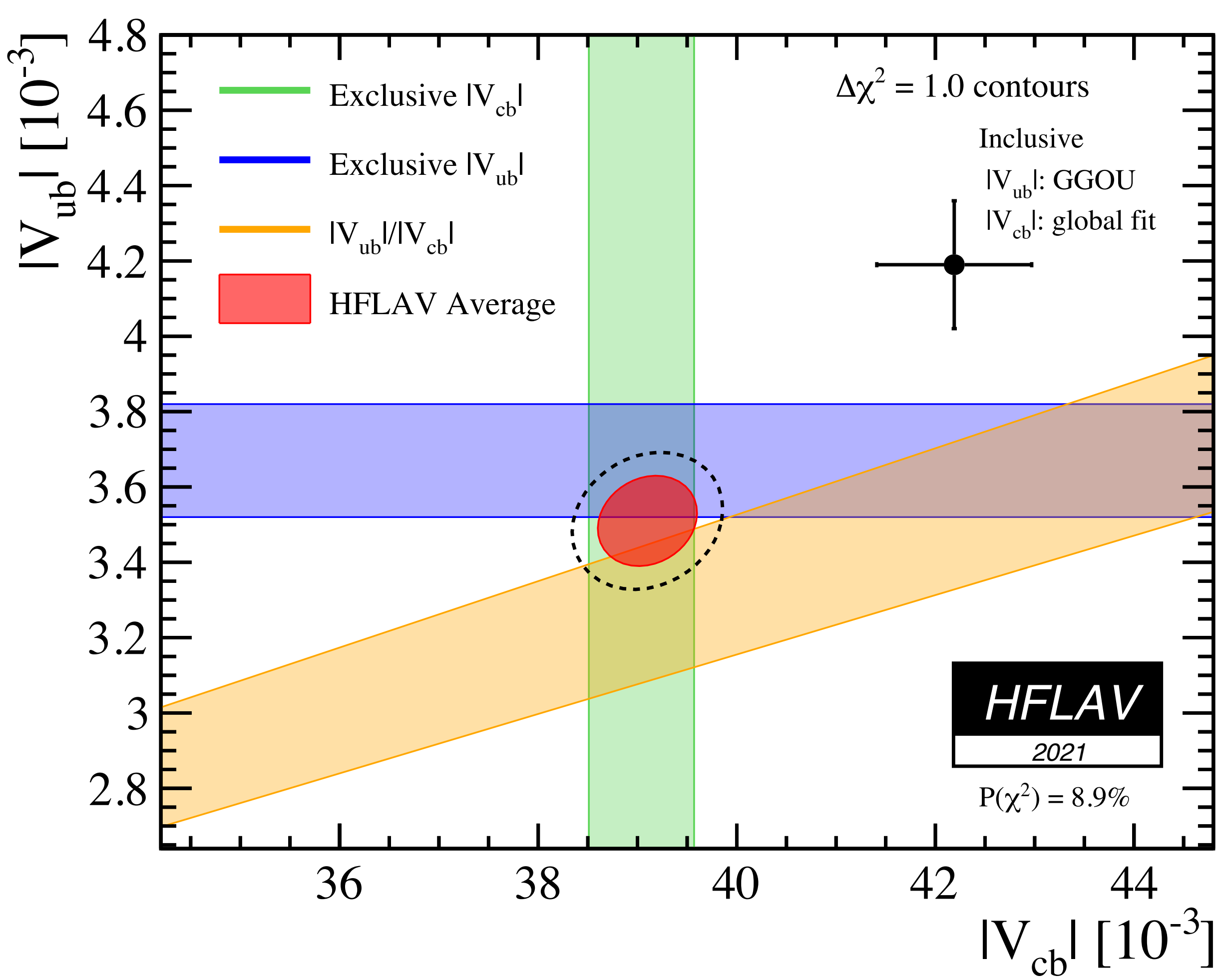
Puzzles…

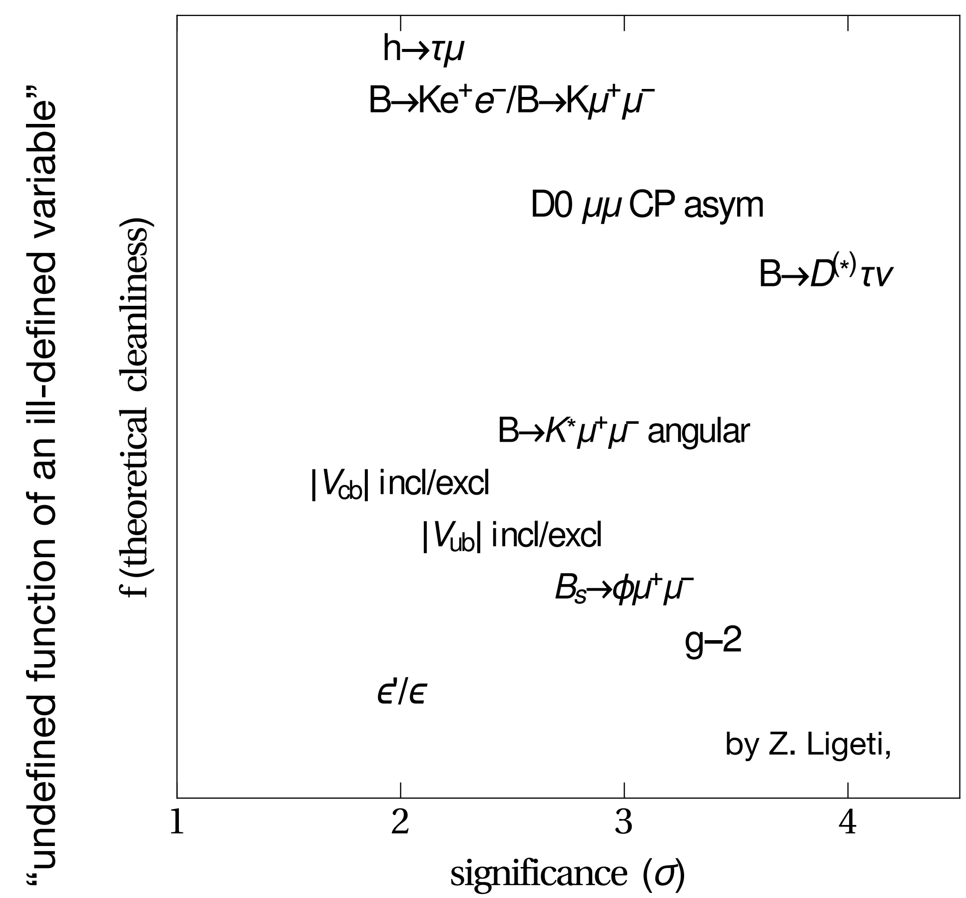
It may look cute, but that might be deceiving…



… Long-standing discrepancy since about a decade







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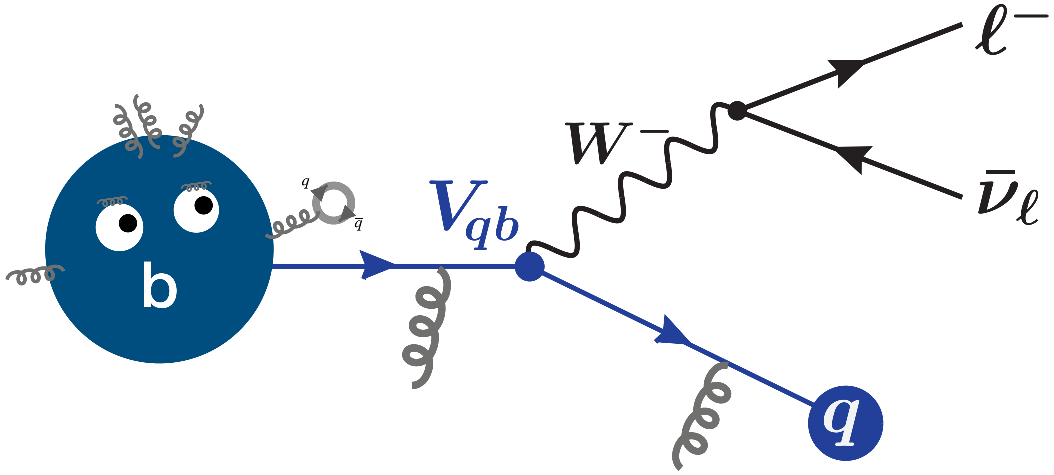
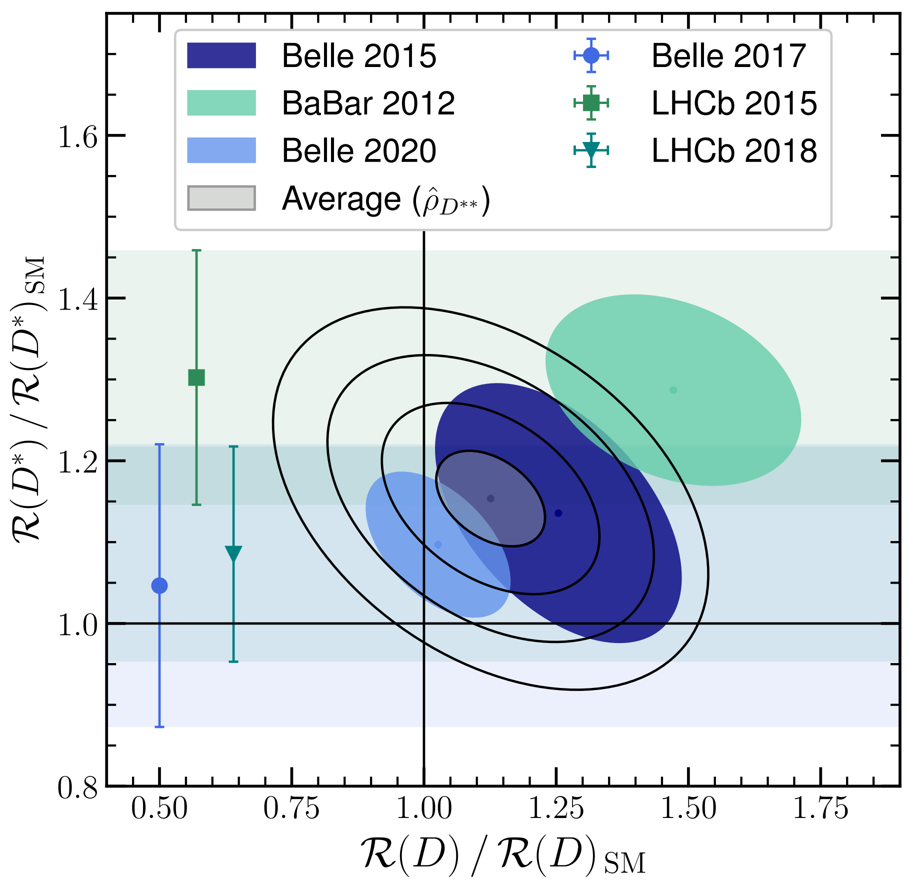


$$R = \frac{b \to q\,\tau\bar{\nu}_\tau}{b \to q\,\ell\bar{\nu}_\ell}$$

$\ell = e, \mu$



Obs.	Current World Av./Data	Current SM Prediction	Significance	
$\mathcal{R}(D)$	0.340 ± 0.030	0.299 ± 0.003	1.2σ	3.1σ
$\mathcal{R}(D^*)$	0.295 ± 0.014	0.258 ± 0.005	2.5σ	
$P_\tau(D^*)$	$-0.38 \pm 0.51^{+0.21}_{-0.16}$	-0.501 ± 0.011	0.2σ	
$F_{L,\tau}(D^*)$	$0.60 \pm 0.08 \pm 0.04$	0.455 ± 0.006	1.6σ	
$\mathcal{R}(J/\psi)$	$0.71 \pm 0.17 \pm 0.18$	0.2582 ± 0.0038	1.8σ	
$\mathcal{R}(\pi)$	1.05 ± 0.51	0.641 ± 0.016	0.8σ	
$\mathcal{R}(D)$	$\mathbf{0.337 \pm 0.030}$	0.299 ± 0.003	1.3σ	$\mathbf{3.6\sigma}$
$\mathcal{R}(D^*)$	$\mathbf{0.298 \pm 0.014}$	0.258 ± 0.005	2.5σ	

SL Analysis Methods

The question of **tagging**:

At e^+e^--B-Factories we can leverage the known initial collision kinematics

E.g. if just one final state particle is missing, then with $Y = X\ell$

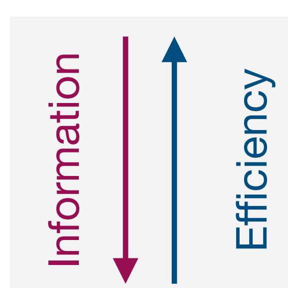
$$\cos\theta_{BY} = \frac{2E_B E_Y - m_B^2 - m_Y^2}{2|\mathbf{p}_B||\mathbf{p}_Y|} \in [-1,1]$$

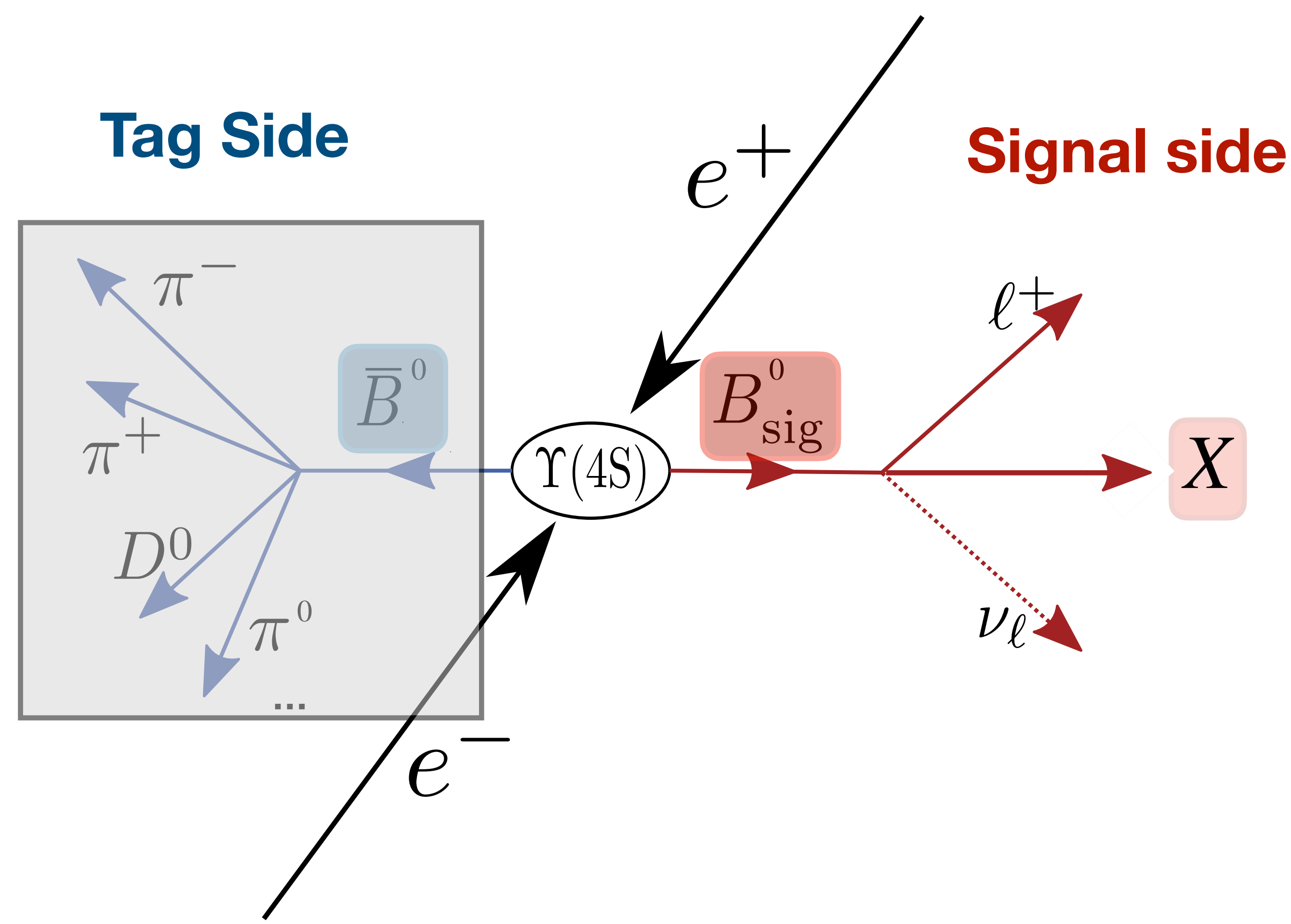
Can gain even more information, if we reconstruct

second B decay $\widehat{=}$ tagging

Idea comes in many flavors:

- inclusive tagging
- SL tagging
- hadronic tagging



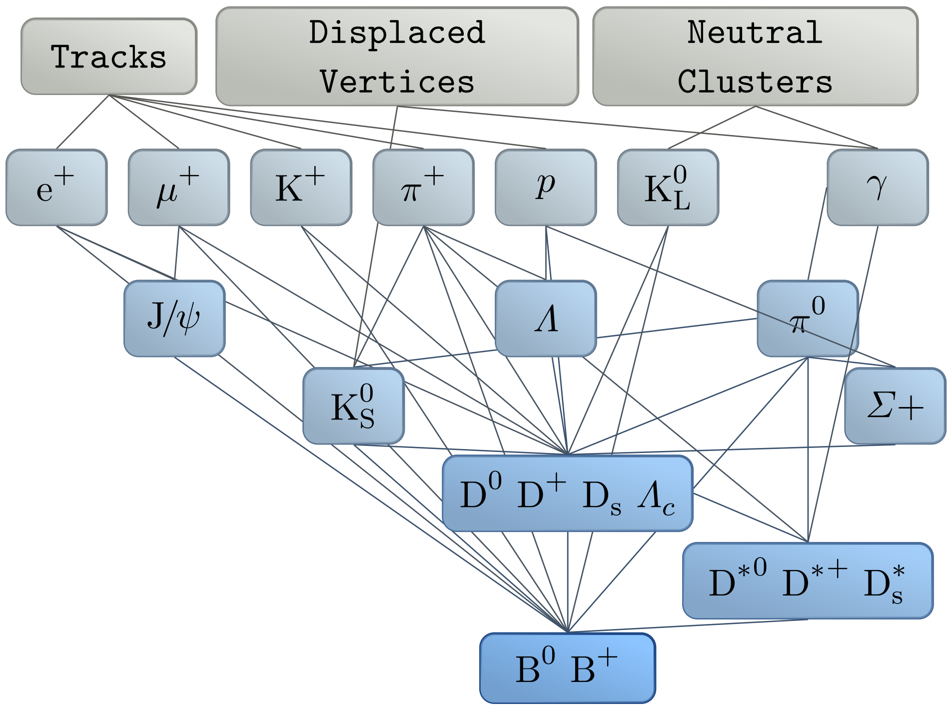


e.g. with hadronic tagging the full event kinematics but the neutrino is reconstructed

$$M_\nu^2 \simeq M_{\text{miss}}^2 = \left(p_{e^+e^-} - p_{B_{\text{tag}}} - p_X - p_\ell\right)^2$$

Tagging in a nutshell

From
arXiv:2008.06096 [hep-ex]



Final Output Score $\mathscr{P}_{\text{tag}}$

Reconstruct B-Mesons in **several stages**:

start with **detector stable particles**; then progress to **simple composite states**; combine the **composite** states to **build** more **complexity**

Each **stage** trains a **Boosted Decision Tree (BDT)** to identify good combinations;

each stage's BDT output is used as input for the next stage
+ all kinematic information
+ (particle identification scores)
+ vertex fit probabilities

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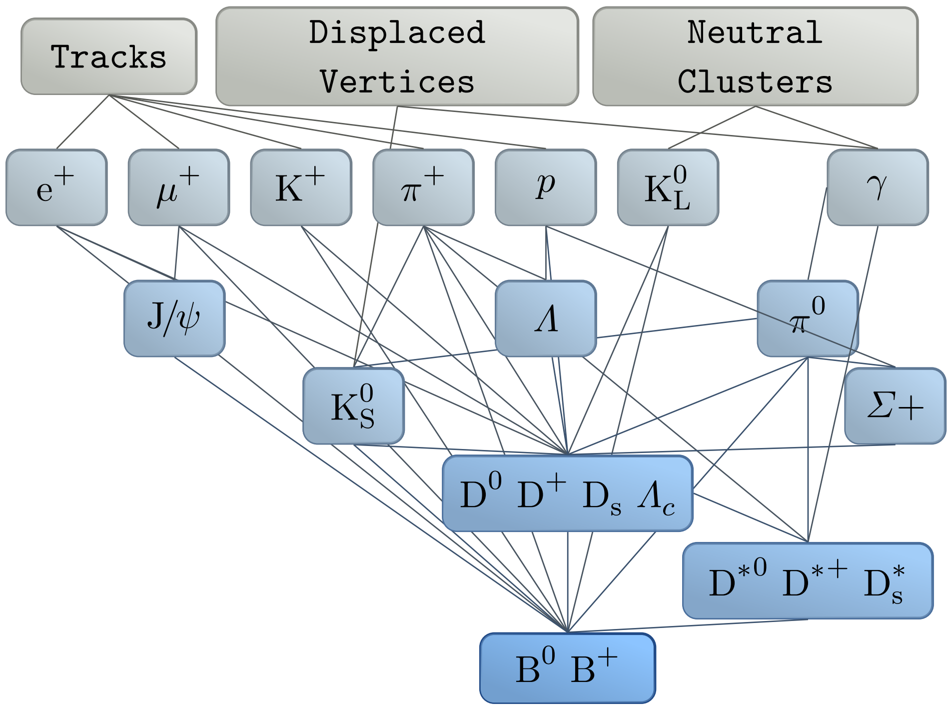

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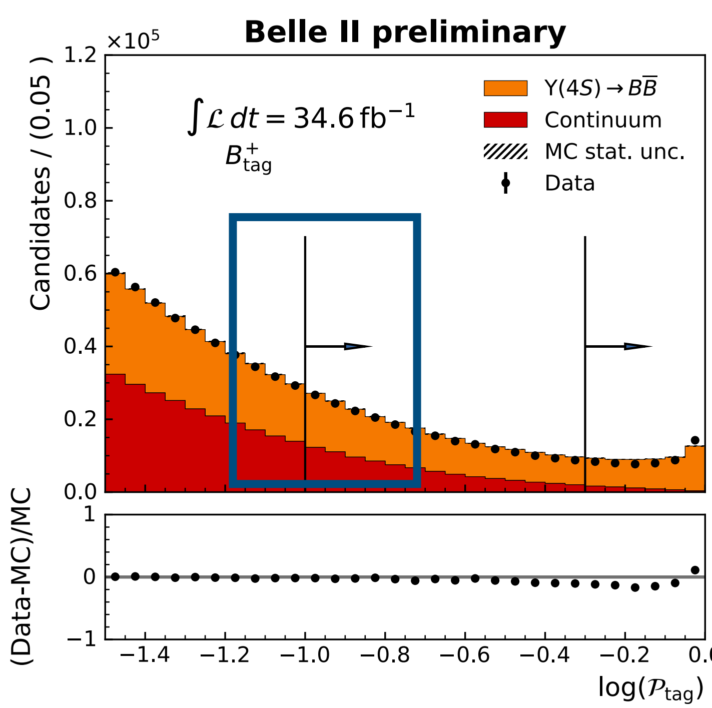

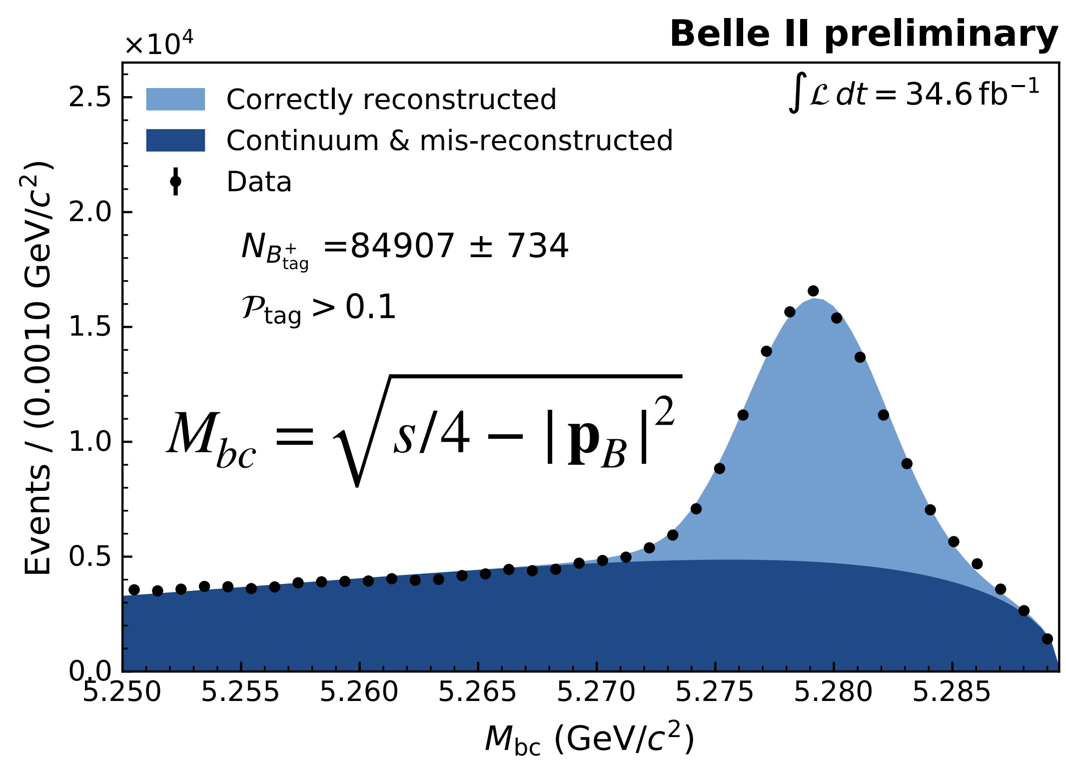

$$M_{bc} = \sqrt{s/4 - |\mathbf{p}_B|^2}$$

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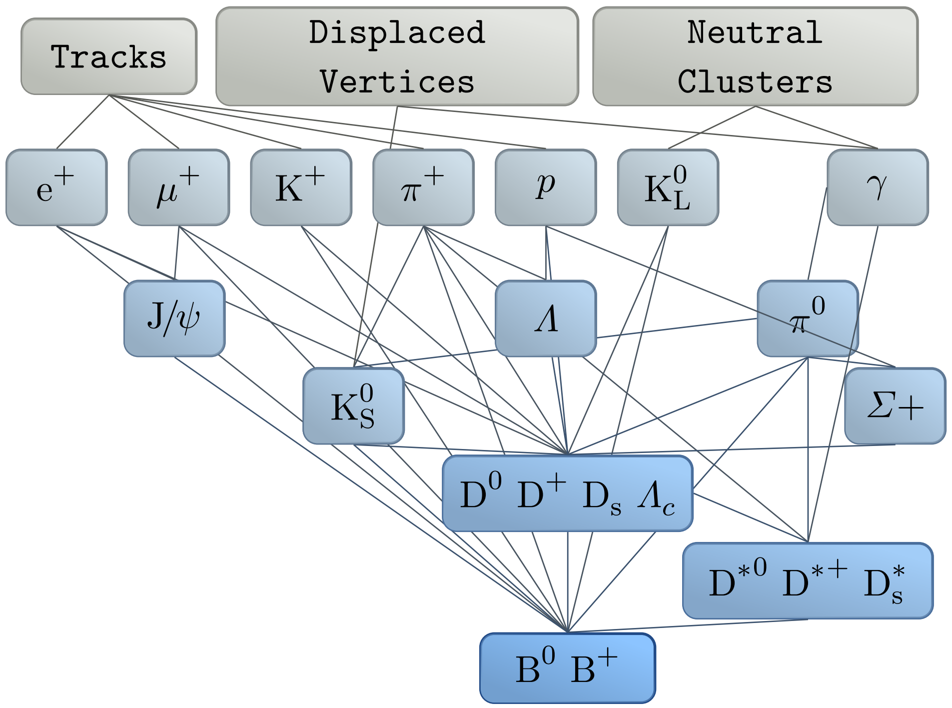

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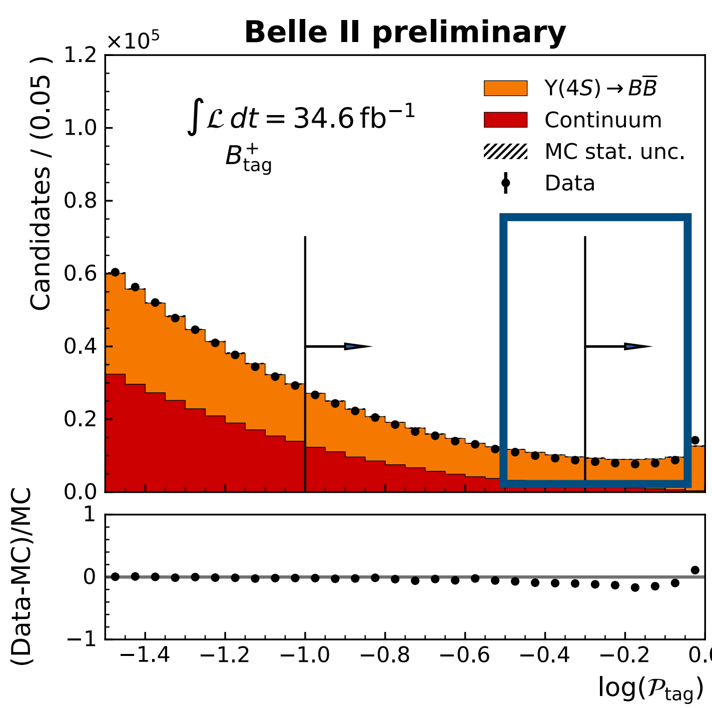
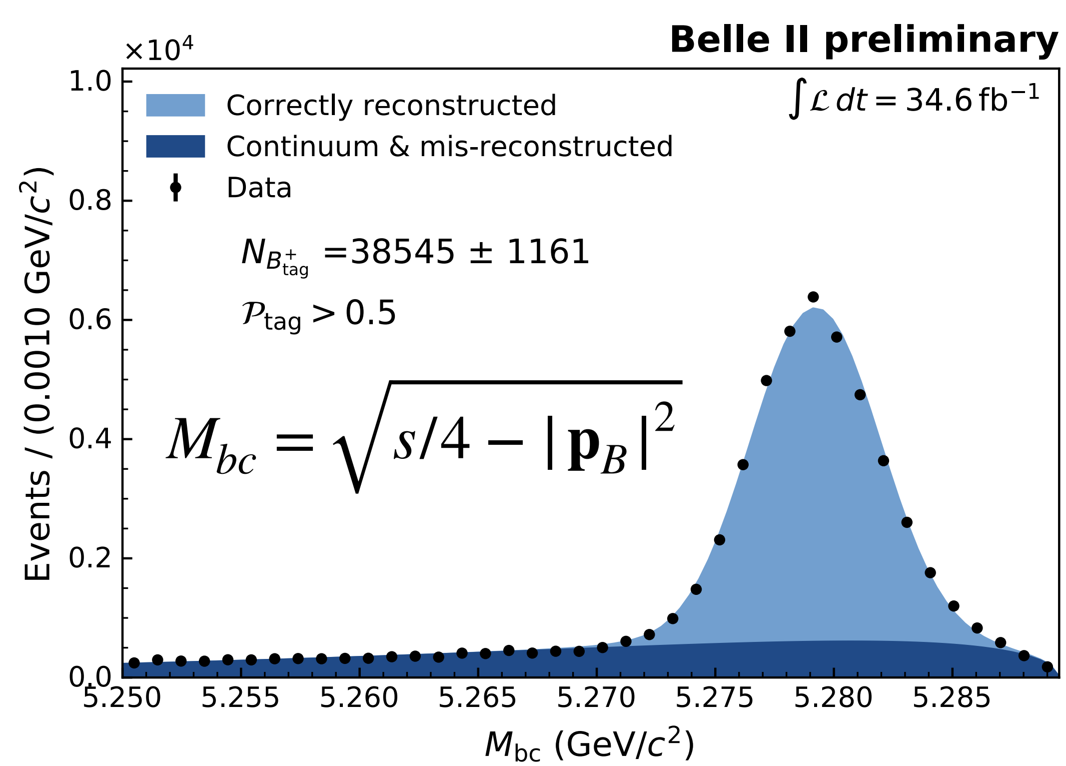
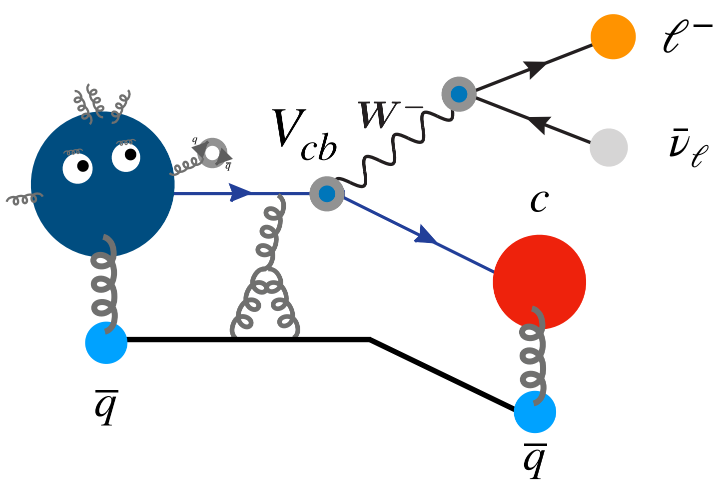


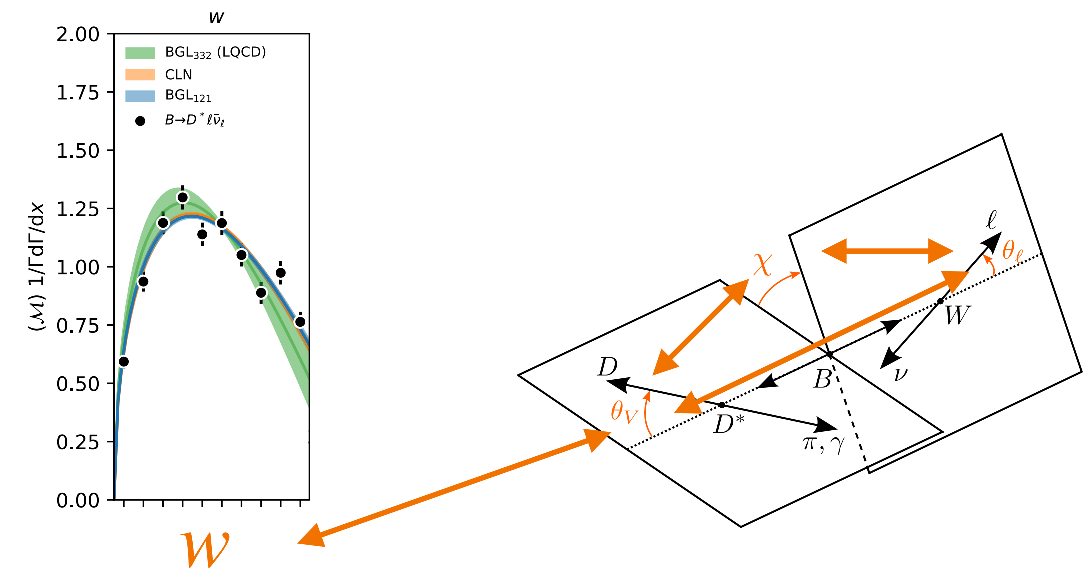


Talk Overview

1. Recent results form **Belle** and **Belle II**

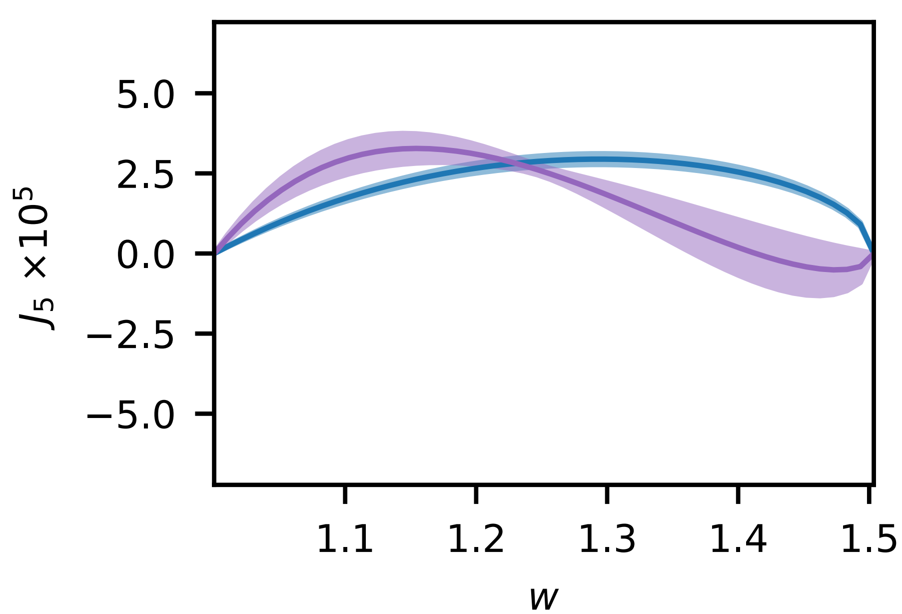


2. From **1D projections** to full angular information



Working around the curse of dimensionality

3. The Potential of full angular fits



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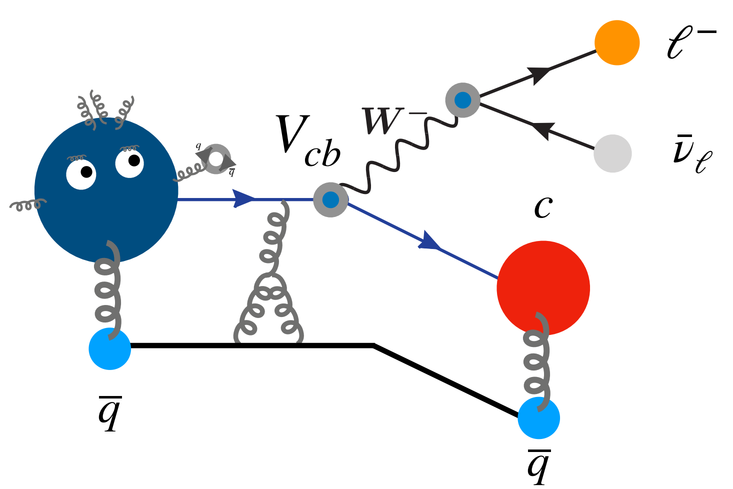

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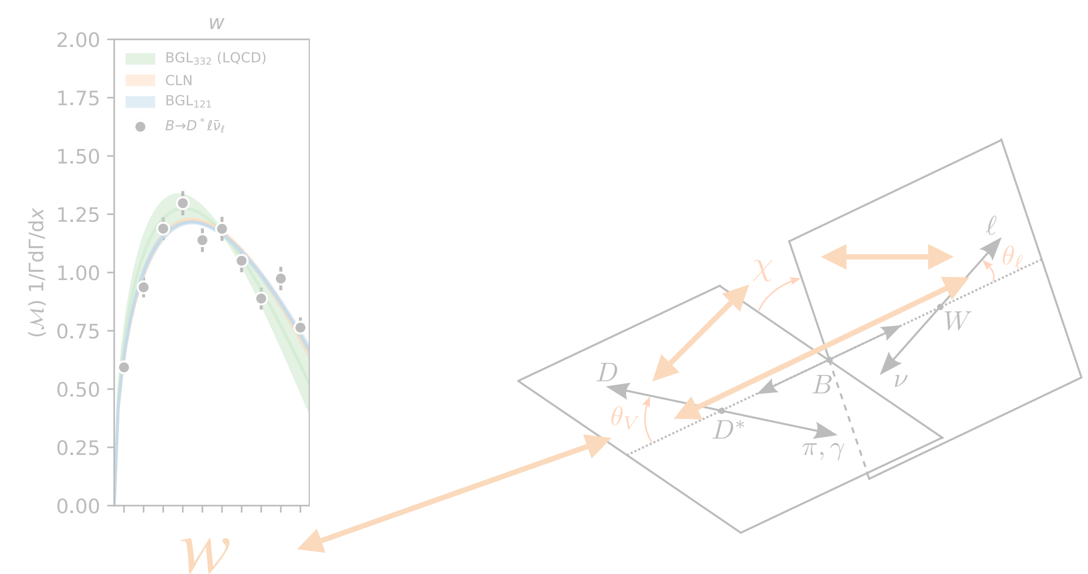

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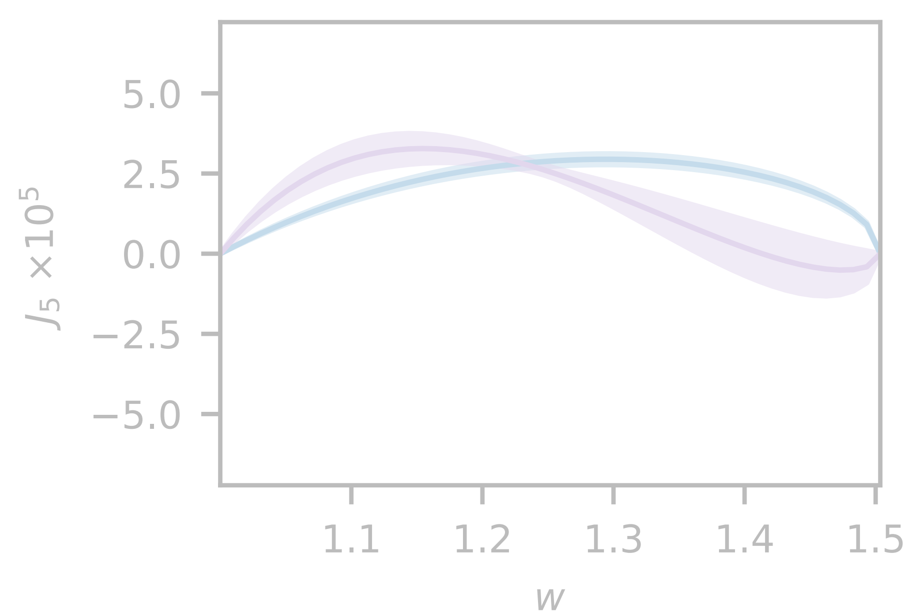


Recent Results Overview

Inclusive

1. Measurements of Lepton **Mass squared moments** in inclusive $B \to X_c \ell \bar{\nu}_\ell$ Decays with the Belle II Experiment [Phys. Rev. D 107, 072002, arXiv:2205.06372]

2. A test of **light-lepton universality** in the rates of inclusive semileptonic B-meson decays at Belle II [Submitted to PRL]

3. First **Simultaneous** Determination of Inclusive and Exclusive $|V_{ub}|$ [Submitted to PRL, arXiv:2303.17309]

Exclusive

4. Measurement of **Differential Distributions** of $B \to D^* \ell \bar{\nu}_\ell$ and Implications on $|V_{cb}|$, [Accepted by PRD], [arXiv:2301.07529]

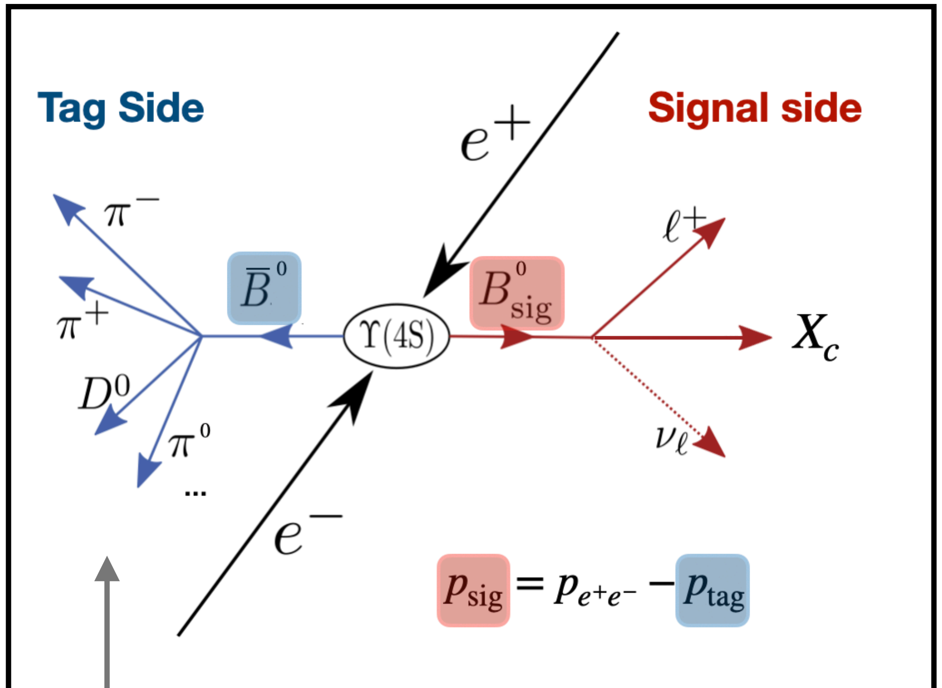
5. Determination of $|V_{cb}|$ using $\overline{B}^0 \to D^{*+} \ell^- \bar{\nu}_\ell$ with **Belle II**, [To be submitted to PRD]

6. Test of **light-lepton universality** in **angular asymmetries** of hadronically tagged $B^0 \to D^{*-}\{e^+, \mu^+\}\,\nu$ decays at Belle II, [To be submitted to PRL]

+ more, e.g. arXiv:2210.04224v2 [hep-ex] or arXiv:2211.09833 [hep-ex] (Phys. Rev. D 107, 092003)

1. Measurements of Lepton **Mass squared moments** in inclusive $B \to X_c \ell \bar{\nu}_\ell$ Decays with the Belle II Experiment [Phys. Rev. D 107, 072002, arXiv:2205.06372]

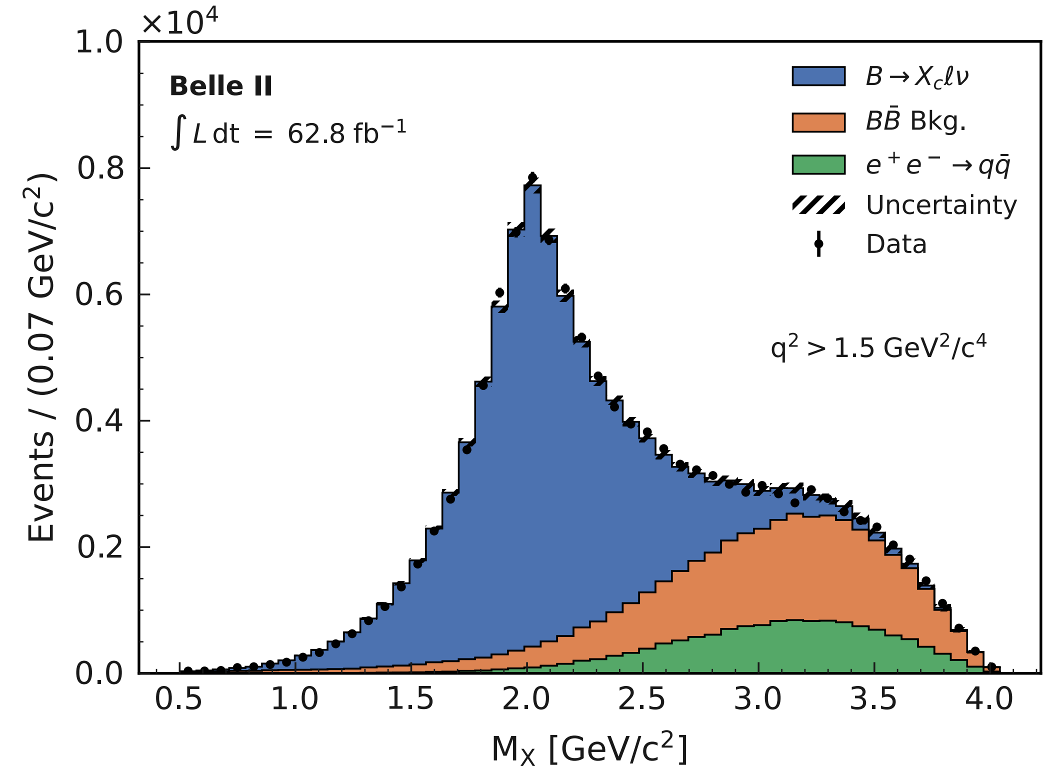
Key-technique: hadronic tagging

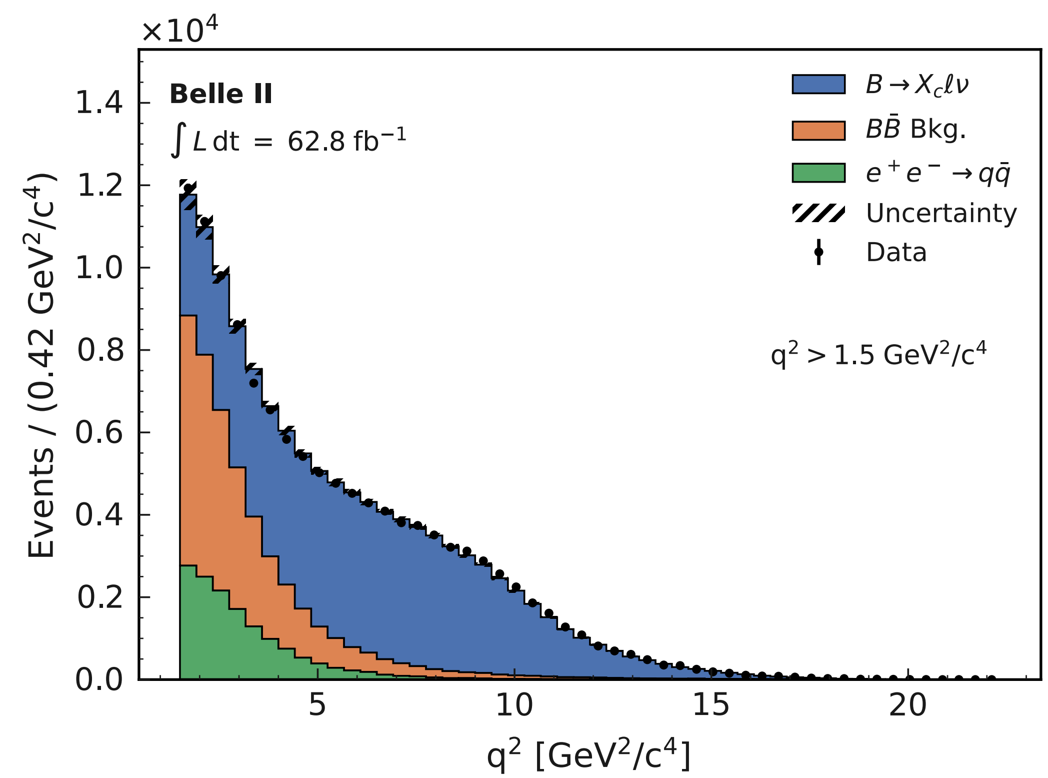


Can identify X_c **constituents**

$$M_X = \sqrt{(p_{X_c})_\mu (p_{X_c})^\mu}$$

$$q^2 = \left(p_{sig} - p_{X_c}\right)^2$$





Improved Hadronic Tagging using **Belle II** algorithm (ca. 2 times more efficient)

[Full Event Interpretation, T. Keck et al, Comp. Soft. Big. Sci 3 (2019), arXiv:1807.08680]

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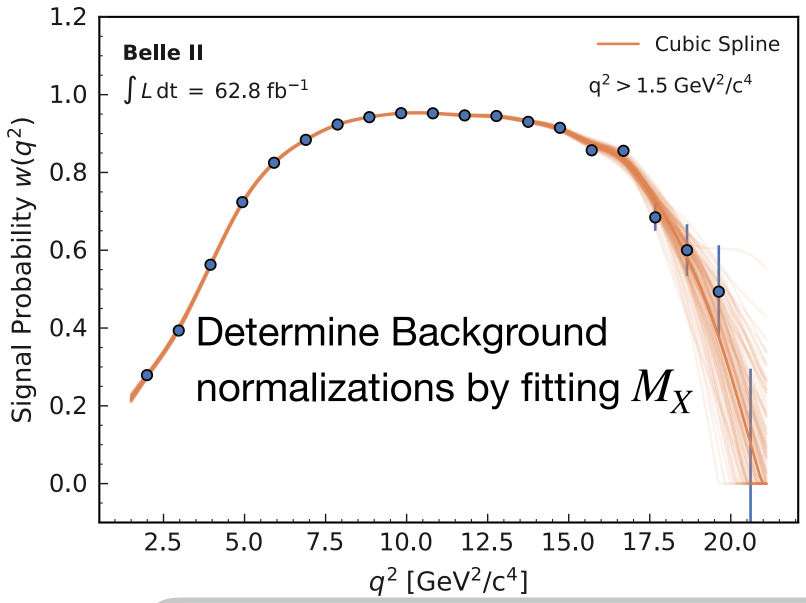

Step #1: Subtract Background

Event-wise **Master-formula**

$$\langle q^{2n} \rangle = \frac{\sum_i^{N_\mathrm{data}} w(q_i^2) \times q_{\mathrm{calib},i}^{2n}}{\sum_j^{N_\mathrm{data}} w(q_j^2)} \times \mathcal{C}_\mathrm{calib} \times \mathcal{C}_\mathrm{gen}$$

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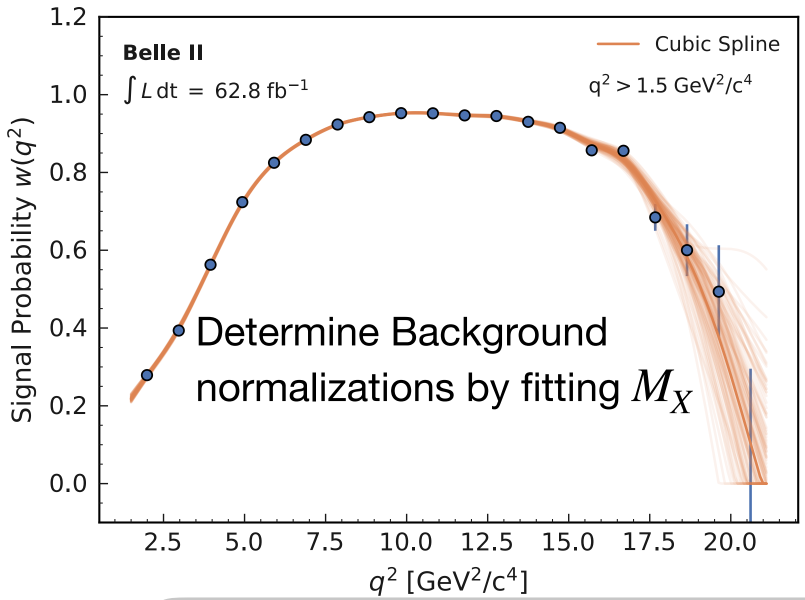

