Status and plan of $B^0 \to K^0\pi^0$ Time-dependent study

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Outline

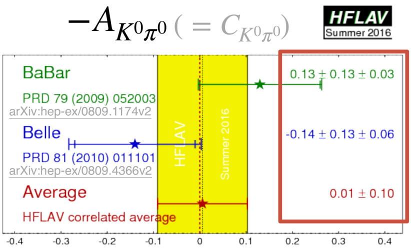
- Motivation & Status at Belle II
- Goal and current development
- 4D $(\Delta E, M_{bc}, \Delta t, C^{'}_{out})$ time-dependent fit
- A_{CP} & S_{CP} measurement
- $B^0 \to J/\psi K^0_S$ control sample study
- B Lifetime, A_{CP} & S_{CP} measurement
- Summary and Plans

Motivation

- In the SM, the decay $B^0 \to K^0\pi^0$ proceeds via $b \to s$ loop diagrams.
- Such FCNC transitions are highly suppressed in the SM and sensitive to non-SM particles appearing in the loops.
- Sum rule relation for $B \to K\pi$ decays

$$I_{K\pi} = \mathcal{A}_{K^+\pi^-} + \mathcal{A}_{K^0\pi^+}\frac{\mathcal{B}(K^0\pi^+)}{\mathcal{B}(K^+\pi^-)}\frac{\tau_{B0}}{\tau_{B+}} - 2\mathcal{A}_{K^+\pi^0}\frac{\mathcal{B}(K^+\pi^0)}{\mathcal{B}(K^+\pi^-)}\frac{\tau_{B0}}{\tau_{B+}} - 2\mathcal{A}_{K^0\pi^0}\frac{\mathcal{B}(K^0\pi^0)}{\mathcal{B}(K^+\pi^-)} = 0$$

Predicting $A_{K^0\pi^0} = -0.17 \pm 0.06$ (Phys.Lett. B627 (2005) 82-8)



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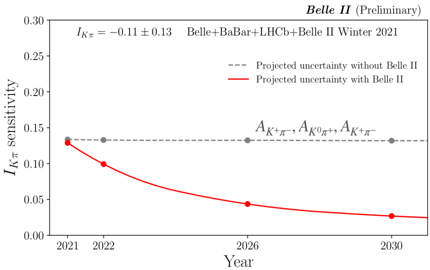
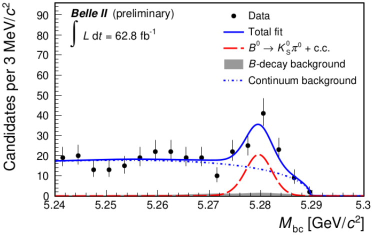