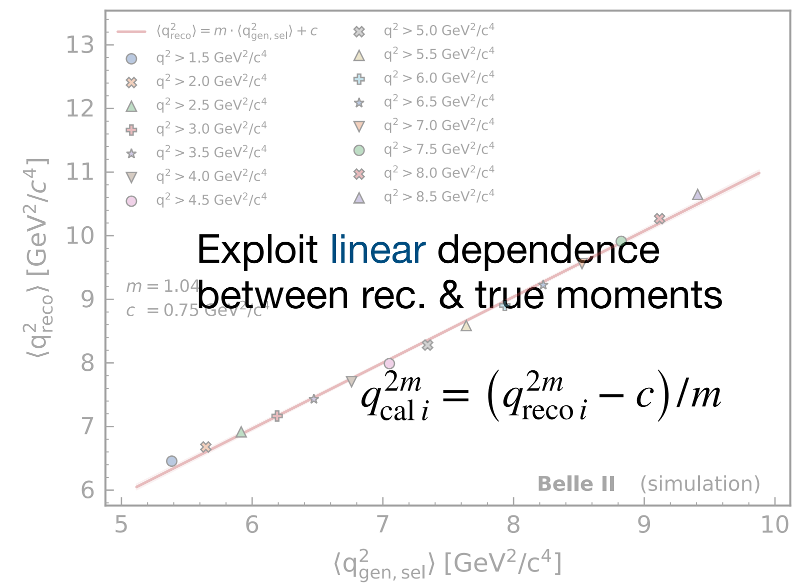

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Step #2: Calibrate moment

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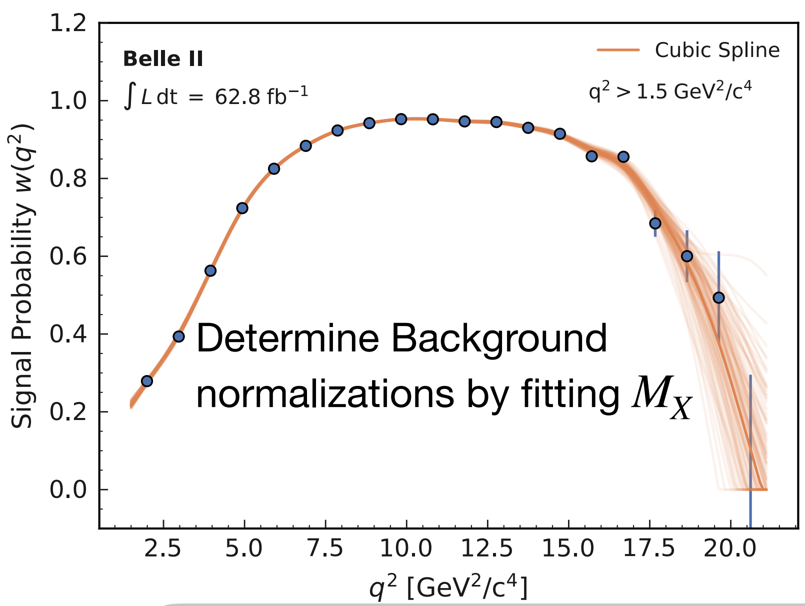

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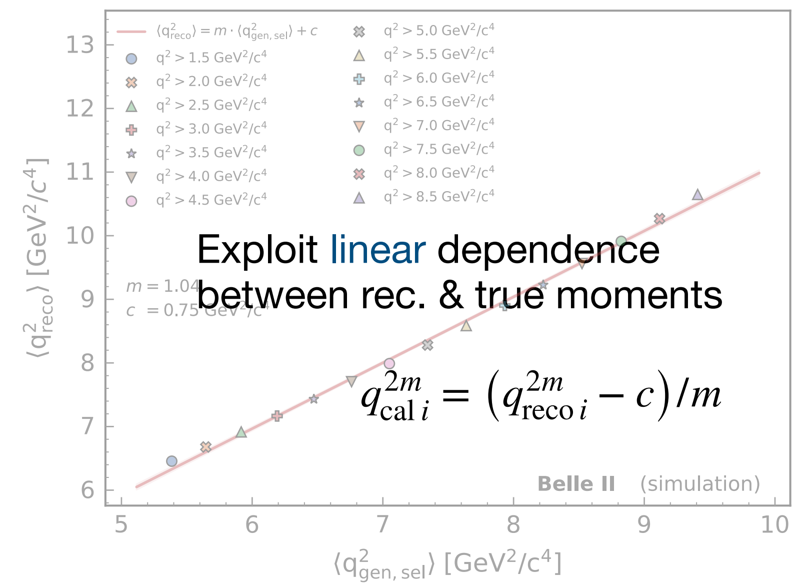

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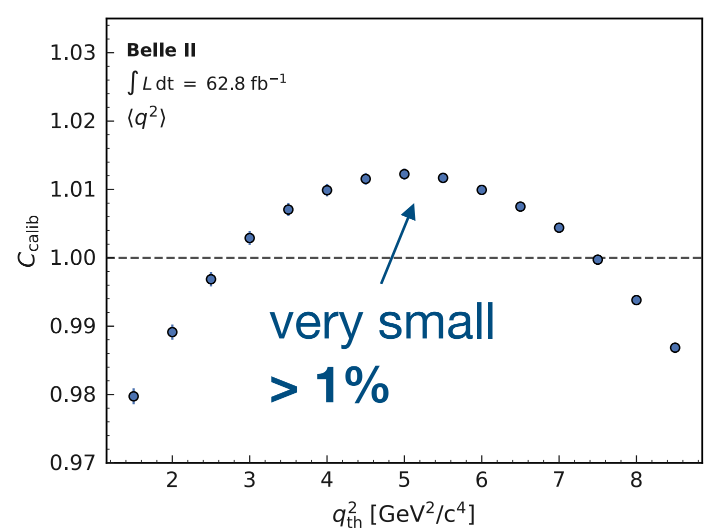
Step #3: If you fail, try again



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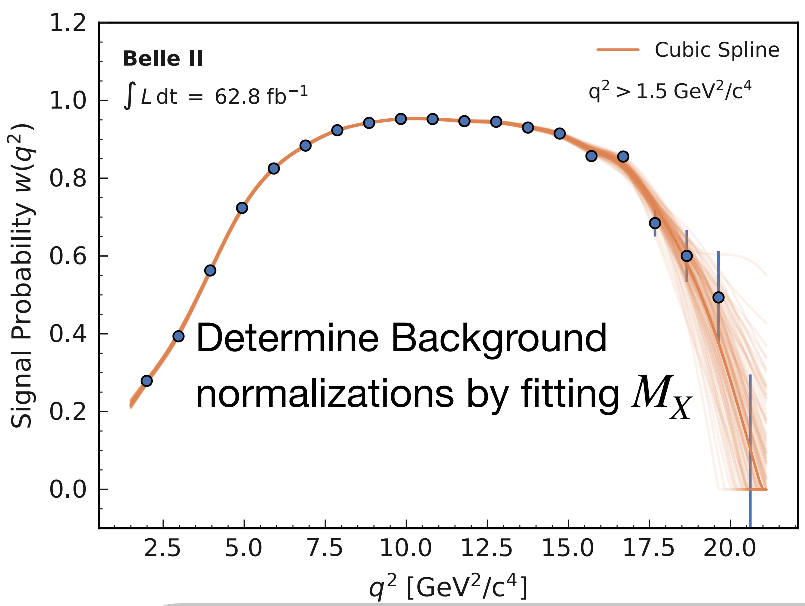
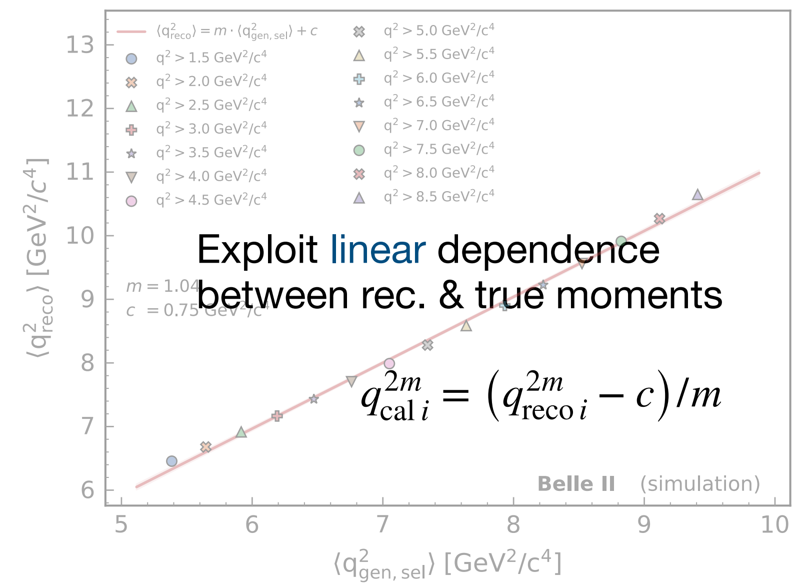


Determine Background normalizations by fitting M_X

Step #1: Subtract Background



Exploit linear dependence between rec. & true moments

$$q^{2m}_{\mathrm{cal}\,i} = \left(q^{2m}_{\mathrm{reco}\,i} - c\right)/m$$

Step #2: Calibrate moment

Event-wise **Master-formula**

$$\langle q^{2n} \rangle = \frac{\sum_i^{N_{\mathrm{data}}} w(q_i^2) \times q^{2n}_{\mathrm{calib},i}}{\sum_j^{N_{\mathrm{data}}} w(q_j^2)} \times \mathcal{C}_{\mathrm{calib}} \times \mathcal{C}_{\mathrm{gen}}$$

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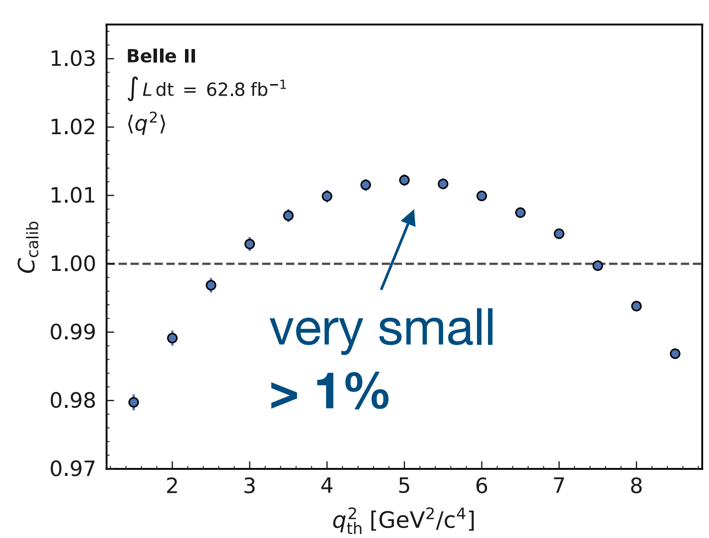
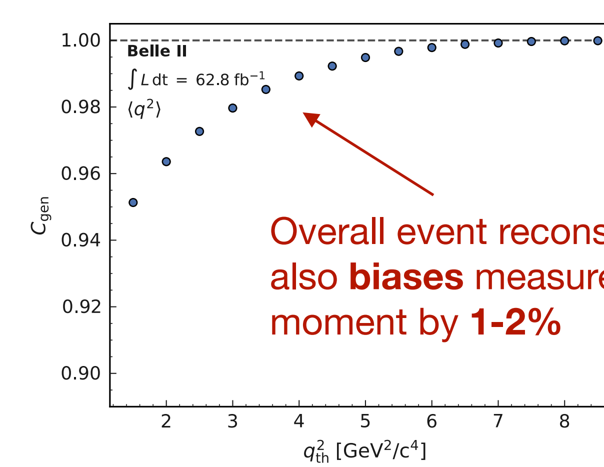
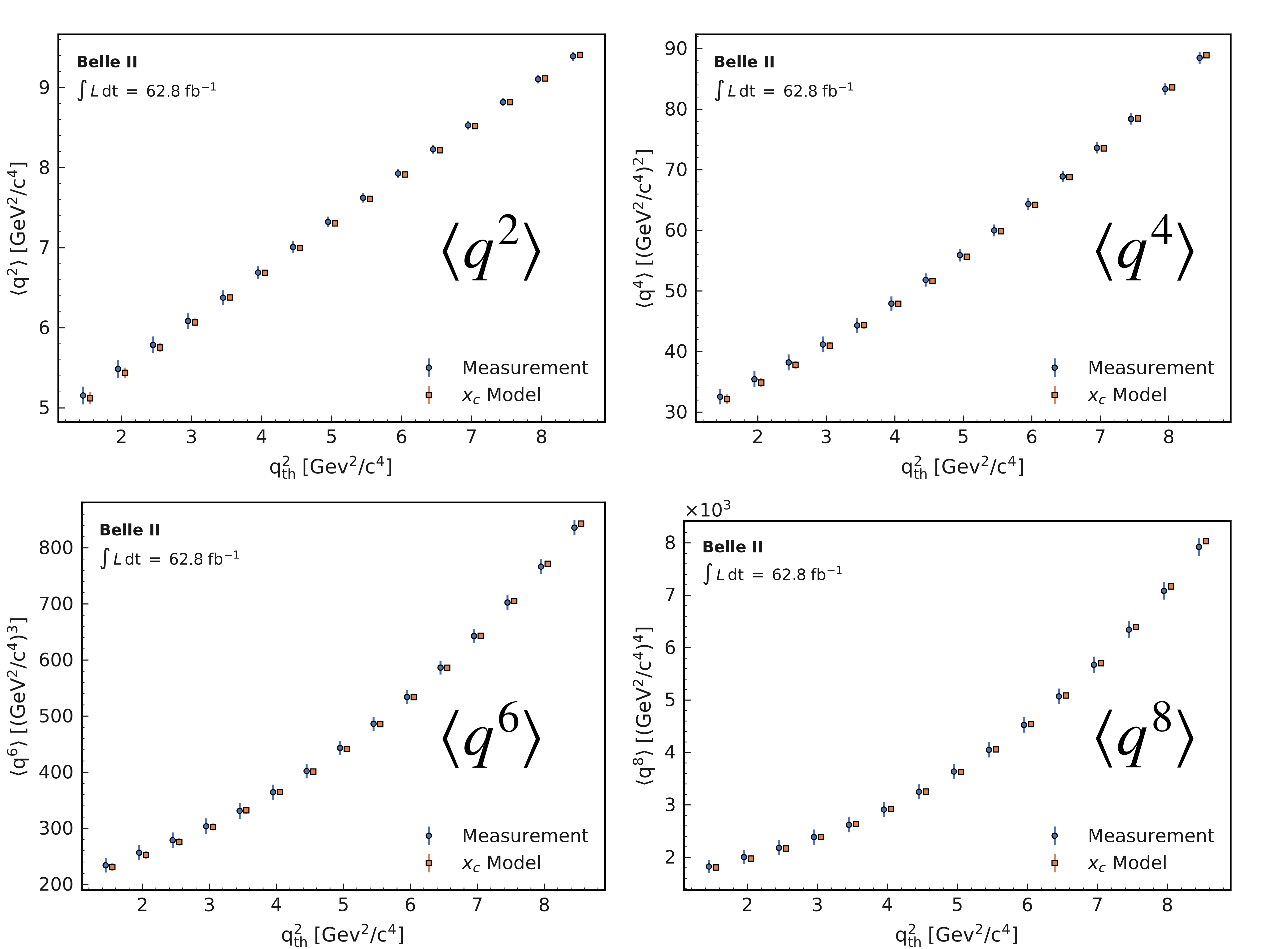


very small
> 1%

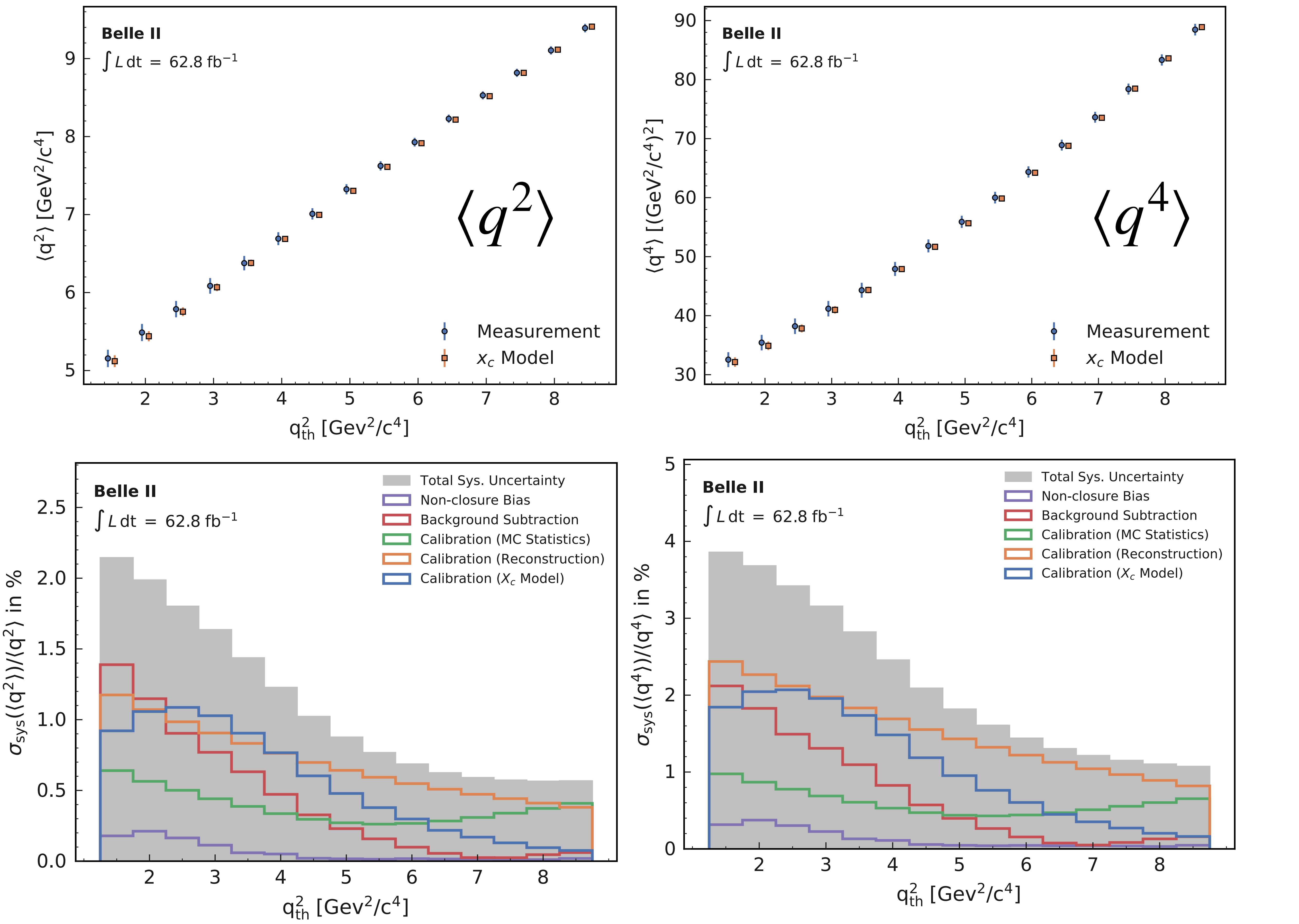
Step #4: Correct for selection effects



Overall event reconstruction itself also **biases** measured moment by **1-2%**



Belle II
$\int L\,dt = 62.8\ \text{fb}^{-1}$
$\langle q^2 \rangle$
$\langle q^2 \rangle$ [GeV^2/c^4]
q^2_{th} [Gev^2/c^4]
Measurement
x_c Model
$\langle q^4 \rangle$
$\langle q^4 \rangle$ [(GeV^2/c^4)^2]
$\langle q^6 \rangle$
$\langle q^6 \rangle$ [(GeV^2/c^4)^3]
$\langle q^8 \rangle$
$\langle q^8 \rangle$ [(GeV^2/c^4)^4]
$\times 10^3$



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Belle II
$\int L\,dt = 62.8\ \mathrm{fb}^{-1}$
Total Sys. Uncertainty
Non-closure Bias
Background Subtraction
Calibration (MC Statistics)
Calibration (Reconstruction)
Calibration (X_c Model)
$\sigma_{sys}(\langle q^2 \rangle)/\langle q^2 \rangle$ in %
q^2_{th} [Gev^2/c^4]
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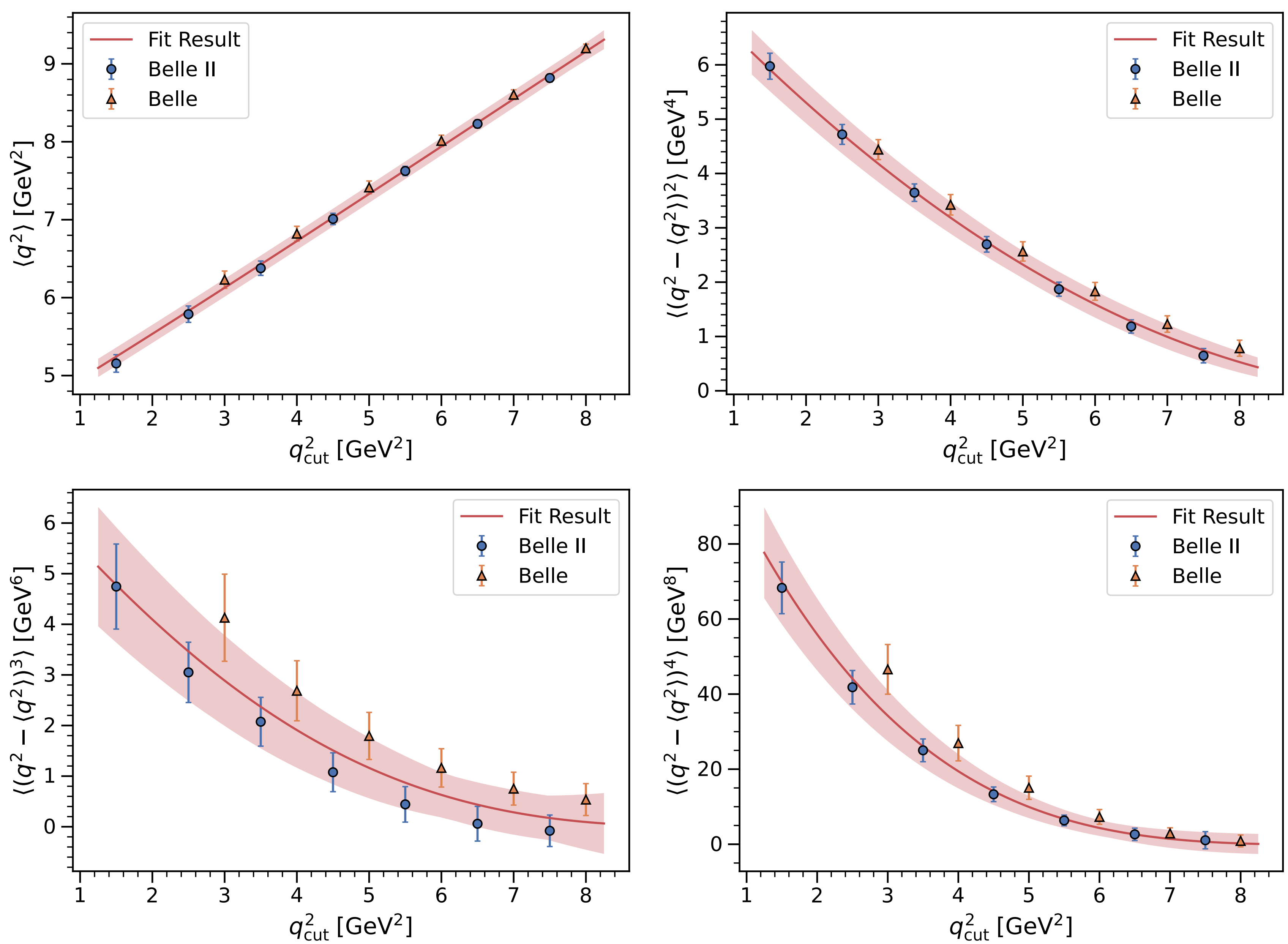
Total Sys. Uncertainty
Non-closure Bias
Background Subtraction
Calibration (MC Statistics)
Calibration (Reconstruction)
Calibration (X_c Model)
$\sigma_{sys}(\langle q^4 \rangle)/\langle q^4 \rangle$ in %
q^2_{th} [Gev^2/c^4]

$|V_{cb}|$ from q^2 mom.

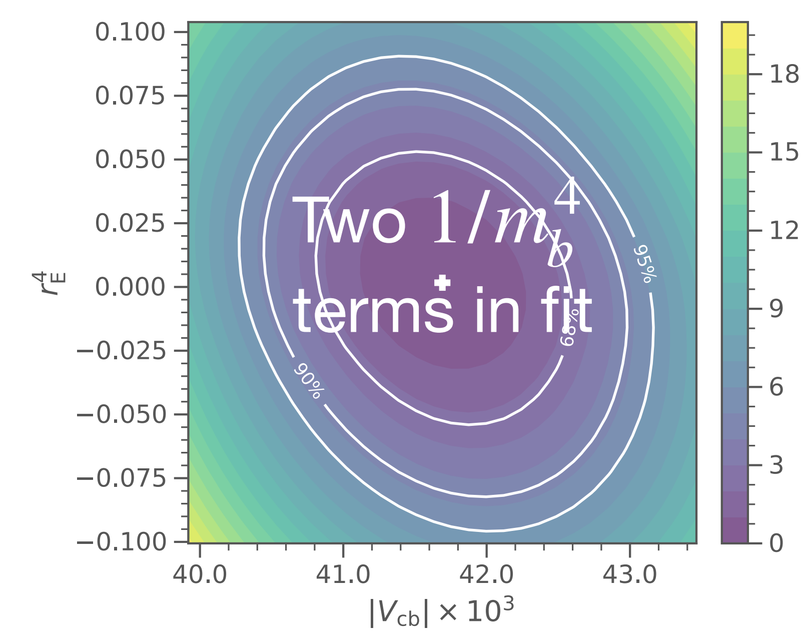
F. Bernlochner, M. Fael, K. Olschwesky, E. Persson,
R. Van Tonder, K. Vos, M. Welsch [JHEP 10 (2022) 068, arXiv:2205.10274]

First extraction of $|V_{cb}|$ from q^2 moments:



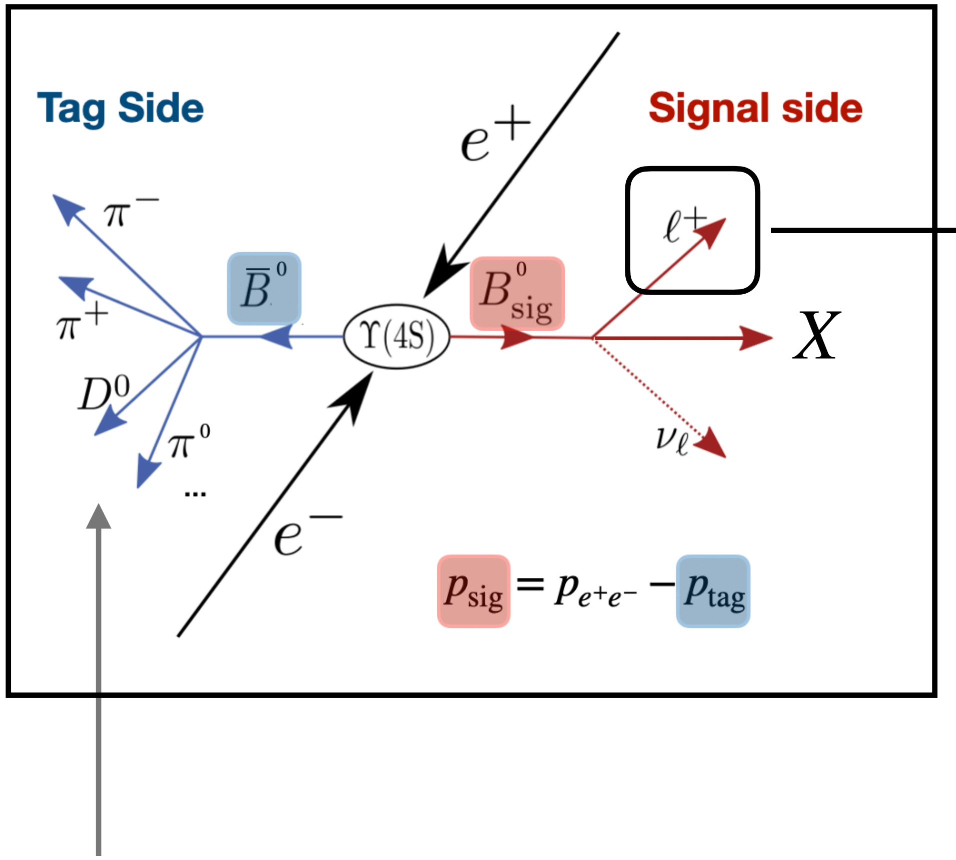
Included corrections
on the mom. predictions

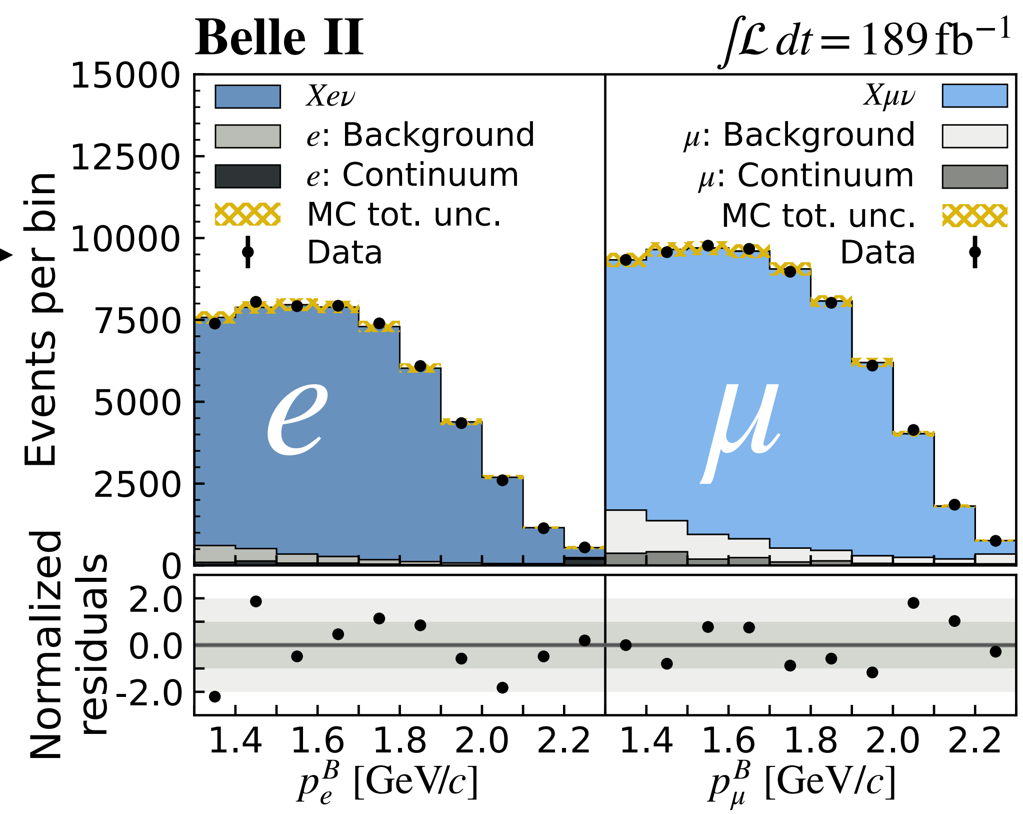
$\langle (q^2)^n \rangle$	tree	α_s	α_s^2	α_s^3
Partonic	✓	✓		
μ_G^2	✓	✓		
ρ_D^3	✓	✓		
$1/m_b^4$	✓			



$$\longrightarrow \quad |V_{cb}| = (41.69 \pm 0.59|_{\mathrm{fit}} \pm 0.23|_{\mathrm{h.o.}}) \cdot 10^{-3} = (41.69 \pm 0.63) \cdot 10^{-3}$$

2. A test of **light-lepton universality** in the rates of inclusive semileptonic B-meson decays at Belle II [Submitted to PRL, arXiv:XYZ]





Hadronic Tagging

Systematic Uncertainties:

Source	Uncertainty [%]
Sample size	0.9
Lepton identification	1.9
$X\,\ell\,\nu$ branching fractions	0.2
$X_c\,\ell\,\nu$ form factors	0.1
Total	2.1

$$R(X_{e/\mu} \mid p_\ell^B > 1.3\ \mathrm{GeV}/c) = 1.005 \pm 0.009\ (\mathrm{stat}) \pm 0.019\ (\mathrm{syst}).$$

$$R(X_{e/\mu}) = 1.007 \pm 0.009\ (\mathrm{stat}) \pm 0.019\ (\mathrm{syst}),$$

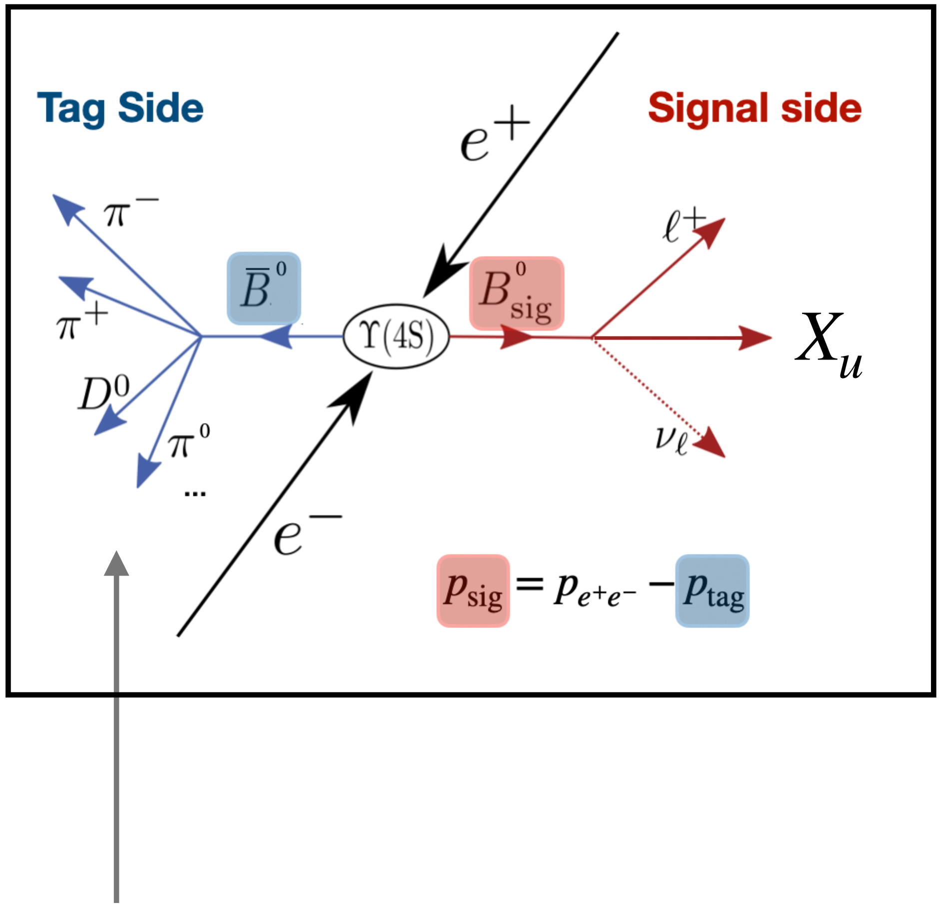
$$R(X_{e/\mu})_{\mathrm{SM}} = 1.006 \pm 0.001$$

M. Rahimi and K. K. Vos, J. High Energ. Phys. 11, 007 (2022).

3.

First **Simultaneous** Determination of Inclusive and Exclusive $|V_{ub}|$

[Submitted to PRL, arXiv:2205.06372]



Belle I **Hadronic Tagging (FR)**

ca. factor of 2 less efficient,
but focus on cleaner tags

Hadronic **tagging** just is **fun**:
Capability to identify **kinematic** and **constituents** of X_u **system**

Charged Tracks — Neutral Clusters

$$p_X = \sum_i \left(\sqrt{m_\pi^2 + |\mathbf{p}_i|^2}, \mathbf{p}_i \right) + \sum_j (E_j, \mathbf{k}_j)$$

$$q^2 = (p_{\rm sig} - p_X)^2$$

$$M_X = \sqrt{(p_X)^\mu (p_X)_\mu}$$

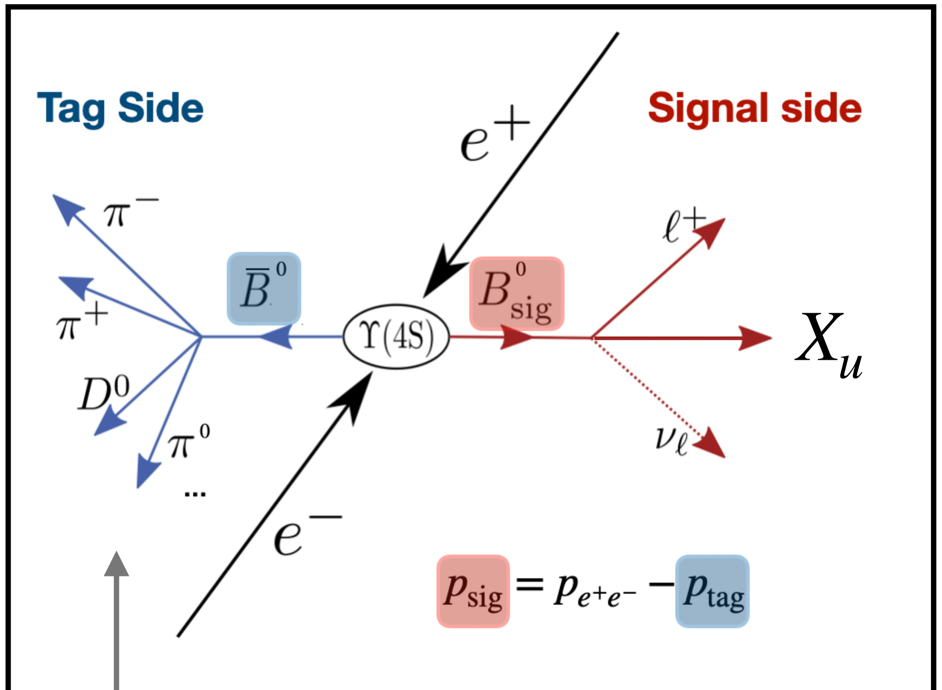
$$m_{\rm miss}^2 = \left(p_{\rm sig} - p_X - p_\ell\right)^2 \approx m_\nu^2 = 0\,{\rm GeV}^2$$

But … this is still a pretty difficult measurement

3.

First **Simultaneous** Determination of Inclusive and Exclusive $|V_{ub}|$

[Submitted to PRL, arXiv:2205.06372]

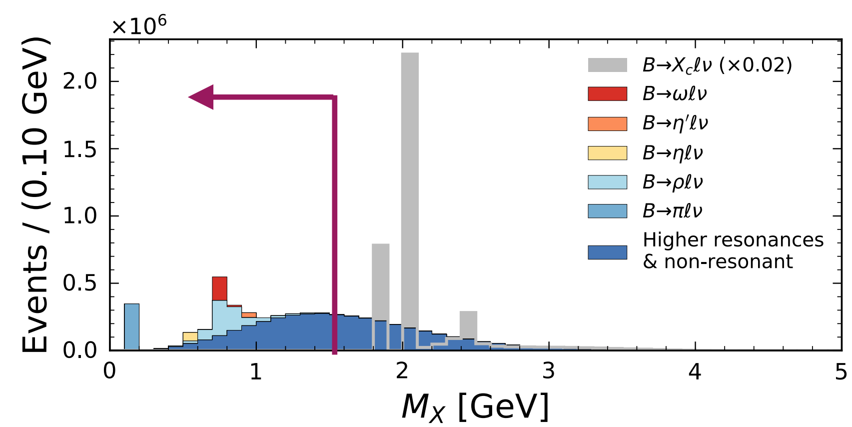


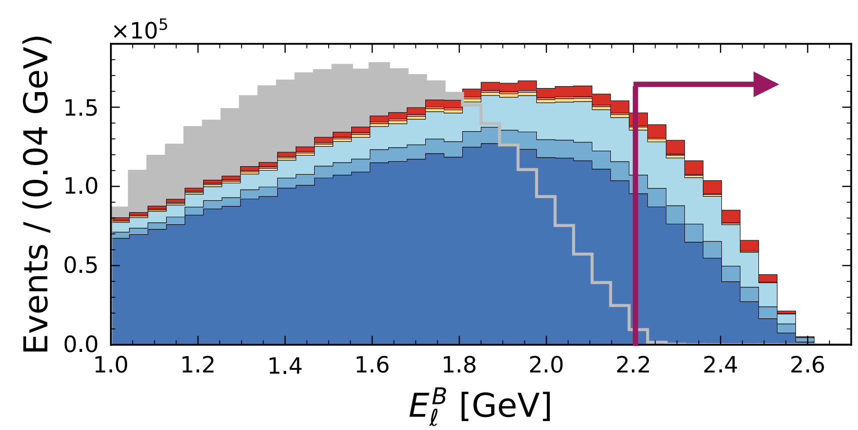
Belle I **Hadronic Tagging (FR)**

ca. factor of 2 less efficient,
but focus on cleaner tags

Inclusive $B \to X_u \ell \bar{\nu}_\ell$ measurements are extremely challenging due to dominant $B \to X_c \ell \bar{\nu}_\ell$ background

Clean **separation** only **possible** in certain **kinematic regions**, e.g. **lepton endpoint** or **low** M_X





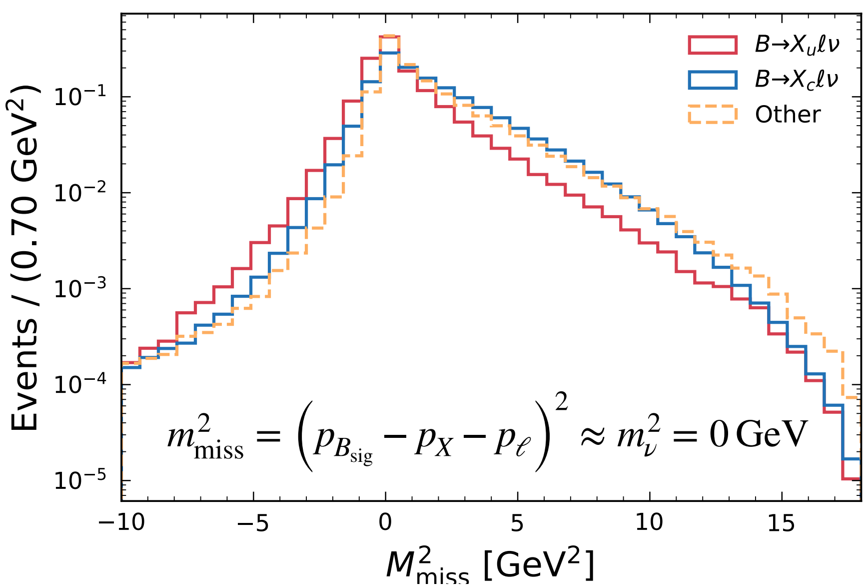
Multivariate Sledgehammer

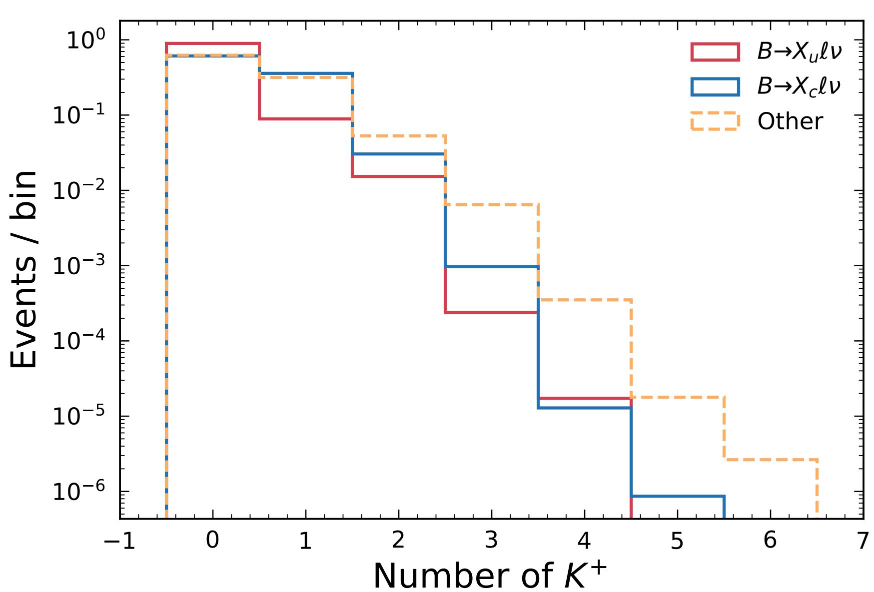
Can exploit that there are differences:

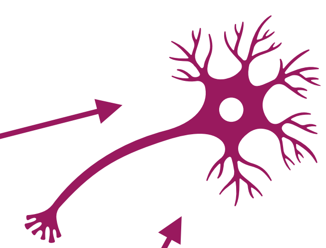
X_u ⟷ X_c

Higher multiplicity
Often come with charged and neutral **Kaons**
D* decays (slow pions)
(Slightly lower E_ℓ)

Direct cuts on m_X, E_ℓ **problematic** (i.e. direct theory / shape-function dependence)

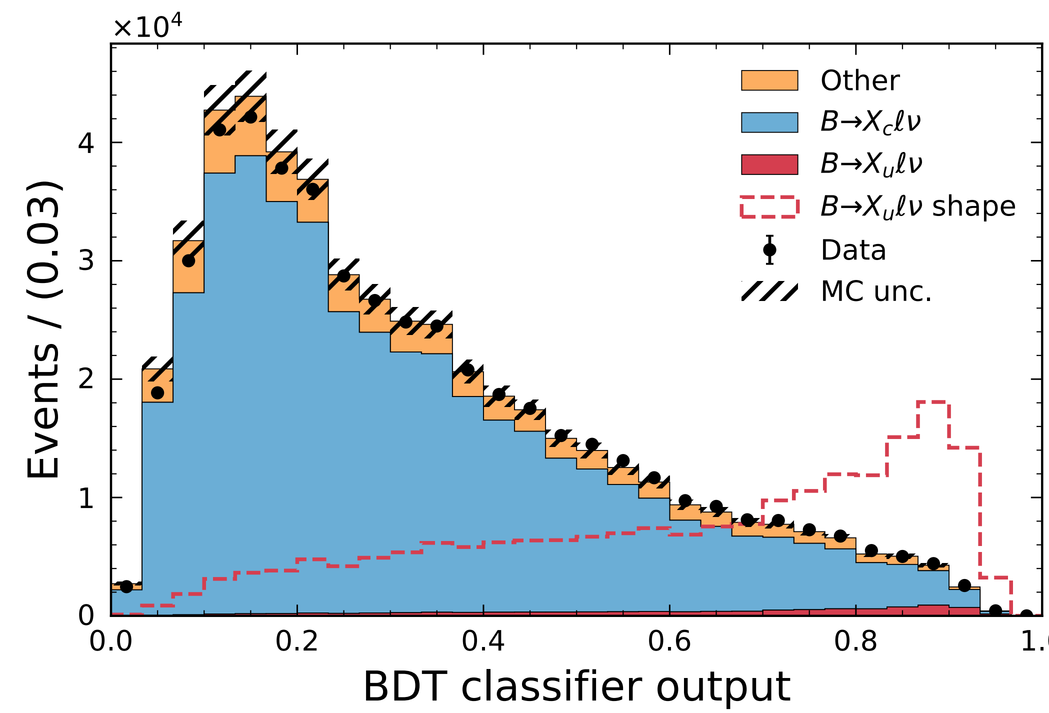






+ 9 other variables

=



Can reject **98.7%** of X_c

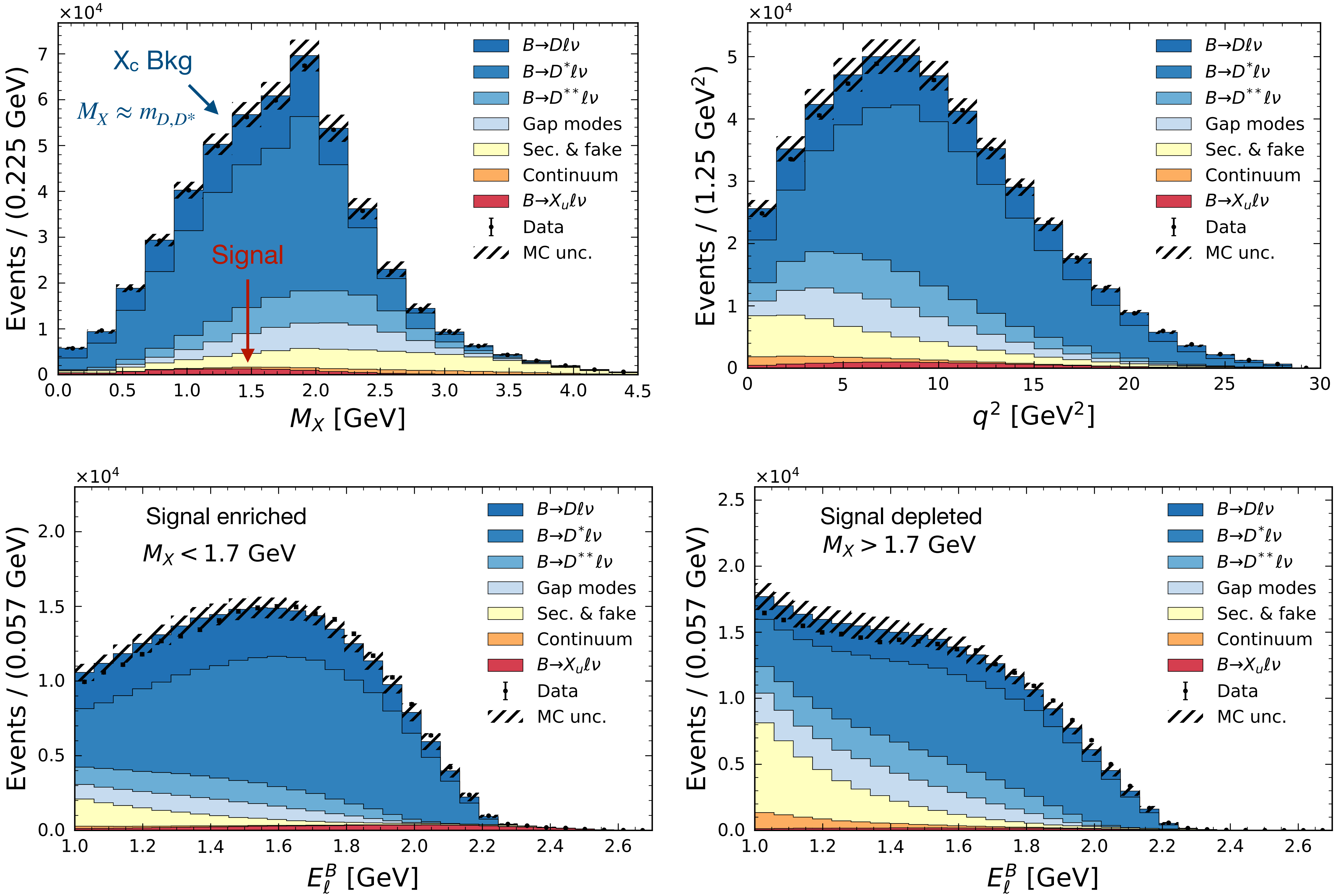
Selection	$B \to X_u\,\ell^+\,\nu_\ell$	$B \to X_c\,\ell^+\,\nu_\ell$	Data
$M_{\rm bc} > 5.27\,{\rm GeV}$	84.8%	83.8%	80.2%
$\mathcal{O}_{\rm BDT} > 0.85$	18.5%	1.3%	1.6%
$\mathcal{O}_{\rm BDT} > 0.83$	21.9%	1.7%	2.1%
$\mathcal{O}_{\rm BDT} > 0.87$	14.5%	0.9%	1.1%

... and retain **18.5%** of X_u

Before BDT selection

Hadronic Mass $M_X = \sqrt{p_X^2}$

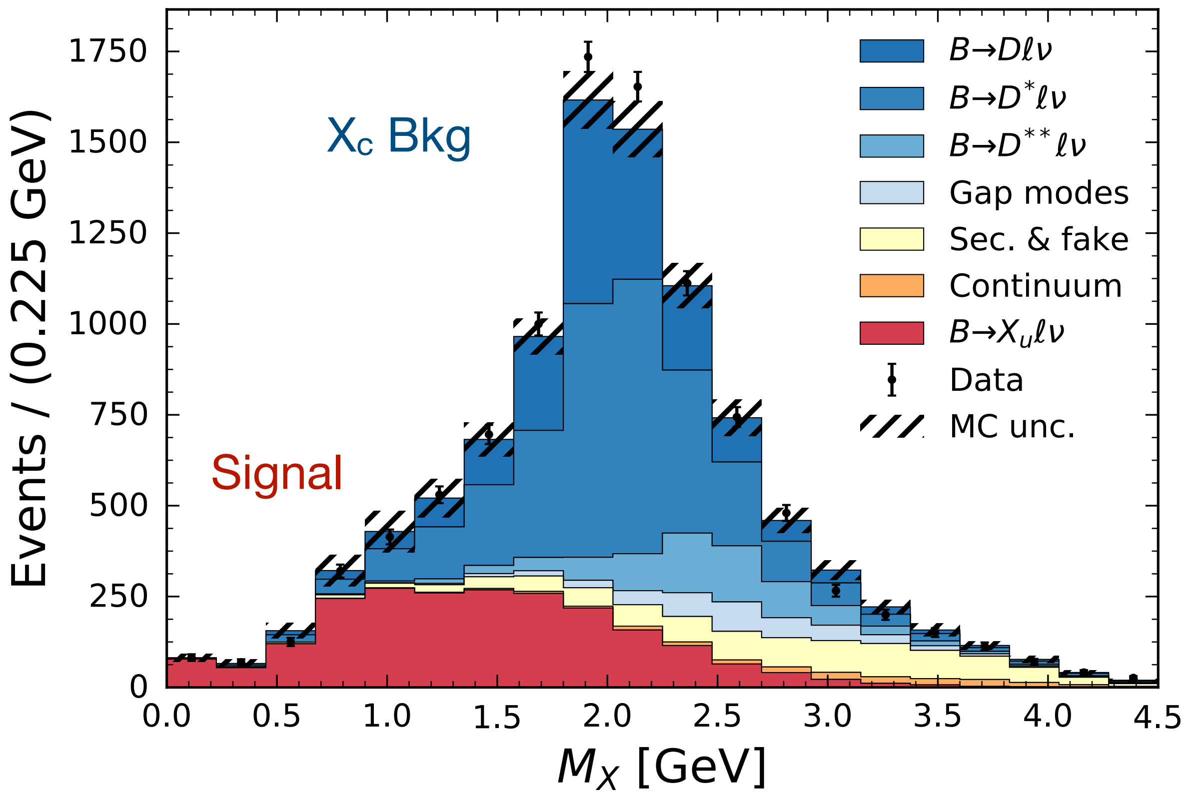
Four-momentum transfer squared $q^2 = (p_B - p_X)^2$



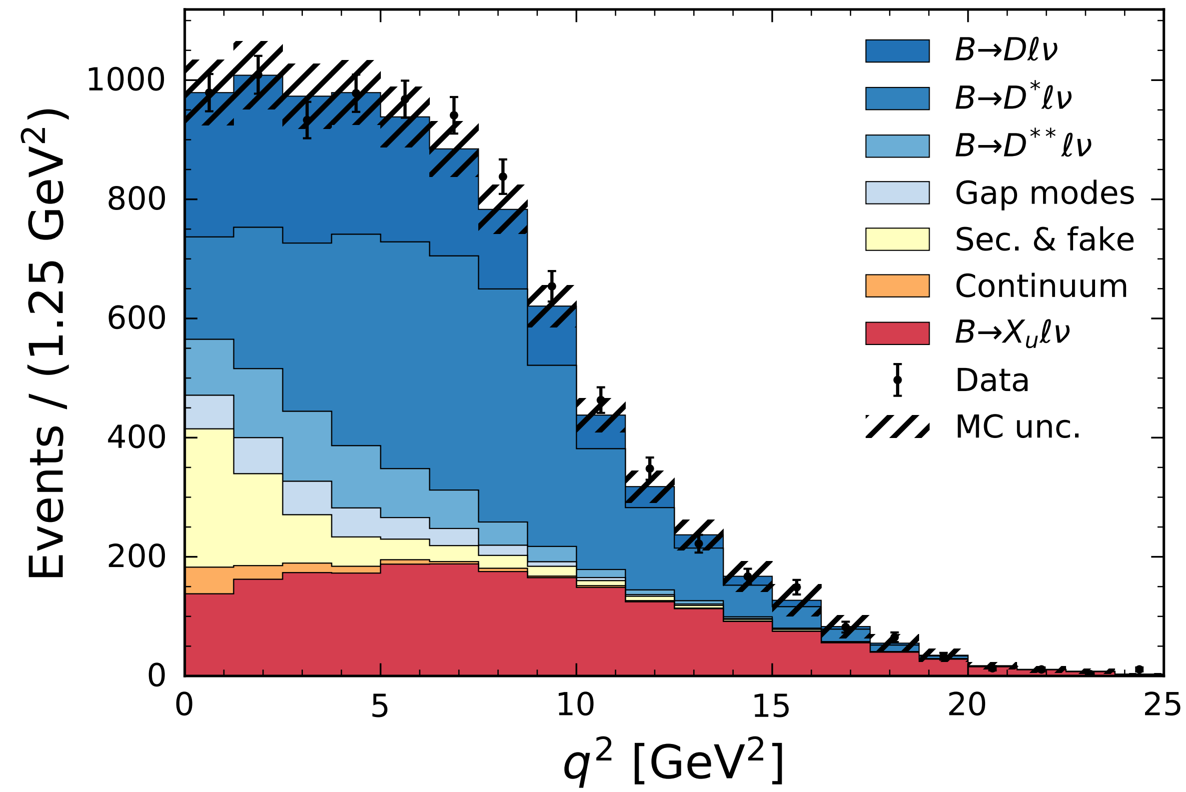
Lepton Energy in signal B rest frame E_ℓ^B

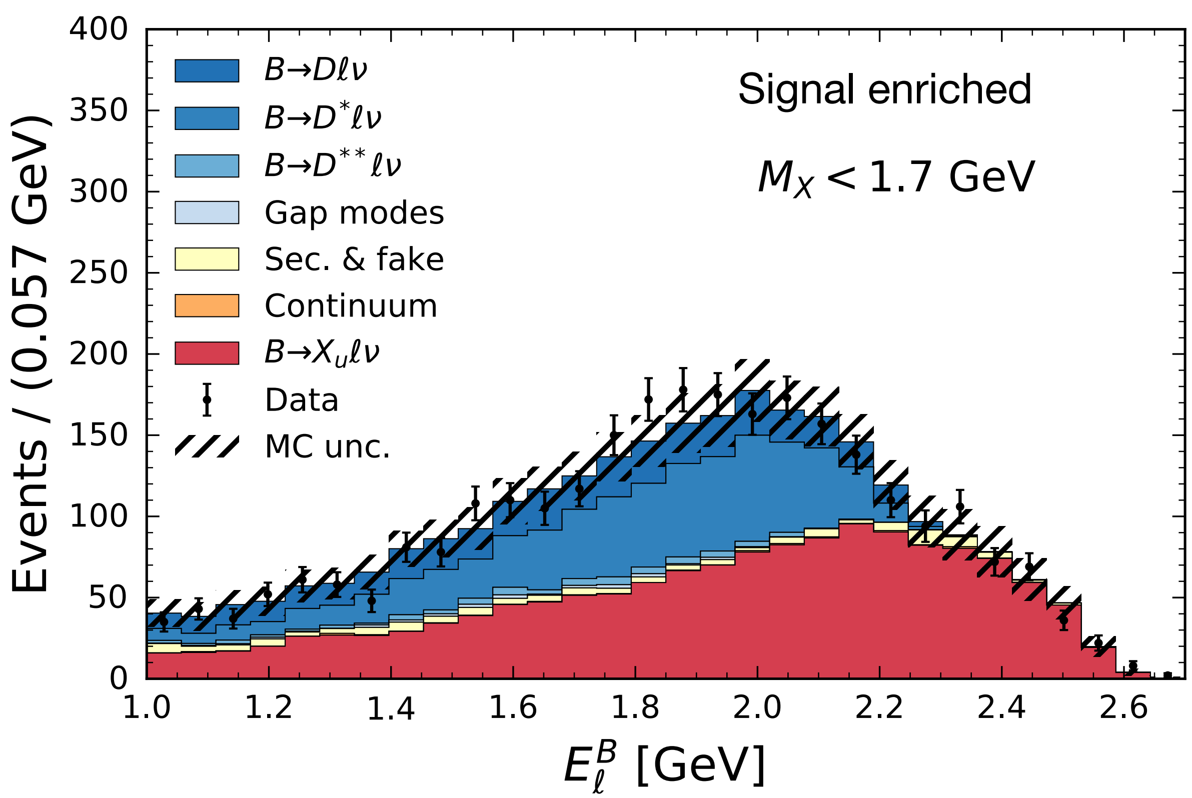
After BDT selection

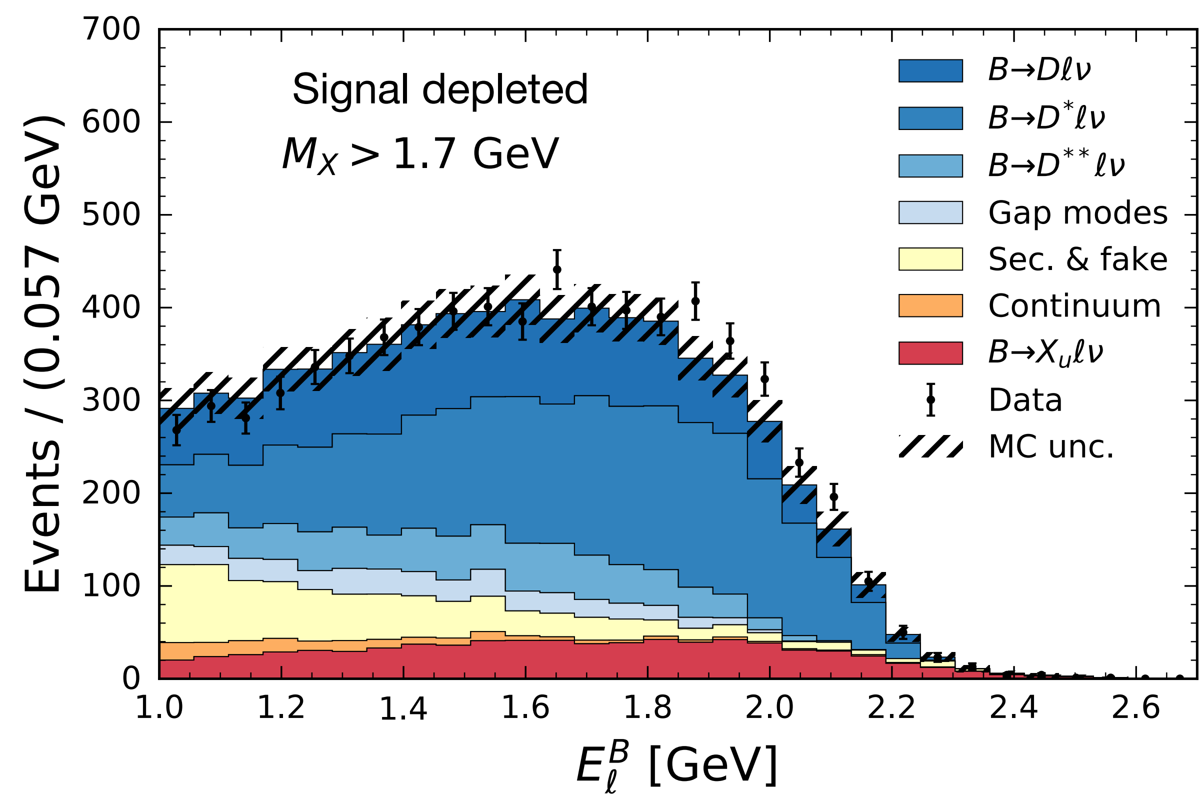
Hadronic Mass $M_X = \sqrt{p_X^2}$



Four-momentum transfer squared $q^2 = (p_B - p_X)^2$





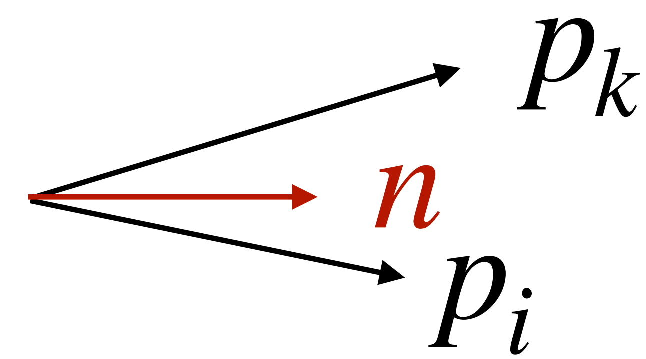


Lepton Energy in signal B restframe E_ℓ^B

First **Simultaneous** Determination of Inclusive and Exclusive $|V_{ub}|$

[Submitted to PRL, arXiv:2205.06372]

New Idea: Exploit that exclusive X_u final states can be separated using the # of charged pions



X_u:

- $n_{\pi^+} = 0 :$ $B \to \pi^0 \ell \bar{\nu}_\ell$
- $n_{\pi^+} = 1 :$ $B \to \pi^+ \ell \bar{\nu}_\ell$
- $n_{\pi^+} = 2 :$ other $B \to X_u \ell \bar{\nu}_\ell$
- $n_{\pi^+} \geq 3 :$

Use **'thrust'**, expect more collimated system for $B \to \pi^0 \ell \bar{\nu}_\ell$ and $B \to \pi^+ \ell \bar{\nu}_\ell$ than for other proceses

$$\max_{|\mathbf{n}|=1} \left(\sum_i |\mathbf{p_i} \cdot \mathbf{n}| / \sum_i |\mathbf{p_i}| \right)$$

q^2

Extraction of **BFs** and $B \to \pi$ **form factors**, in 2D fit of $q^2 : n_{\pi^+}$

M_X

Use high M_X to constrain $B \to X_c \ell \bar{\nu}_\ell$

3. First **Simultaneous** Determination of Inclusive and Exclusive $|V_{ub}|$

[Submitted to PRL, arXiv:2205.06372]

2D Categories :

For fit link

$B \to \pi^0 \ell \bar{\nu}_\ell$

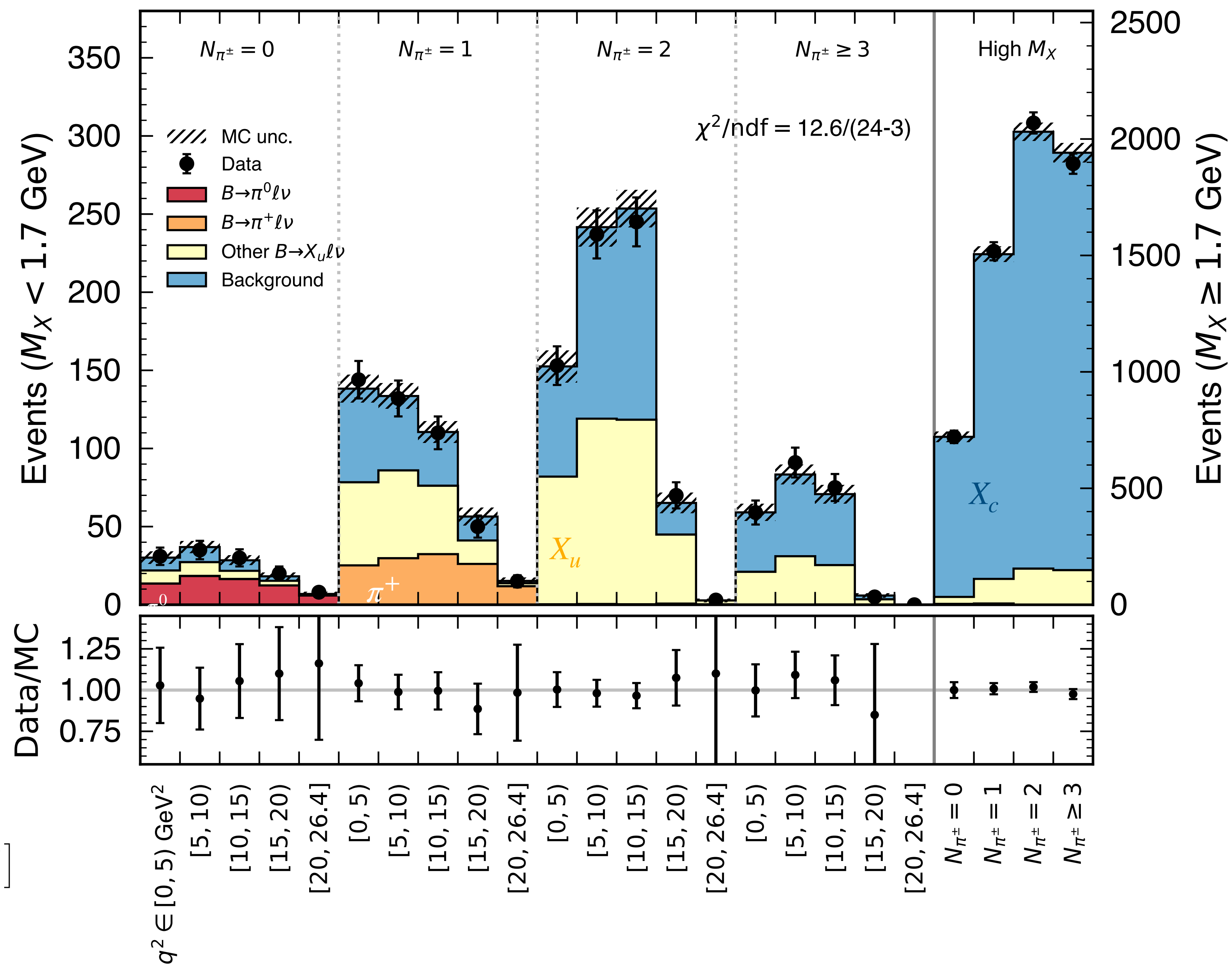
$B \to \pi^+ \ell \bar{\nu}_\ell$

assuming **isospin**

Float BCL $B \to \pi$ FF constrained to **FLAG 2022** WA [Eur.Phys.J.C 82 (2022) 10, 869]

$$f_+(q^2) = \frac{1}{1 - q^2/m_{B^*}^2} \sum_{n=0}^{N^+-1} a_n^+ \left[z^n - (-1)^{n-N^+} \frac{n}{N^+} z^{N^+} \right]$$

$$f_0(q^2) = \sum_{n=0}^{N^0-1} a_n^0 z^n , \quad (3)$$



$$\rightarrow \quad \mathcal{B}(\overline{B}^0 \to \pi^+ \ell^- \bar{\nu}_\ell) = (1.43 \pm 0.19 \pm 0.13) \times 10^{-4} ,$$
$$\Delta\mathcal{B}(B \to X_u \ell \bar{\nu}_\ell) = (1.40 \pm 0.14 \pm 0.23) \times 10^{-3} ,$$

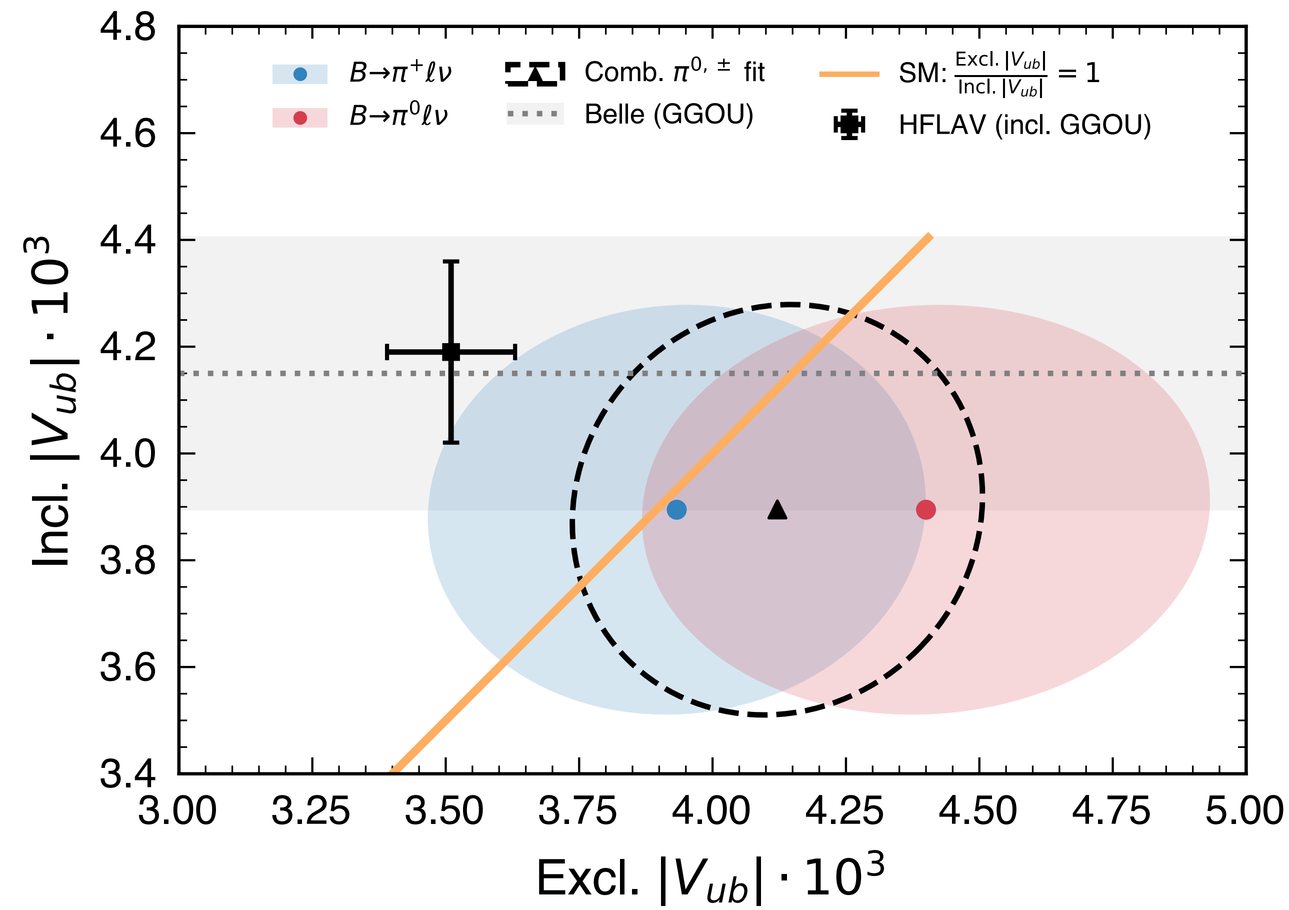
$\rho = 0.10$

(Note that $B \to X_u \ell \bar{\nu}_\ell$ of course contains $B \to \pi \ell \bar{\nu}_\ell$)

Two sets of results:

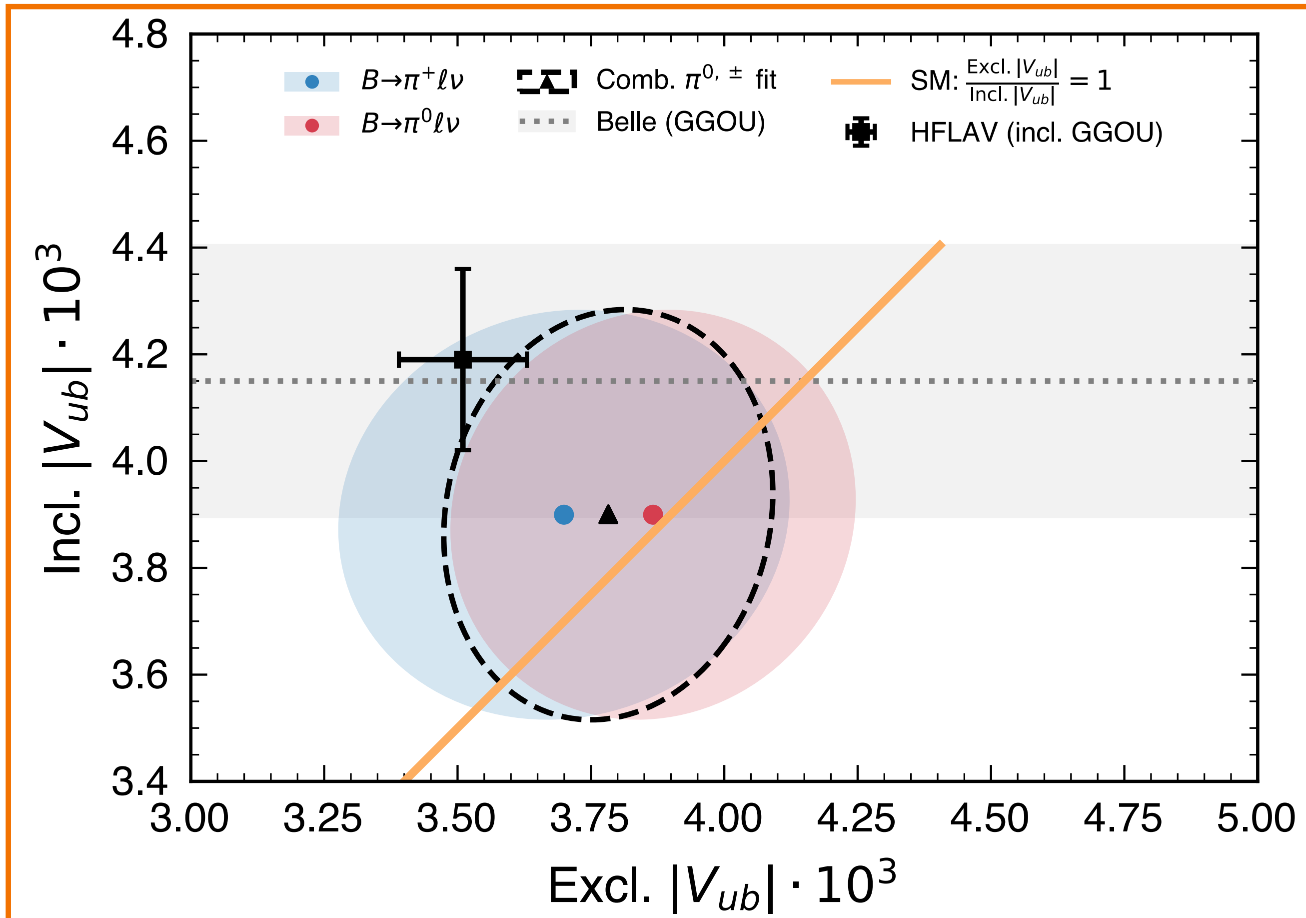
1) FLAG 2022

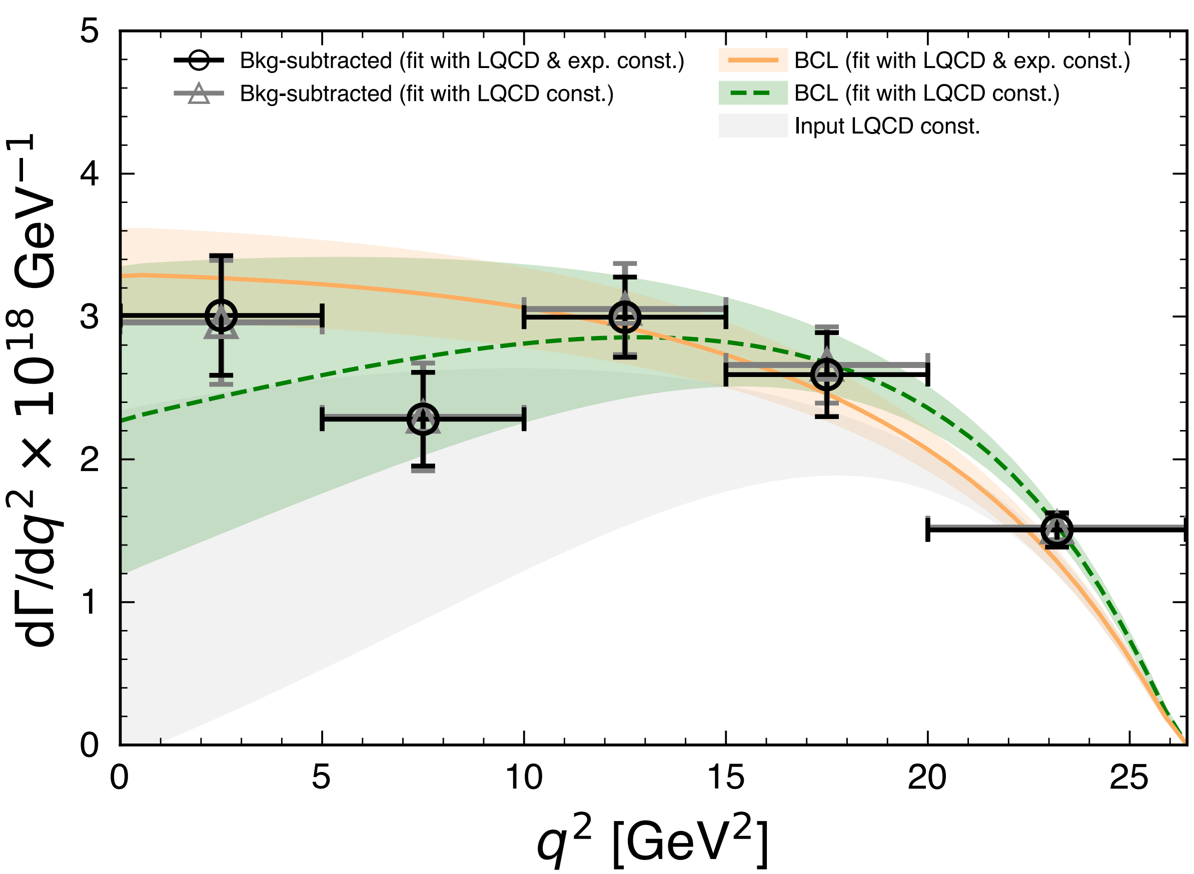
$$\left|V_{ub}^{\text{excl.}}\right| / \left|V_{ub}^{\text{incl.}}\right| = 1.06 \pm 0.14\,,$$



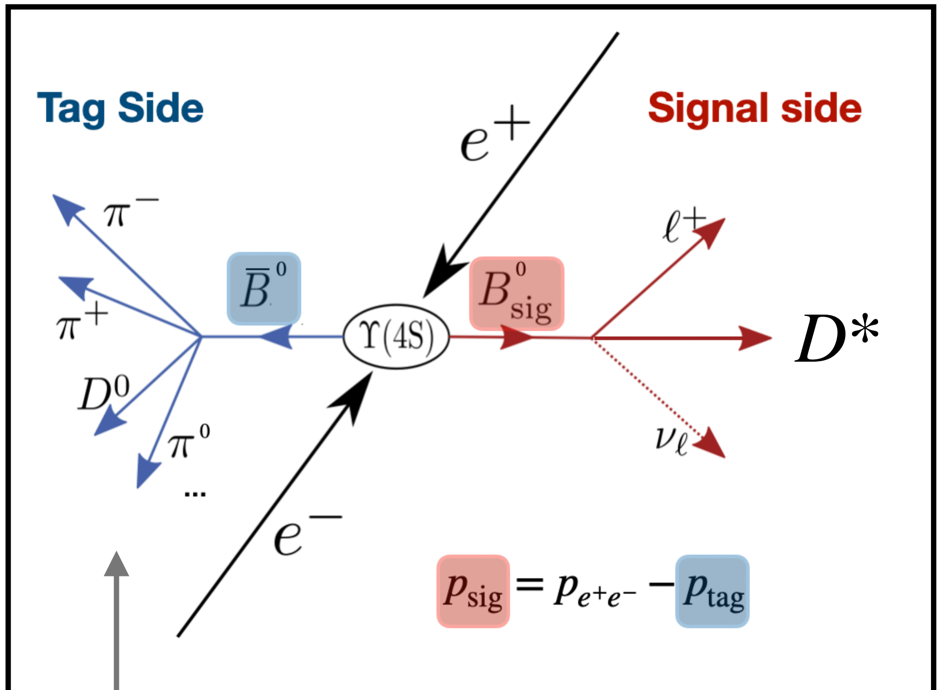
2) FLAG 2022 + all experimental information on $B \to \pi$ FF

$$\left|V_{ub}^{\text{excl.}}\right| / \left|V_{ub}^{\text{incl.}}\right| = 0.97 \pm 0.12\,,$$





4. Measurement of **Differential Distributions** of $B \to D^* \ell \bar{\nu}_\ell$ and Implications on $|V_{cb}|$, **[Accepted by PRD]**, [arXiv:2301.07529], **Available on HepData**

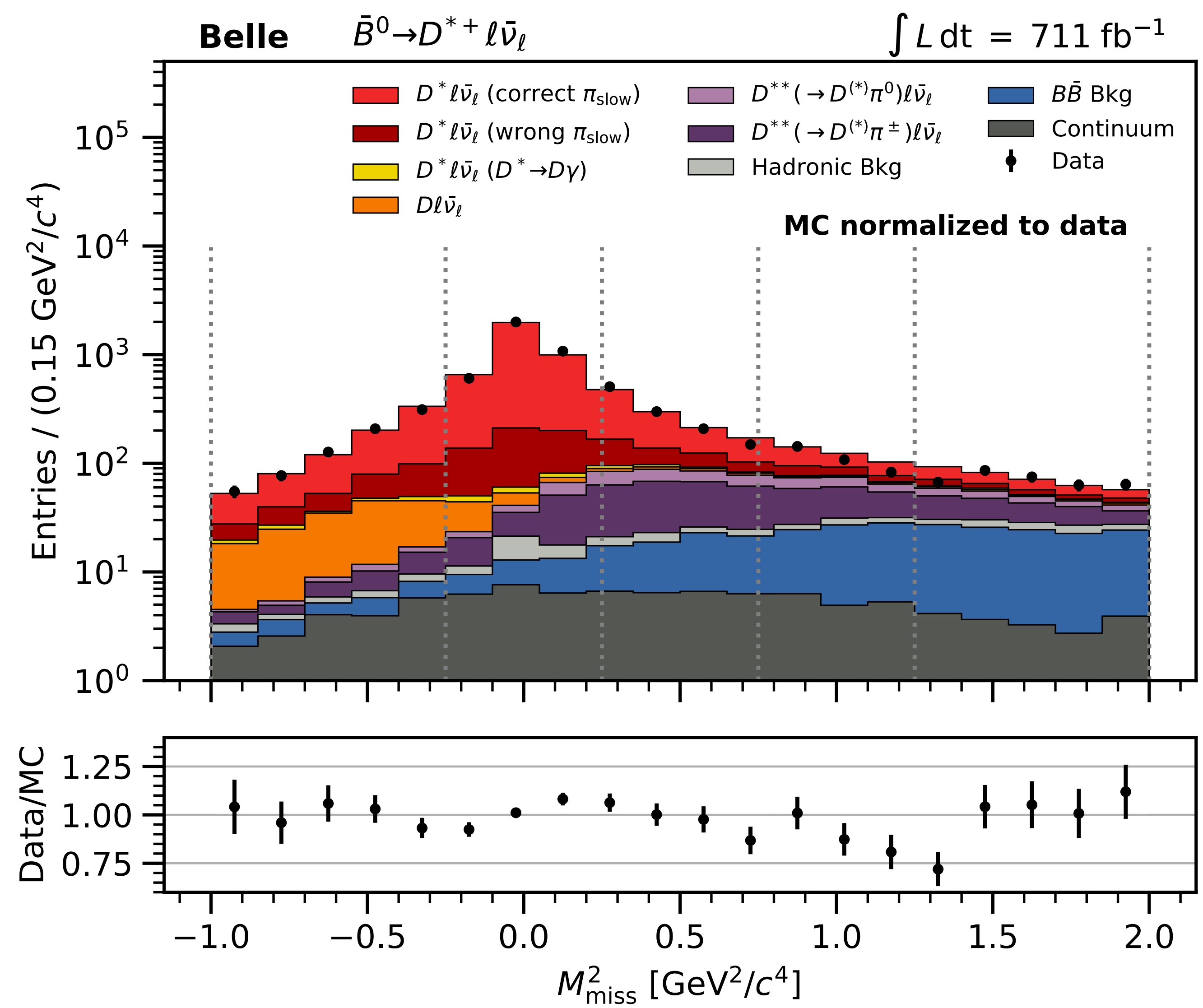


Belle II **Hadronic Tagging (FEI) applied to Belle data**

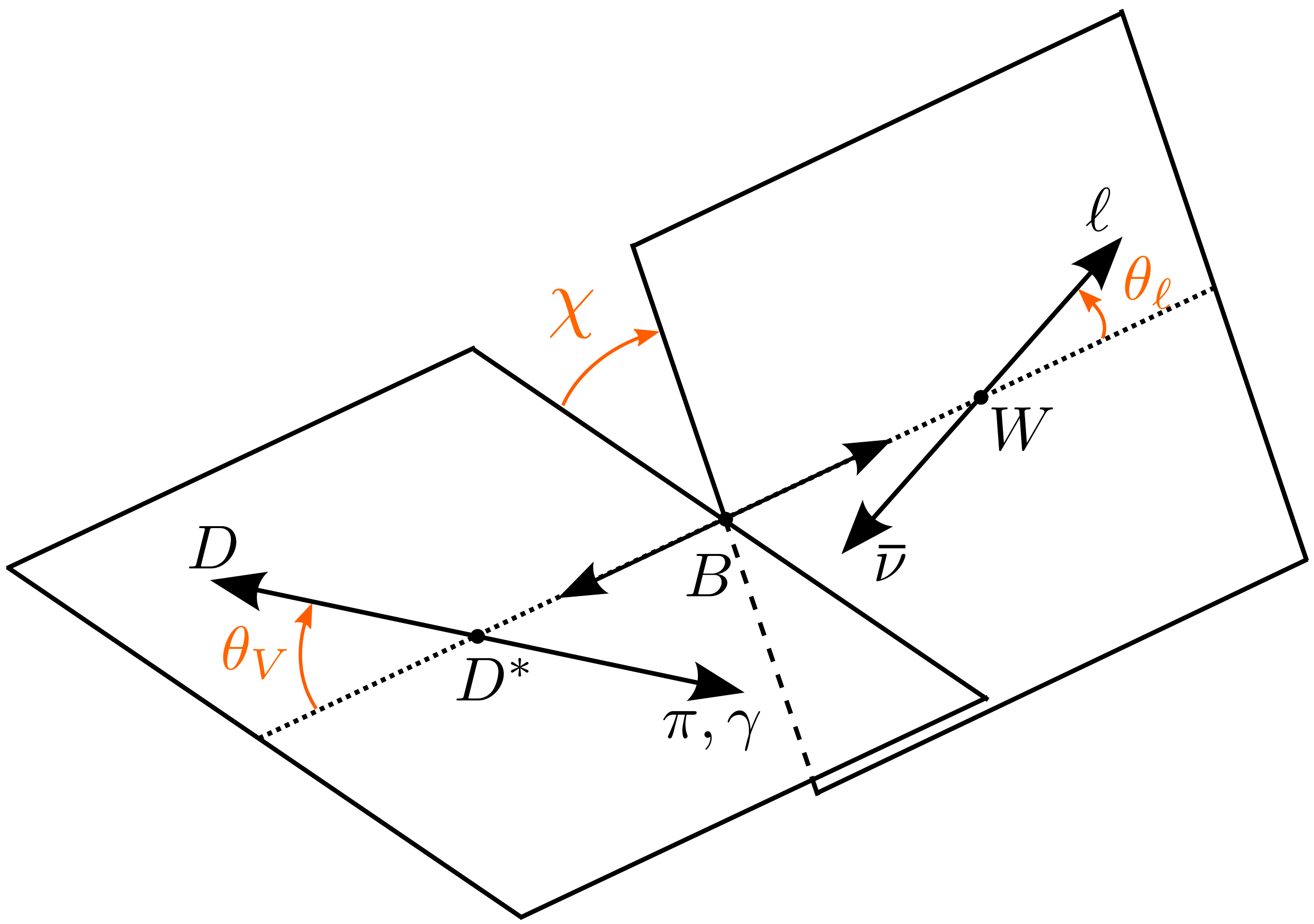
Target $B^\pm$ and $B^0/\overline{B}^0$ and decays with **slow pions**

Very clean sample; signal extraction **using**

$$M^2_{\rm miss} = \left(p_{e^+e^-} - p_{B_{\rm tag}} - p_{D^*} - p_\ell\right)^2$$



Focus on **1D projections** of recoil parameter and decay angles:

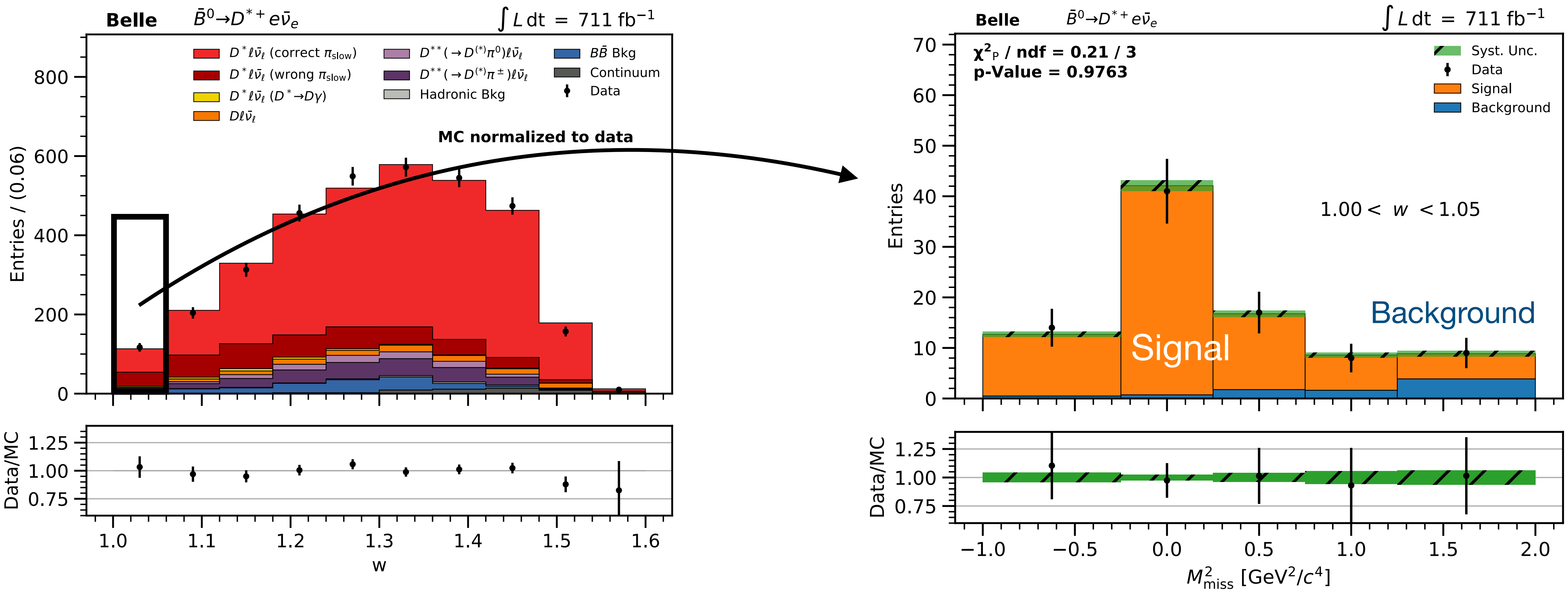


$+ \, w/q^2$

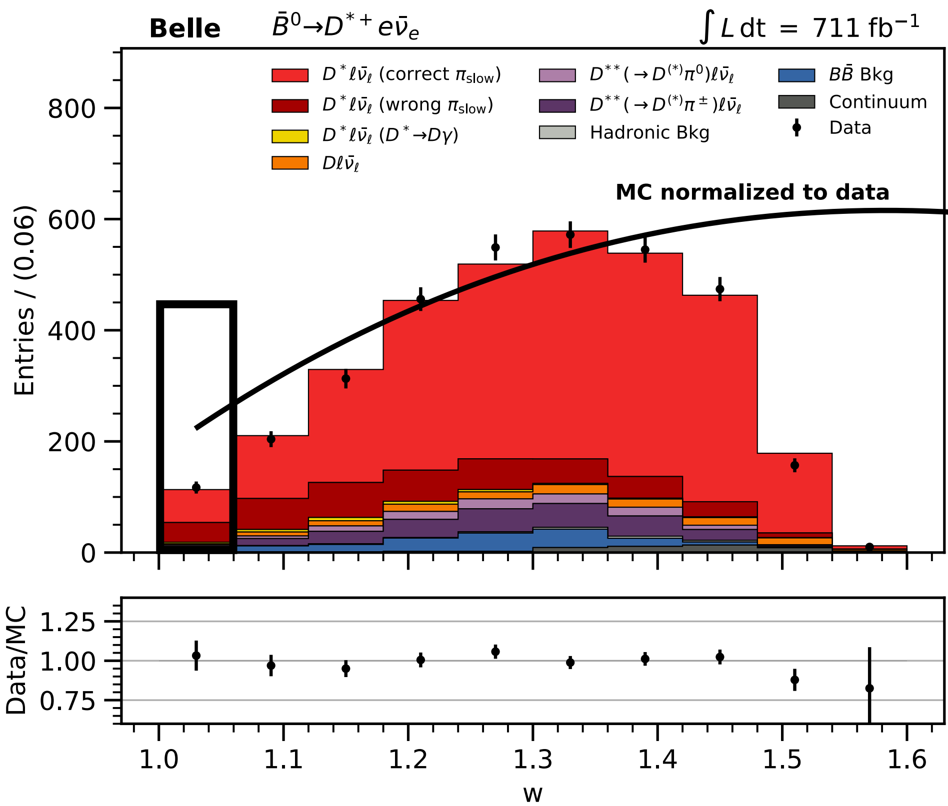
Provide full experimental covariance matrix for simultaneous analysis

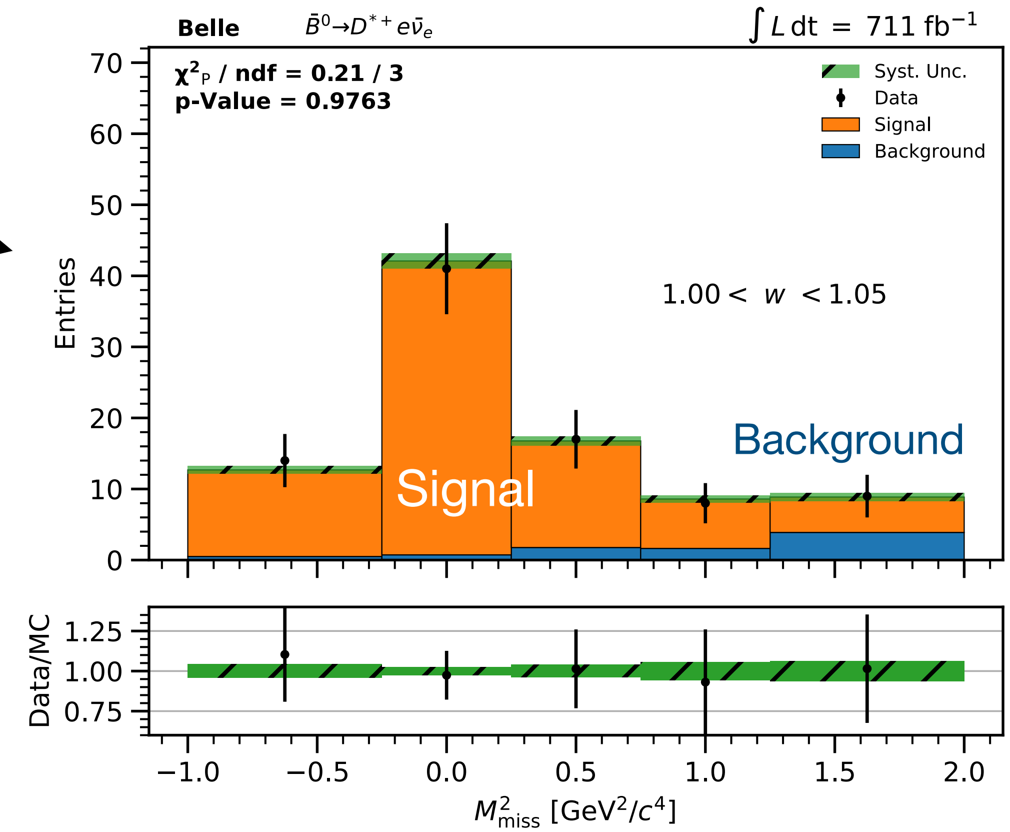
Overall efficiency is **very challenging** to determine due to **tagging**;
focus on decay shapes

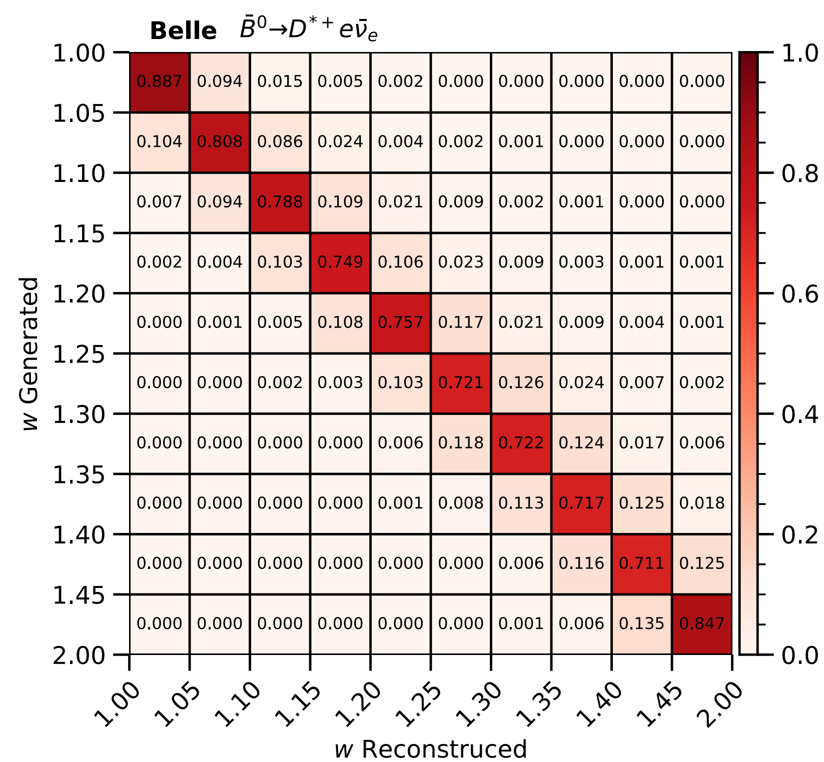
Focus on **1D projections** of recoil parameter and decay angles:



Focus on **1D projections** of recoil parameter and decay angles:







"True"

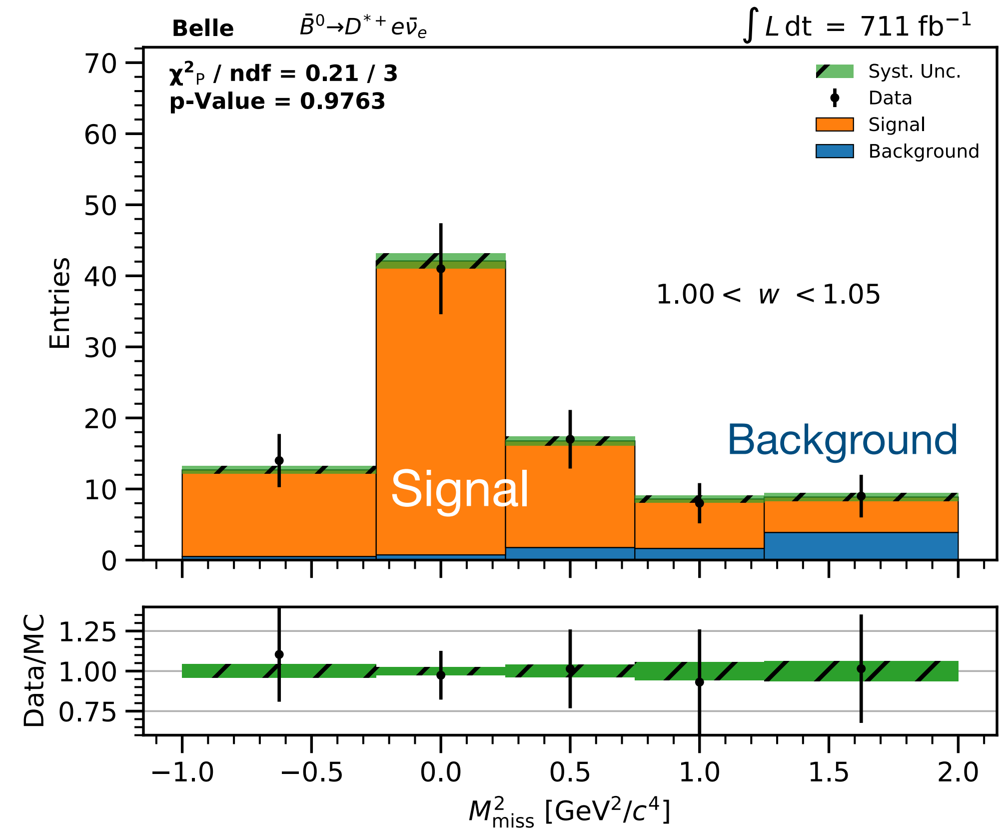
"Reconstructed"

Reverse detector migration using **matrix inversion**

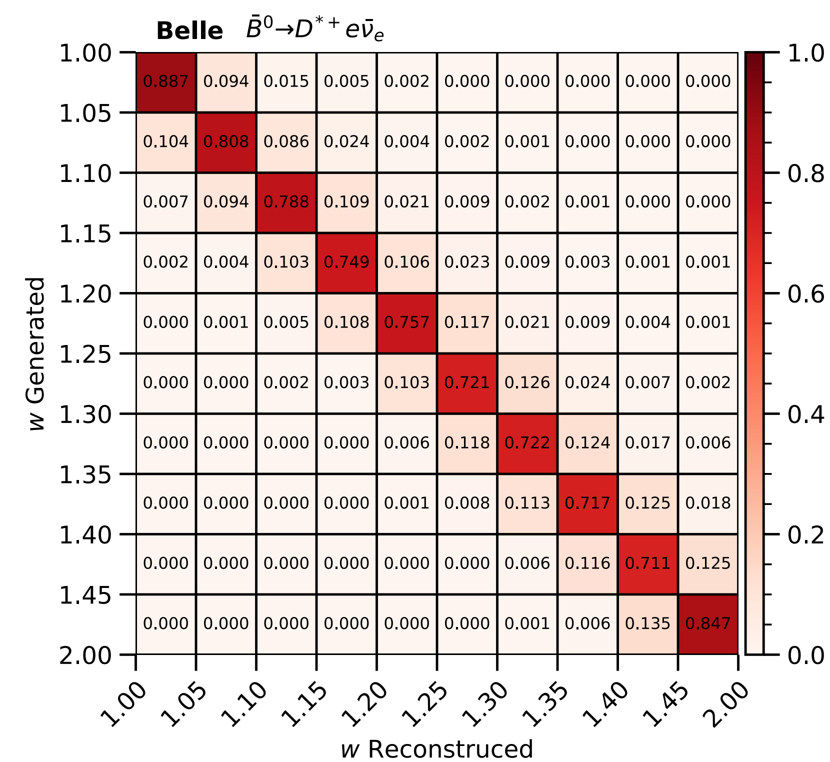
$$\hat{\vec{\mu}} = R^{-1}\hat{\vec{n}}\,,$$

$$R_{ij} = P(\text{reco bin } i \,|\, \text{generated bin } j)\,.$$

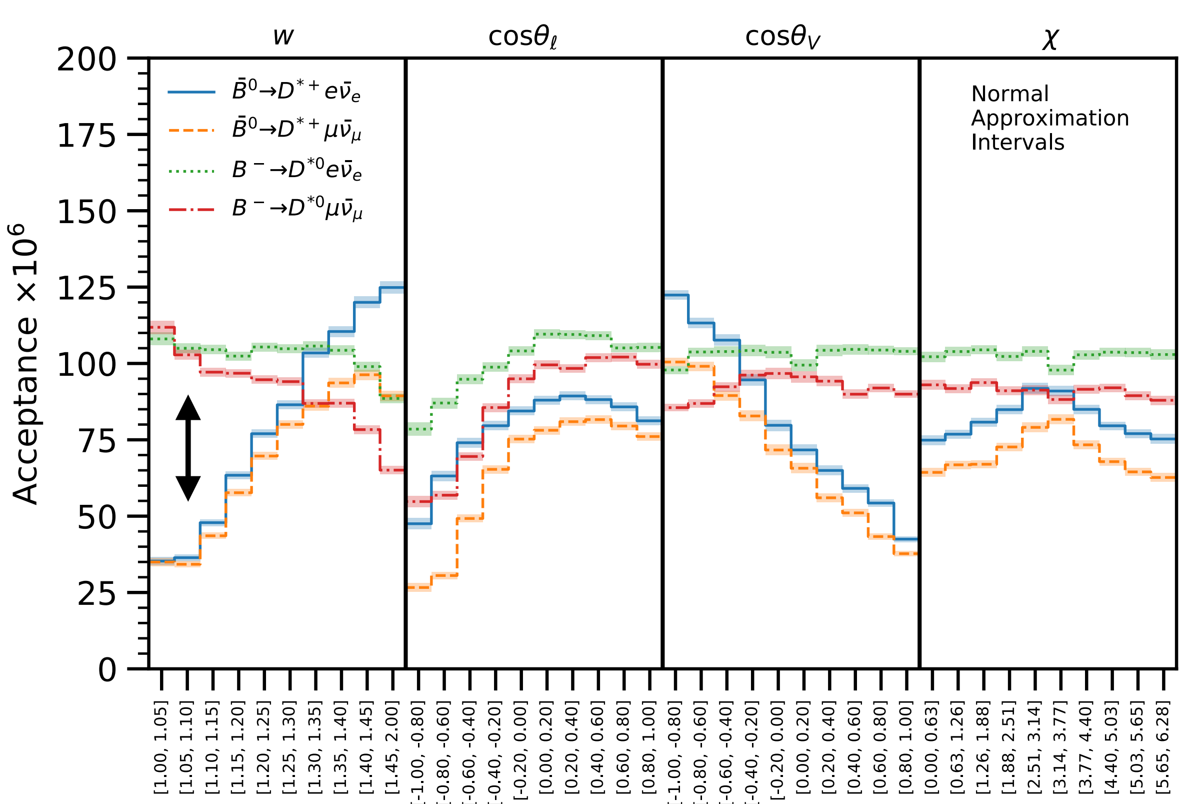
Focus on **1D projections** of recoil parameter and decay angles:



Correct for **acceptance**
and **efficiency** effects



"True"

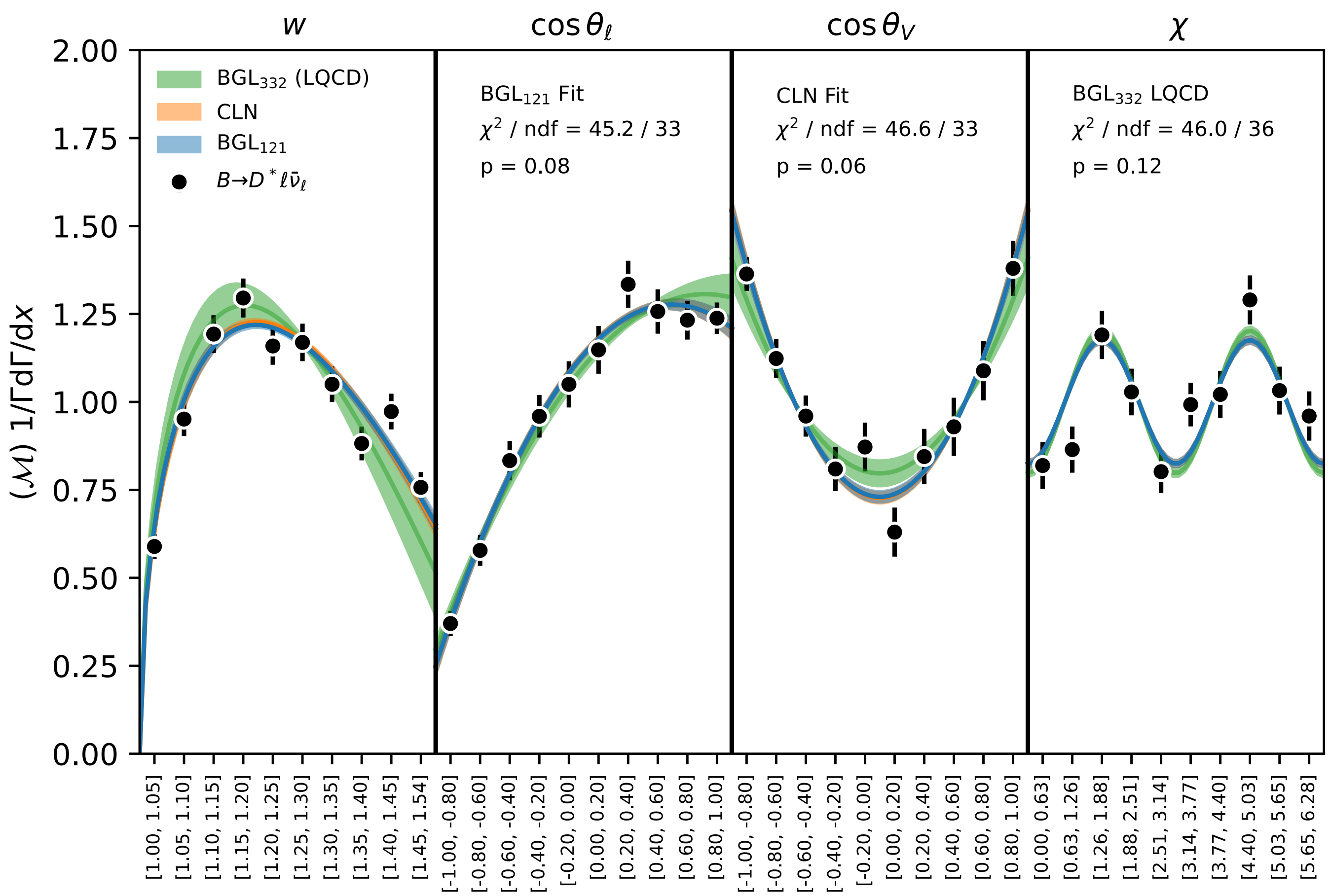


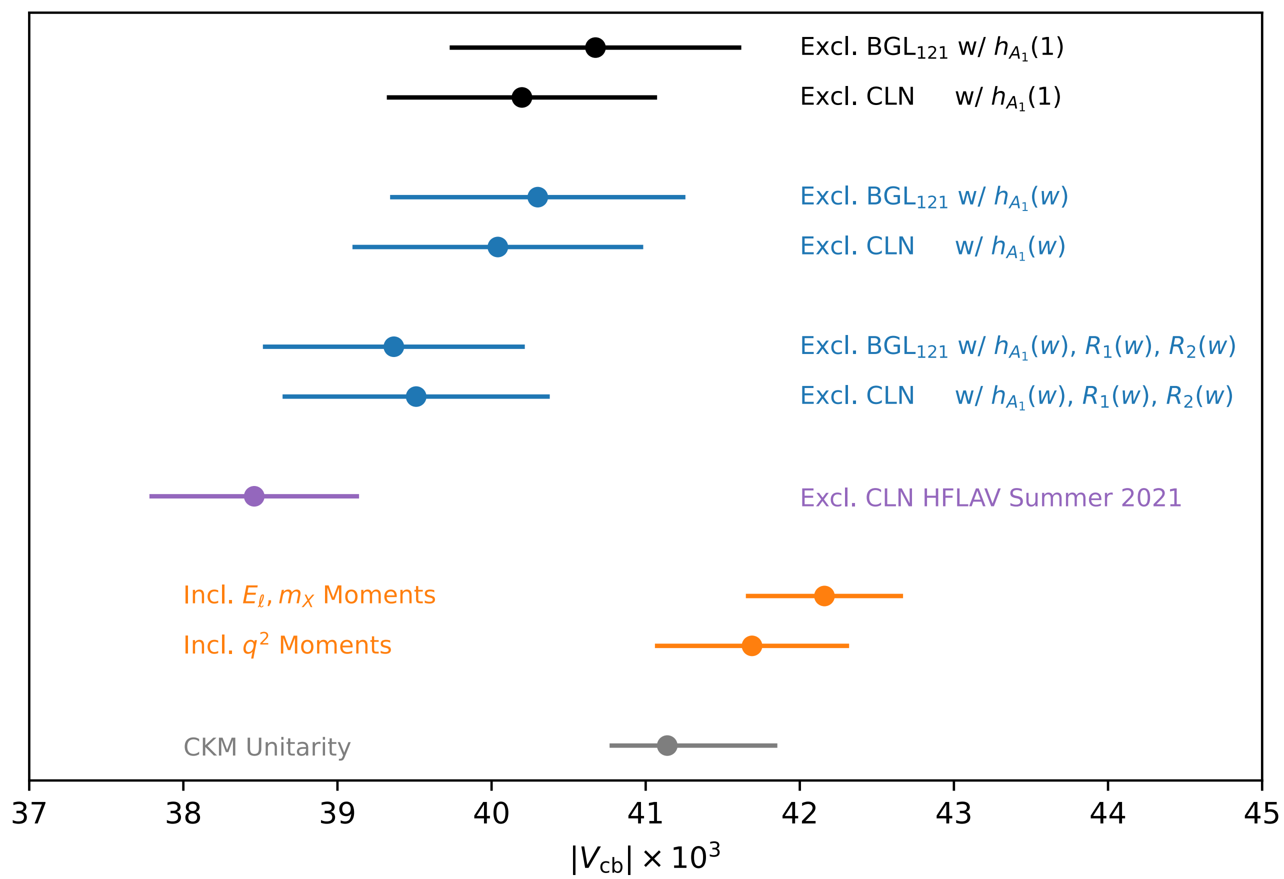
"Reconstructed"

Provide **4 x 40 bins** plus average (careful, only **36 dof**) ;

Some of the (many) **results**:

BGL truncation order determined using **Nested Hypothesis Test**





$$R_{e\mu} = \frac{\mathcal{B}(B \to D^* e \bar{\nu}_e)}{\mathcal{B}(B \to D^* \mu \bar{\nu}_\mu)} = 0.993 \pm 0.023 \pm 0.023 \,,$$

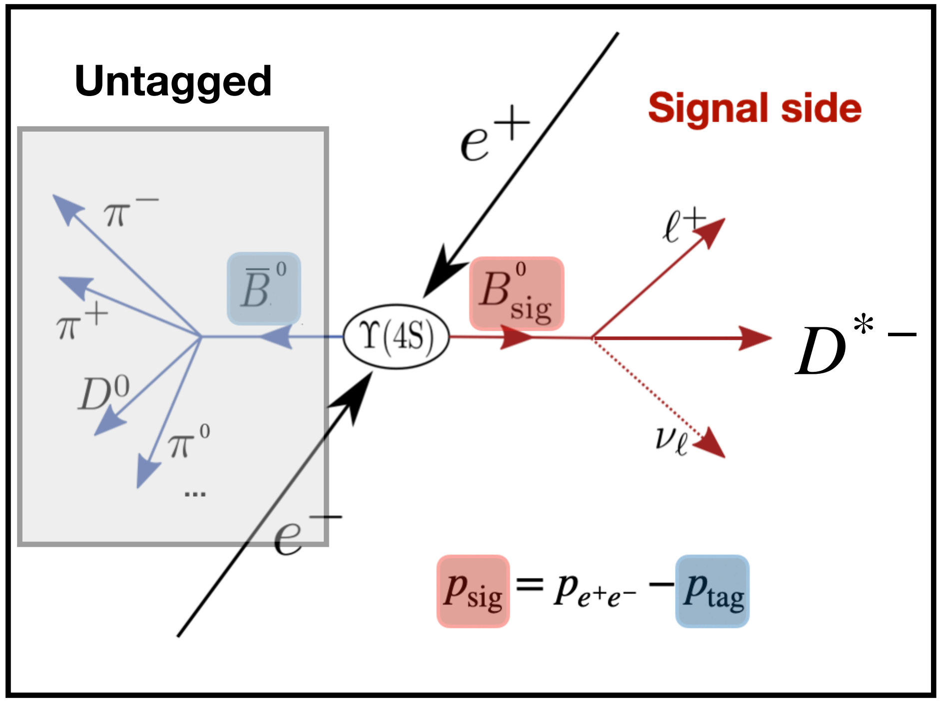
	$\lvert V_{cb}\rvert$	χ^2	dof	N	$\lvert\rho_{max}\rvert$
BGL$_{111}$	40.4 ± 0.8	45.6	34	3	0.70
BGL$_{112}$	40.9 ± 0.9	43.4	33	4	0.98
BGL$_{121}$	40.7 ± 0.9	45.2	33	4	0.60
BGL$_{122}$	41.5 ± 1.1	42.3	32	5	0.98
BGL$_{131}$	38.1 ± 1.7	41.7	32	5	0.98
BGL$_{132}$	39.0 ± 1.6	37.5	31	6	0.98
BGL$_{211}$	39.7 ± 1.0	42.7	33	4	0.99
BGL$_{212}$	40.4 ± 1.0	39.3	32	5	0.99

...

5.

Determination of $|V_{cb}|$ using $\overline{B}^0 \to D^{*+}\ell^- \bar{\nu}_\ell$ with Belle II,

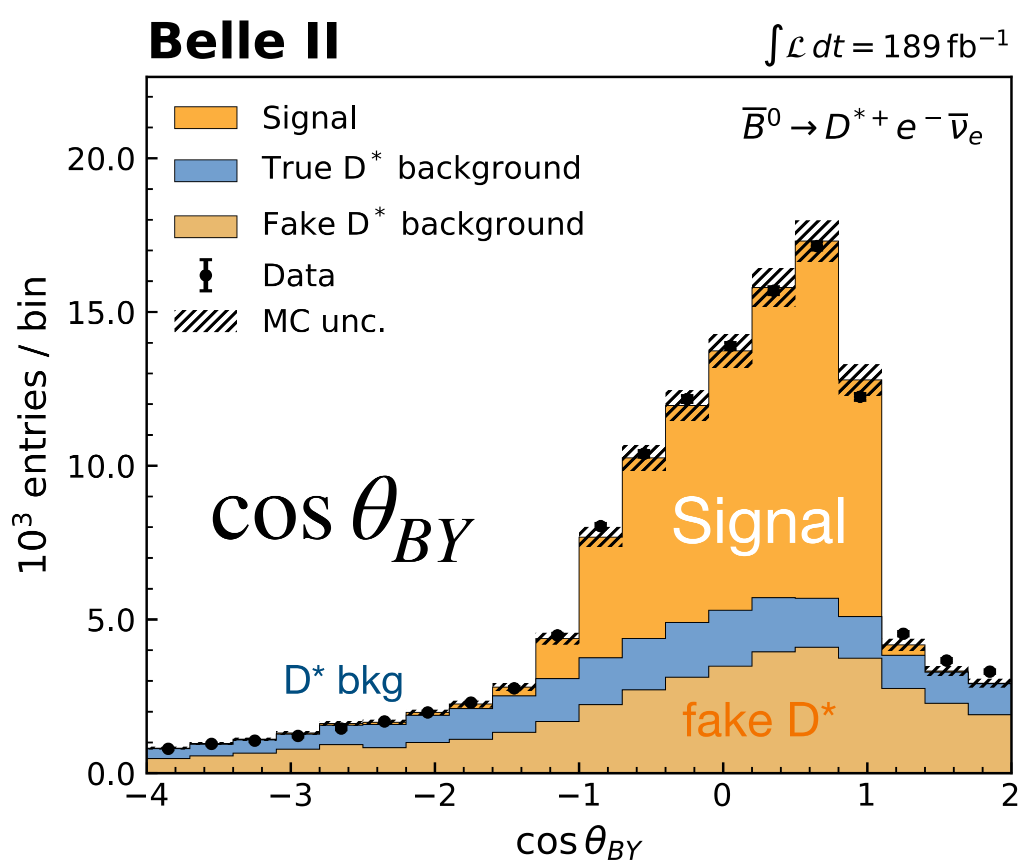
[To be submitted to PRD]

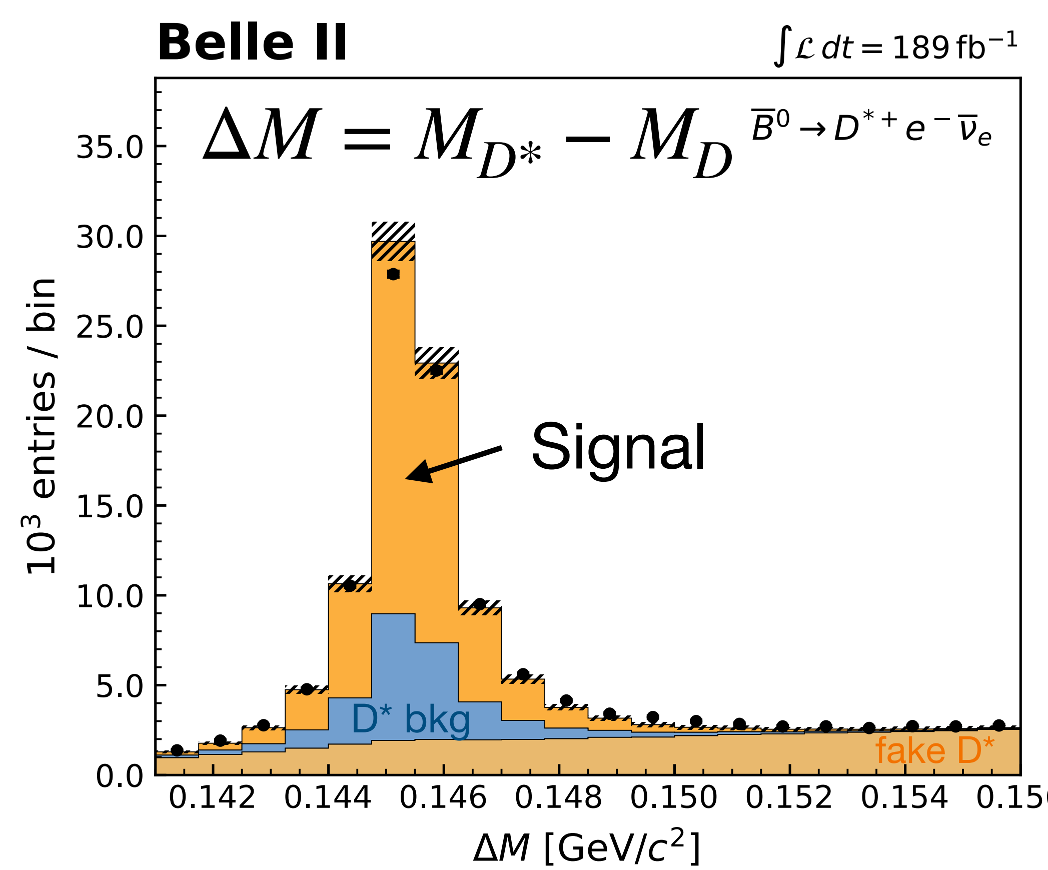


Untagged analysis focussing on experimentally **cleanest mode**:

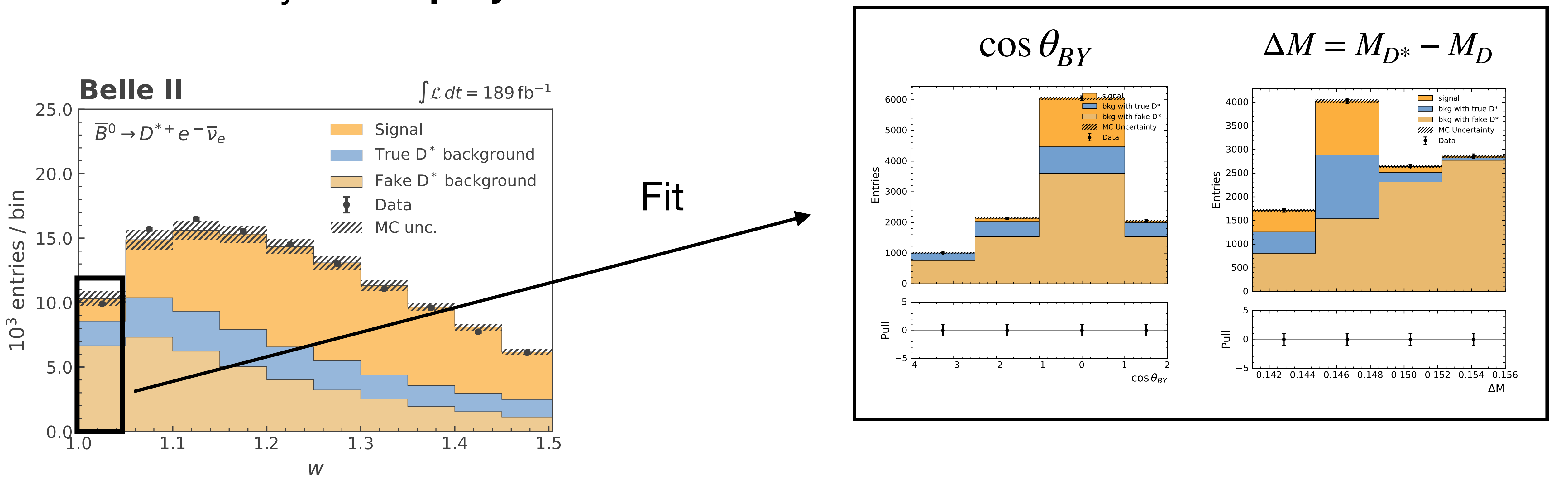
$$\overline{B}^0 \to D^{*+}\ell^- \bar{\nu}_\ell$$
$$\hookrightarrow D^{*+} \to D^0 + \pi^+$$
$$\hookrightarrow D^0 \to K^-\pi^+$$

Extraction in **2D fit**:

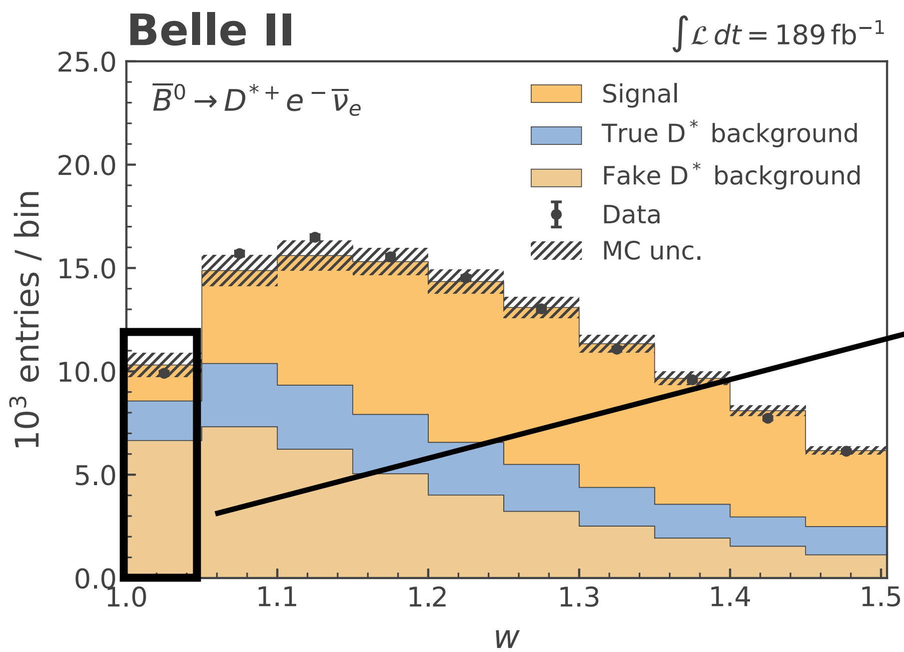




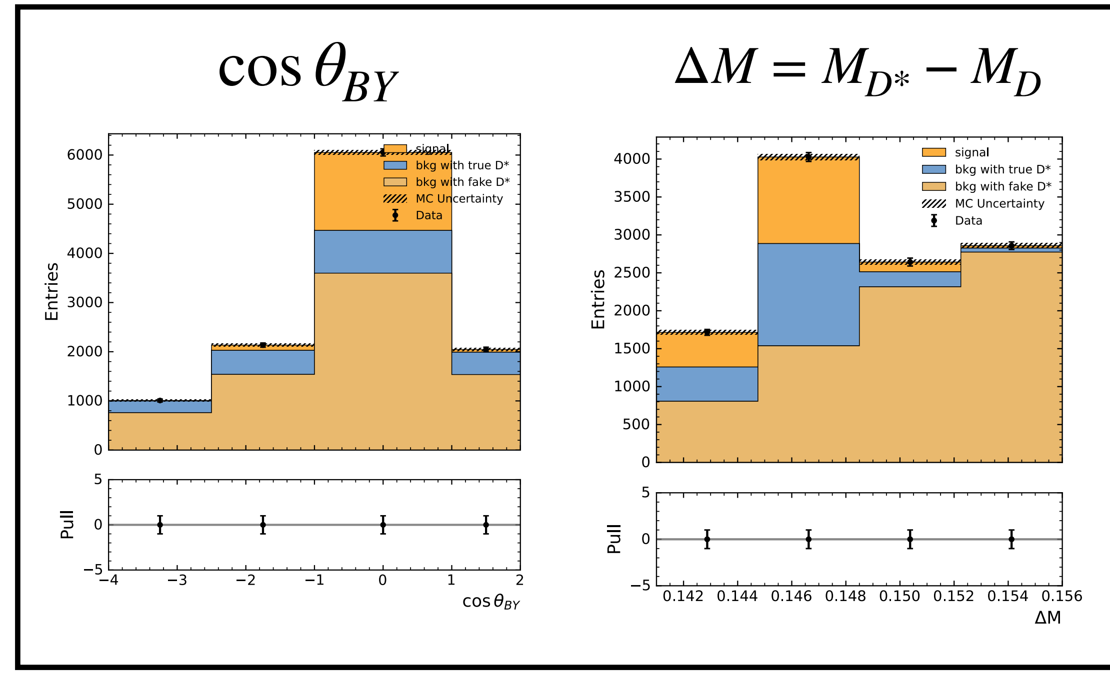
Also focus initially on **1D projections**:



Also focus initially on **1D projections**:

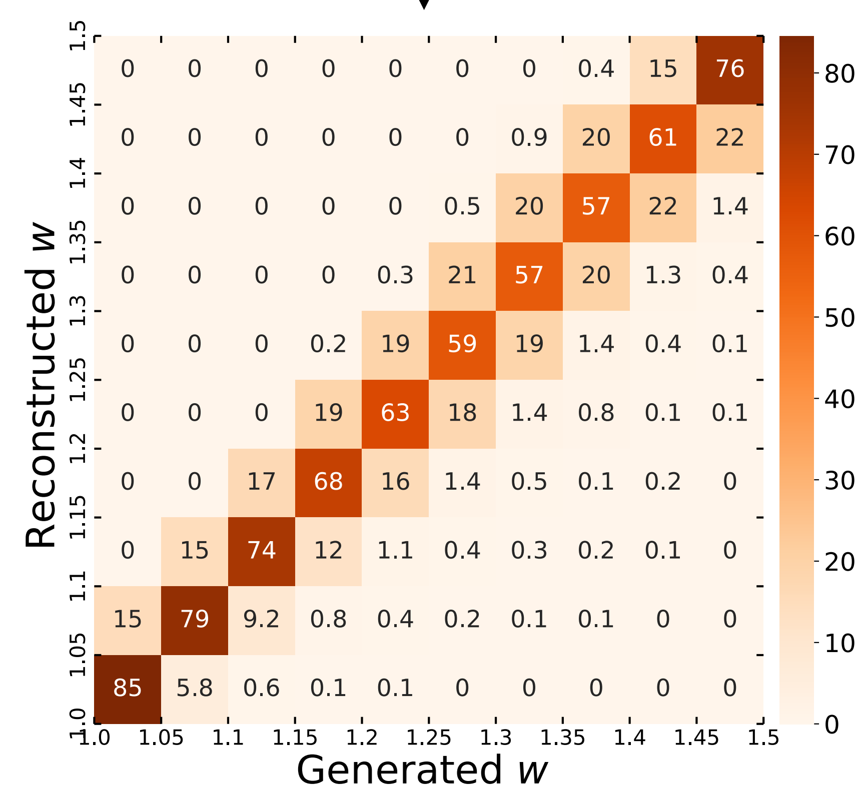


Fit



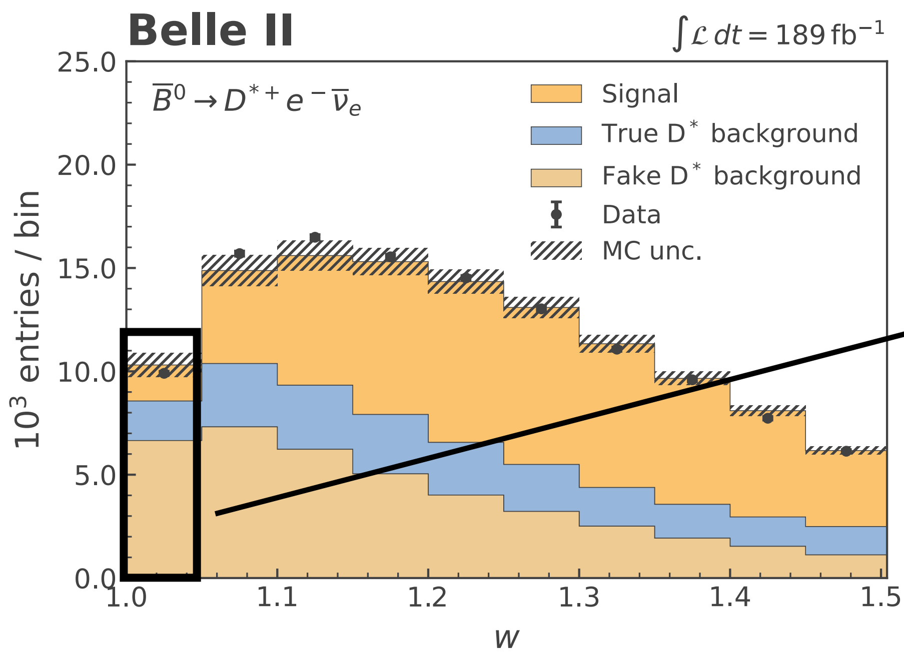
Correct for migration effects:

“Reco”

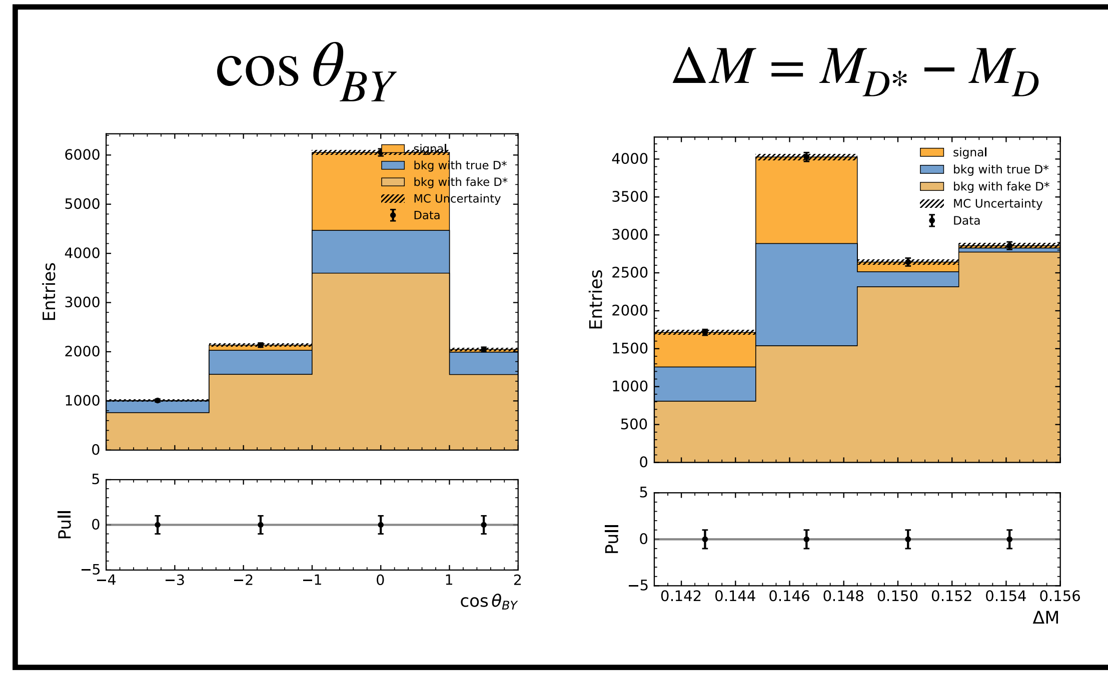


“True”

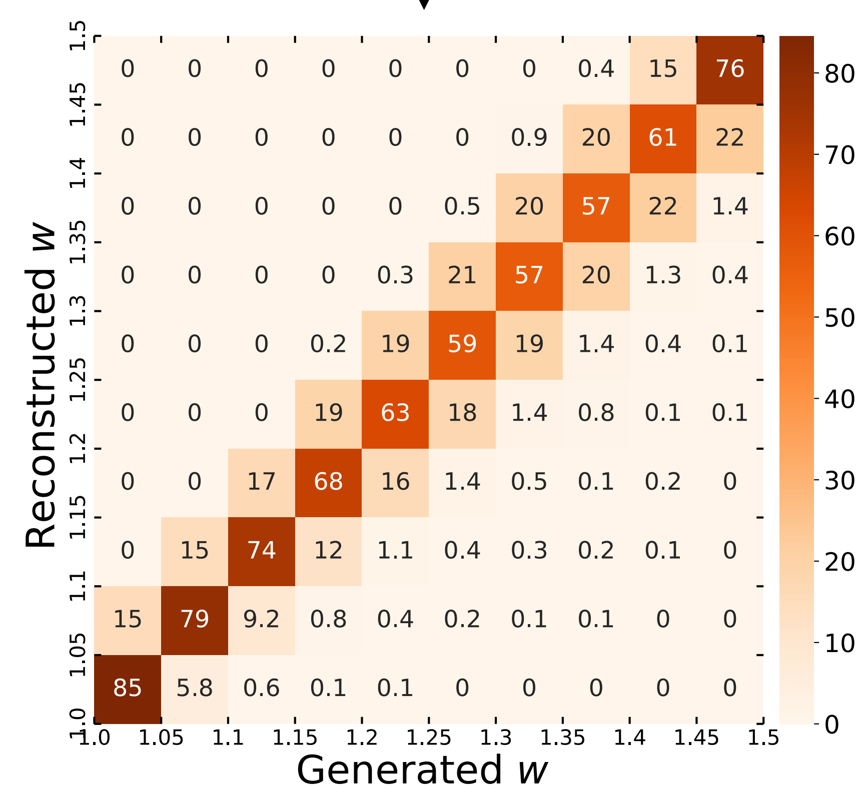
Also focus initially on **1D projections**:



Fit



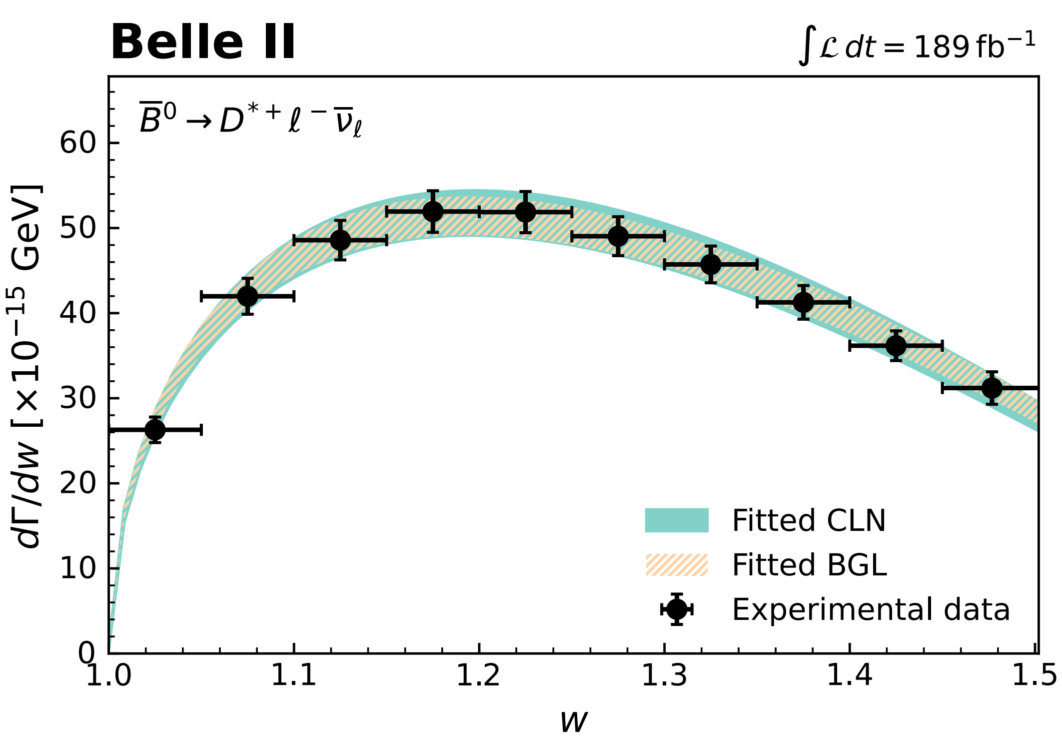
Correct for migration effects:

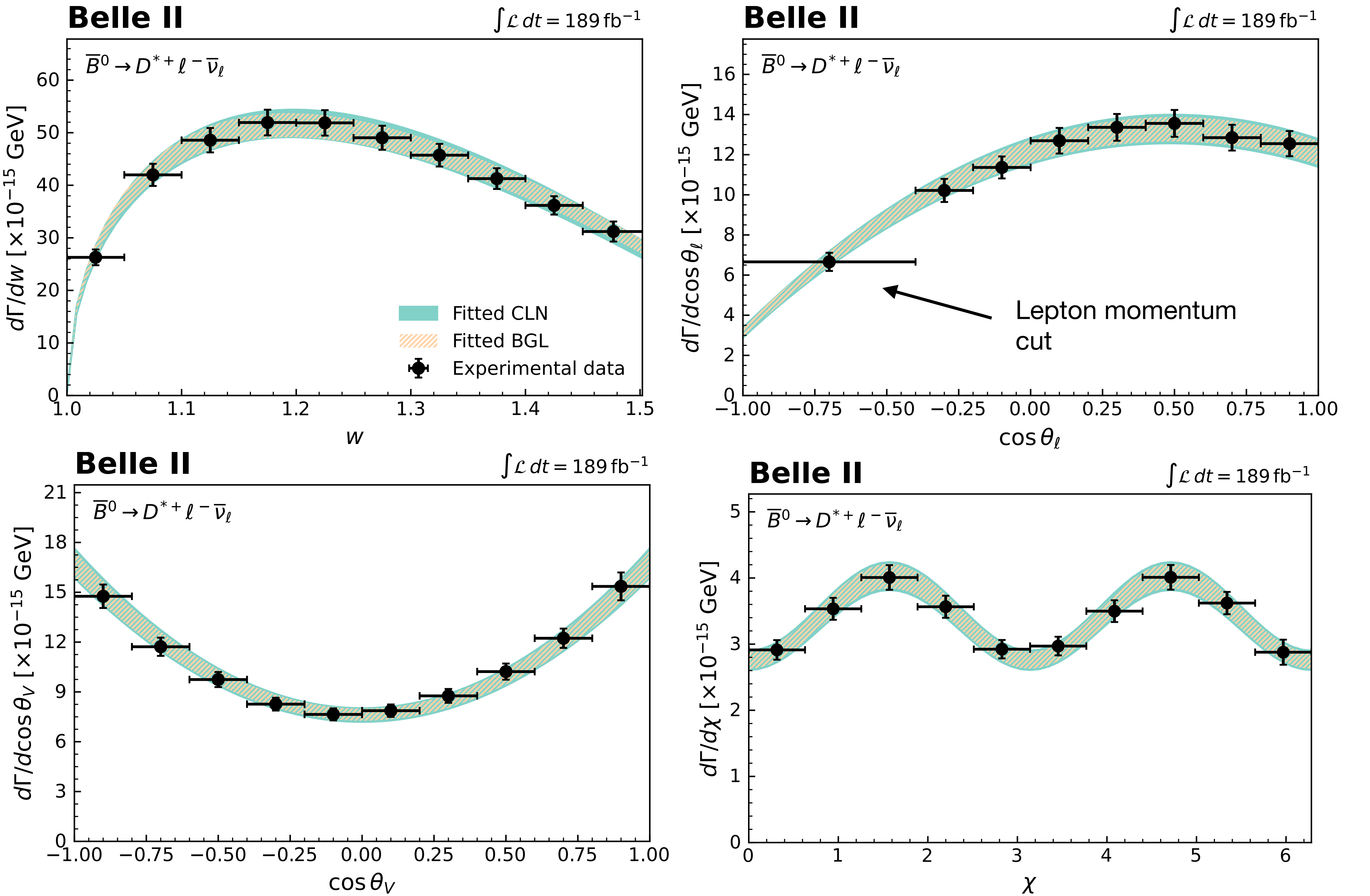


"Reco"

"True"

Correct for acceptance & efficiency





$$|V_{cb}|_{\mathrm{CLN}} = (40.2 \pm 0.3 \pm 0.9 \pm 0.6) \times 10^{-3}\,,$$

$$|V_{cb}|_{\mathrm{BGL}} = (40.6 \pm 0.3 \pm 1.0 \pm 0.6) \times 10^{-3}.$$

← **BGL truncation** order determined using **Nested Hypothesis Test**

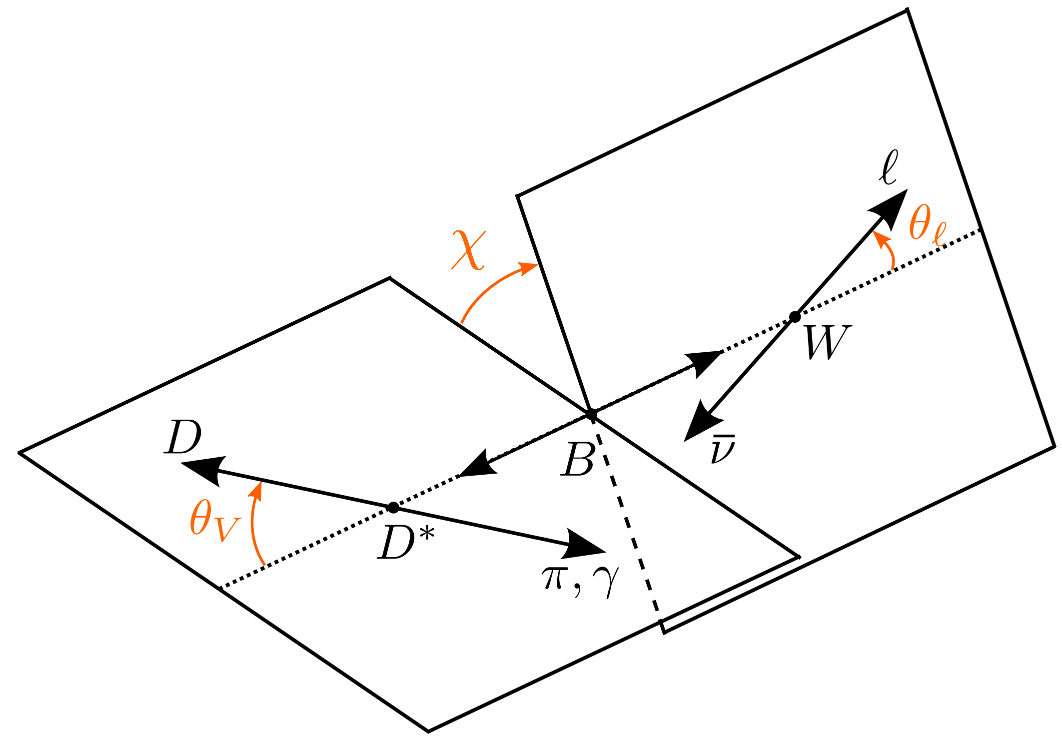
(n_a, n_b, n_c)	$\lvert V_{cb}\rvert \times 10^3$	$\rho_{\max}$	χ^2	Ndf	p-value
(1, 1, 2)	40.2 ± 1.1	0.28	40.5	32	14%
(2, 1, 2)	40.1 ± 1.1	0.97	38.6	31	16%
(1, 2, 2)	**40.6 ± 1.2**	**0.57**	**39.1**	**31**	**15%**
(1, 1, 3)	40.1 ± 1.1	0.97	40	31	13%
(2, 2, 2)	40.2 ± 1.3	0.99	38.6	30	13%
(1, 3, 2)	39.8 ± 1.3	0.98	37.6	30	16%
(1, 2, 3)	40.5 ± 1.2	0.97	39	30	13%

6. Test of **light-lepton universality** in **angular asymmetries** of hadronically tagged $B^0 \to D^{*-}\{e^+, \mu^+\}\,\nu$ decays at Belle II, [To be submitted to PRL]

Construct **asymmetries**:

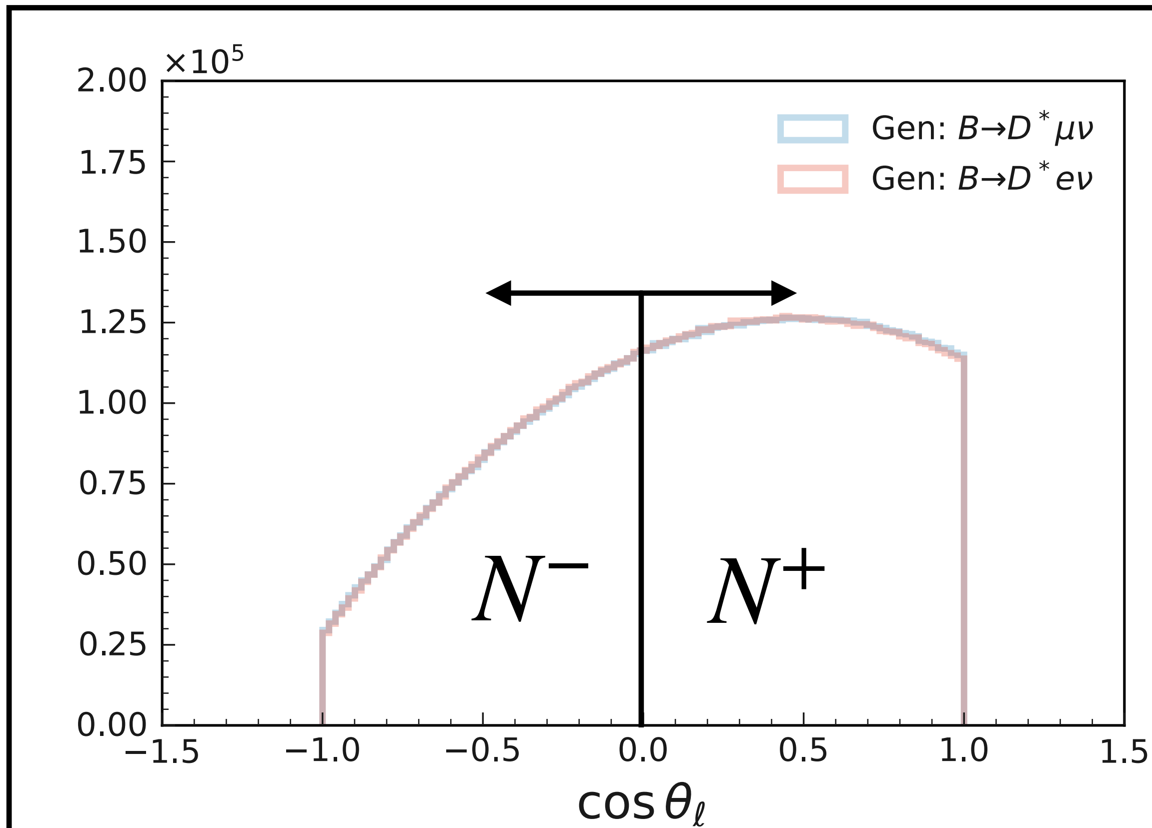
$$\mathcal{A}(w) = \left(\frac{\mathrm{d}\Gamma}{\mathrm{d}w}\right)^{-1} \left[\int_0^1 - \int_{-1}^0\right] dX \frac{\mathrm{d}\Gamma}{\mathrm{d}w\mathrm{d}X},$$

$$\begin{aligned} A_{\mathrm{FB}} &: \mathrm{d}X \to \mathrm{d}(\cos\theta_l) \\ S_3 &: \mathrm{d}X \to \mathrm{d}(\cos 2\chi) \\ S_5 &: \mathrm{d}X \to \mathrm{d}(\cos\chi\cos\theta_V) \\ S_7 &: \mathrm{d}X \to \mathrm{d}(\sin\chi\cos\theta_V) \\ S_9 &: \mathrm{d}X \to \mathrm{d}(\sin 2\chi) \end{aligned}$$



E.g. forward-backward asymmetry in $\cos\theta_\ell$

$$A_{\mathrm{FB}} = \frac{N^+ - N^-}{N^+ + N^+}$$



6.

Test of **light-lepton universality** in **angular asymmetries** of hadronically tagged $B^0 \to D^{*-}\{e^+, \mu^+\}\,\nu$ decays at Belle II, [To be submitted to PRL]

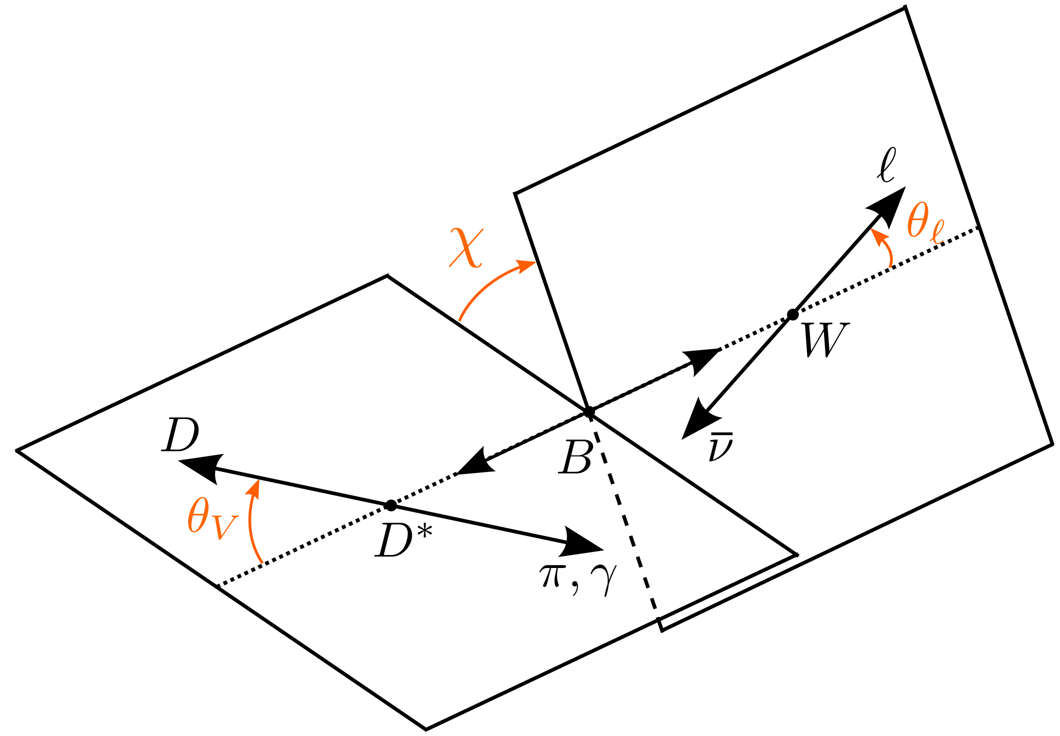
Construct **asymmetries**:

$$\mathcal{A}(w) = \left(\frac{\mathrm{d}\Gamma}{\mathrm{d}w}\right)^{-1} \left[\int_0^1 - \int_{-1}^0\right] dX \frac{\mathrm{d}\Gamma}{\mathrm{d}w\mathrm{d}X},$$

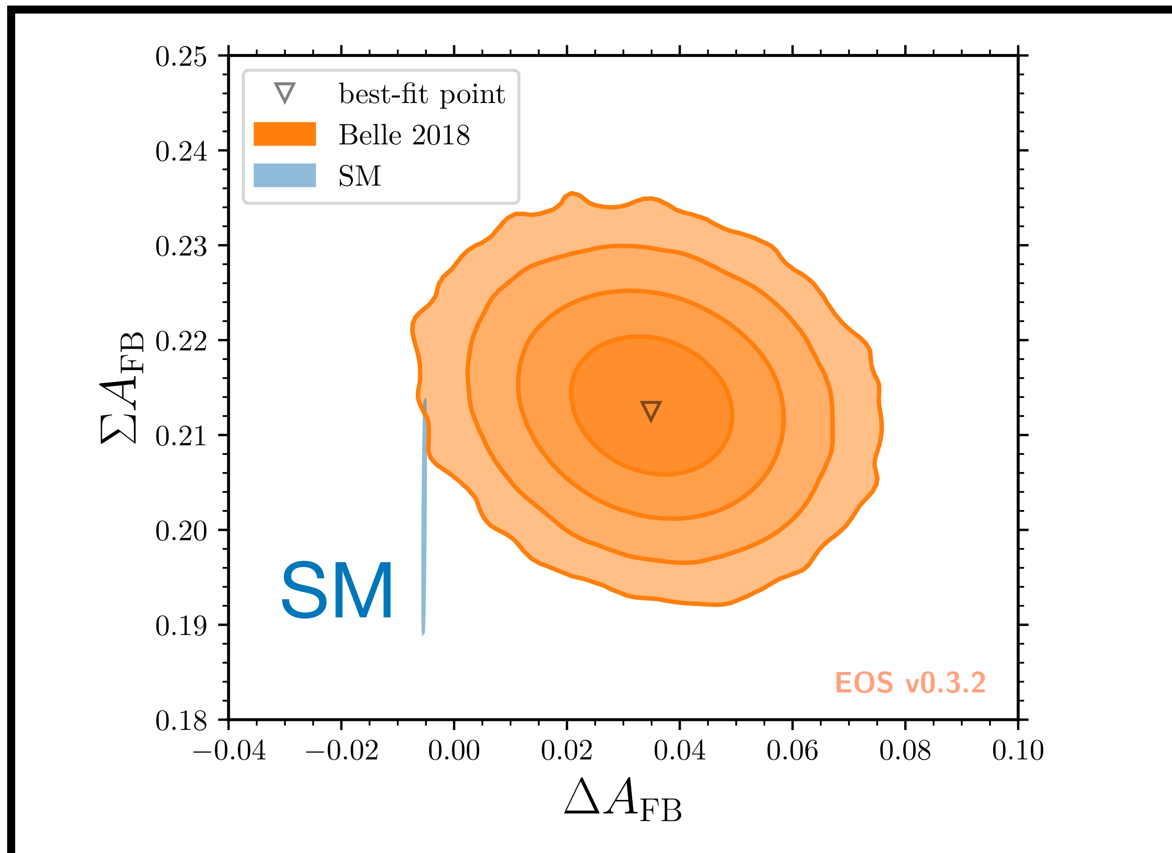
$$A_{\mathrm{FB}} : \mathrm{d}X \to \mathrm{d}(\cos\theta_l)$$
$$S_3 : \mathrm{d}X \to \mathrm{d}(\cos 2\chi)$$
$$S_5 : \mathrm{d}X \to \mathrm{d}(\cos\chi\cos\theta_V)$$
$$S_7 : \mathrm{d}X \to \mathrm{d}(\sin\chi\cos\theta_V)$$
$$S_9 : \mathrm{d}X \to \mathrm{d}(\sin 2\chi)$$

E.g. forward-backward asymmetry in $\cos\theta_\ell$

$$A_{\mathrm{FB}} = \frac{N^+ - N^-}{N^+ + N^+}$$

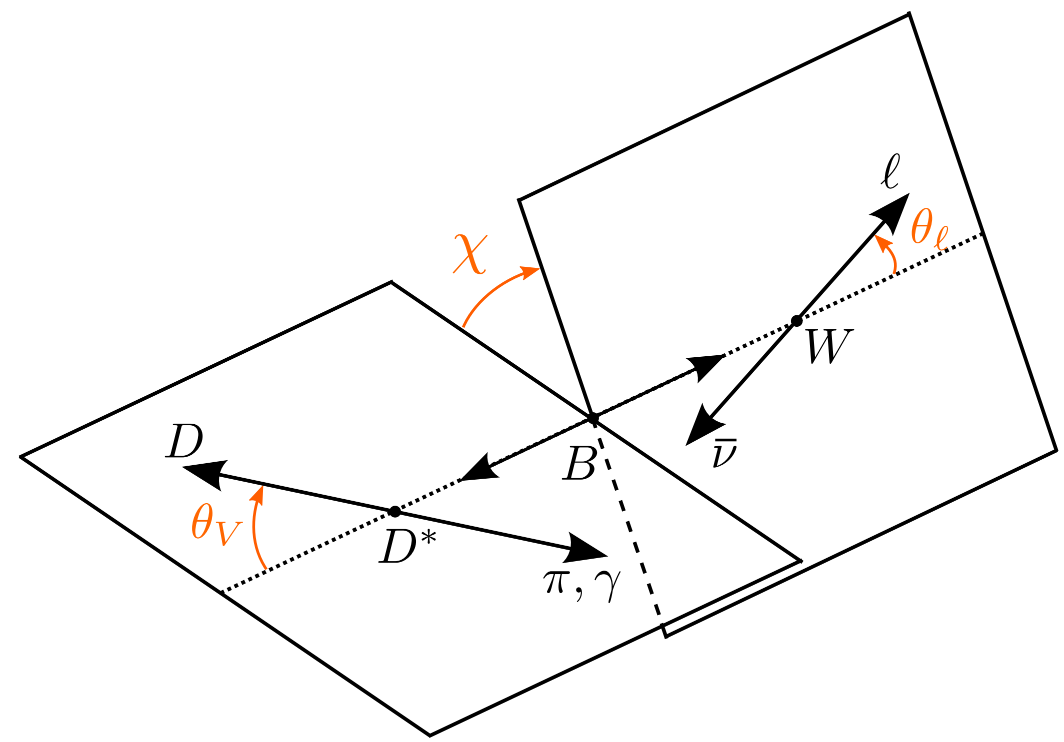


Bobeth et al. [*Eur.Phys.J.C* 81 (2021) 11, 984]

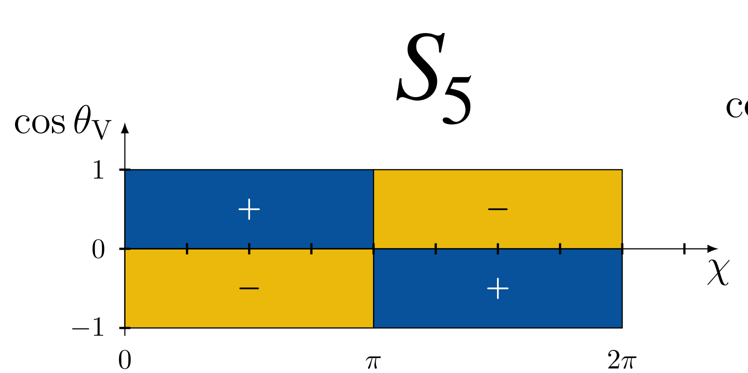


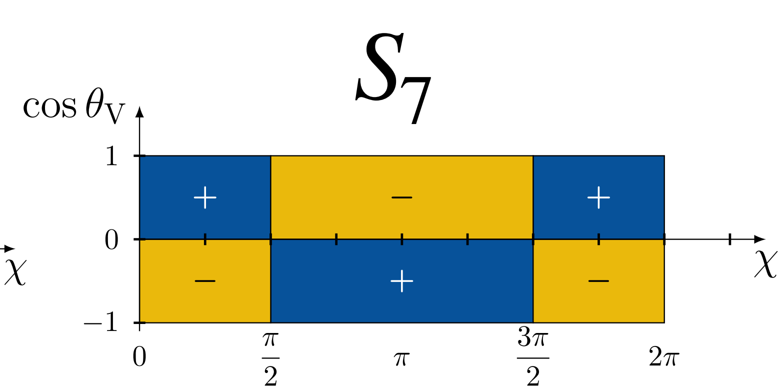
6. Test of **light-lepton universality** in **angular asymmetries** of hadronically tagged $B^0 \to D^{*-}\{e^+, \mu^+\}\,\nu$ decays at Belle II, [To be submitted to PRL]

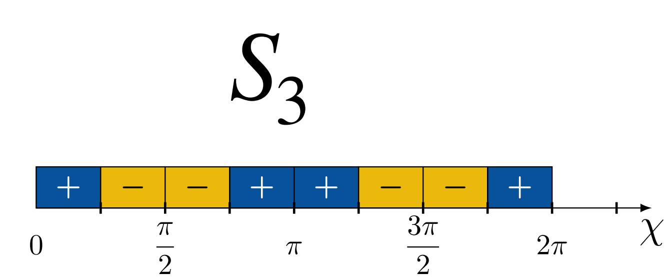
$$\mathcal{A}(w) = \left(\frac{\mathrm{d}\Gamma}{\mathrm{d}w}\right)^{-1} \left[\int_0^1 - \int_{-1}^0\right] dX \frac{\mathrm{d}\Gamma}{\mathrm{d}w\mathrm{d}X},$$

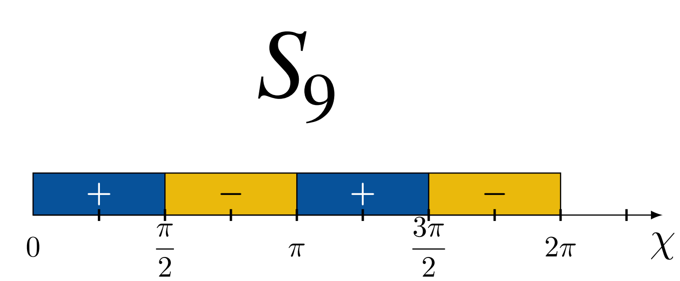


$$A_{\mathrm{FB}} : \mathrm{d}X \to \mathrm{d}(\cos\theta_l)$$
$$S_3 : \mathrm{d}X \to \mathrm{d}(\cos 2\chi)$$
$$S_5 : \mathrm{d}X \to \mathrm{d}(\cos\chi\cos\theta_V)$$
$$S_7 : \mathrm{d}X \to \mathrm{d}(\sin\chi\cos\theta_V)$$
$$S_9 : \mathrm{d}X \to \mathrm{d}(\sin 2\chi)$$









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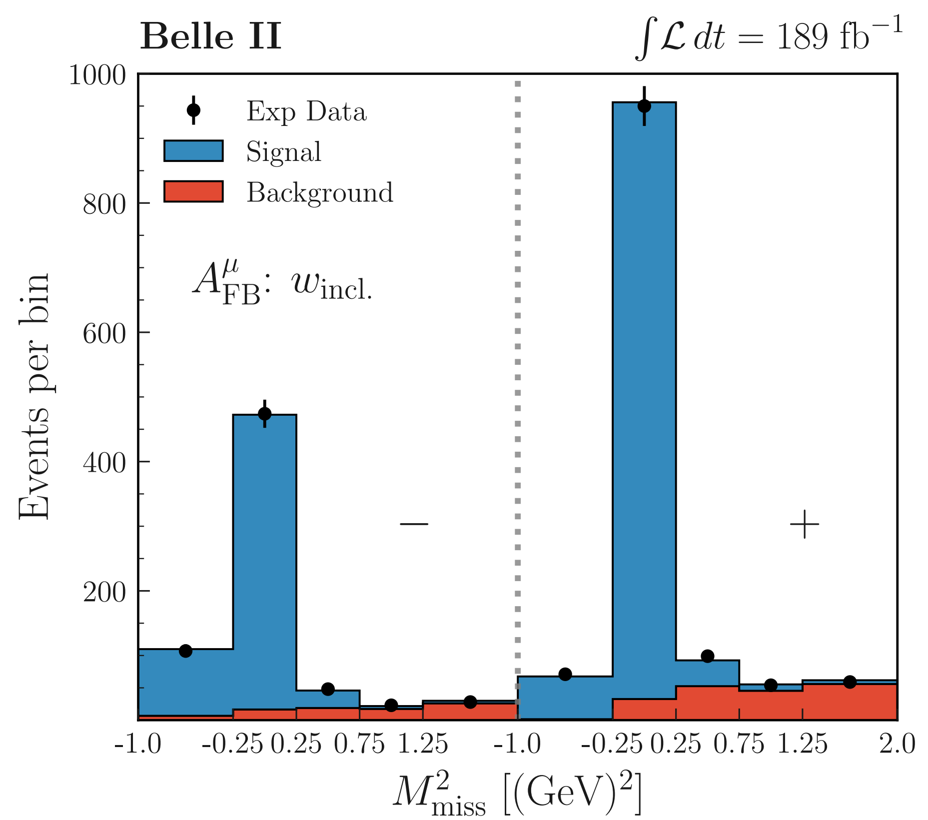
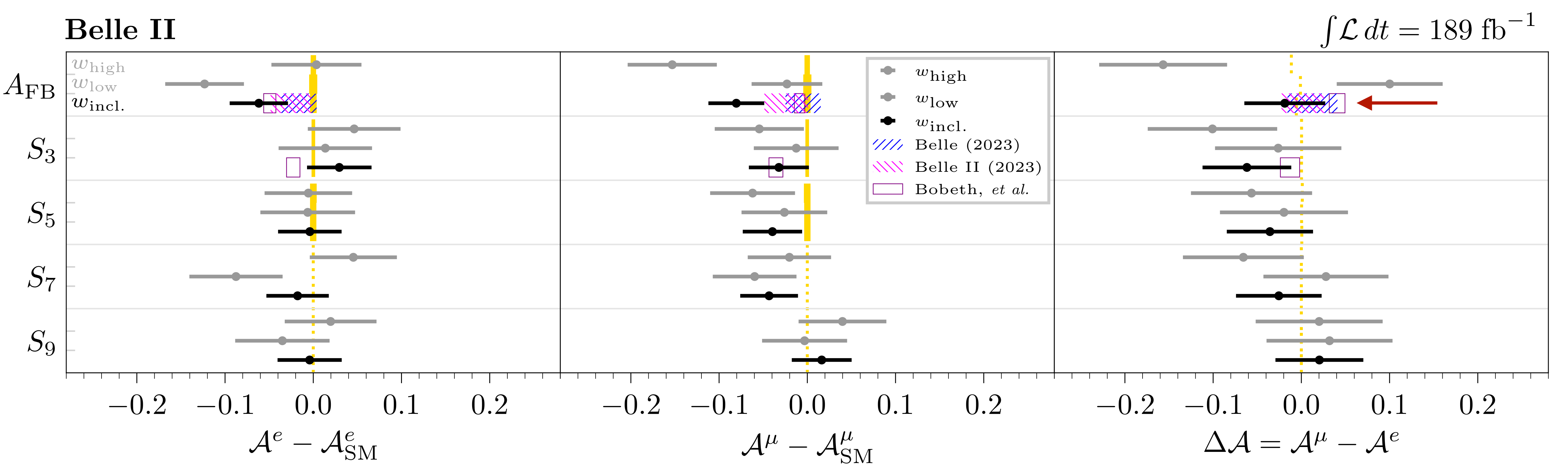


Can also split these **asymmetries** further into w **bins** :

$$w \in [1, w_{\mathrm{max}}]$$

$$w \in [1, 1.275]$$

$$w \in [1.275, w_{\mathrm{max}}]$$



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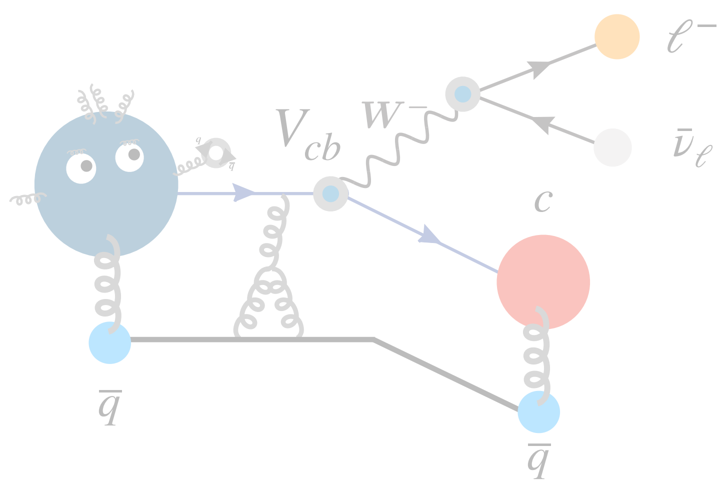

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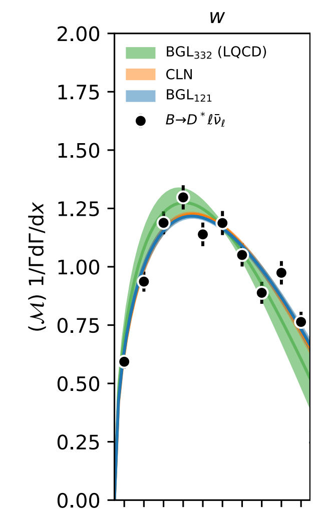

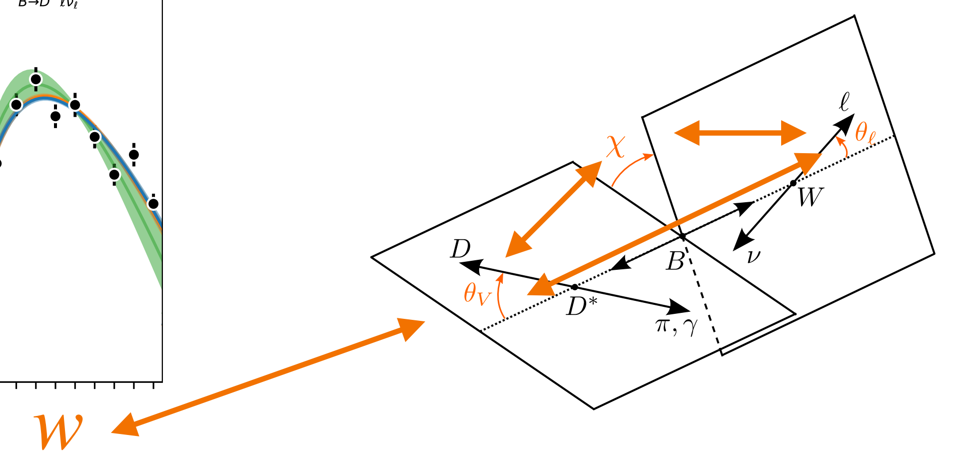

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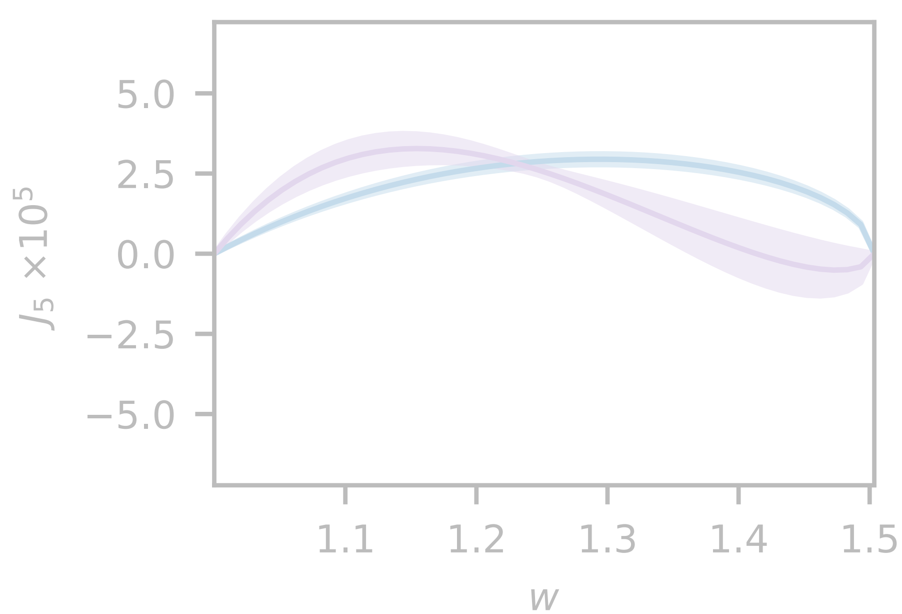


Possible Strategies

Publish either container that allows later reinterpretation

(includes final selected data, MC, etc.)

opendata CERN

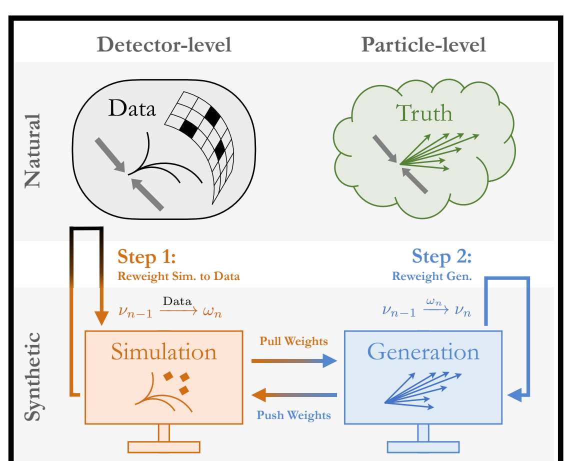
Very ambitious, but great goal!

- Not everybody agrees and not everybody agrees to what extent

Publish ND or unbinned unfolded measurements

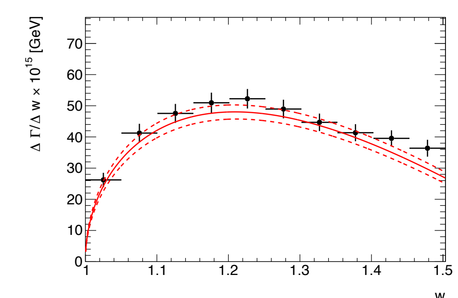
Very challenging, binned: curse of dimensionality (5D measurement essentially)

Unbinned unfolding cool new idea, beats high dimensionality



Omnifold: unbinned unfolding
Phys. Rev. Lett. 124, 182001 (2020)

Publish 1D Measurements of partial BFs



Belle started doing this in 2017

Followed up in 2018 and 2023

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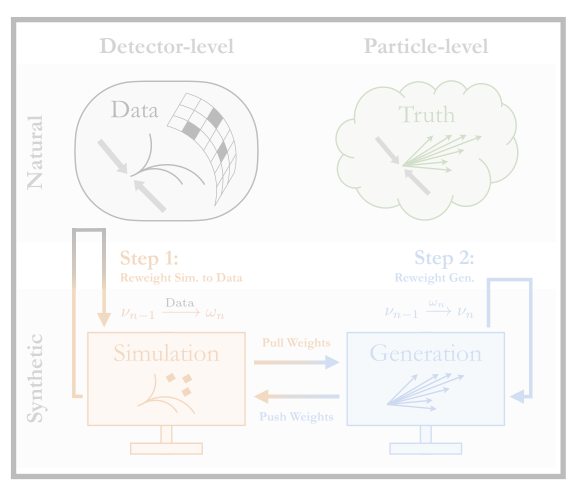
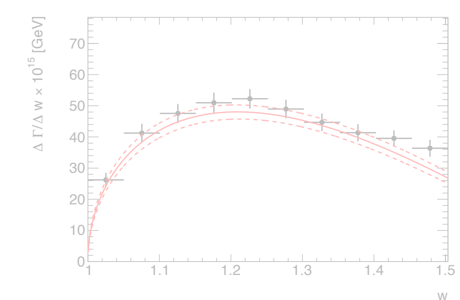


Omnifold: unbinned unfolding
Phys. Rev. Lett. 124, 182001 (2020)

Somewhere in **between**?

Without loosing too much interesting information?

Publish 1D Measurements of partial BFs



Belle started doing this in 2017

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Full Angular Information **without** going to 4D

Full angular information can be encoded into **12 coefficients** :

$$\frac{\mathrm{d}\Gamma}{\mathrm{d}q^2\,\mathrm{d}\cos\theta_V\,\mathrm{d}\cos\theta_\ell\,\mathrm{d}\chi} = \frac{G_F^2\,\left|V_{cb}\right|^2 m_B^3}{2\pi^4}$$

$$\times\Big\{ J_{1s}\sin^2\theta_V + J_{1c}\cos^2\theta_V$$
$$+ (J_{2s}\sin^2\theta_V + J_{2c}\cos^2\theta_V)\cos 2\theta_\ell$$
$$+ J_3\sin^2\theta_V\sin^2\theta_\ell\,\cos 2\chi$$
$$+ J_4\sin 2\theta_V\sin 2\theta_\ell\,\cos\chi + J_5\sin 2\theta_V\sin\theta_\ell\,\cos\chi$$
$$+ (J_{6s}\sin^2\theta_V + J_{6c}\cos^2\theta_V)\cos\theta_\ell$$
$$+ J_7\sin 2\theta_V\sin\theta_\ell\,\sin\chi + J_8\sin 2\theta_V\sin 2\theta_\ell\,\sin\chi$$
$$+ J_9\sin^2\theta_V\sin^2\theta_\ell\,\sin 2\chi\Big\}\,.$$

→ Each of these coefficients is a function of $q^2 \sim w$

↓

With some smart folding, one can "easily" determine them

Based on the ideas of:
JHEP 05 (2013) 043
JHEP 05 (2013) 137
Phys. Rev. D 90, 094003 (2014)
http://cds.cern.ch/record/1605179

8 Coefficients relevant in massless limit & SM

How can we measure these coefficients?

Step 1: bin up phase-space in $q^2 \sim w$ in however many bins you can afford

How can we measure these coefficients?

Step 1: bin up phase-space in $q^2 \sim w$ in however many bins you can afford

Step 2: Determine the # of signal events in specific phase-space regions

The coefficients are related to a weighted sun of events in a given q^2 bin

$$J_i = \frac{1}{N_i} \sum_{j=1}^{8} \sum_{k,l=1}^{4} \eta_{ij}^{\chi} \, \eta_{ik}^{\theta_\ell} \, \eta_{il}^{\theta_V} \left[\chi^i \otimes \theta_\ell^j \otimes \theta_V^k \right]$$

Normalization Factor — Weights — Phase space region

J_i	η_i^χ	$\eta_i^{\theta_\ell}$	$\eta_i^{\theta_V}$	normalization N_i
J_{1s}	$\{+\}$	$\{+,a,a,+\}$	$\{-,c,c,-\}$	$2\pi(1)2$
J_{1c}	$\{+\}$	$\{+,a,a,+\}$	$\{+,d,d,+\}$	$2\pi(1)(2/5)$
J_{2s}	$\{+\}$	$\{-,b,b,-\}$	$\{-,c,c,-\}$	$2\pi(-2/3)2$
J_{2c}	$\{+\}$	$\{-,b,b,-\}$	$\{+,d,d,+\}$	$2\pi(-2/3)(2/5)$
J_3	$\{+,-,-,+,+,-,-,+\}$	$\{+\}$	$\{+\}$	$4(4/3)^2$
J_4	$\{+,+,-,-,-,-,+,+\}$	$\{+,+,-,-\}$	$\{+,+,-,-\}$	$4(4/3)^2$
J_5	$\{+,+,-,-,-,-,+,+\}$	$\{+\}$	$\{+,+,-,-\}$	$4(\pi/2)(4/3)$
J_{6s}	$\{+\}$	$\{+,+,-,-\}$	$\{-,c,c,-\}$	$2\pi(1)2$
J_{6c}	$\{+\}$	$\{+,+,-,-\}$	$\{+,d,d,+\}$	$2\pi(1)(2/5)$
J_7	$\{+,+,+,+,-,-,-,-\}$	$\{+\}$	$\{+,+,-,-\}$	$4(\pi/2)(4/3)$
J_8	$\{+,+,+,+,-,-,-,-\}$	$\{+,+,-,-\}$	$\{+,+,-,-\}$	$4(4/3)^2$
J_9	$\{+,+,-,-,+,+,-,-\}$	$\{+\}$	$\{+\}$	$4(4/3)^2$

$a = 1 - 1/\sqrt{2}, b = a\sqrt{2}, c = 2\sqrt{2} - 1, d = 1 - 4\sqrt{2}/5$

FB, Z. Ligeti, S. Turczyk, Phys. Rev. D 90, 094003 (2014)

E.g. for J_3: **Split** χ into **2 Regions**

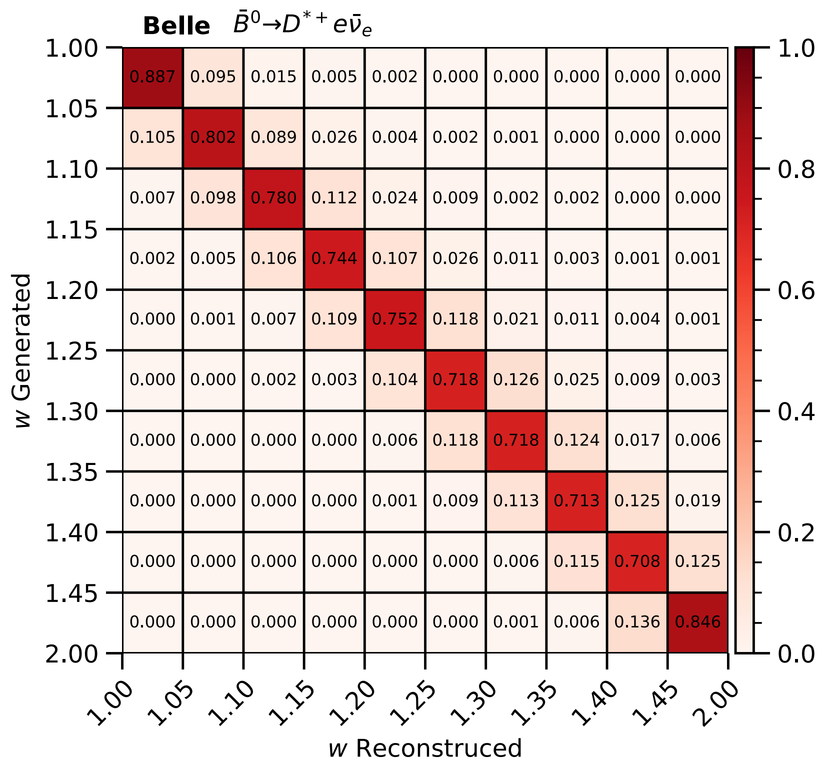
$'+' : \chi \in [0,\pi/4], [3/4\pi,5/4\pi], [7/4\pi,2\pi]$ — $\tilde{N}_+$

$'-' : \chi \in [\pi/4,3/4\pi], [5/4\pi,7/4\pi]$ — $\tilde{N}_-$

Step 3: Reverse Migration and Acceptance Effects

Resolution effects: events with a **given "true" value** of $\{q^2, \cos\theta_\ell, \cos\theta_V, \chi\}$ can **fall into different reconstructed** bins

E.g. w migration matrix



arXiv:2301.07529 **[hep-ex]**

$$\begin{pmatrix} \tilde{N}_+^{q_1^2} \\ \tilde{N}_-^{q_1^2} \\ \tilde{N}_+^{q_2^2} \\ \tilde{N}_-^{q_2^2} \\ \tilde{N}_+^{q_3^2} \\ \tilde{N}_-^{q_3^2} \\ \tilde{N}_+^{q_4^2} \\ \tilde{N}_-^{q_4^2} \\ \tilde{N}_+^{q_5^2} \\ \tilde{N}_-^{q_5^2} \end{pmatrix} = \mathscr{M} \begin{pmatrix} N_+^{q_1^2} \\ N_-^{q_1^2} \\ N_+^{q_2^2} \\ N_-^{q_2^2} \\ N_+^{q_3^2} \\ N_-^{q_3^2} \\ N_+^{q_4^2} \\ N_-^{q_4^2} \\ N_+^{q_5^2} \\ N_-^{q_5^2} \end{pmatrix}$$

← Unfolded yields

Bkg subtracted yields

Migration matrix

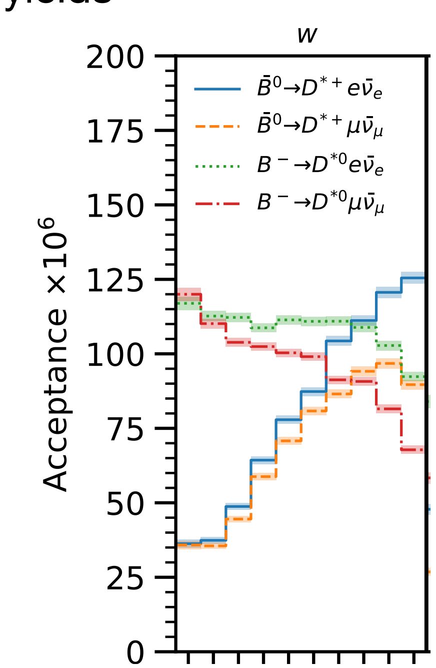
Acceptance x Efficiency Corrections:

$$N_+^{q_i^2} \cdot e^{-1}_{\mathrm{eff},+,q_i^2} = n_+^{q_i^2}$$

$$N_-^{q_i^2} \cdot e^{-1}_{\mathrm{eff},-,q_i^2} = n_-^{q_i^2}$$

← Acceptance / Eff. corrected yields

Unfolded yields



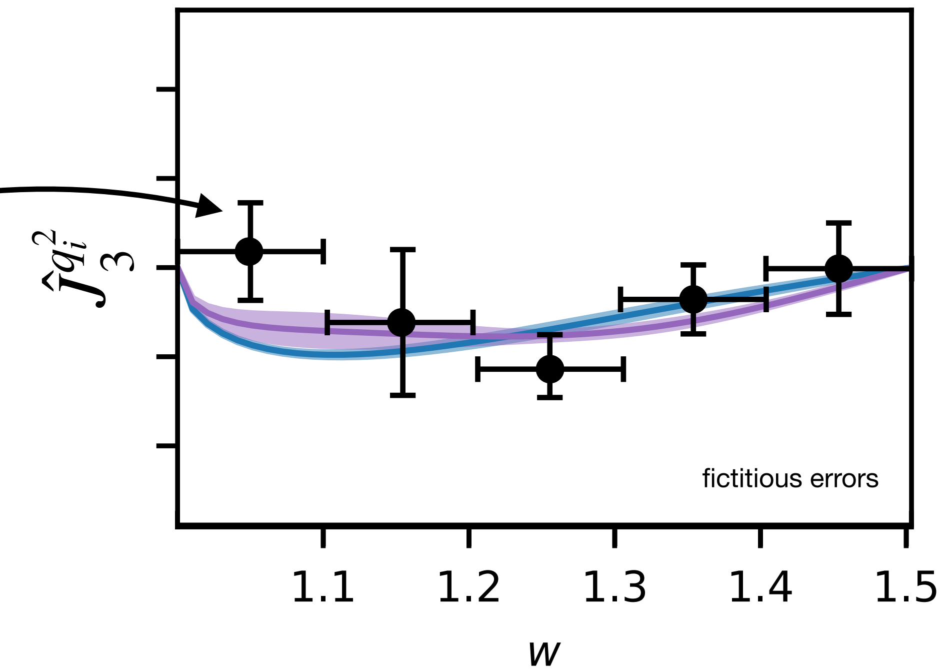
arXiv:2301.07529 **[hep-ex]**

Step 4: Calculate J_i for a given w/q^2 bin

$$\begin{matrix} n_+^{q_i^2} \\ n_-^{q_i^2} \end{matrix} \rightarrow \hat{J}_3^{q_i^2} = \frac{1}{\Gamma} \times \frac{n_+^{q_i^2} - n_-^{q_i^2}}{4(4/3)^2}$$

Normalization

$$\Gamma = \frac{8}{9}\pi\left(3\sum_i J_{1c}^{q_i^2} + 6\sum_i J_{1s}^{q_i^2} - \sum_i J_{2c}^{q_i^2} - 2\sum_i J_{2s}^{q_i^2}\right)$$



More **involved** for the **other** coefficients: need full experimental covariance between all measured w/q^2 bins and coefficients (statistical overlap, systematics)

SM:

$$\{J_{1s}^{q_i^2}, J_{1c}^{q_i^2}, J_{2s}^{q_i^2}, J_{2c}^{q_i^2}, J_3^{q_i^2}, J_4^{q_i^2}, J_5^{q_i^2}, J_{6s}^{q_i^2}\}$$

e.g. **5 x 8** = **40 coefficients**

or full thing (SM + NP) with **5 x 12** = **60 coefficients**

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FB, Z. Ligeti, S. Turczyk, Phys. Rev. D 90, 094003 (2014)

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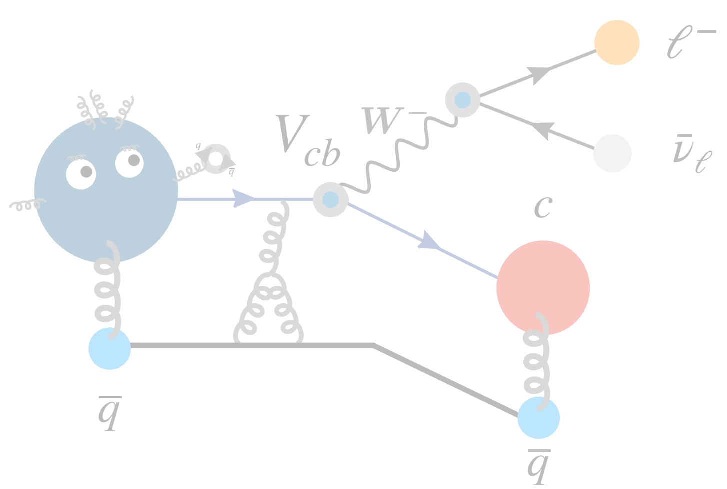
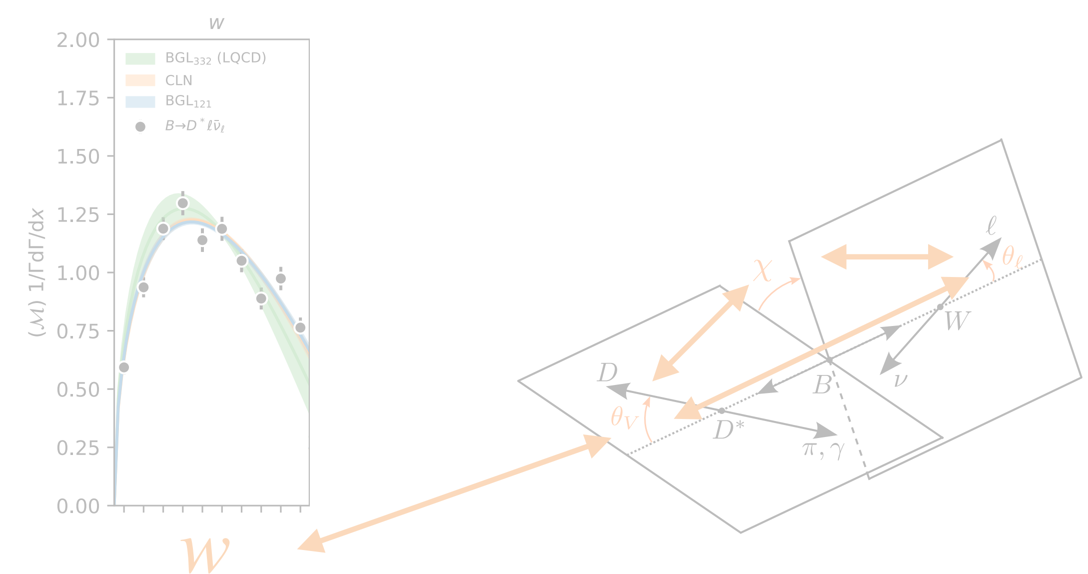
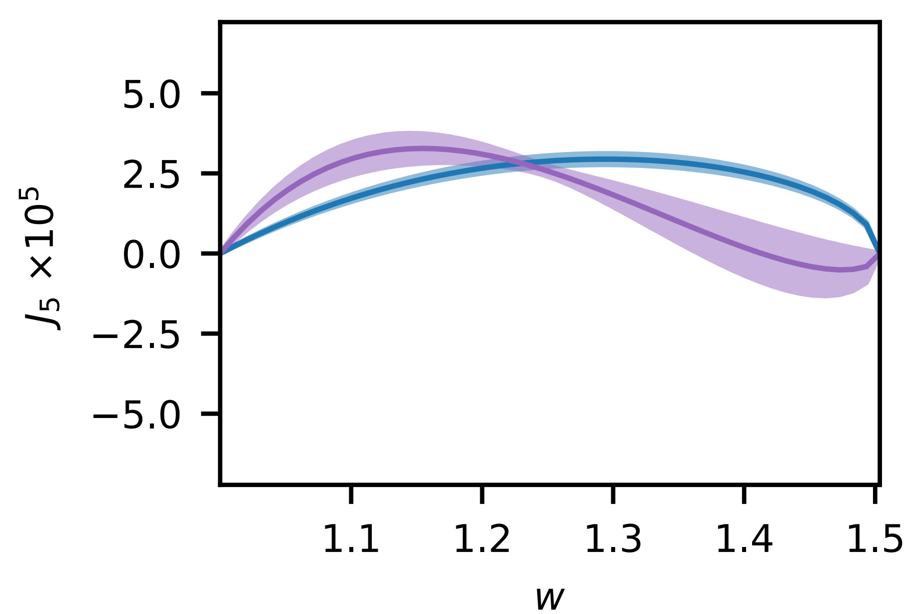


2. From **1D projections** to full angular information



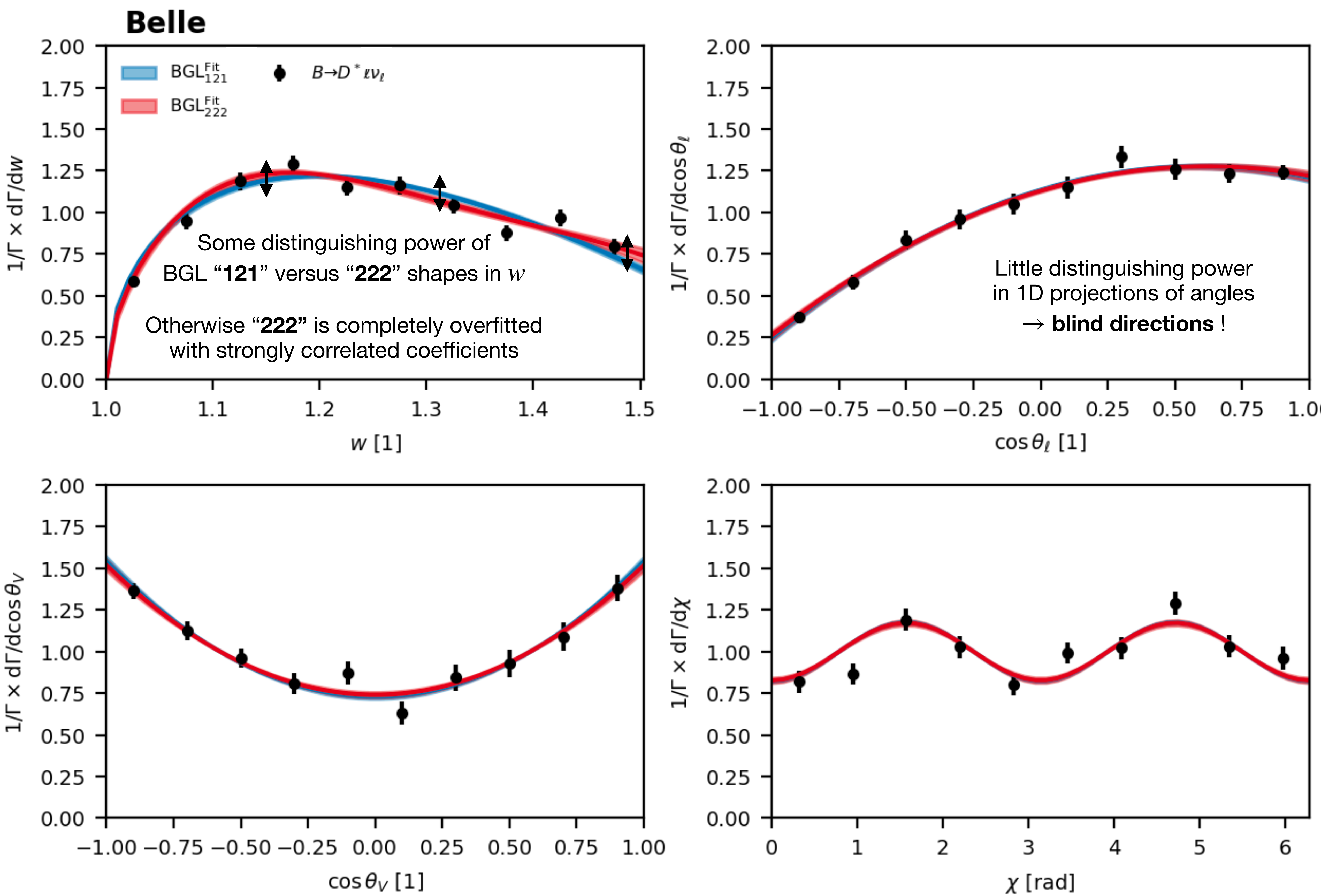
Working around the curse of dimensionality

3. The Potential of full angular fits



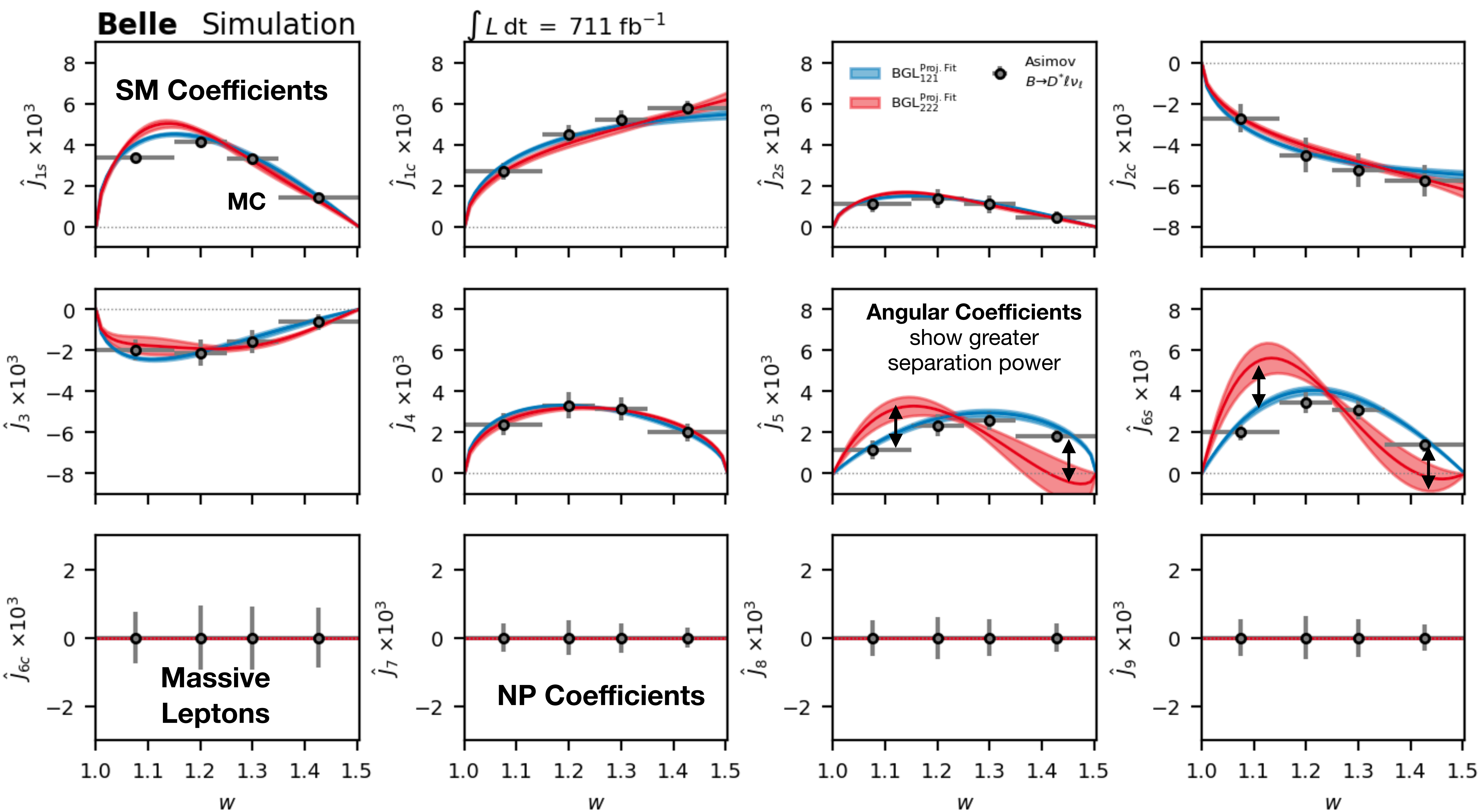
1D versus Full Angular Sensitivities

Errors and central values from 1D projection fits of arXiv:2301.07529 (Table XVI)



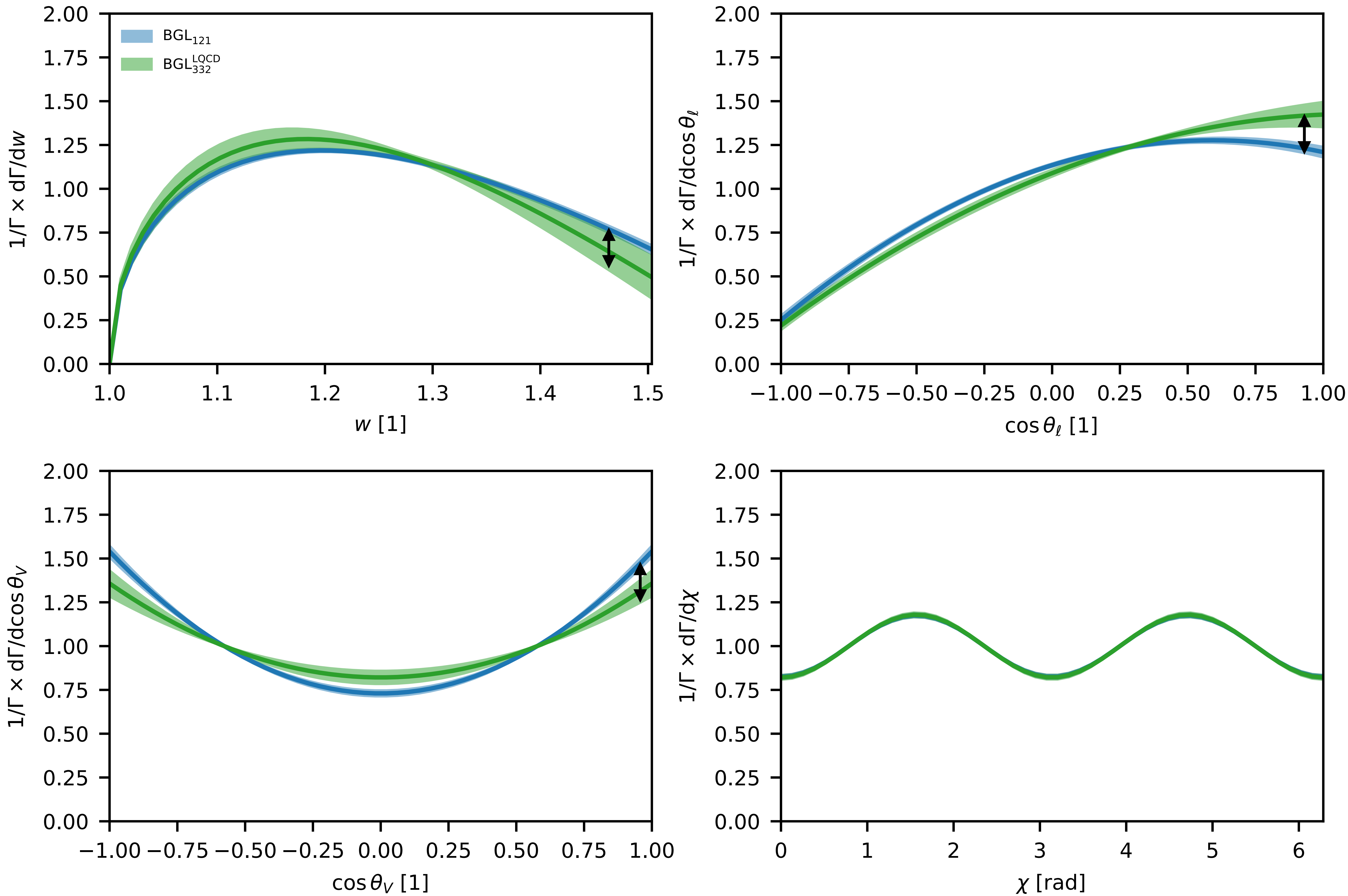
1D versus Full Angular Sensitivities

Errors and central values from 1D projection fit of arXiv:2301.07529 (Table XVI)
Data points: **Asimov Fit using MC (!)**



1D versus Full Angular Sensitivities

BGL121 1D projection fit of arXiv:2301.07529 (Table XVI) or **FNAL/MILC prediction** [arXiv:2105.14019]



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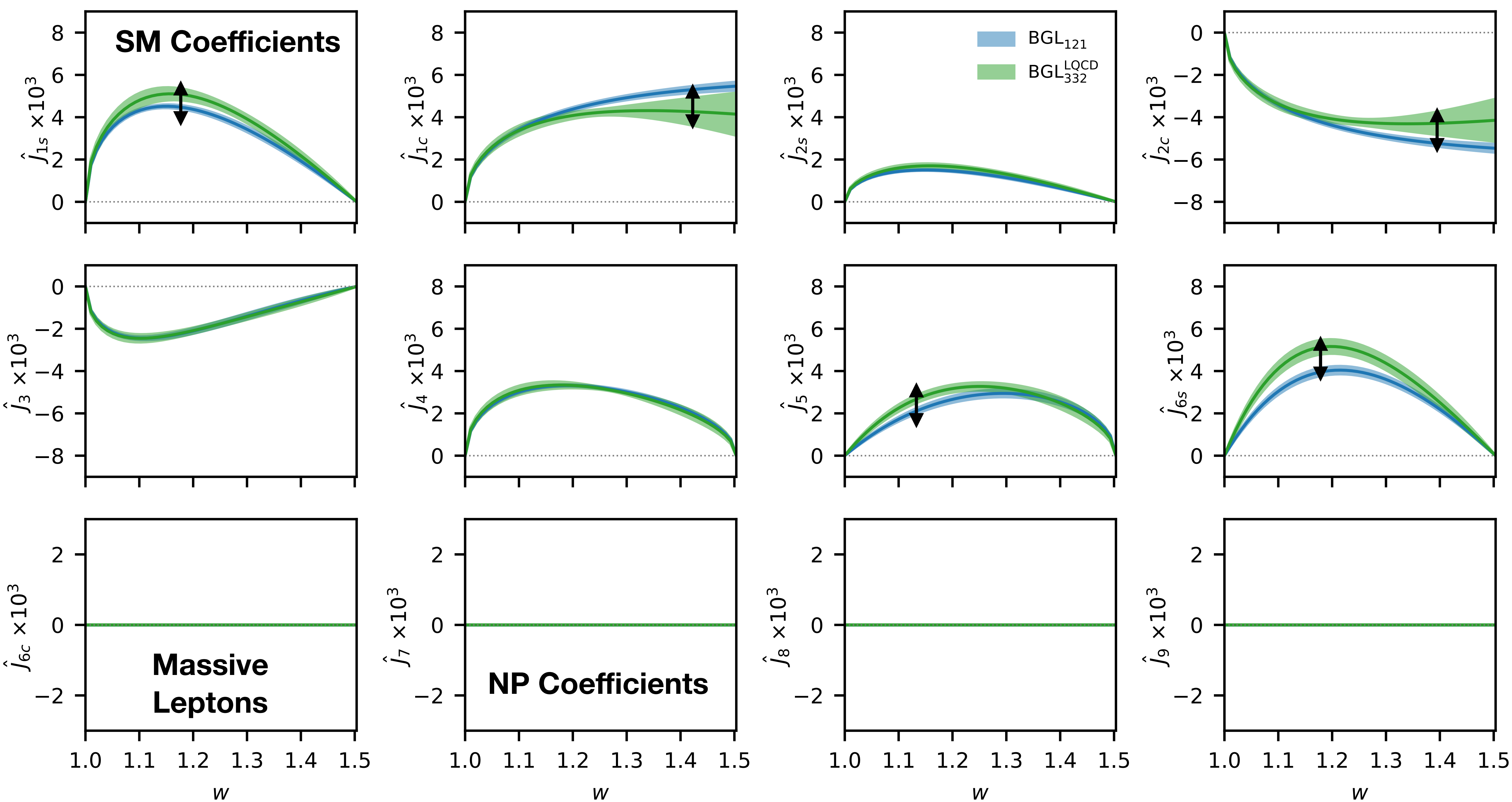


Angular Coefficients also will allow us to better **investigate** what is going on with **lattice** versus **data tensions**..

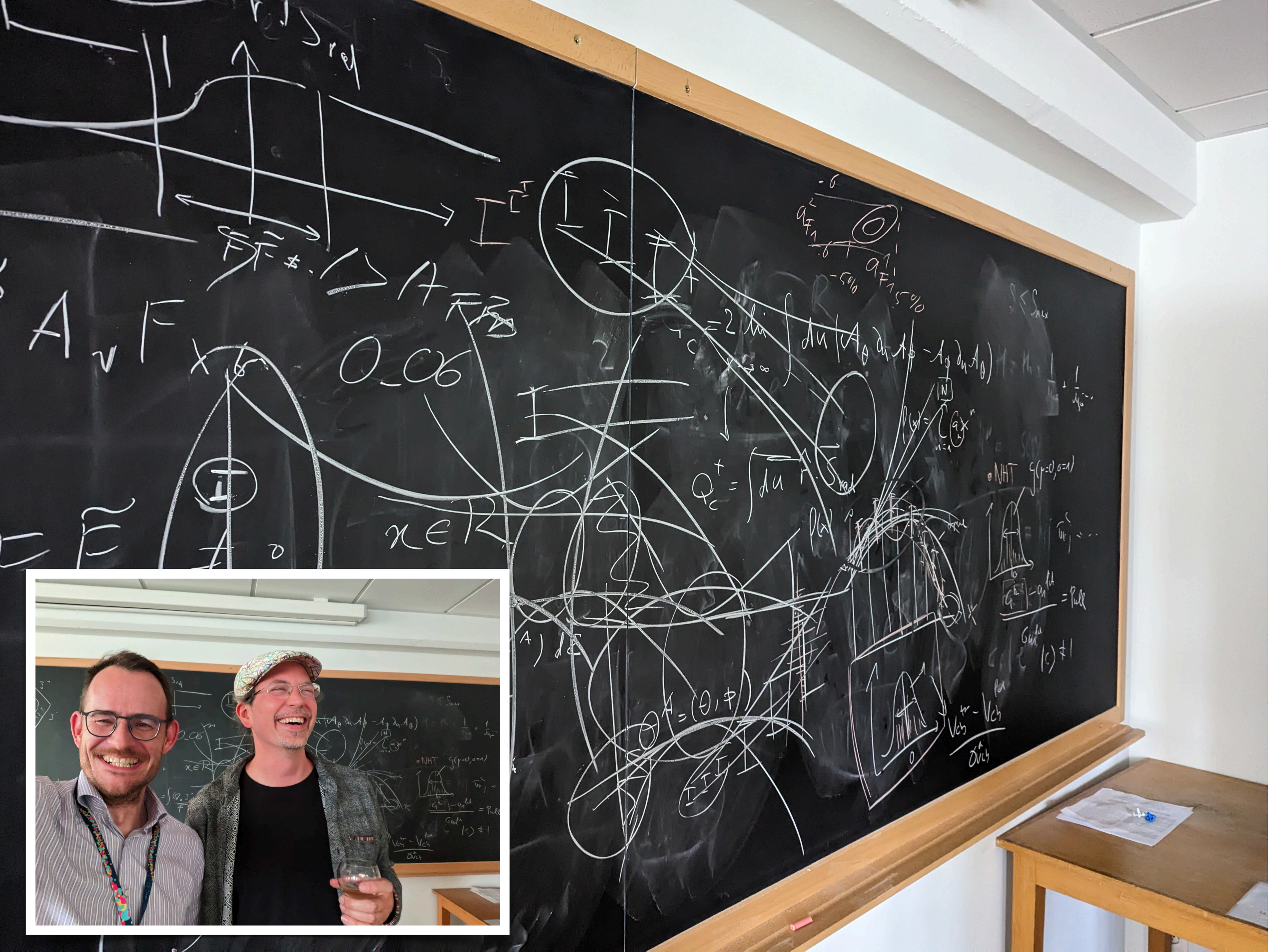
Some closing thoughts

Number of excitingdevelopments are happening:

- Many exciting new results from Belle and Belle II

↓

More to come…


0.06
NHT
= Pull

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>
> Paolo Gambino — Challenges in Semileptonic B Decays 2022

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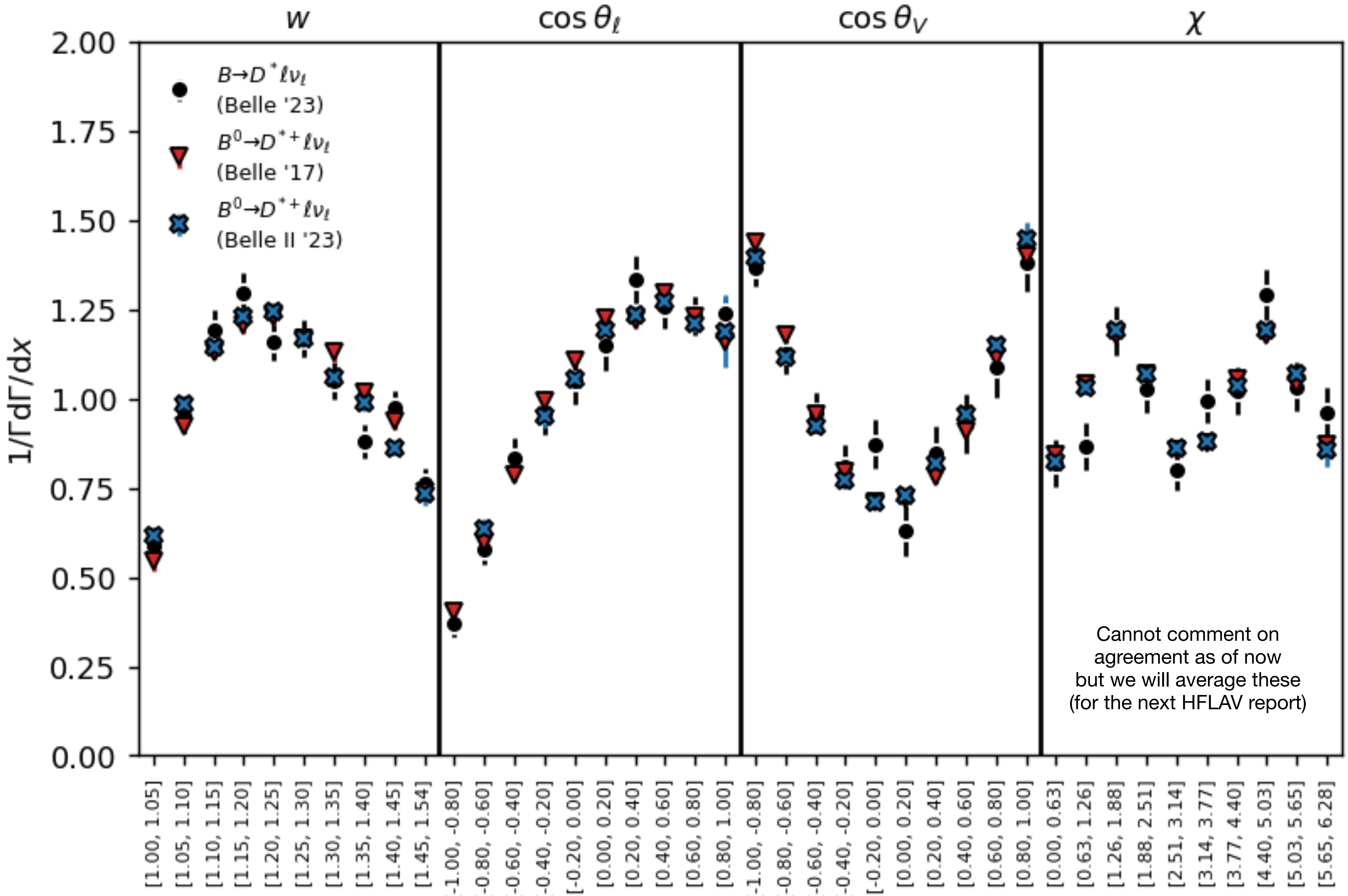
Thank you for your attention

More Information

Florian Bernlochner (florian.bernlochner@uni-bonn.de)



w
cos θ_ℓ
cos θ_V
χ
1/ΓdΓ/dx
B→D*ℓν_ℓ (Belle '23)
B⁰→D*⁺ℓν_ℓ (Belle '17)
B⁰→D*⁺ℓν_ℓ (Belle II '23)
Cannot comment on agreement as of now but we will average these (for the next HFLAV report)

Why is it important to measure $|V_{ub}|$ & $|V_{cb}|$?

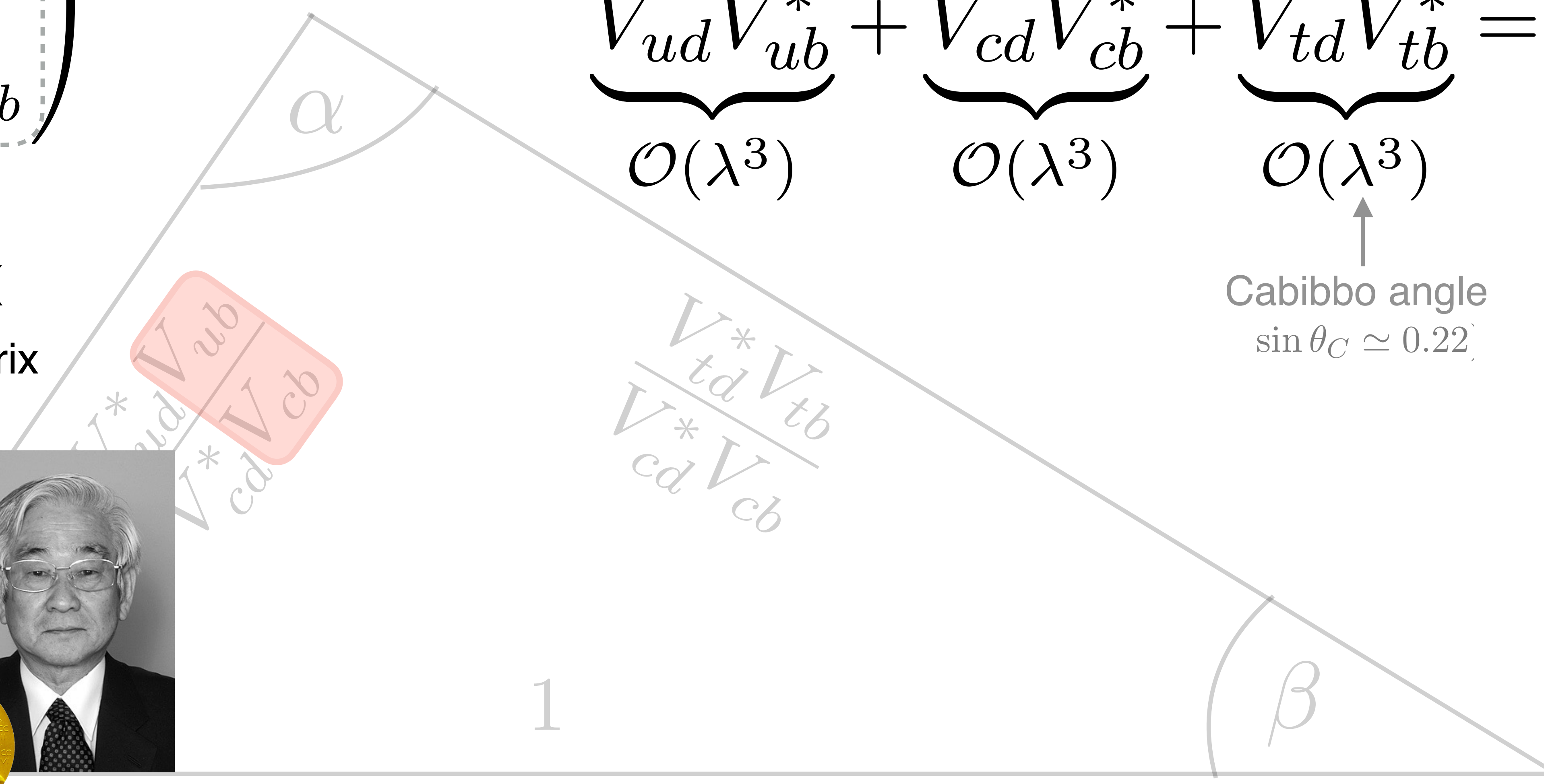
$$
\begin{array}{c} \\ u \\ c \\ t \end{array}
\begin{array}{c} \quad d \quad\quad s \quad\quad b \\ \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \end{array}
$$

Unitarity $CC^\dagger = 1$

Overconstrain Unitarity condition
→ Potent test of Standard Model

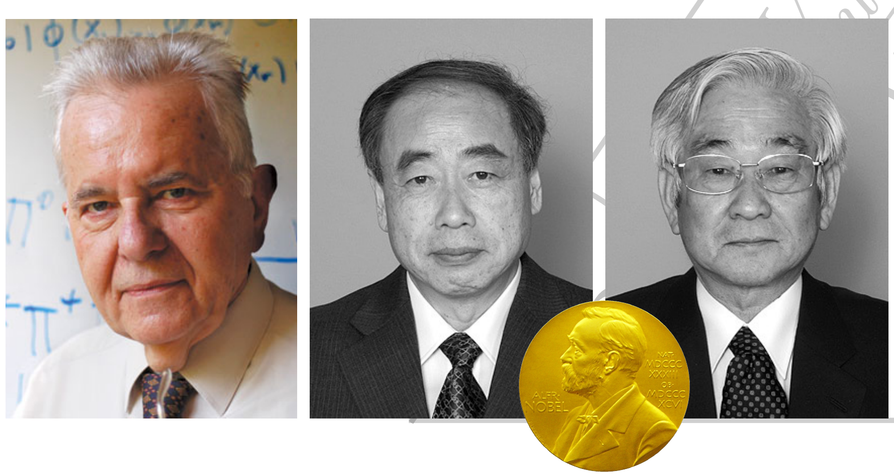
$$
\underbrace{V_{ud}V_{ub}^*}_{\mathcal{O}(\lambda^3)} + \underbrace{V_{cd}V_{cb}^*}_{\mathcal{O}(\lambda^3)} + \underbrace{V_{td}V_{tb}^*}_{\mathcal{O}(\lambda^3)} = 0
$$

Cabibbo angle
$\sin\theta_C \simeq 0.22$



CKM Matrix

SM: Unitary 3x3 Matrix



Nobel prize 2008

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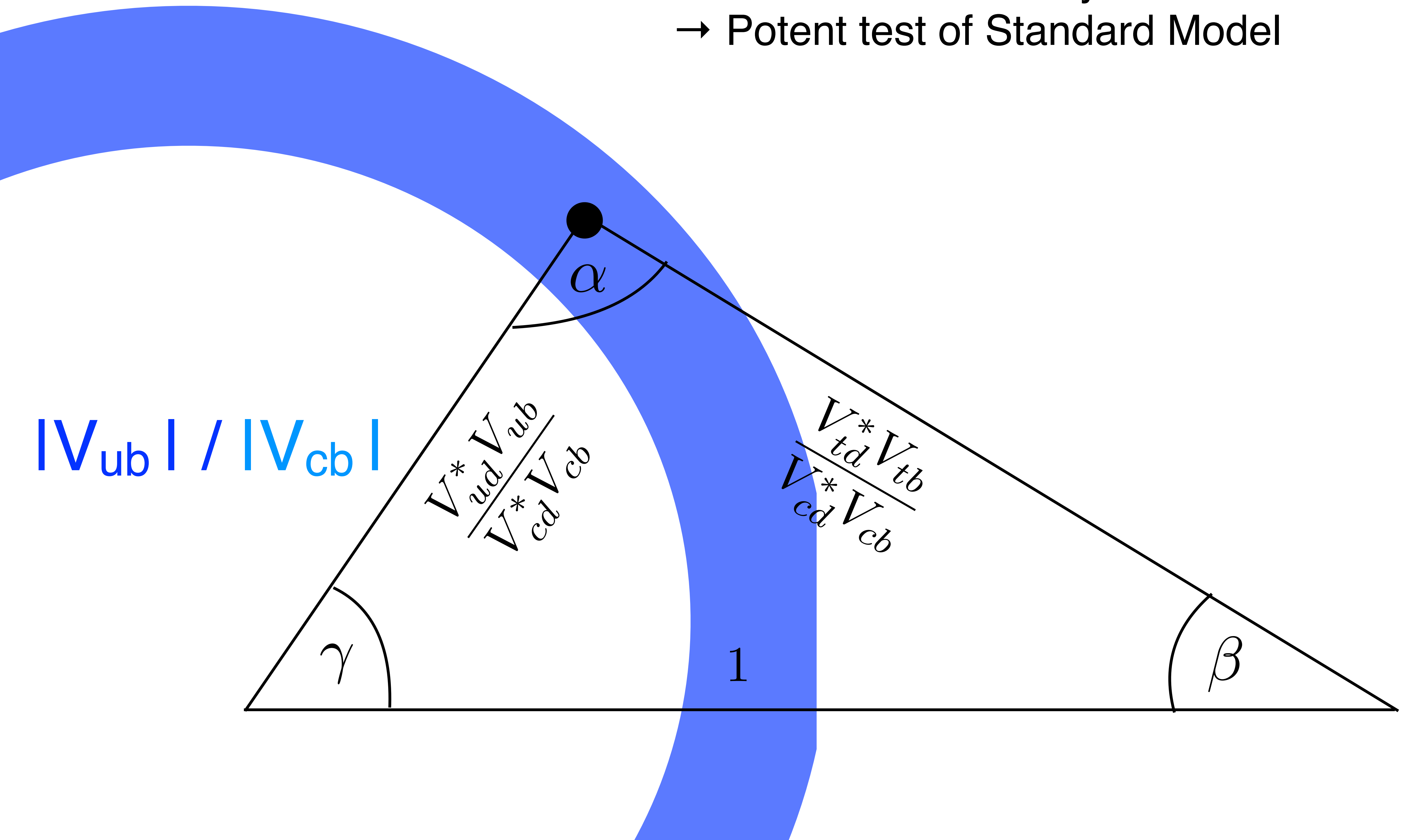

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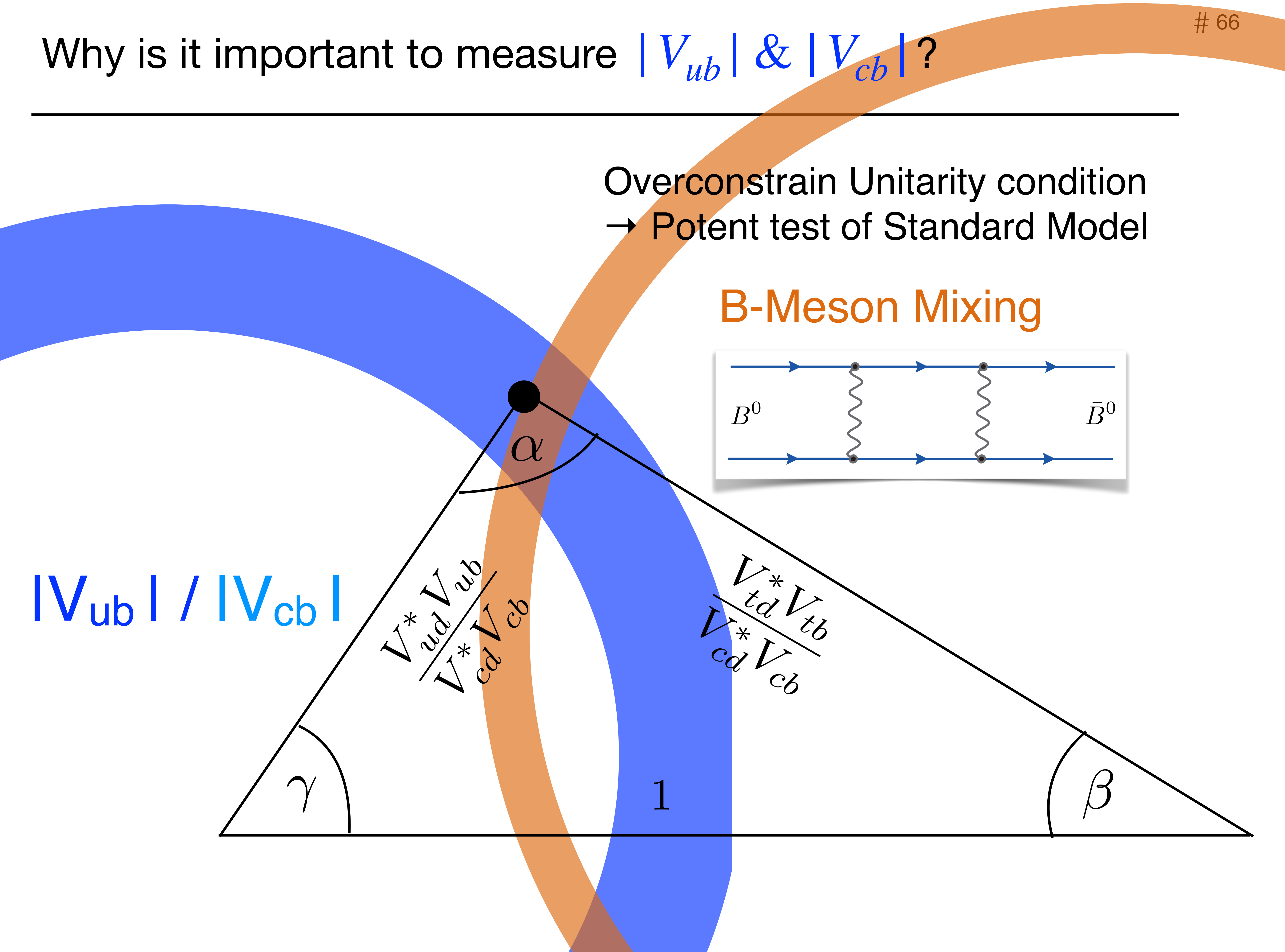
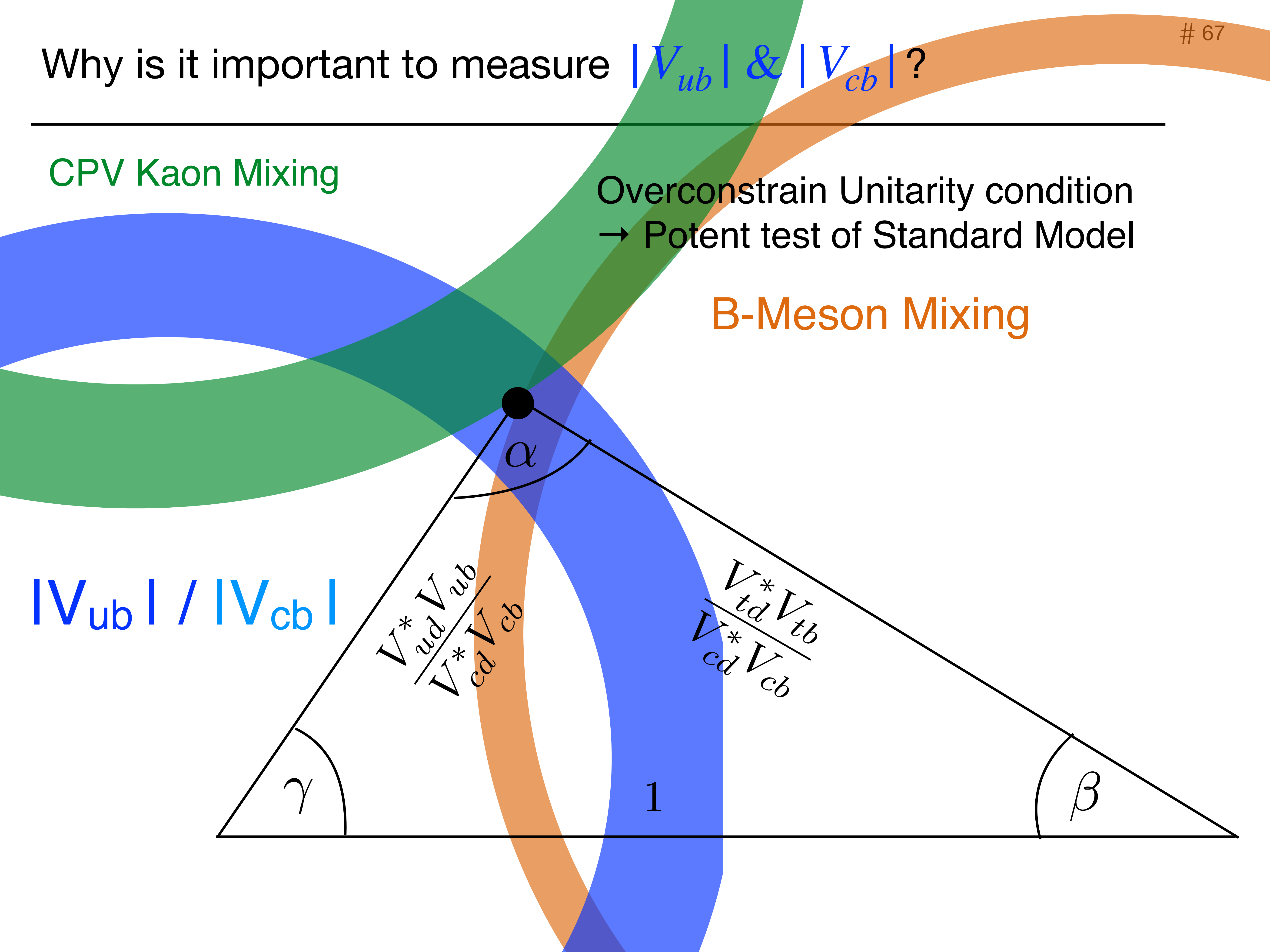


Why is it important to measure $|V_{ub}|$ & $|V_{cb}|$?

Overconstrain Unitarity condition
→ Potent test of Standard Model

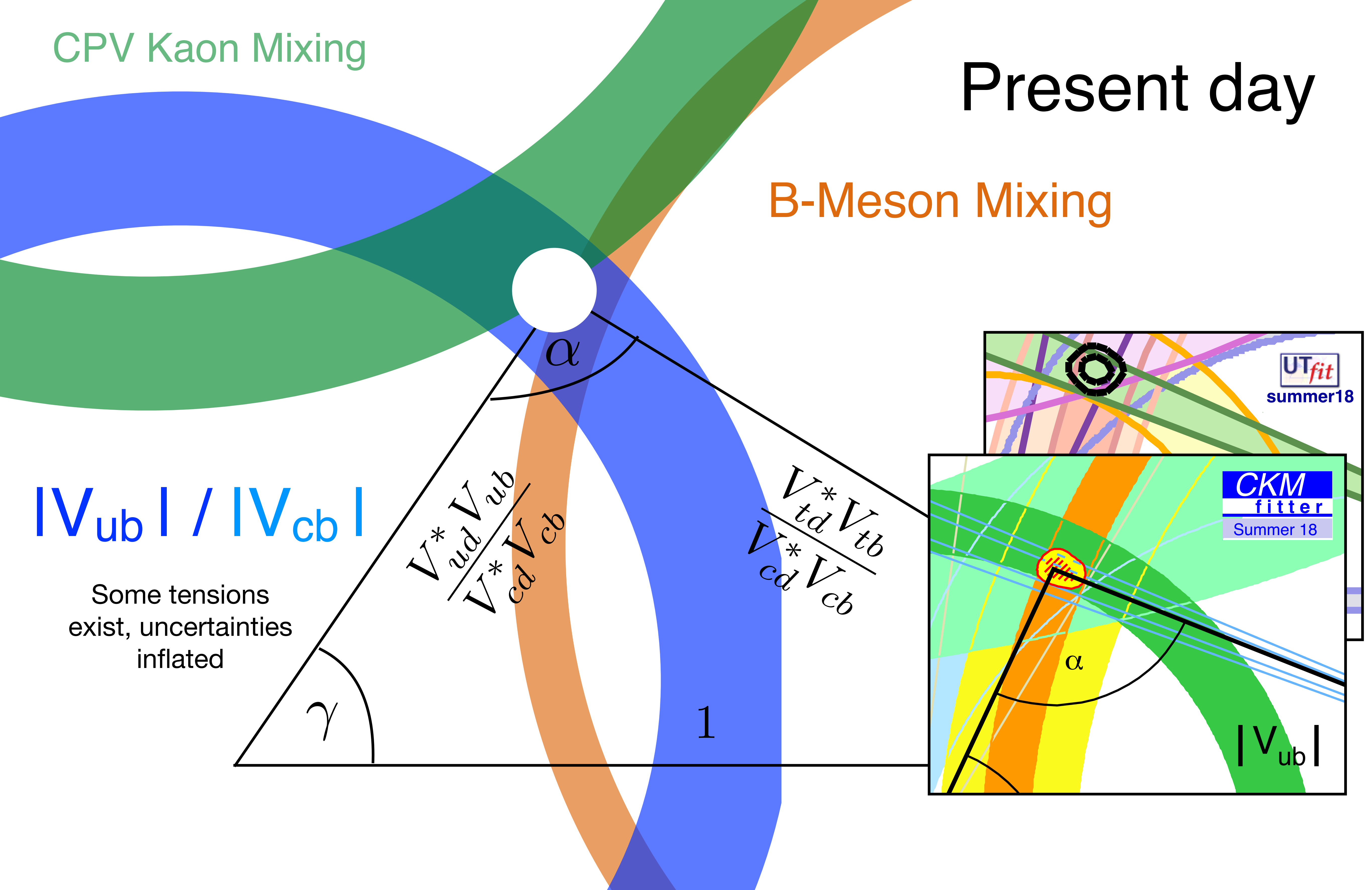


Why is it important to measure $|V_{ub}|$ & $|V_{cb}|$?

CPV Kaon Mixing

Present day

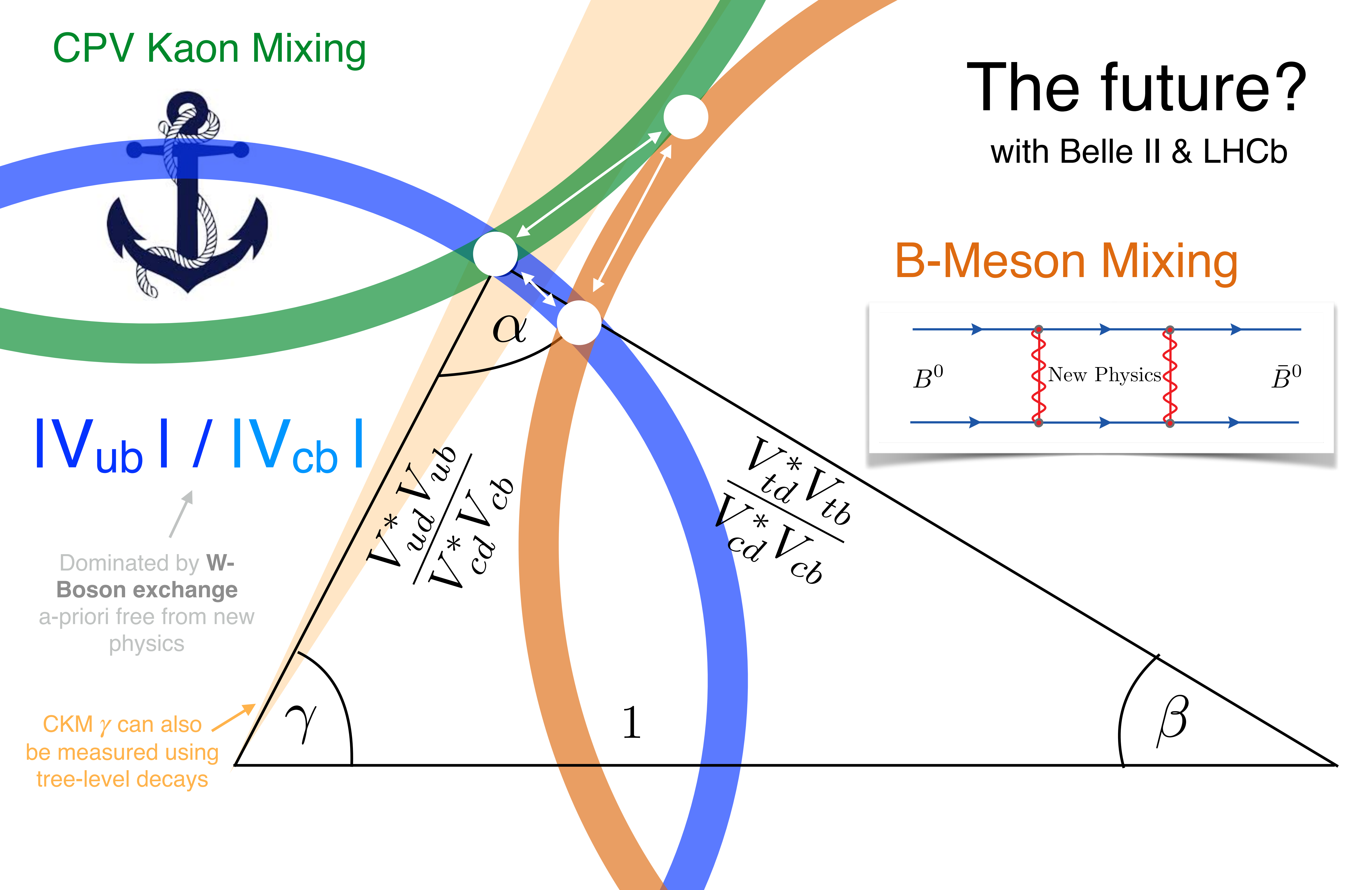
B-Meson Mixing



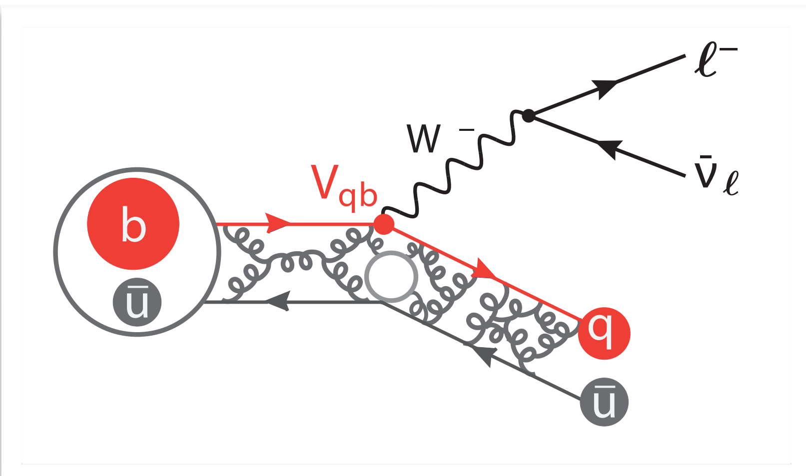
$|V_{ub}|$ / $|V_{cb}|$

Some tensions exist, uncertainties inflated

Why is it important to measure $|V_{ub}|$ & $|V_{cb}|$?



How do we study SL decays to obtain e.g. $|V_{ub}|$ & $|V_{cb}|$?



Inclusive $|V_{ub}|$

$$B \to X_u \ell \bar{\nu}_\ell$$

+ Fermi Motion / Shape Function

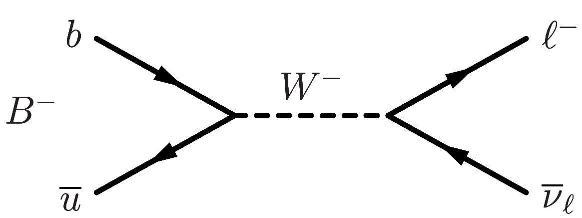
Inclusive $|V_{cb}|$

$$B \to X_c \ell \bar{\nu}_\ell$$

Operator Product Expansion

$$\mathcal{B} = |V_{qb}|^2 \left[\Gamma(b \to q\, \ell\, \bar{\nu}_\ell) + 1/m_{c,b} + \alpha_s + \ldots \right]$$

Leptonic $|V_{ub}|$



$$\mathcal{B} \propto |V_{ub}|^2 f_B^2\, m_\ell^2$$

B-Meson decay constant

Exclusive $|V_{ub}|$

$$B \to \pi, \rho, \omega\, \ell \bar{\nu}_\ell,\ \Lambda_b \to p \mu \bar{\nu}_\mu$$

$$B_s \to K \mu\, \bar{\nu}_\mu$$

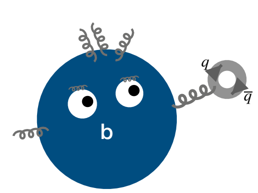
Exclusive $|V_{cb}|$

$$B_{(s)} \to D_{(s)}^{(*)} \ell \bar{\nu}_\ell$$

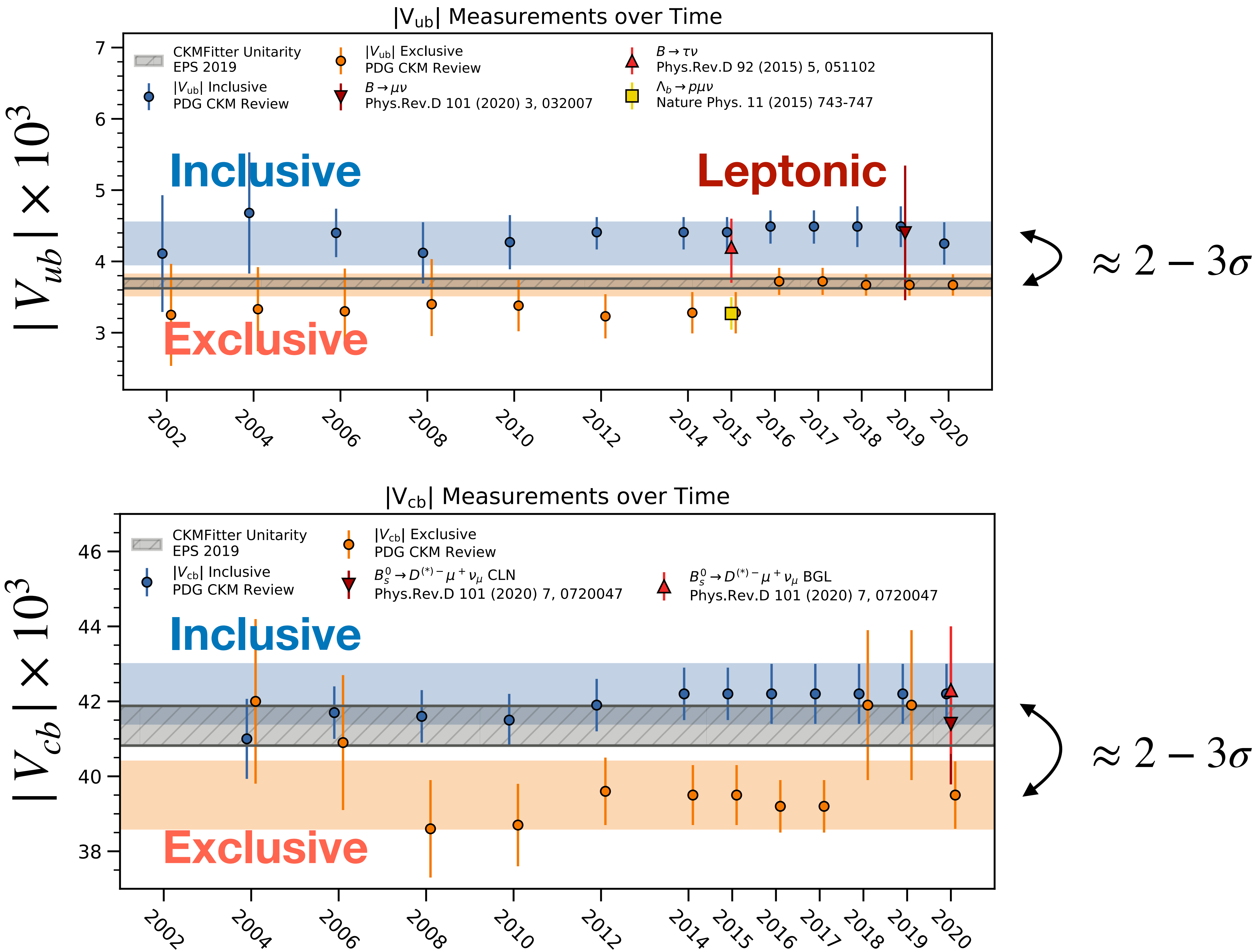
$$\mathcal{B} \propto |V_{qb}|^2 f^2$$

Form Factors

$$\langle B | H_\mu | P \rangle = (p + p')_\mu\, f_+$$



How are we doing?



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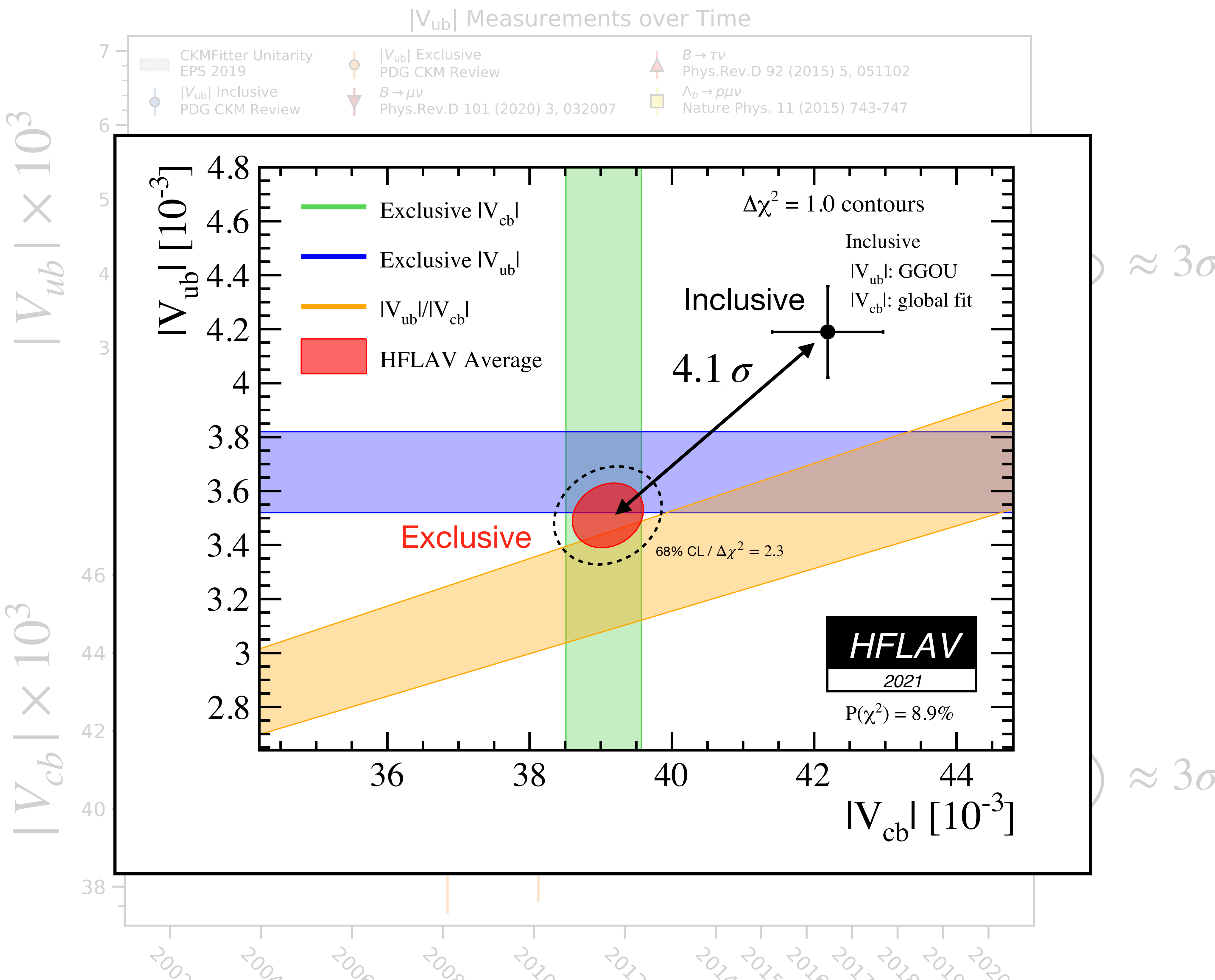
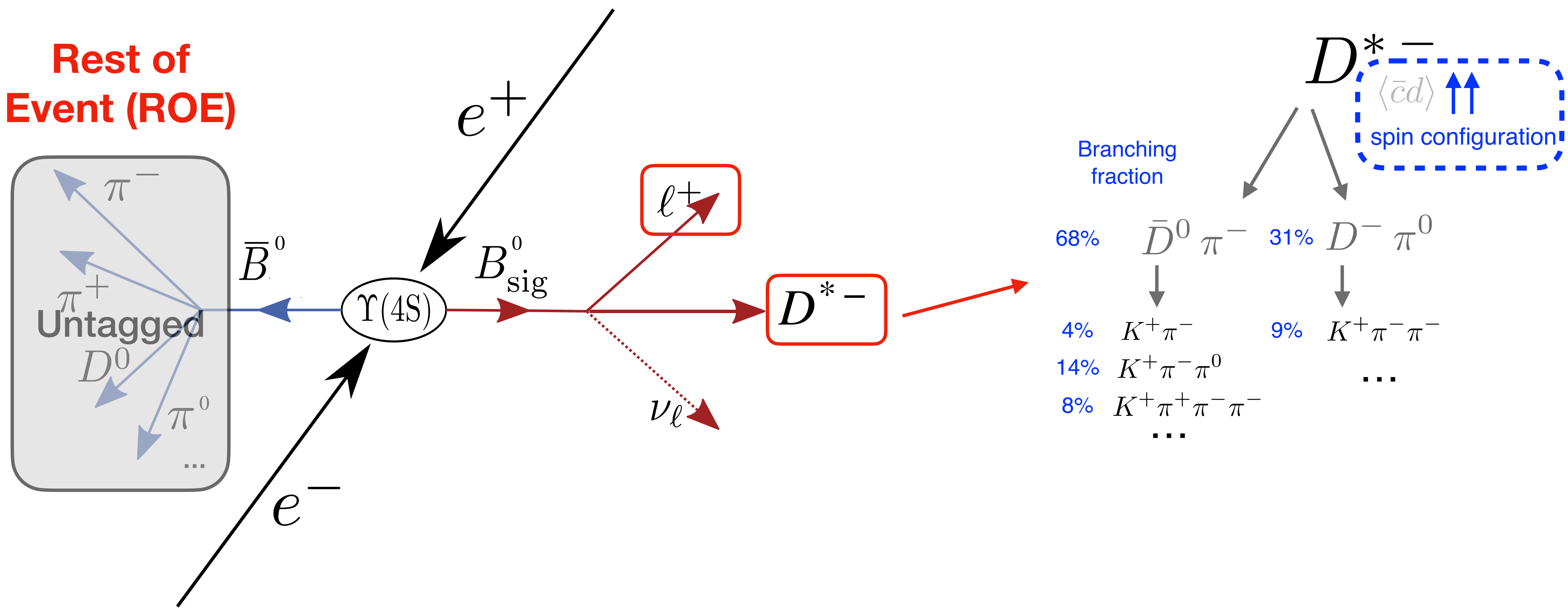


Untagged measurements of $B \to D^{(*)}\ell\bar{\nu}_\ell$



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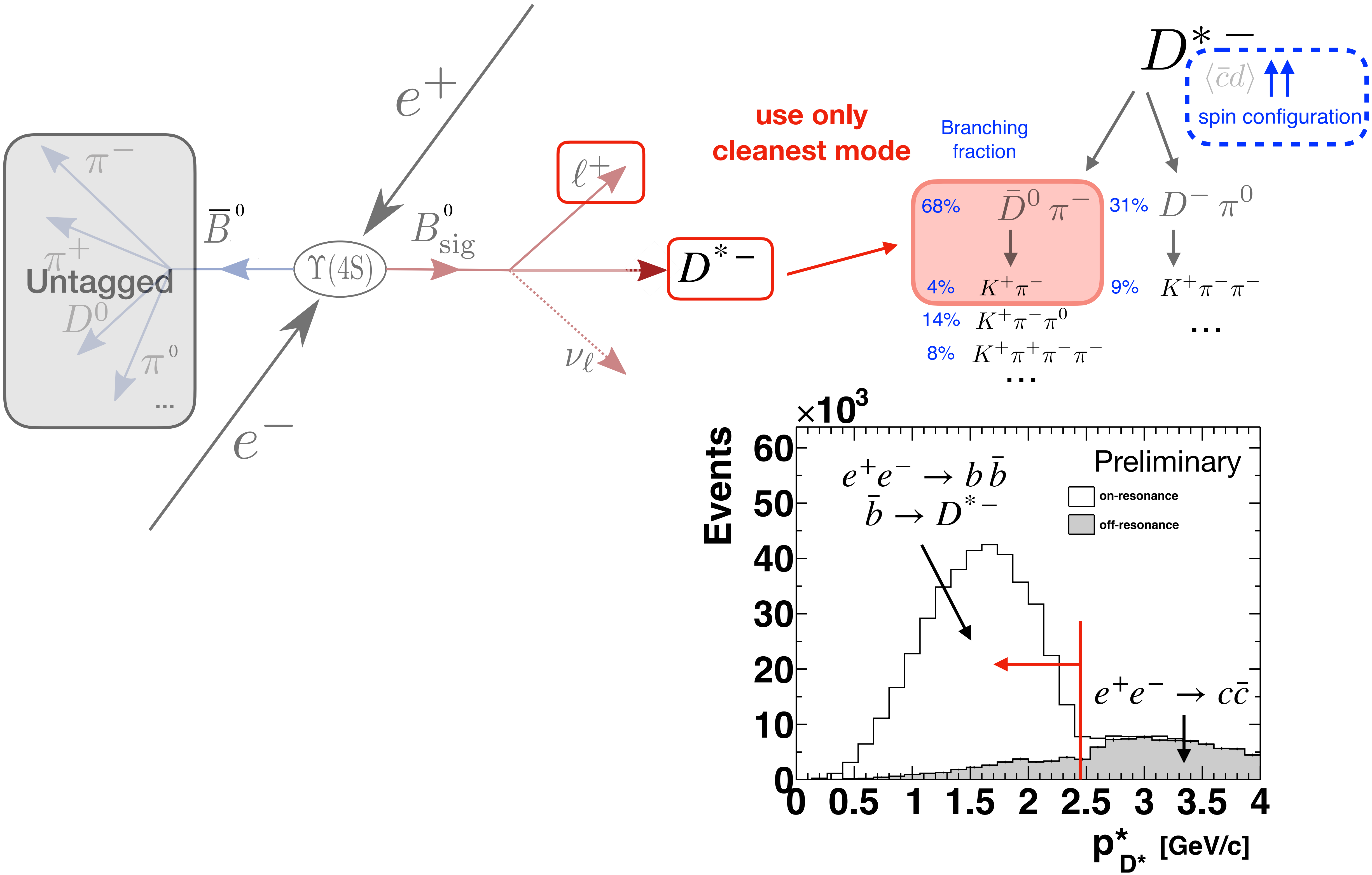

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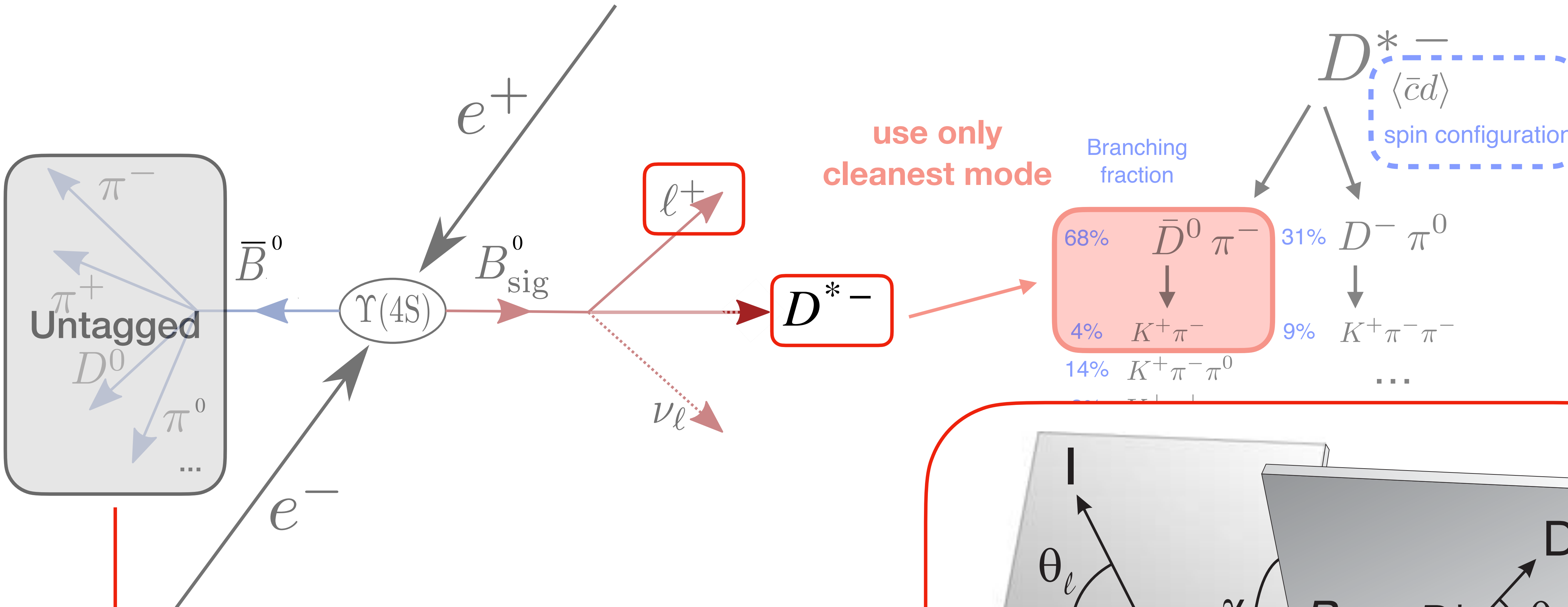
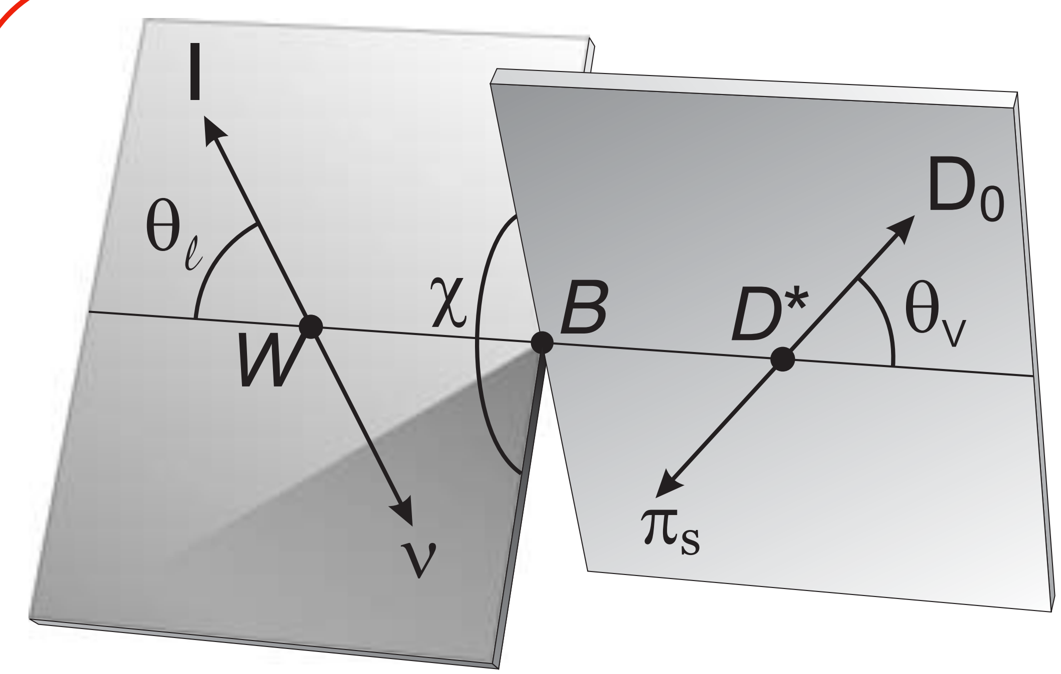


$$\vec{p}_{\rm incl.} = \sum_i \vec{p}_i \ ,$$

Reconstruct ROE to estimate B_{sig}^0 momentum

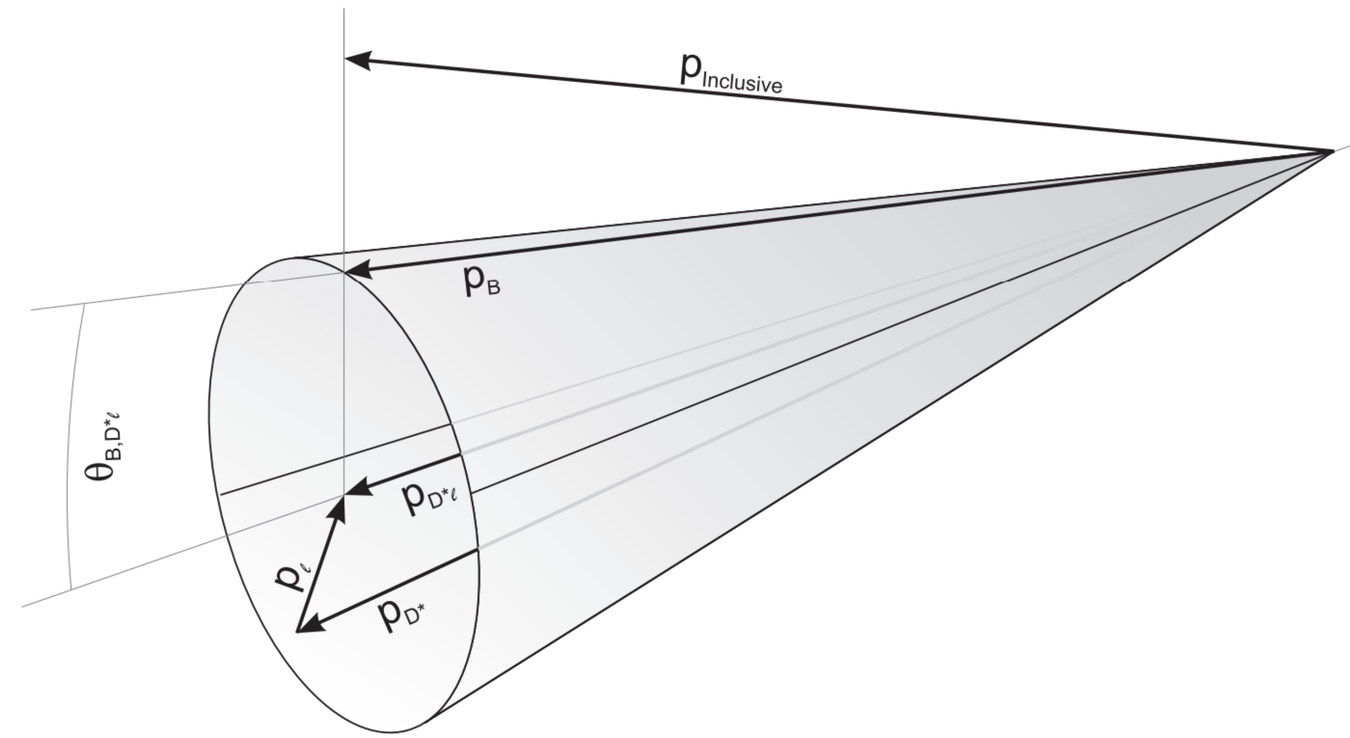
$$\vec{p}_{B_{sig}} = -\vec{p}_{\rm incl}$$



$$w \sim q^2 = (p_B - p_{D^*})^2$$

Alternative Reconstruction Methods

Can exploit that the B meson lies on a **cone**, whose opening angle is fully determined by properties of visible particles:



$$\cos\theta_{B,D^*\ell} = \frac{2E_B E_{D^*\ell} - m_B^2 - m_{D^*\ell}^2}{2|\mathbf{p}_B||\mathbf{p}_{D\ell}|}$$

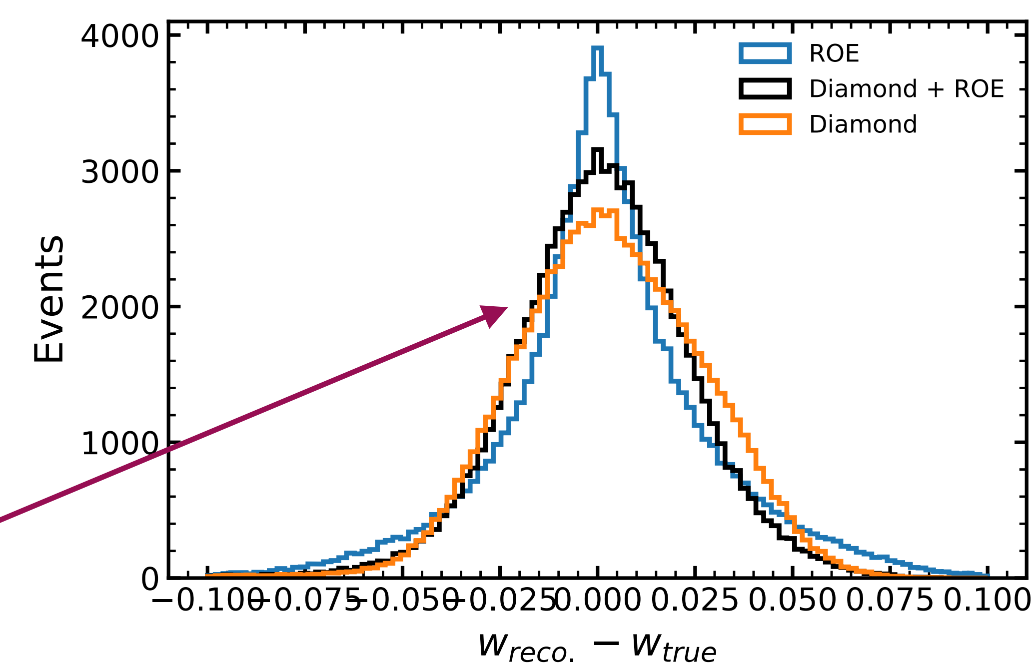
Can use this to estimate B meson direction building a weighted average on the cone

$$(E^B, p_B^x, p_B^y, p_B^z) = (\sqrt{s}/2, |\mathbf{p}_B|\sin\theta_{BY}\cos\phi, |\mathbf{p}_B|\sin\theta_{BY}\sin\phi, |\mathbf{p}_B|\cos\theta_{BY})$$

with weights according to $w_i = \sin^2\theta_i$ with θ denoting the polar angle

(following the angular distribution of $\Upsilon(4S) \to B\bar{B}$)

One can also **combine** both estimates



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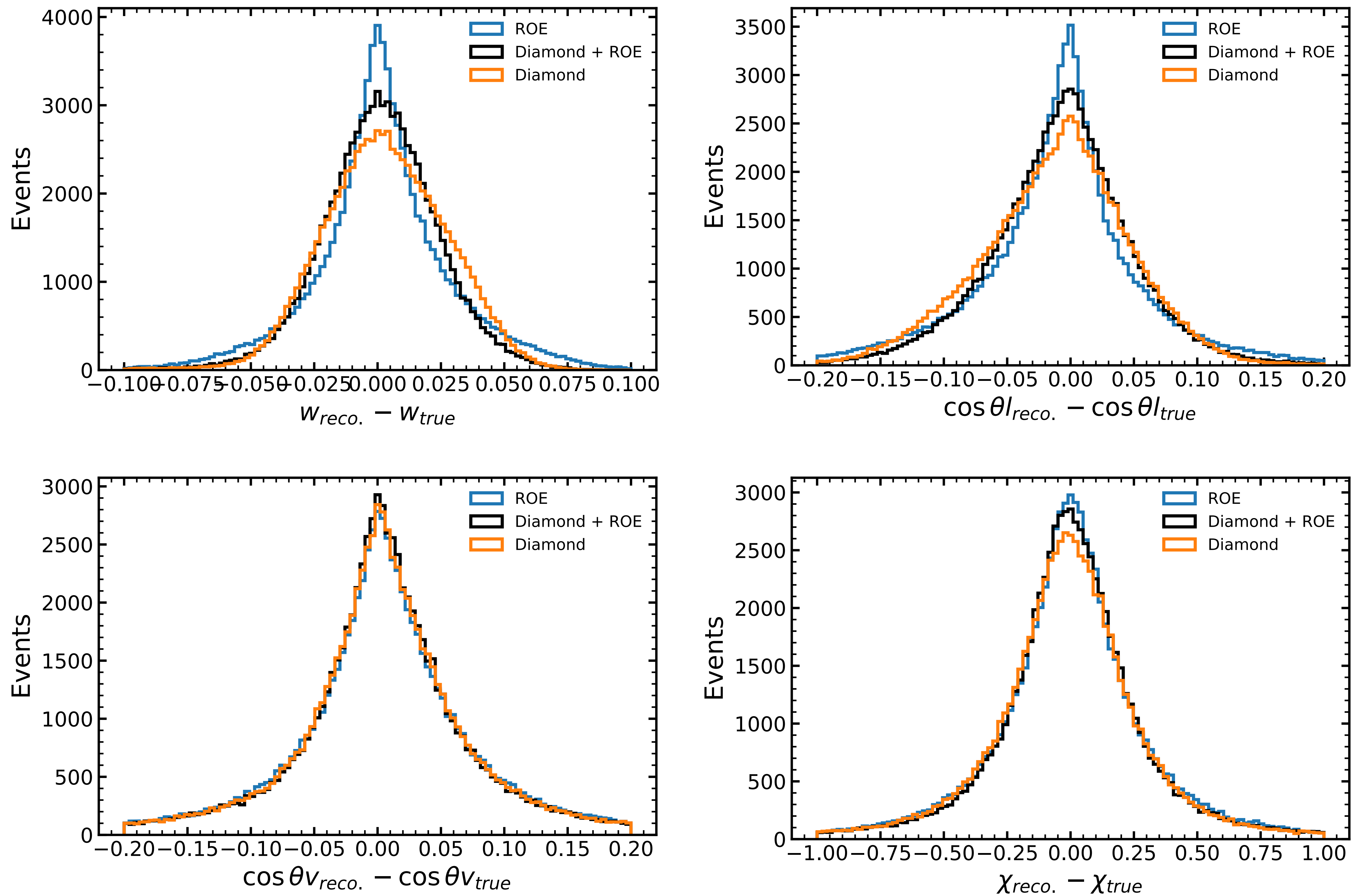


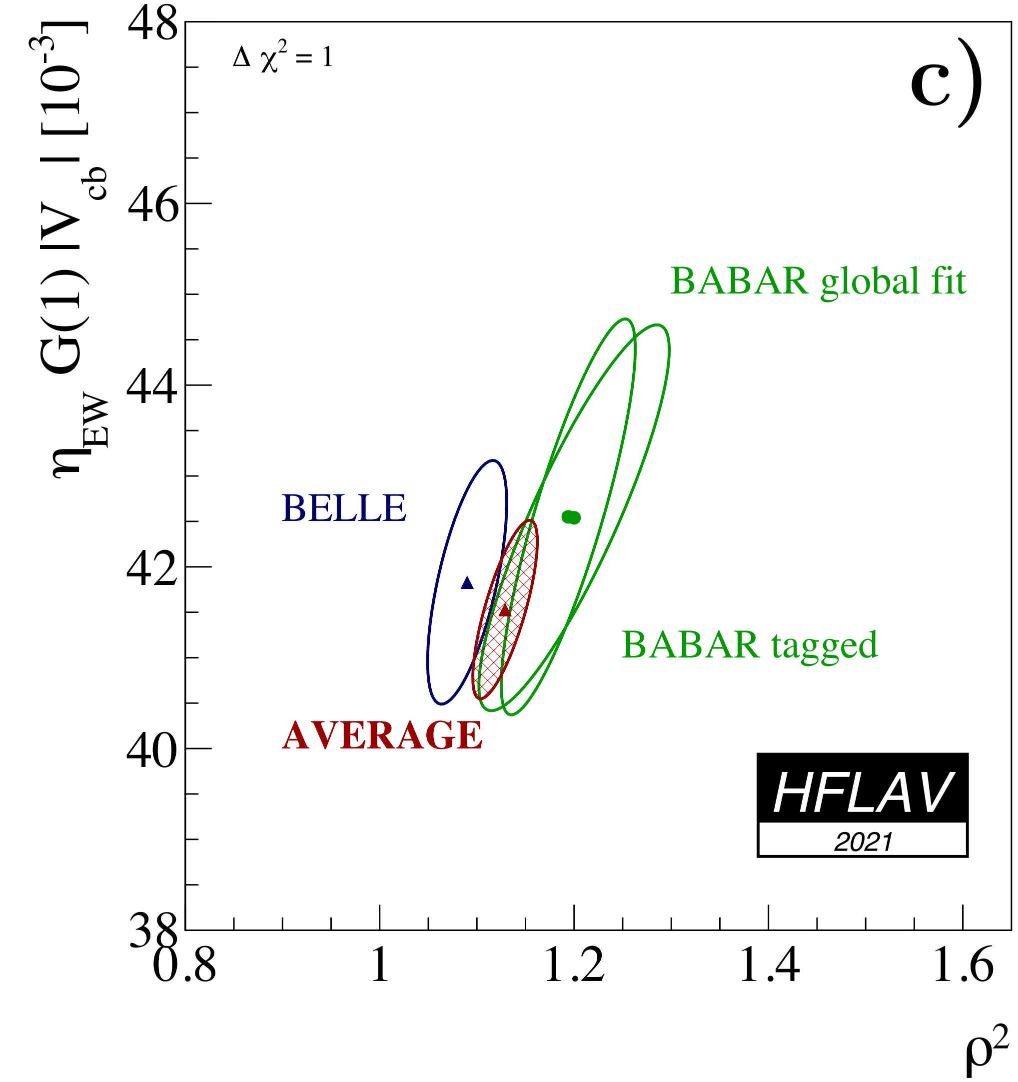
More than a decade of $B \to D^{(*)}\ell\bar{\nu}_\ell$ is “lost” :-(

For $B \to D^{(*)}\ell\bar{\nu}_\ell$ traditionally **single form factor** parametrization (Caprini-Lellouch-Neubert, **CLN**) was used. Nucl.Phys. B530 (1998) 153–181

Measurements directly determined the parameters and quoted these with correlations.

Problem: Theory knowledge advances; **today more** general parametrization are preferred (**BGL**, …)

Experiment	$\eta_{\rm EW}\mathcal{F}(1)\lvert V_{cb}\rvert[10^{-3}]$ (rescaled) $\eta_{\rm EW}\mathcal{F}(1)\lvert V_{cb}\rvert[10^{-3}]$ (published)	ρ^2 (rescaled) ρ^2 (published)
ALEPH [497]	$31.38 \pm 1.80_{\rm stat} \pm 1.24_{\rm syst}$ $31.9 \pm 1.8_{\rm stat} \pm 1.9_{\rm syst}$	$0.488 \pm 0.226_{\rm stat} \pm 0.146_{\rm syst}$ $0.37 \pm 0.26_{\rm stat} \pm 0.14_{\rm syst}$
CLEO [501]	$40.16 \pm 1.24_{\rm stat} \pm 1.54_{\rm syst}$ $43.1 \pm 1.3_{\rm stat} \pm 1.8_{\rm syst}$	$1.363 \pm 0.084_{\rm stat} \pm 0.087_{\rm syst}$ $1.61 \pm 0.09_{\rm stat} \pm 0.21_{\rm syst}$
OPAL excl [498]	$36.20 \pm 1.58_{\rm stat} \pm 1.47_{\rm syst}$ $36.8 \pm 1.6_{\rm stat} \pm 2.0_{\rm syst}$	$1.198 \pm 0.206_{\rm stat} \pm 0.153_{\rm syst}$ $1.31 \pm 0.21_{\rm stat} \pm 0.16_{\rm syst}$
OPAL partial reco [498]	$37.44 \pm 1.20_{\rm stat} \pm 2.32_{\rm syst}$ $37.5 \pm 1.2_{\rm stat} \pm 2.5_{\rm syst}$	$1.090 \pm 0.137_{\rm stat} \pm 0.297_{\rm syst}$ $1.12 \pm 0.14_{\rm stat} \pm 0.29_{\rm syst}$
DELPHI partial reco [499]	$35.52 \pm 1.41_{\rm stat} \pm 2.29_{\rm syst}$ $35.5 \pm 1.4_{\rm stat}\ {}^{+2.3}_{-2.4\rm syst}$	$1.139 \pm 0.123_{\rm stat} \pm 0.382_{\rm syst}$ $1.34 \pm 0.14_{\rm stat}\ {}^{+0.24}_{-0.22\rm syst}$
DELPHI excl [500]	$35.87 \pm 1.69_{\rm stat} \pm 1.95_{\rm syst}$ $39.2 \pm 1.8_{\rm stat} \pm 2.3_{\rm syst}$	$1.070 \pm 0.141_{\rm stat} \pm 0.153_{\rm syst}$ $1.32 \pm 0.15_{\rm stat} \pm 0.33_{\rm syst}$
Belle [502]	$34.82 \pm 0.15_{\rm stat} \pm 0.55_{\rm syst}$ $35.06 \pm 0.15_{\rm stat} \pm 0.56_{\rm syst}$	$1.106 \pm 0.031_{\rm stat} \pm 0.008_{\rm syst}$ $1.106 \pm 0.031_{\rm stat} \pm 0.007_{\rm syst}$
BABAR excl [503]	$33.37 \pm 0.29_{\rm stat} \pm 0.97_{\rm syst}$ $34.7 \pm 0.3_{\rm stat} \pm 1.1_{\rm syst}$	$1.182 \pm 0.048_{\rm stat} \pm 0.029_{\rm syst}$ $1.18 \pm 0.05_{\rm stat} \pm 0.03_{\rm syst}$
BABAR D^{*0} [507]	$34.55 \pm 0.58_{\rm stat} \pm 1.06_{\rm syst}$ $35.9 \pm 0.6_{\rm stat} \pm 1.4_{\rm syst}$	$1.124 \pm 0.058_{\rm stat} \pm 0.053_{\rm syst}$ $1.16 \pm 0.06_{\rm stat} \pm 0.08_{\rm syst}$
BABAR global fit [509]	$35.45 \pm 0.20_{\rm stat} \pm 1.08_{\rm syst}$ $35.7 \pm 0.2_{\rm stat} \pm 1.2_{\rm syst}$	$1.171 \pm 0.019_{\rm stat} \pm 0.060_{\rm syst}$ $1.21 \pm 0.02_{\rm stat} \pm 0.07_{\rm syst}$
Average	$\mathbf{35.00 \pm 0.11_{\rm stat} \pm 0.34_{\rm syst}}$	$\mathbf{1.121 \pm 0.014_{\rm stat} \pm 0.019_{\rm syst}}$



Old measurements **cannot be updated** the underlying distributions were not provided but only the result of the fit.

Obviously we should **avoid** this in the future.

The emergence of beyond zero-recoil lattice:

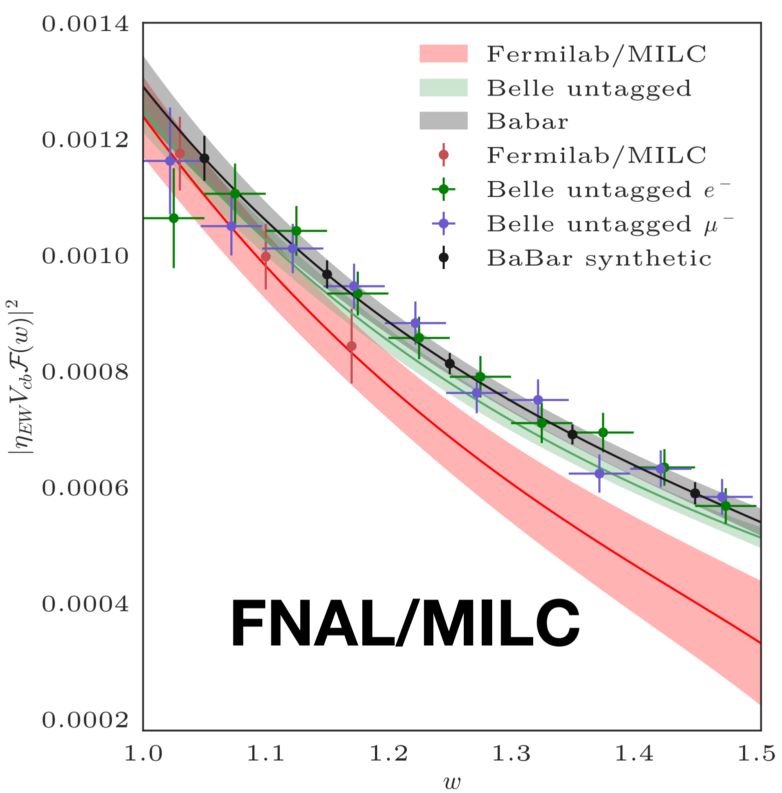
Very exciting times:

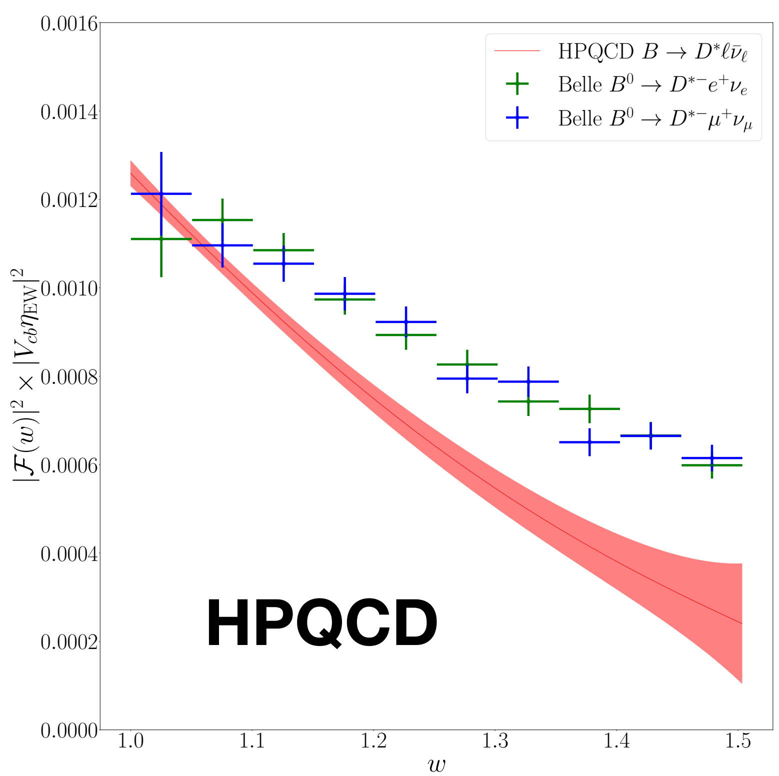
A. Bazavov et al. [FNAL/MILC] [Eur. Phys. J. C 82, 1141 (2022), arXiv:2105.14019]

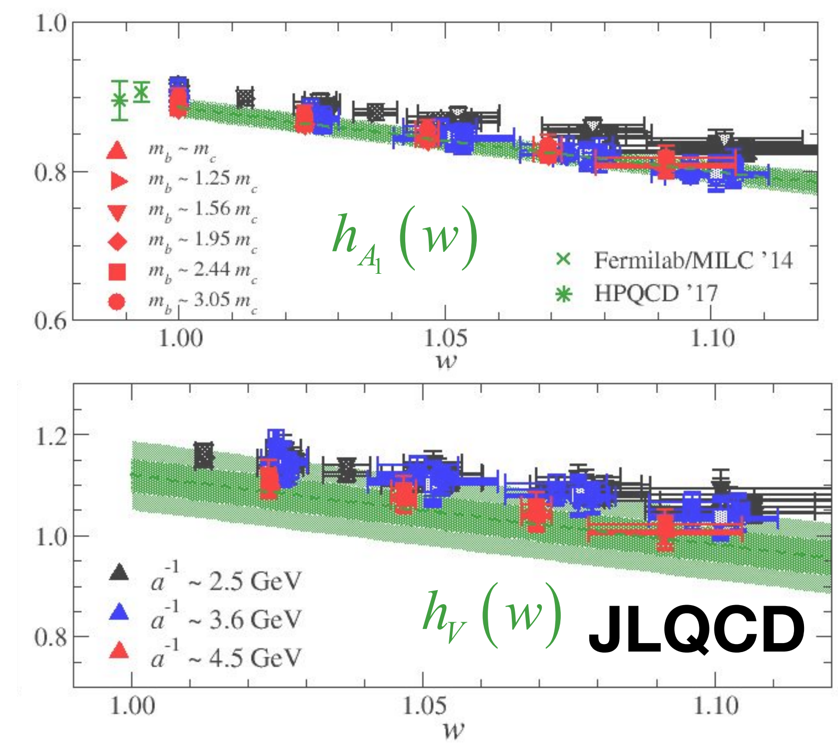
J. Harrison & T.H. Davies [HPQCD] [arXiv:2304.03137 [hep-lat]]

After more than 10 years in the making, we have beyond zero recoil LQCD predictions for $B \to D^* \ell \bar{\nu}_\ell$

Three groups: One published, One freshly on arxiv, One preliminary :







Tension with measured shapes …

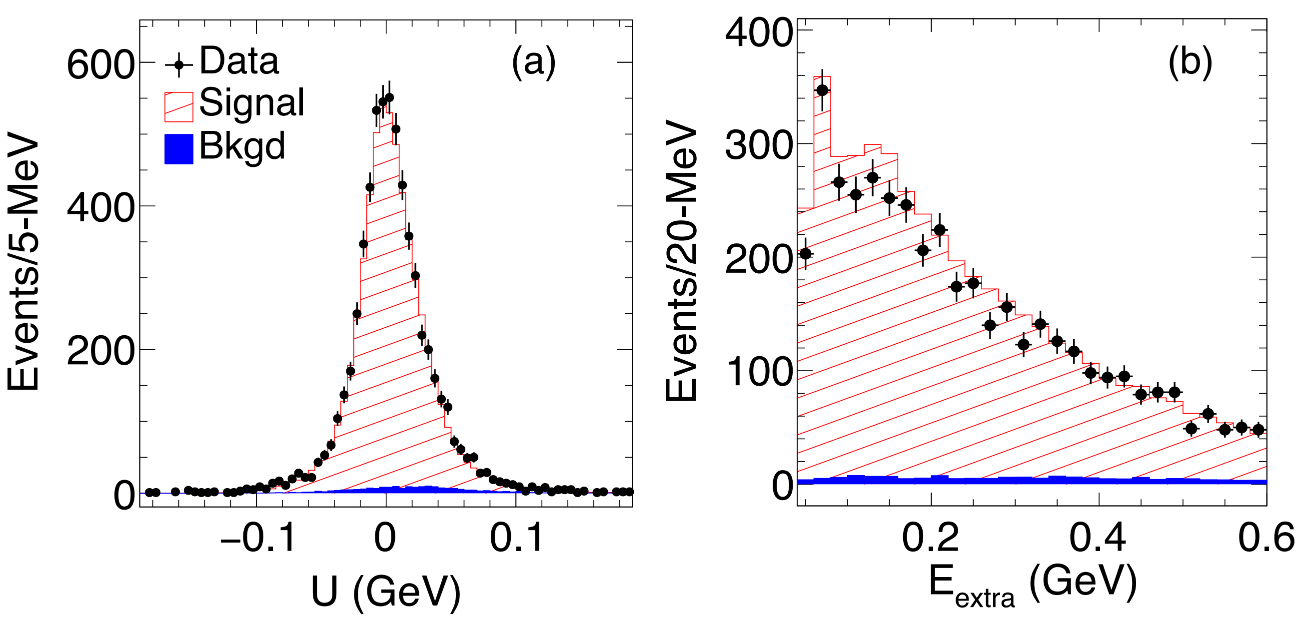
BGL is much better, model independent

So is it ok to just present results with Boyd Grinstein Lebed (**BGL**) ?

BGL looks great:

- it removes the relation between slope and curvature on the leading form factor; **data can pull it.**
- Slop and curvature of the form factor ratios $R_{1/2}$ are not constrained, **data can pull it.**

Beautiful **unbinned 4D fit (!)** from BaBar [Phys. Rev. Lett. 123, 091801 (2019)]



$a_0^f \times 10^2$	$a_1^f \times 10^2$	$a_1^{F_1} \times 10^2$	$a_0^g \times 10^2$	$a_1^g \times 10^2$	$\|V_{cb}\| \times 10^3$
1.29 ±0.03	1.63 ±1.00	0.03 ±0.11	2.74 ±0.11	8.33 ±6.67	38.36 ±0.90

TABLE I. The $N = 1$ BGL expansion results of this analysis, including systematic uncertainties.

$\rho^2_{D^*}$	$R_1(1)$	$R_2(1)$	$\|V_{cb}\| \times 10^3$
0.96 ± 0.08	1.29 ± 0.04	0.99 ± 0.04	38.40 ± 0.84

TABLE II. The CLN fit results from this analysis, including systematic uncertainties.

Truncation Order

Model independence is a **step forward**, but **choices have** to be **made** here as well..

$$g(z) = \frac{1}{P_g(z)\phi_g(z)} \sum_{n=0}^{N} a_n z^n, \qquad f(z) = \frac{1}{P_f(z)\phi_f(z)} \sum_{n=0}^{N} b_n z^n, \qquad \mathcal{F}_1(z) = \frac{1}{P_{\mathcal{F}_1}(z)\phi_{\mathcal{F}_1}(z)} \sum_{n=0}^{N} c_n z^n,$$

One Problem you face as an experimentalist: where do you truncate?

Truncate too soon:

- Model dependence in extracted result for $|V_{cb}|$?

Truncate too late:

- Unnecessarily increase variance on $|V_{cb}|$?

Is there an **ideal** truncation order?

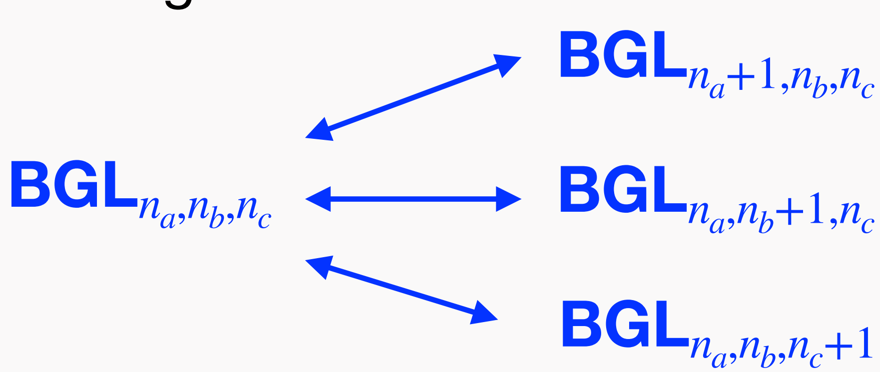
What about **additional constraints**?

Nested Hypothesis Tests or Saturation Constraints

Z. Ligeti, D. Robinson, M. Papucci, FB
[arXiv:1902.09553, PRD100,013005 (2019)]

Use a **nested hypothesis test (NHT)** to determine optimal truncation order

Challenge nested fits



Test statistics & Decision boundary

$\Delta\chi^2 = \chi^2_N - \chi^2_{N+1}$ $\qquad$ $\Delta\chi^2 > 1$

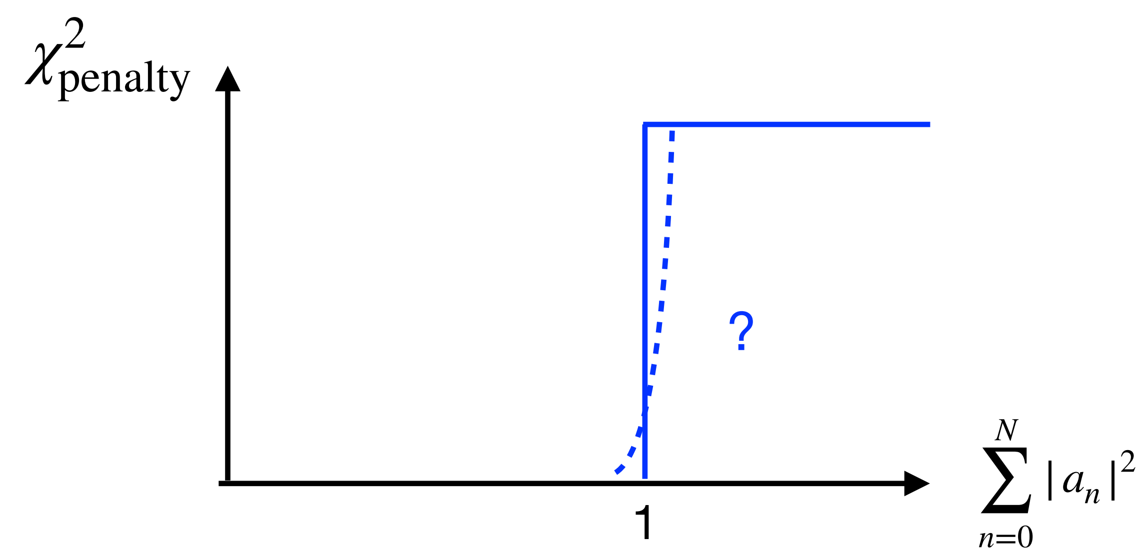
Distributed like a χ^2-distribution with 1 dof (Wilk's theorem)

Gambino, Jung, Schacht
[arXiv:1905.08209, PLB]

Constrain contributions from higher order coefficients using **unitarity bounds**

$$\sum_{n=0}^{N} |a_n|^2 \leq 1 \qquad \sum_{n=0}^{N} \left(|b_n|^2 + |c_n|^2 \right) \leq 1$$

e.g. $\quad \chi^2 \to \chi^2 + \chi^2_{\text{penalty}}$



Nesting Procedure

Steps:

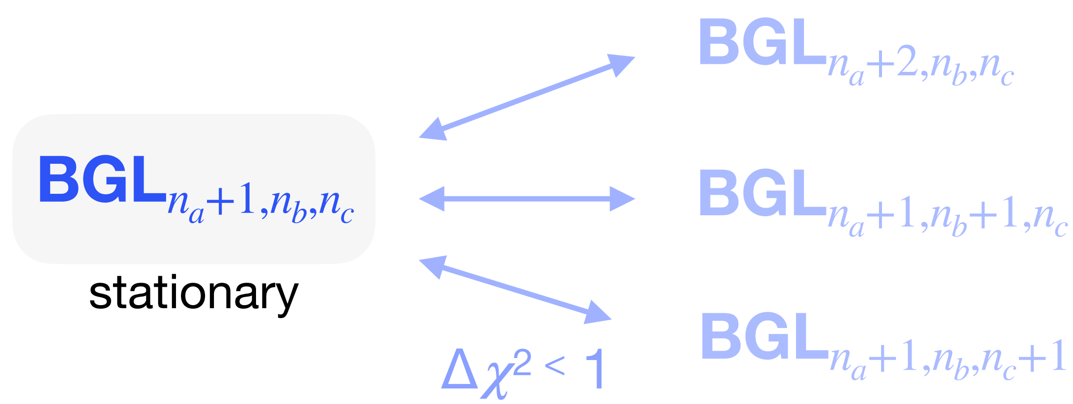
1 Carry out nested fits with one parameter added

2 Accept descendant over parent fit, if $\Delta\chi^2 > 1$

3 Repeat 1 and 2 until you find **stationary** points

4 If multiple **stationary** points remain, choose the one with smallest N, then smallest χ^2

5 Reject scenarios that **produce strong correlations** (= blind directions)



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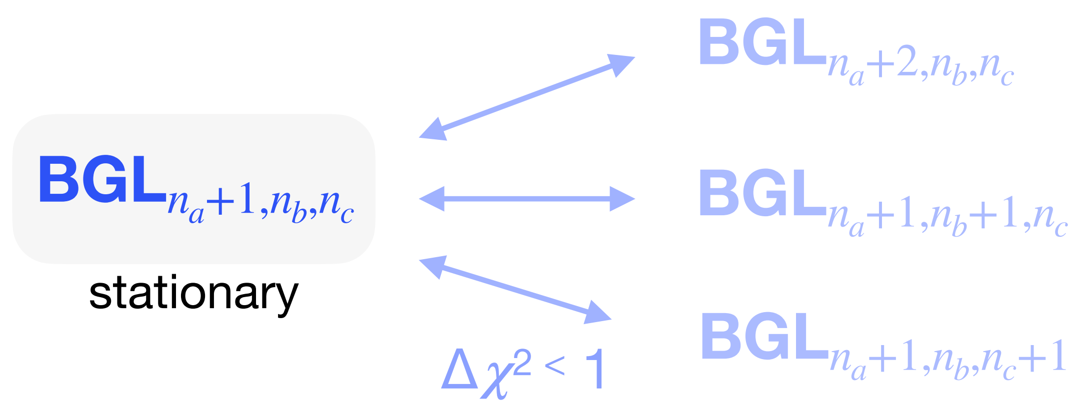

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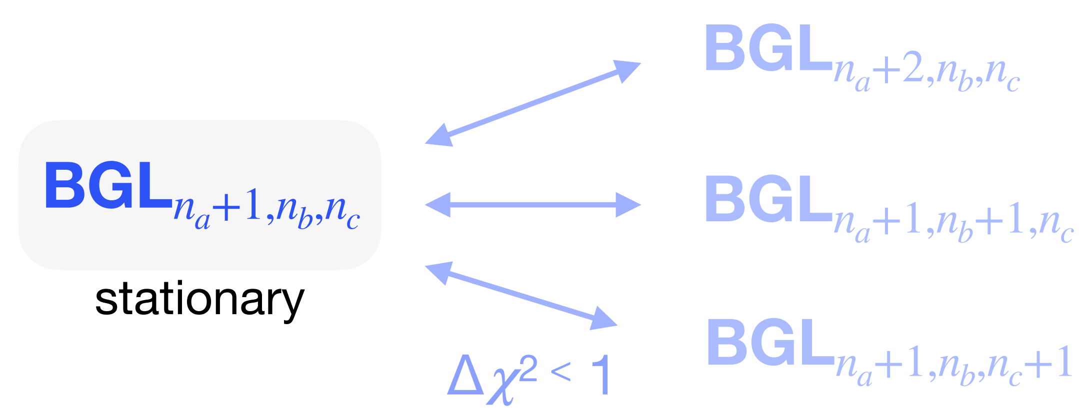

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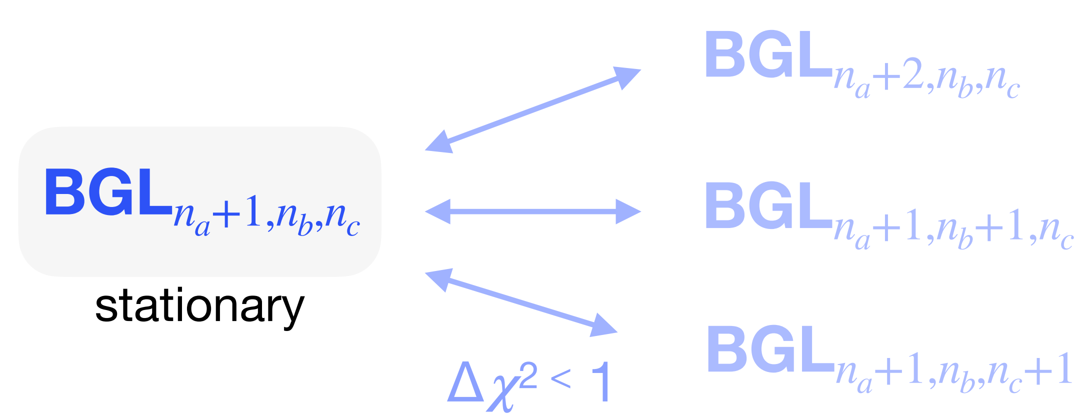

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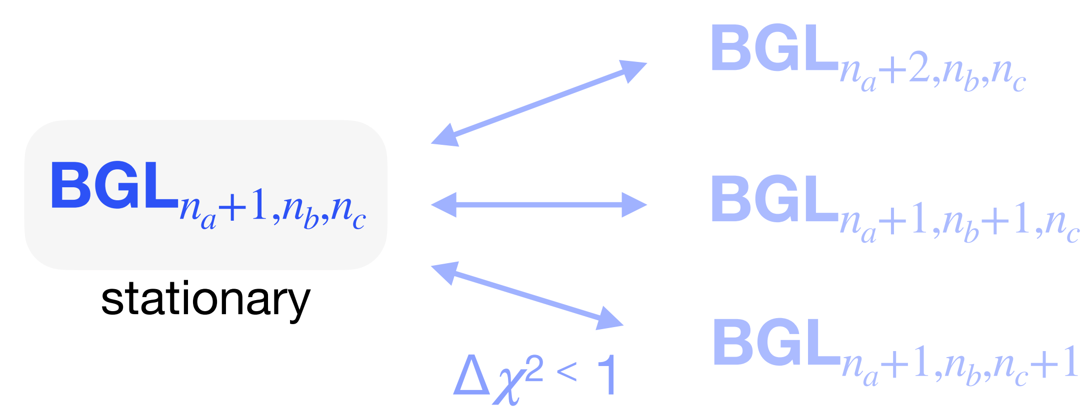


Toy study to illustrate possible bias

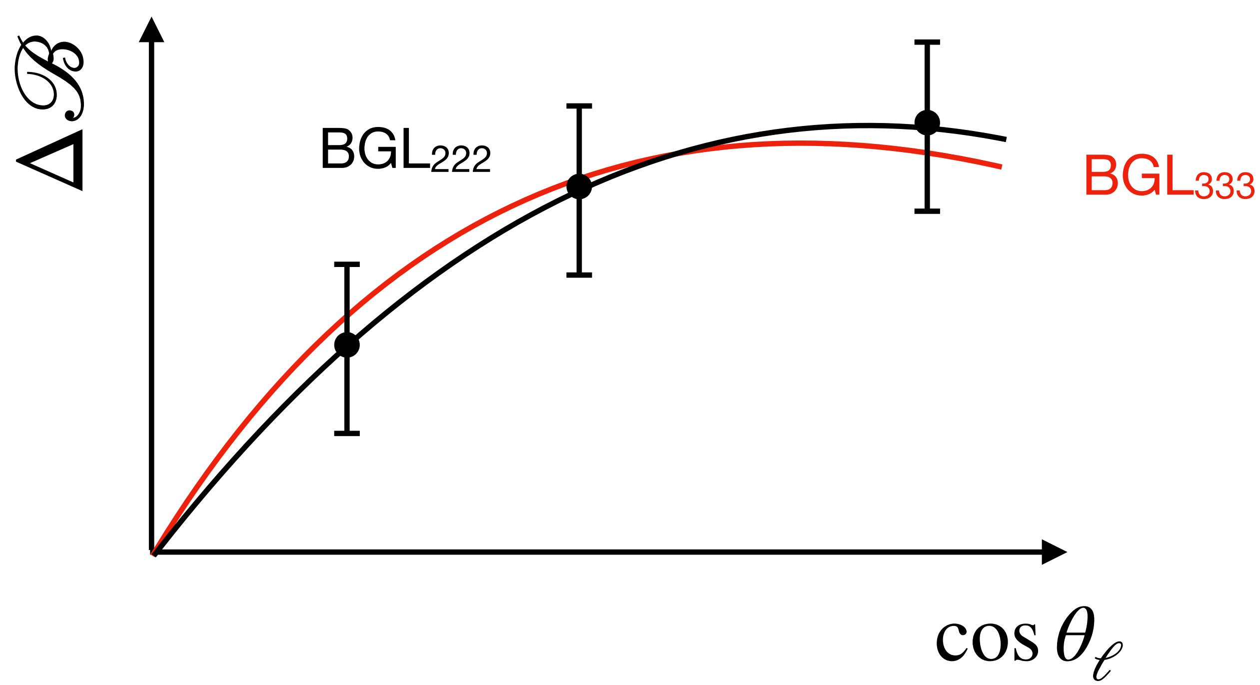
Use the central values of the **BGL_{222} fit** as a starting point to add **fine structure**

fit = fit to prel. 2017 Belle data

	'1-times'	'10-times'
Parameter	Value $\times\ 10^2$	Value $\times\ 10^2$
$\tilde{a}_2$	2.6954	26.954
$\tilde{b}_2$	-0.2040	-2.040
$\tilde{c}_3$	0.5350	5.350

Create a "true" higher order Hypothesis of order **BGL_{333}**

Has fine structure element the **current data cannot resolve**



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Has fine structure element the **current data cannot resolve**

Toy Test

Produce **ensemble** of toy measurements using **meas. covariance** & **BGL_{333}** central values

Each toy is fitted to build the descendant tree and carry out a **NHT** to select its preferred **$BGL n_a n_b n_c$**

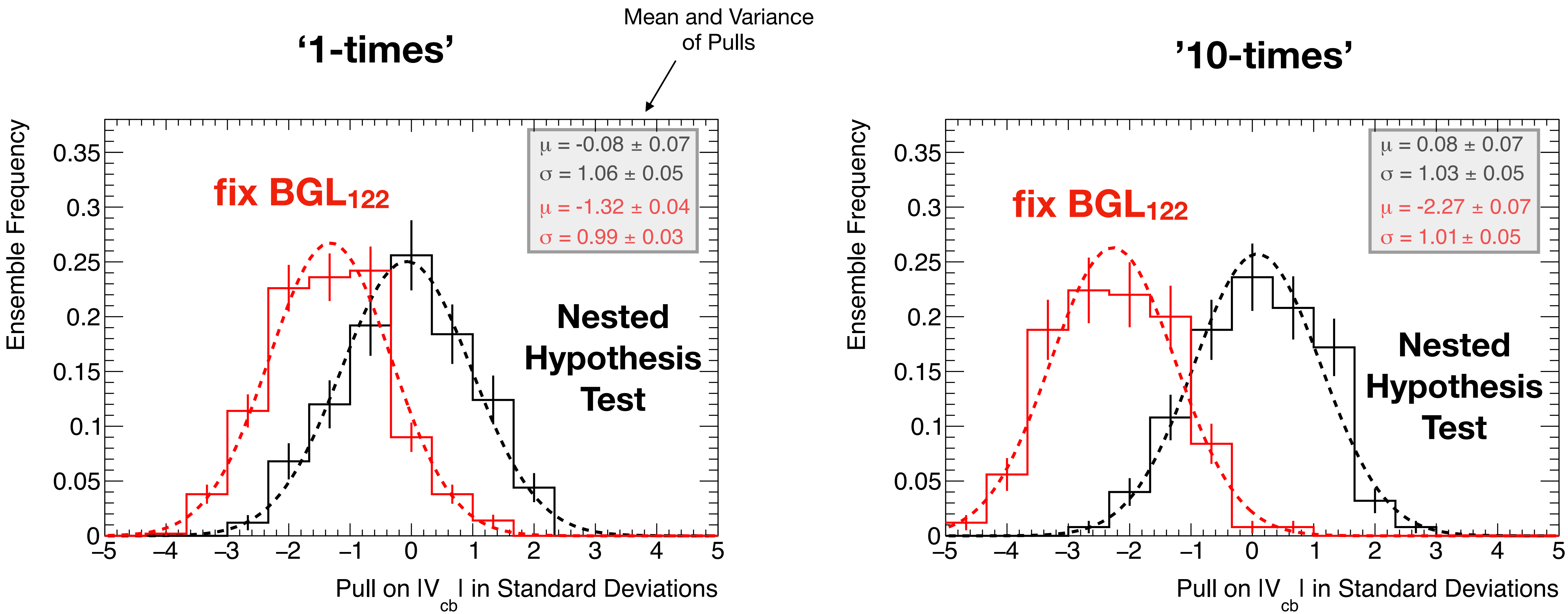
Construct Pulls

$$\textbf{Pull} = \frac{\left|V_{cb}\right|_{\textbf{true}} - \left|V_{cb}\right|_{\textbf{toy}}}{\Delta \left|V_{cb}\right|_{\textbf{toy}}}$$

As calculated from selected $BGL n_a n_b n_c$ fit of each toy

If methodology unbiased, should follow a standard normal distribution (mean 0, width 1)

Bias



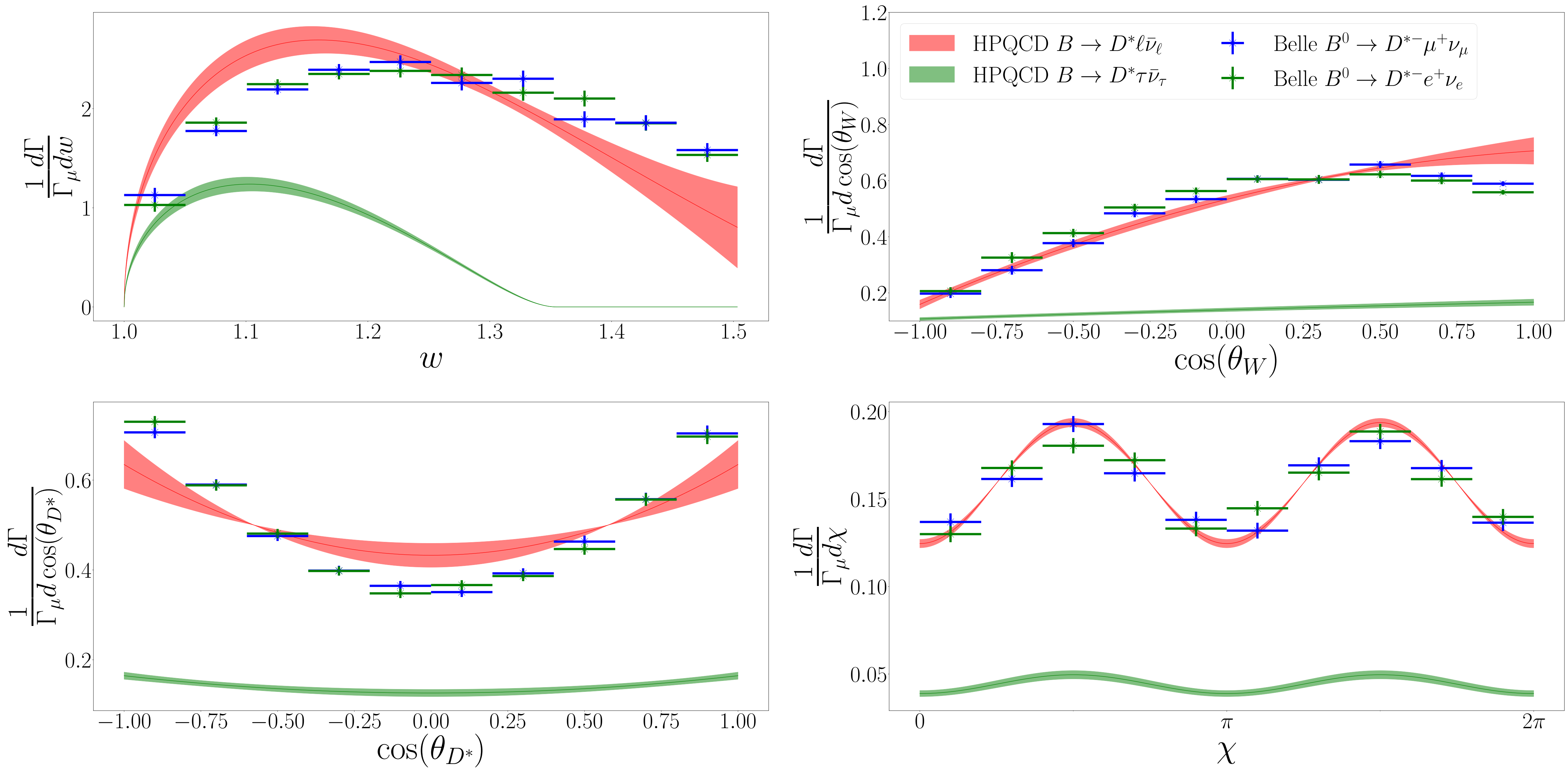
→ Procedure produces **unbiased** $|V_{cb}|$ values, just picking a given hypothesis (BGL_{122}) **does not**

Relative Frequency of selected Hypothesis:

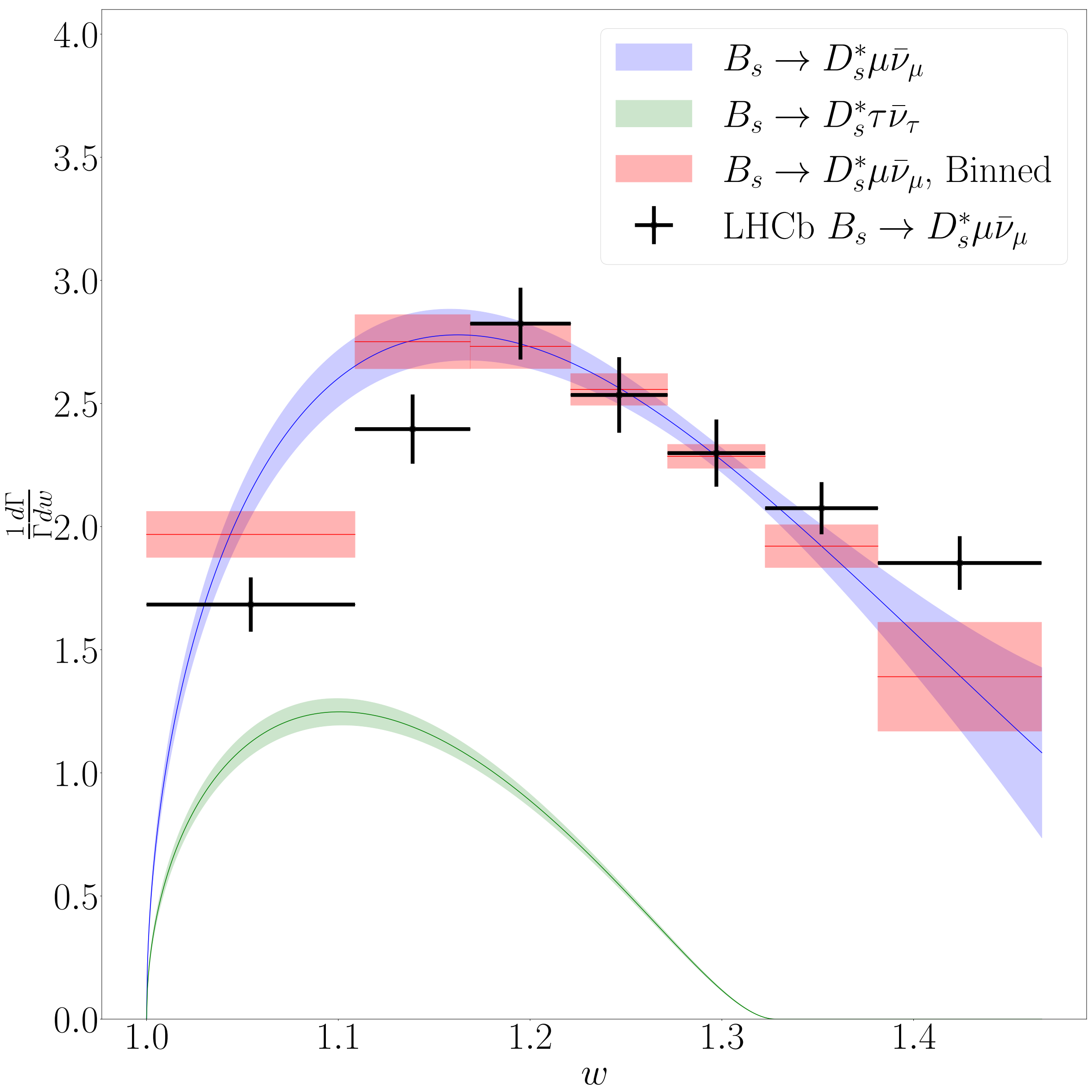
	BGL_{122}	BGL_{212}	BGL_{221}	BGL_{222}	BGL_{223}	BGL_{232}	BGL_{322}	BGL_{233}	BGL_{323}	BGL_{332}	BGL_{333}
1-times	6%	0%	37%	27%	6%	6%	11%	0%	2%	4%	0.4%
10-times	0%	0%	8%	38%	14%	8%	16%	3%	4%	8%	1%

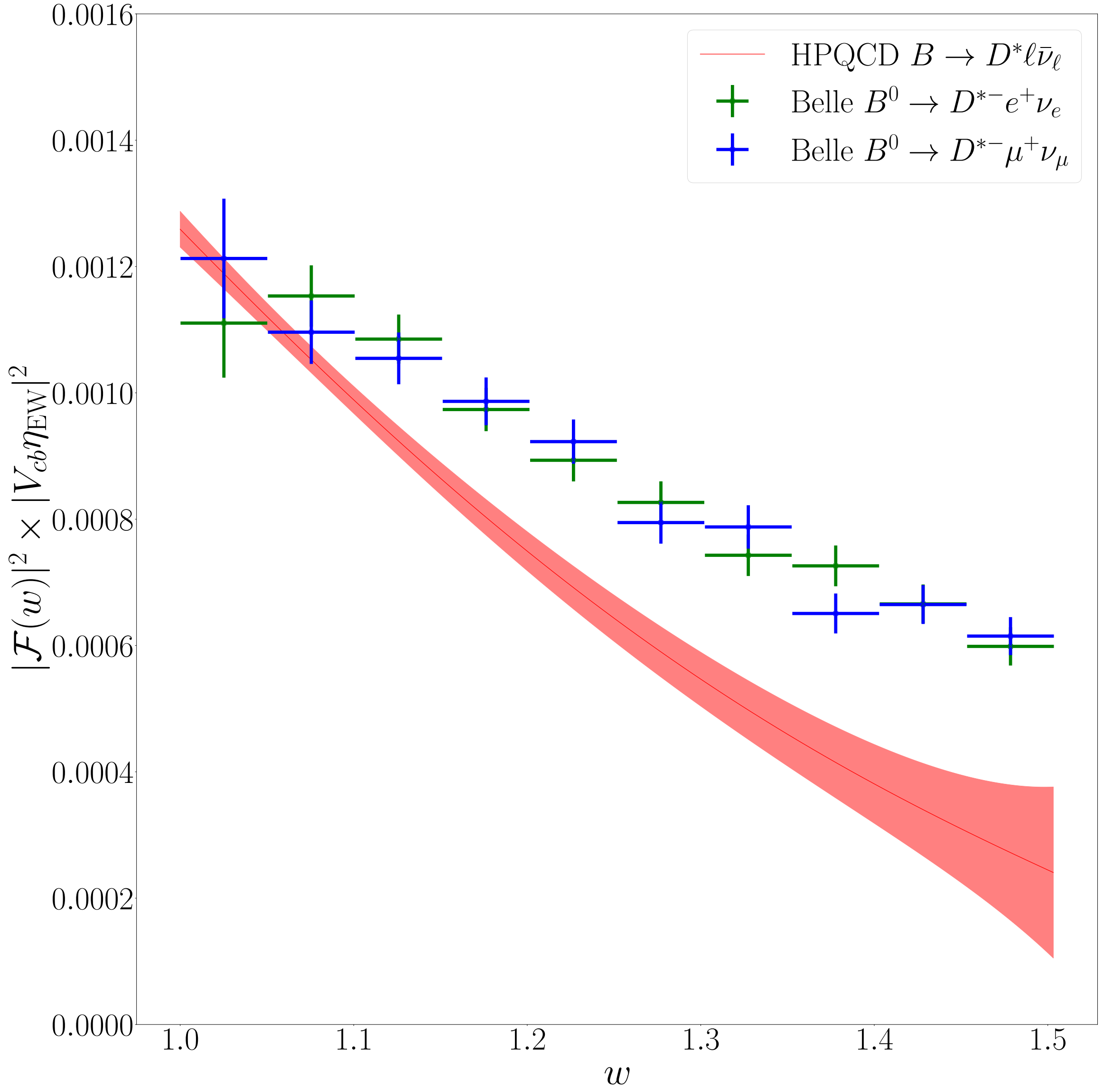
New HPQCD

J. Harrison & T.H. Davies [HPQCD] [arXiv:2304.03137 [hep-lat]]



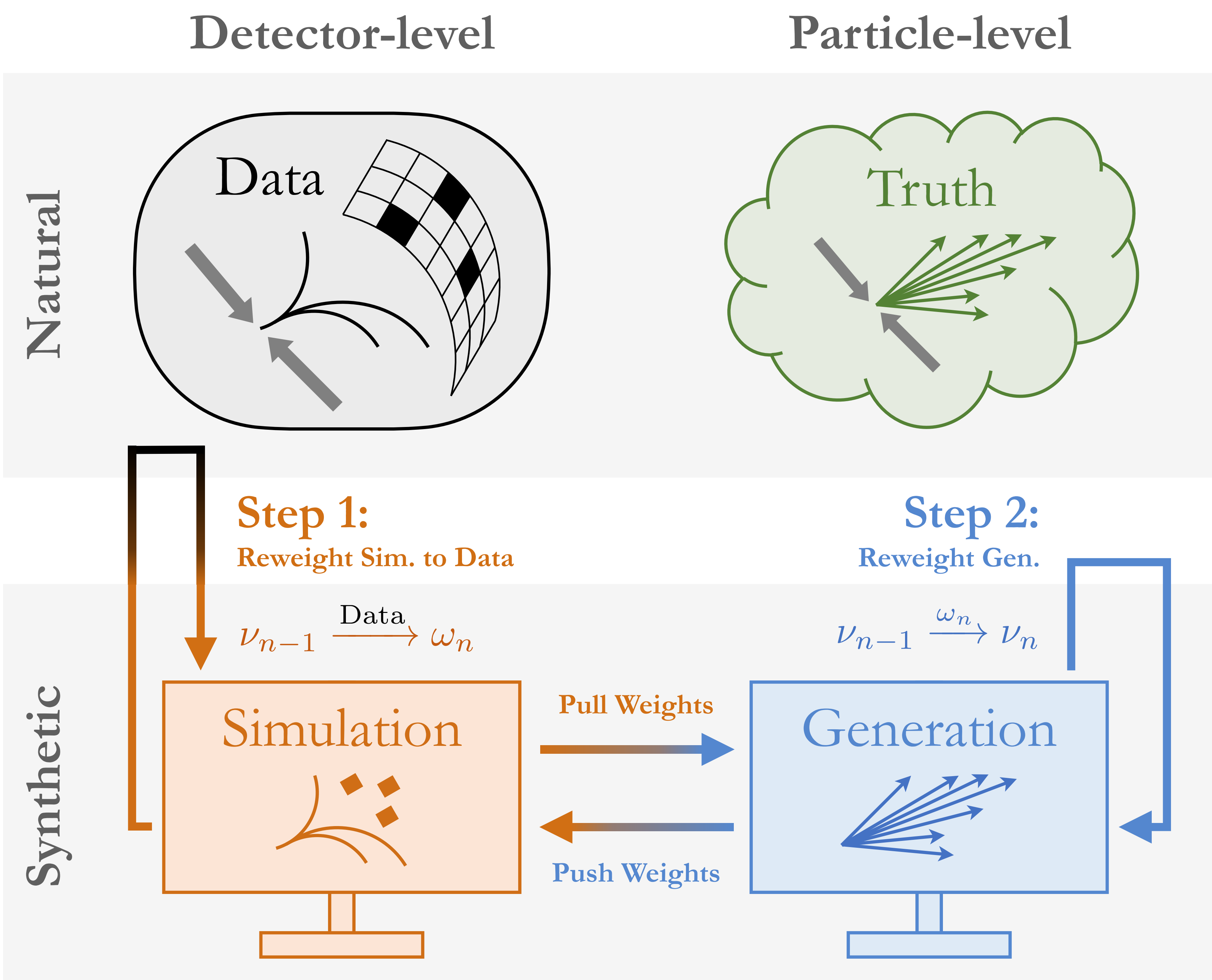
Is it meaningful to combine LQCD and data that do not agree in shape?
What does this mean for our $|V_{cb}|$ values? Can we trust $\mathscr{F}(1)$?





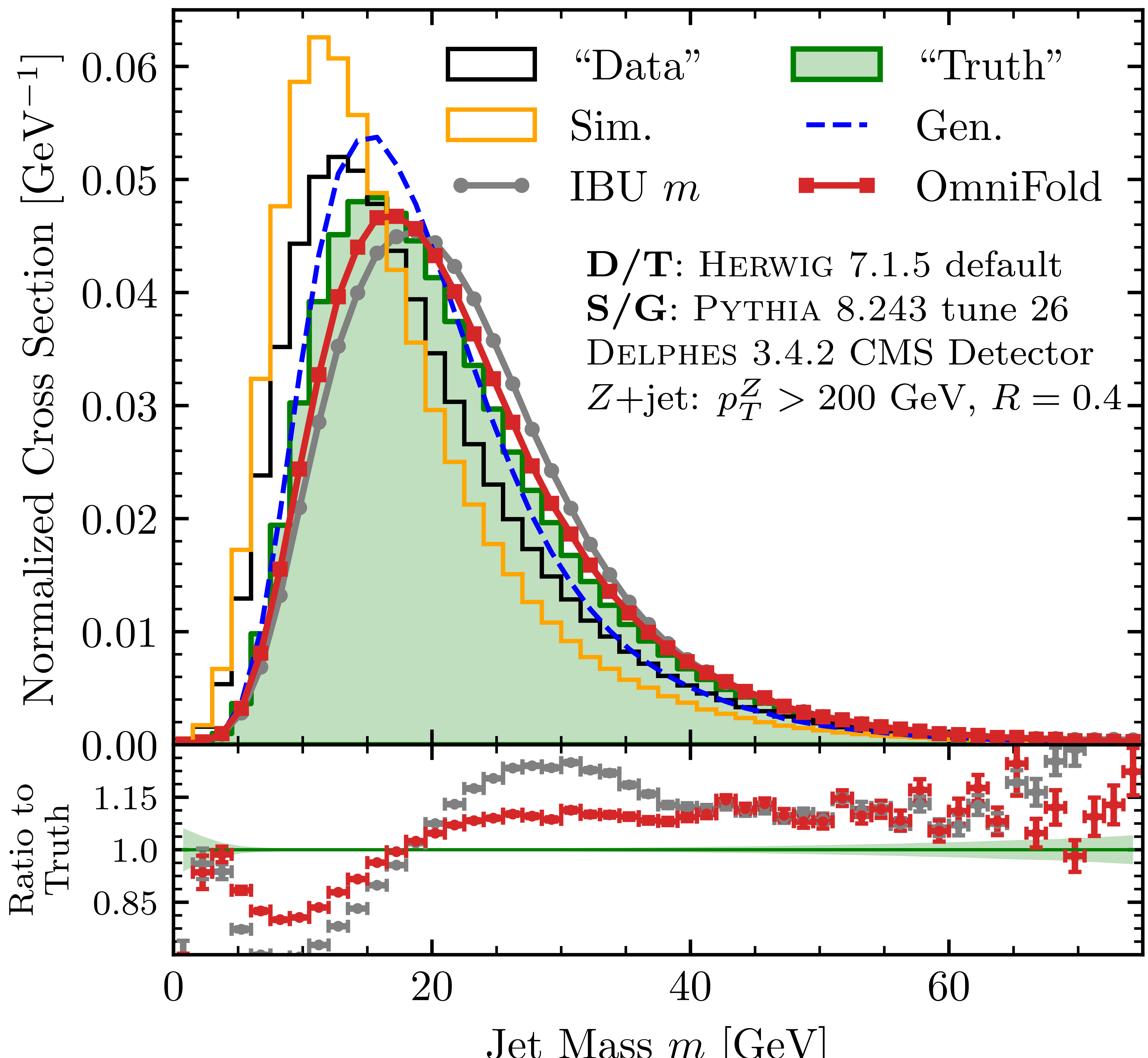
Same data / MC disagreement?

Omnifold



$$p^{(n)}_{\text{unfolded}}(t) = \nu_n(t)\, p_{\text{Gen.}}(t).$$

- UniFold: A single observable as input. This is an unbinned version of IBU.
- MultiFold: Many observables as input. Here, we use the six jet substructure observables in Fig. 2 to derive the detector response.
- OmniFold: The full event (or jet) as input, using the full phase space information.



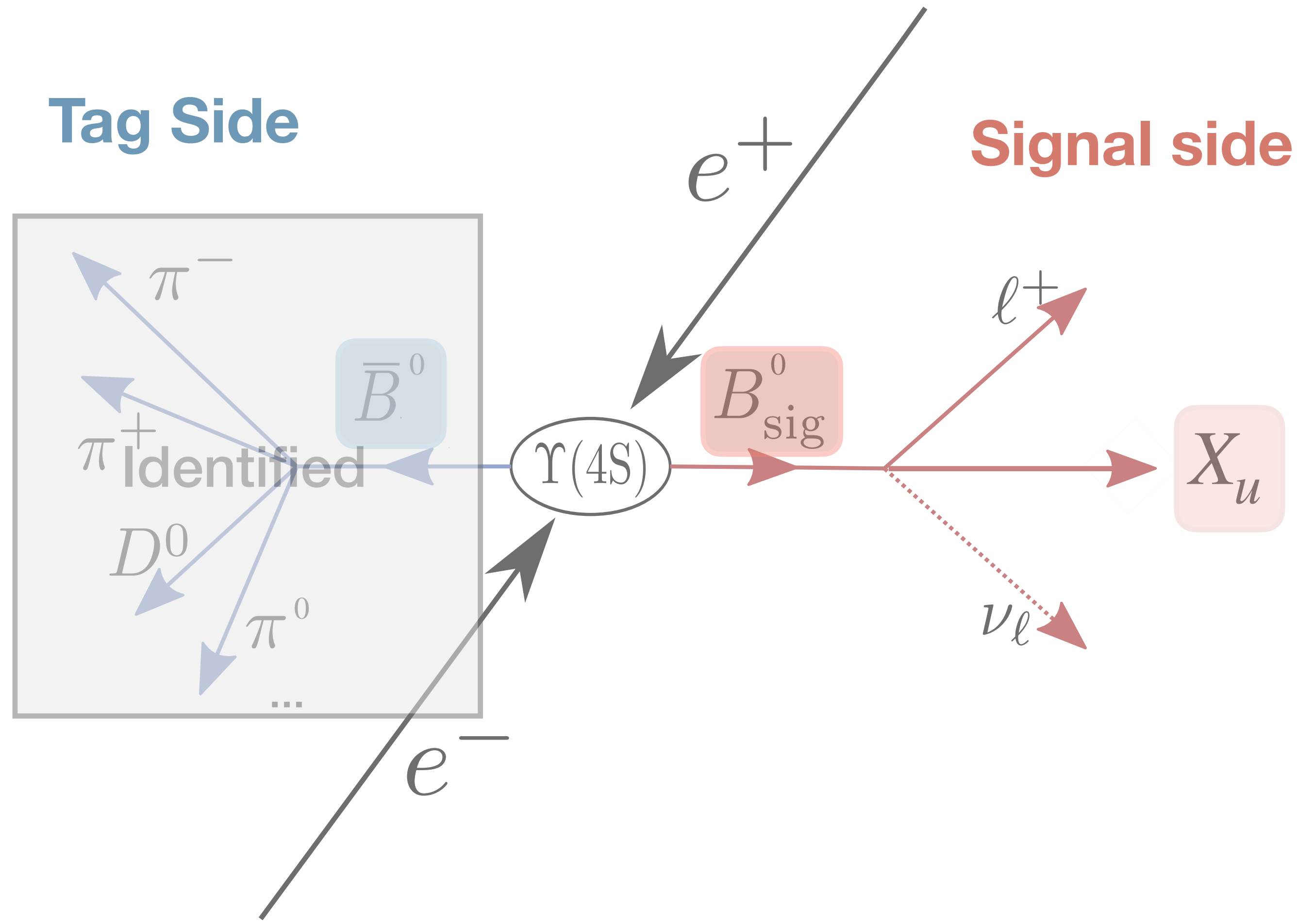
"Data"
"Truth"
Sim.
Gen.
IBU m
OmniFold
D/T: Herwig 7.1.5 default
S/G: Pythia 8.243 tune 26
Delphes 3.4.2 CMS Detector
Z+jet: $p_T^Z > 200$ GeV, $R = 0.4$
Normalized Cross Section [GeV^{-1}]
0.00
0.01
0.02
0.03
0.04
0.05
0.06
Ratio to Truth
0.85
1.0
1.15
0
20
40
60
Jet Mass m [GeV]

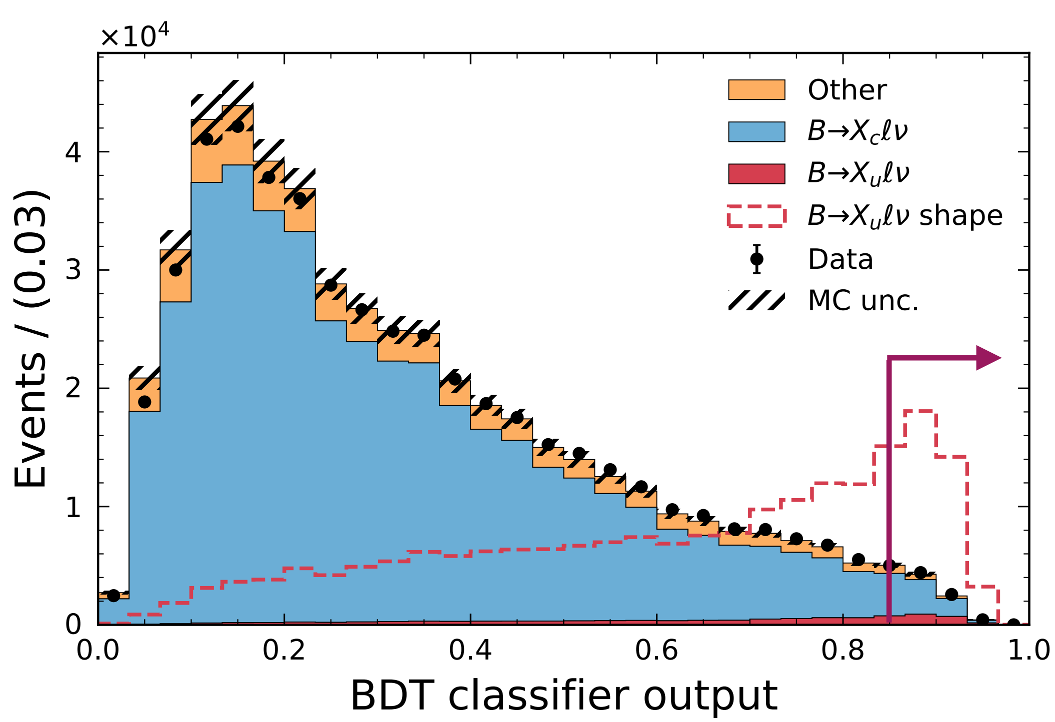
Measurement of **partial** branching fractions of inclusive $B \to X_u \ell \bar{\nu}_\ell$ decays with hadronic tagging [PRD 104, 012008 (2021), arXiv:2102.00020]

Use **full Belle** data set of **711/fb**

Hadronic tagging with neural networks (ca. 0.2-0.3% efficiency)

Use **machine learning** (BDTs) to suppress backgrounds with 11 training features, e.g. m^2_{miss}, #K±, #Ks, etc.





Charged Tracks — Neutral Clusters

$$p_X = \sum_i \left(\sqrt{m_\pi^2 + |\mathbf{p}_i|^2}, \mathbf{p}_i \right) + \sum_j (E_j, \mathbf{k}_j)$$

$$q^2 = (p_{\text{sig}} - p_X)^2 \qquad M_X = \sqrt{(p_X)^\mu (p_X)_\mu}$$

$$m^2_{\text{miss}} = \left(p_{\text{sig}} - p_X - p_\ell \right)^2 \approx m_\nu^2 = 0\,\text{GeV}^2$$

Fit kinematic distributions and measure partial BF

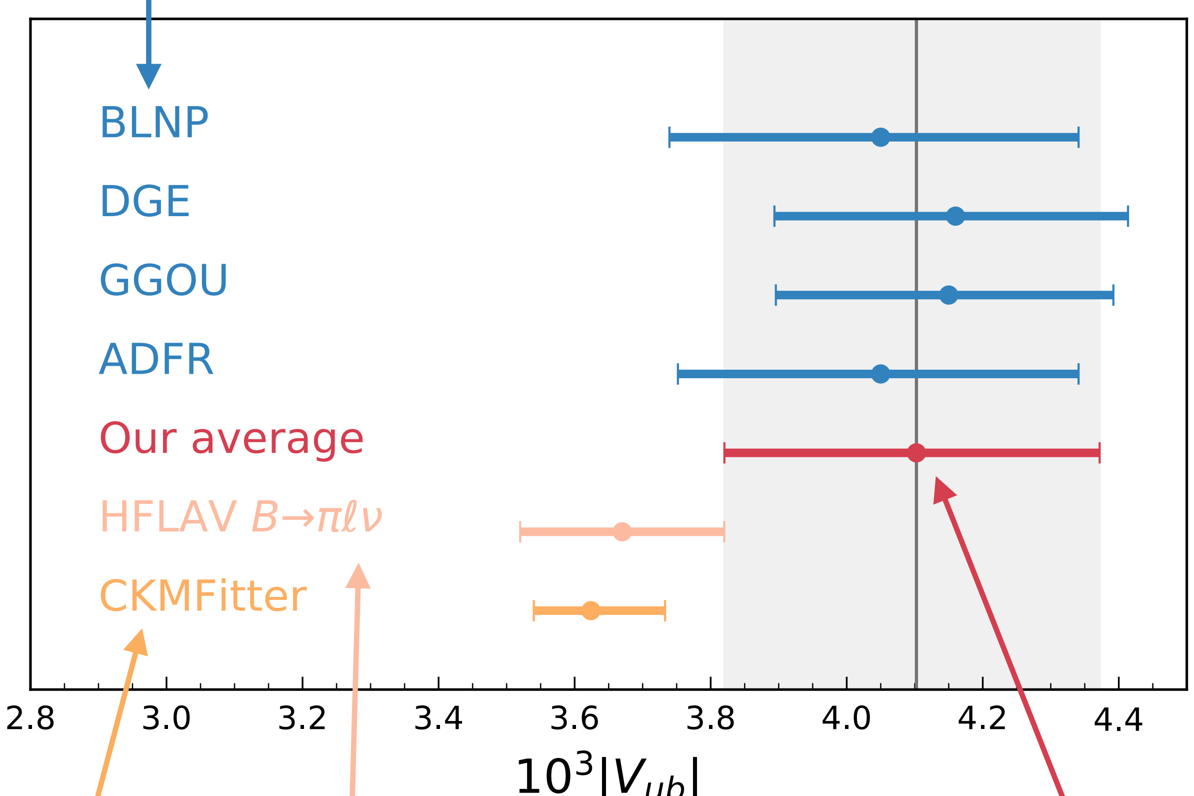
$$|V_{ub}| = \sqrt{\frac{\Delta\mathcal{B}(B \to X_u\,\ell^+\,\nu_\ell)}{\tau_B \cdot \Delta\Gamma(B \to X_u\,\ell^+\,\nu_\ell)}}$$

3 phase-space regions

Phase-space region
$M_X < 1.7\,\mathrm{GeV}$
$M_X < 1.7\,\mathrm{GeV},\ q^2 > 8\,\mathrm{GeV}^2$
$E_\ell^B > 1\,\mathrm{GeV}$

4 predictions of the partial rate

Result for most inclusive region with $E_\ell^B > 1\,\mathrm{GeV}$

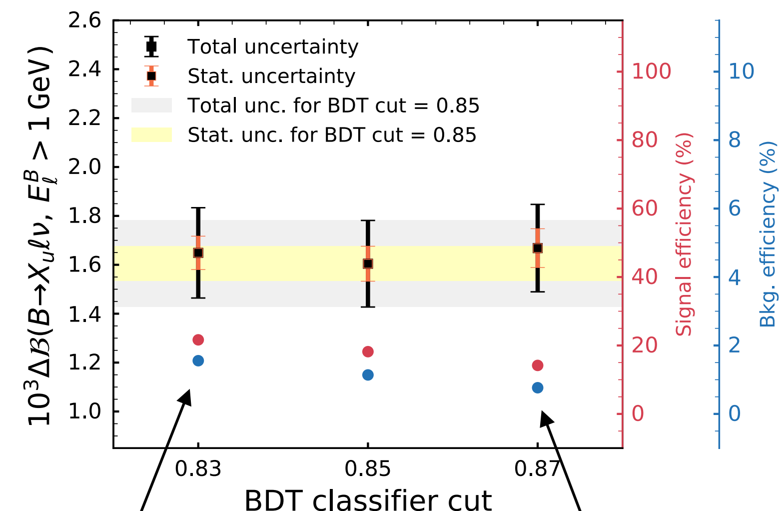


Exclusive Average for $B \to \pi\ell\bar{\nu}_\ell$:
$|V_{ub}| = (3.67 \pm 0.09 \pm 0.12) \times 10^{-3}$

CKM Unitarity:
$|V_{ub}| = \left(3.62^{+0.11}_{-0.08}\right) \times 10^{-3}$

Arithmetic average:
$|V_{ub}| = (4.10 \pm 0.09 \pm 0.22 \pm 0.15) \times 10^{-3}$

Stability as a function of BDT cut:



33% more Bkg

37% less Bkg

Measurement of **differential** branching fractions of inclusive $B \to X_u \ell \bar{\nu}_\ell$ decays with hadronic tagging [Phys. Rev. Lett. 127, 261801 (2021), arXiv:2107.13855]

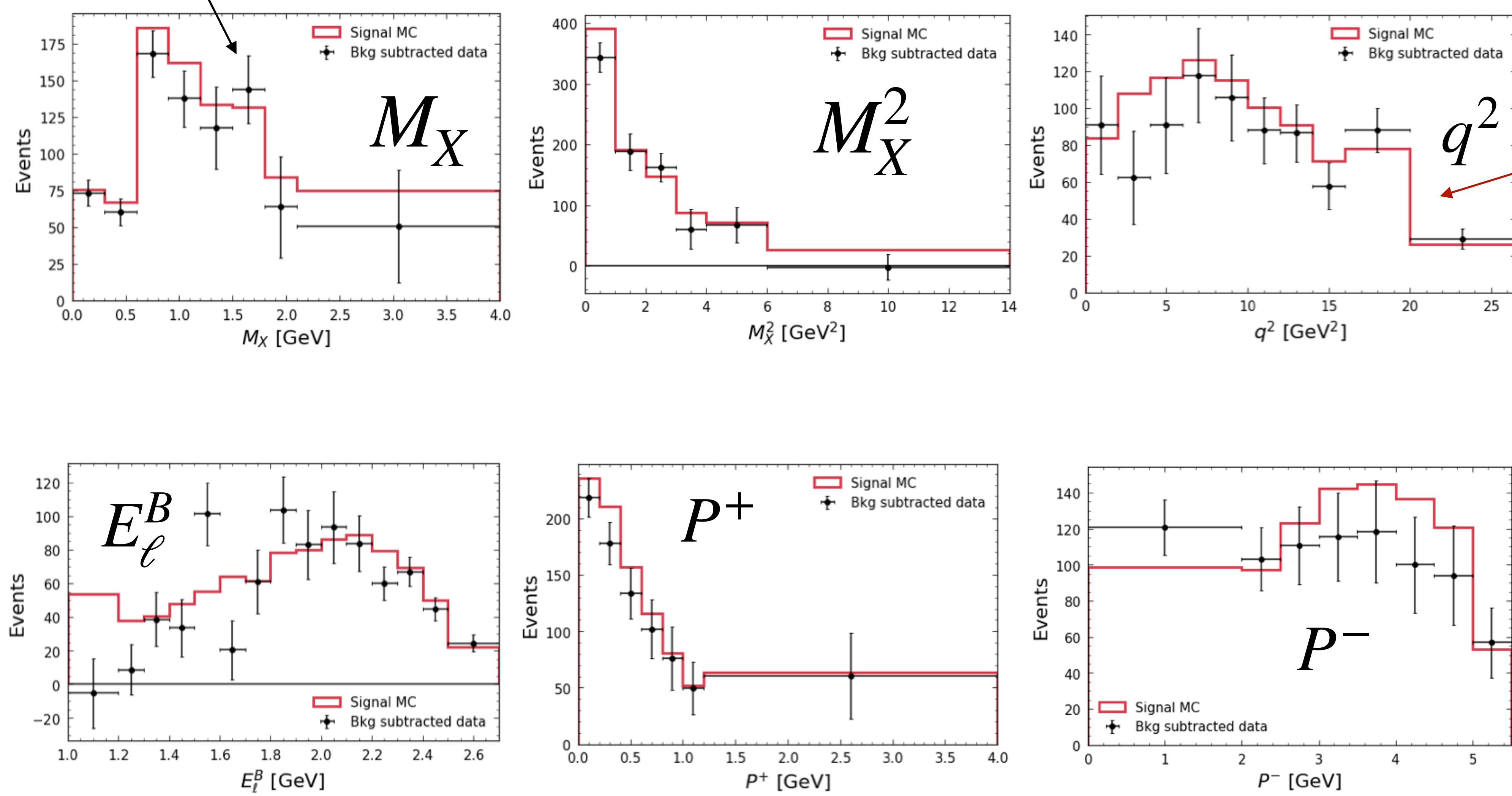
Measurement of **6 kinematic** variables **characterizing** $B \to X_u \ell \bar{\nu}_\ell$ in $E_\ell^B > 1\,\mathrm{GeV}$ region of PS

Selection and reconstruction **analogous** to **partial BF** measurement

Apply **additional selections** to **improve resolution and background shape uncertainties**

Bkg. subtracted data

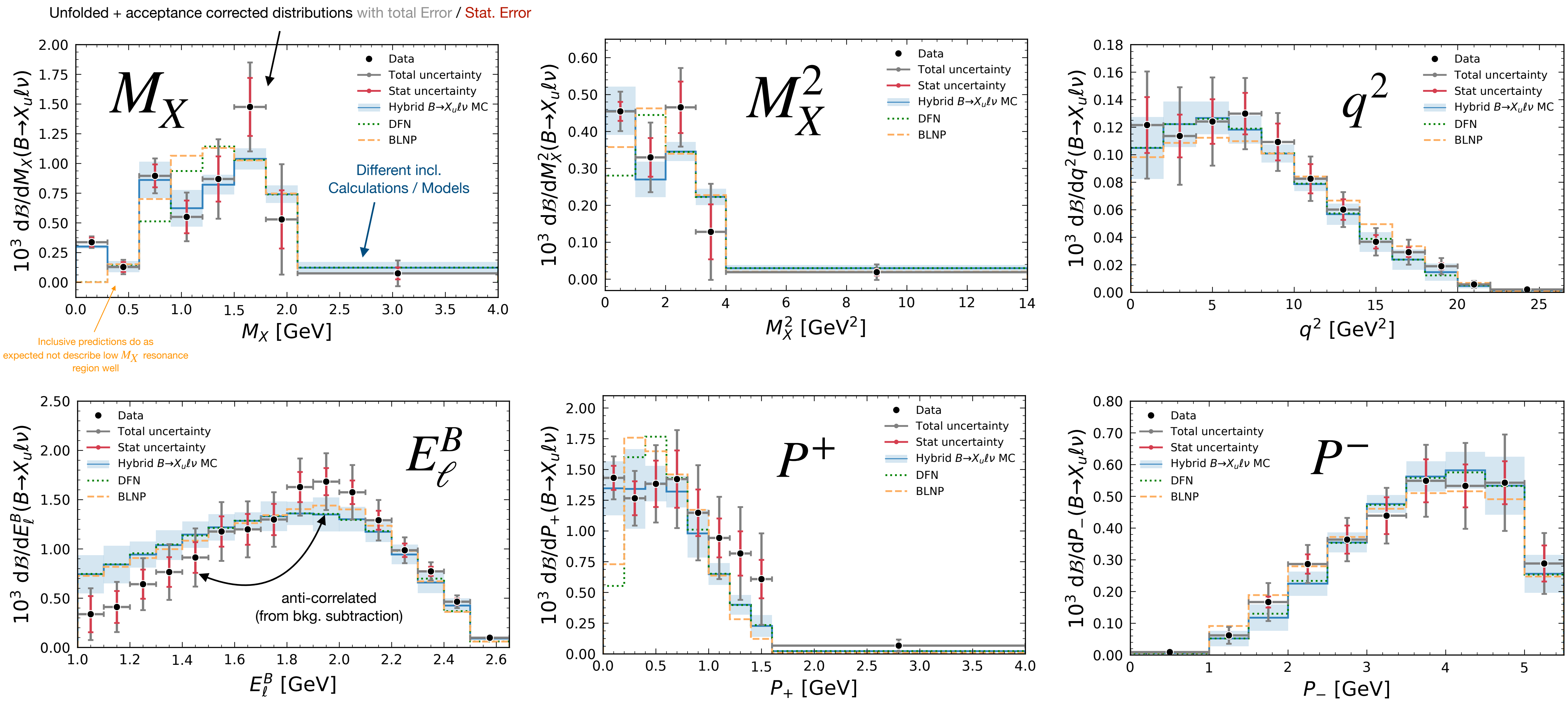
Background subtraction via coarse M_X fit:



Overlaid **signal MC** (hybrid $B \to X_u \ell \bar{\nu}_\ell$)

light-cone momenta: $P_\pm = E_X \mp |P_X|$

Differential Spectra

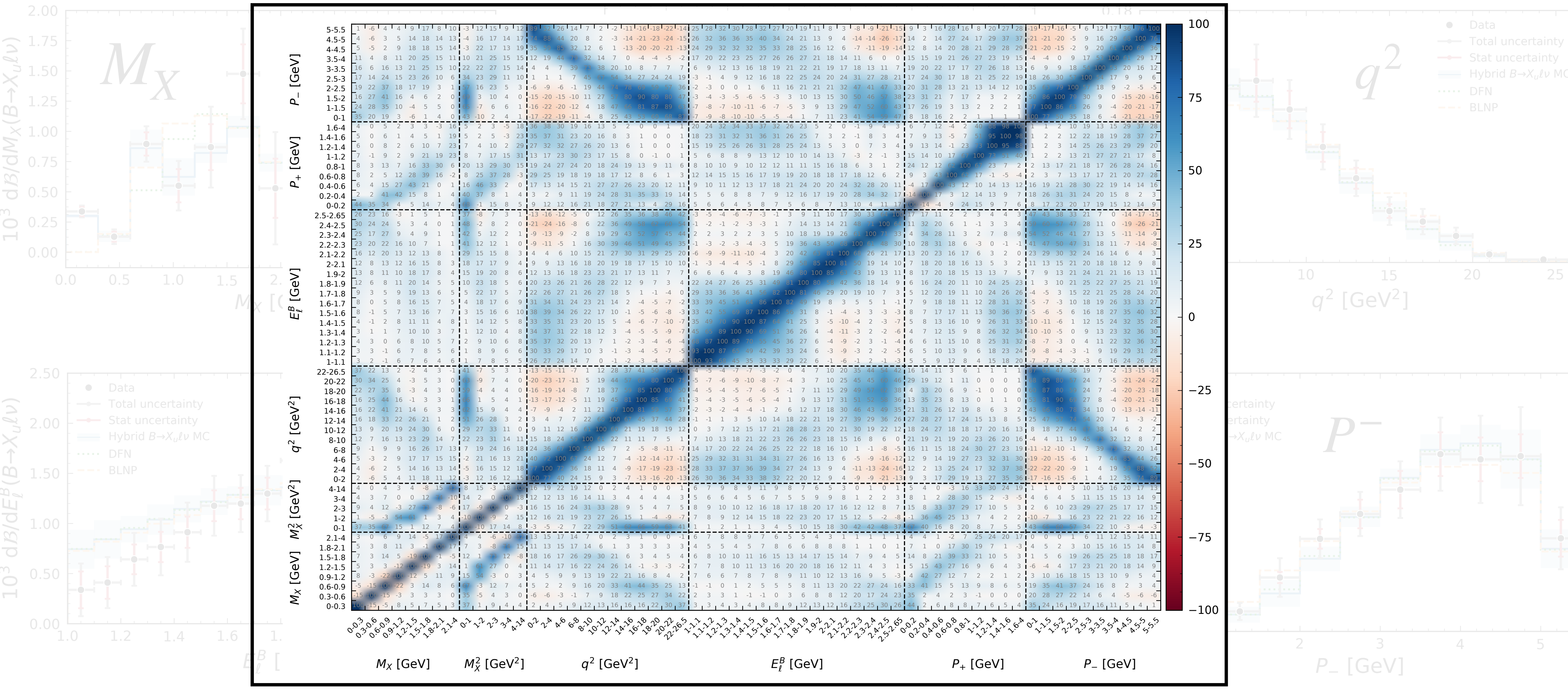


χ^2	E_ℓ^B	M_X	M_X^2	q^2	P_+	P_-
n.d.f.	16	8	5	12	9	10
Hybrid	13.5	2.5	2.6	4.5	1.7	5.2
DFN	16.2	63.2	13.1	18.5	29.3	6.1
BLNP	16.5	61.0	6.3	20.6	23.6	13.7

Agreement
(w/o theory uncertainties)

Differential Spectra

Full **experimental correlations**



↓

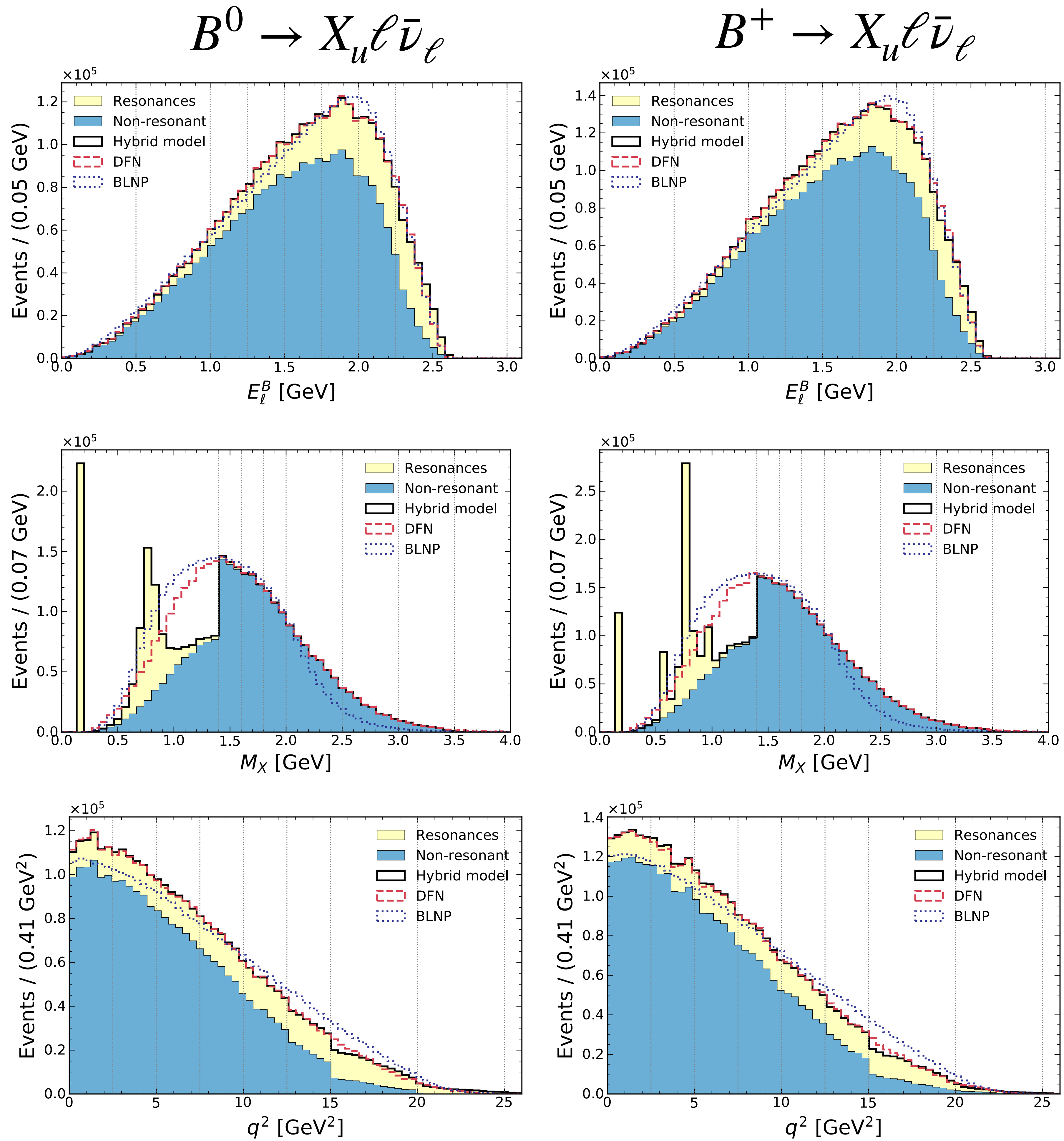
Can be used for future shape-function independent $|V_{ub}|$ determinations

NNVub

P. Gambino, K. Healey, C. Mondino, Phys. Rev. D 94, 014031 (2016), [arXiv:1604.07598]

SIMBA - Spectral Analysis of Inclusive B-Meson

F. Bernlochner, H. Lacker, Z. Ligeti, I. Stewart, F. Tackmann, K. Tackmann Phys. Rev. Lett. 127, 102001 (2021) [arXiv:2007.04320]

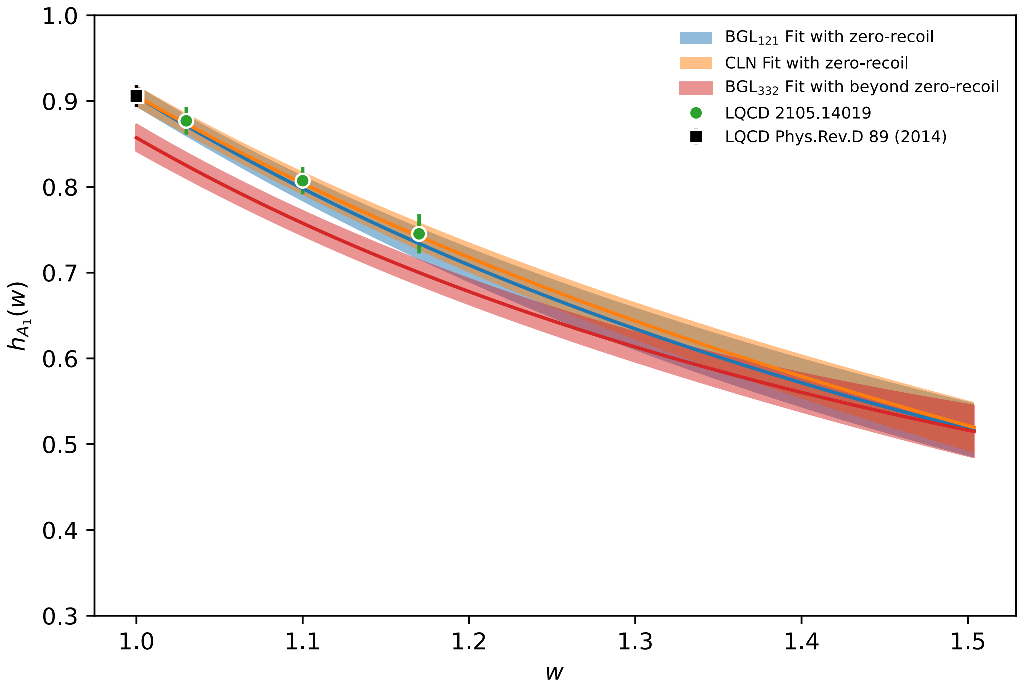

$B^0 \to X_u \ell \bar{\nu}_\ell$
$B^+ \to X_u \ell \bar{\nu}_\ell$
Resonances
Non-resonant
Hybrid model
DFN
BLNP
×10^5
Events / (0.05 GeV)
E_ℓ^B [GeV]
Events / (0.07 GeV)
M_X [GeV]
Events / (0.41 GeV^2)
q^2 [GeV^2]

$\bar{B} \to X_c \ell \bar{\nu}_\ell$ modelling

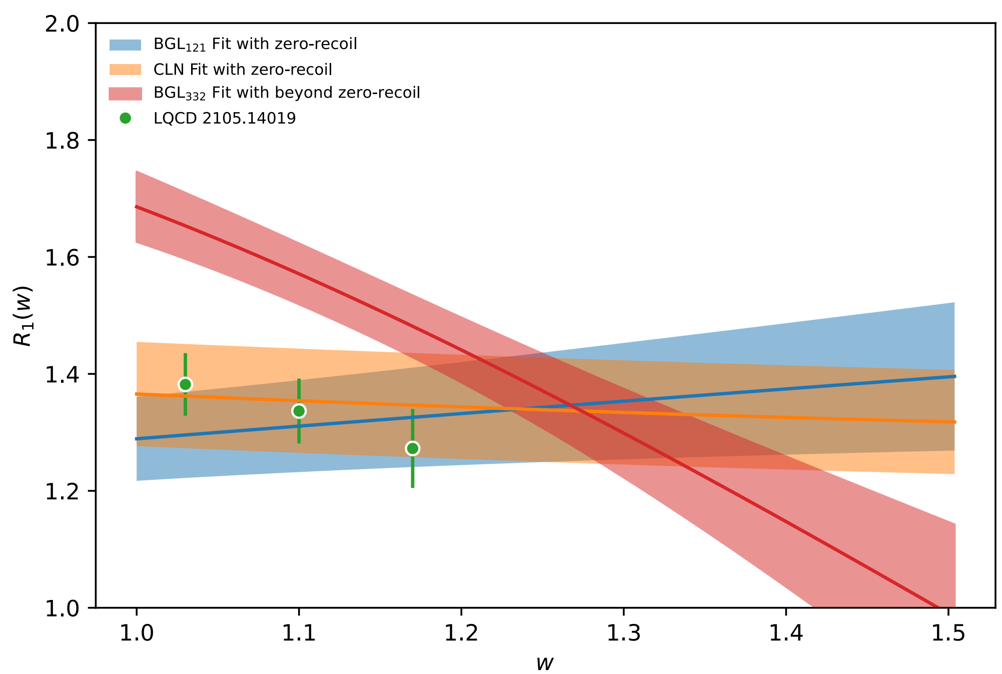
- **Update** excl. branching ratios to PDG 2020 and the masses and widths of D** decays
- **Generate** additional MC samples to fill the **gap** between the exclusive & inclusive measurement (assign 100% BR uncertainty in systematics covariance matrix)

	BR	B^+	B^0
	$B \to X_c \ell^+ \nu_\ell$		
D, D*	$B \to D \ell^+ \nu_\ell$	$(2.5 \pm 0.1) \times 10^{-2}$	$(2.3 \pm 0.1) \times 10^{-2}$
D, D*	$B \to D^* \ell^+ \nu_\ell$	$(5.4 \pm 0.1) \times 10^{-2}$	$(5.1 \pm 0.1) \times 10^{-2}$
D**	$B \to D_0^* \ell^+ \nu_\ell$ $(\hookrightarrow D\pi)$	$(0.420 \pm 0.075) \times 10^{-2}$	$(0.390 \pm 0.069) \times 10^{-2}$
D**	$B \to D_1^* \ell^+ \nu_\ell$ $(\hookrightarrow D^*\pi)$	$(0.423 \pm 0.083) \times 10^{-2}$	$(0.394 \pm 0.077) \times 10^{-2}$
D**	$B \to D_1 \ell^+ \nu_\ell$ $(\hookrightarrow D^*\pi)$	$(0.422 \pm 0.027) \times 10^{-2}$	$(0.392 \pm 0.025) \times 10^{-2}$
D**	$B \to D_2^* \ell^+ \nu_\ell$ $(\hookrightarrow D^*\pi)$	$(0.116 \pm 0.011) \times 10^{-2}$	$(0.107 \pm 0.010) \times 10^{-2}$
D**	$B \to D_2^* \ell^+ \nu_\ell$ $(\hookrightarrow D\pi)$	$(0.178 \pm 0.024) \times 10^{-2}$	$(0.165 \pm 0.022) \times 10^{-2}$
D**	$\rho(D_2^* \to D^*\pi, D_2^* \to D\pi) = 0.693$		
Gap	$B \to D_1 \ell^+ \nu_\ell$ $(\hookrightarrow D\pi\pi)$	$(0.242 \pm 0.100) \times 10^{-2}$	$(0.225 \pm 0.093) \times 10^{-2}$
Gap	$B \to D\pi\pi \ell^+ \nu_\ell$	$(0.06 \pm 0.06) \times 10^{-2}$	$(0.06 \pm 0.06) \times 10^{-2}$
Gap	$B \to D^*\pi\pi \ell^+ \nu_\ell$	$(0.216 \pm 0.102) \times 10^{-2}$	$(0.201 \pm 0.095) \times 10^{-2}$
Gap	$B \to D\eta \ell^+ \nu_\ell$	$(0.396 \pm 0.396) \times 10^{-2}$	$(0.399 \pm 0.399) \times 10^{-2}$
Gap	$B \to D^*\eta \ell^+ \nu_\ell$	$(0.396 \pm 0.396) \times 10^{-2}$	$(0.399 \pm 0.399) \times 10^{-2}$
	$B \to X_c \ell^+ \nu_\ell$	$(10.8 \pm 0.4) \times 10^{-2}$	$(10.1 \pm 0.4) \times 10^{-2}$

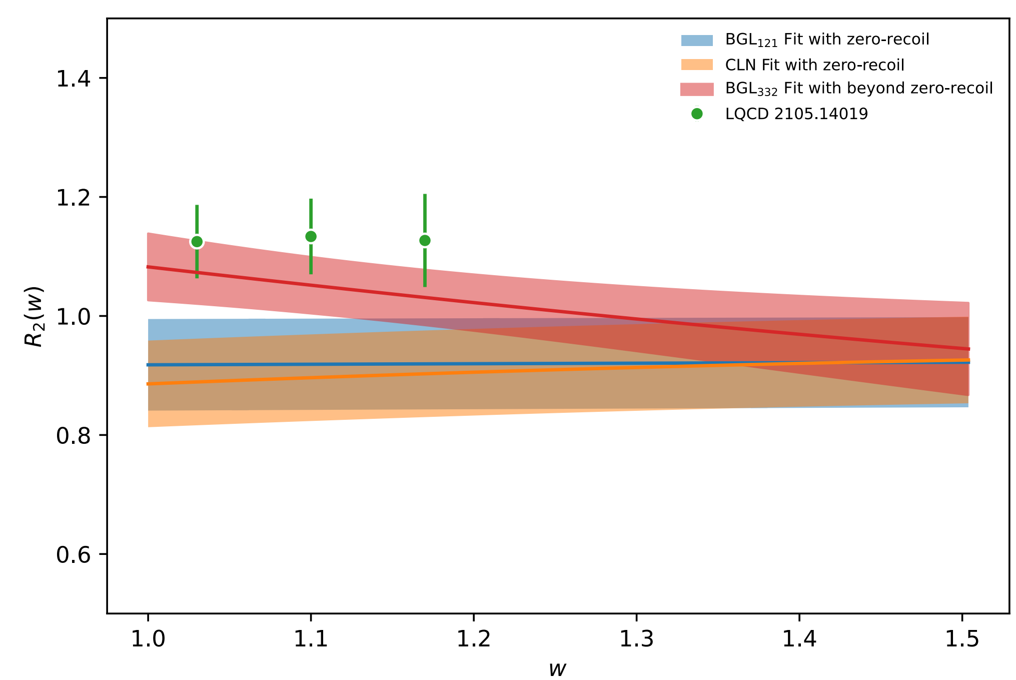
BR	B^+	B^0
$B \to D_0^* \ell^+ \nu_\ell$ $(\hookrightarrow D\pi\pi)$	$(0.03 \pm 0.03) \times 10^{-2}$	$(0.03 \pm 0.03) \times 10^{-2}$
$B \to D_1^* \ell^+ \nu_\ell$ $(\hookrightarrow D\pi\pi)$	$(0.03 \pm 0.03) \times 10^{-2}$	$(0.03 \pm 0.03) \times 10^{-2}$
$B \to D_0^*\pi\pi \ell^+ \nu_\ell$ $(\hookrightarrow D^*\pi\pi)$	$(0.108 \pm 0.051) \times 10^{-2}$	$(0.101 \pm 0.048) \times 10^{-2}$
$B \to D_1^*\pi\pi \ell^+ \nu_\ell$ $(\hookrightarrow D^*\pi\pi)$	$(0.108 \pm 0.051) \times 10^{-2}$	$(0.101 \pm 0.048) \times 10^{-2}$
$B \to D_0^* \ell^+ \nu_\ell$ $(\hookrightarrow D\eta)$	$(0.396 \pm 0.396) \times 10^{-2}$	$(0.399 \pm 0.399) \times 10^{-2}$
$B \to D_1^* \ell^+ \nu_\ell$ $(\hookrightarrow D^*\eta)$	$(0.396 \pm 0.396) \times 10^{-2}$	$(0.399 \pm 0.399) \times 10^{-2}$



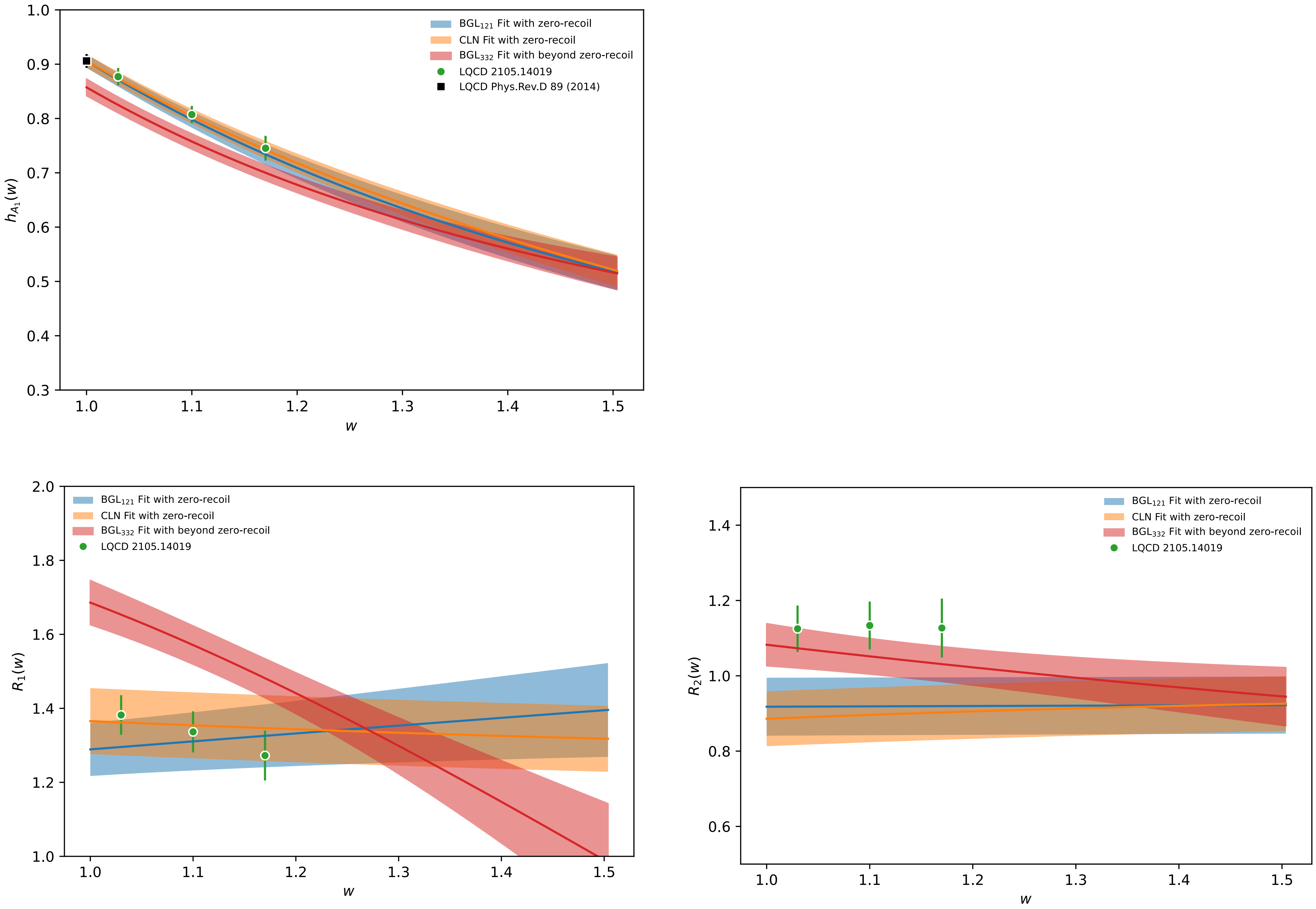
BGL121 Fit with zero-recoil
CLN Fit with zero-recoil
BGL332 Fit with beyond zero-recoil
LQCD 2105.14019
LQCD Phys.Rev.D 89 (2014)
$h_{A_1}(w)$
w



BGL121 Fit with zero-recoil
CLN Fit with zero-recoil
BGL332 Fit with beyond zero-recoil
LQCD 2105.14019
$R_1(w)$
w



BGL121 Fit with zero-recoil
CLN Fit with zero-recoil
BGL332 Fit with beyond zero-recoil
LQCD 2105.14019
$R_2(w)$
w



$h_{A_1}(w)$
w
BGL_{121} Fit with zero-recoil
CLN Fit with zero-recoil
BGL_{332} Fit with beyond zero-recoil
LQCD 2105.14019
LQCD Phys.Rev.D 89 (2014)
$R_1(w)$
w
BGL_{121} Fit with zero-recoil
CLN Fit with zero-recoil
BGL_{332} Fit with beyond zero-recoil
LQCD 2105.14019
$R_2(w)$
w
BGL_{121} Fit with zero-recoil
CLN Fit with zero-recoil
BGL_{332} Fit with beyond zero-recoil
LQCD 2105.14019

	Values	Correlations						
$\|V_{cb}\| \times 10^3$	39.8 ± 1.1	1	−0.16	0.02	−0.1	−0.61	−0.16	0.11
$a_0 \times 10^3$	28.3 ± 1.0	−0.16	1	−0.09	−0.2	0.17	0.11	−0.03
$a_1 \times 10^3$	-45.9 ± 65.7	0.02	−0.09	1	−0.85	−0.04	−0.09	0.14
a_2	-4.8 ± 2.4	−0.1	−0.2	−0.85	1	0.12	0.13	−0.17
$b_0 \times 10^3$	13.3 ± 0.2	−0.61	0.17	−0.04	0.12	1	0.11	−0.13
$c_1 \times 10^3$	-3.2 ± 1.4	−0.16	0.11	−0.09	0.13	0.11	1	−0.91
$c_2 \times 10^3$	59.1 ± 29.9	0.11	−0.03	0.14	−0.17	−0.13	−0.91	1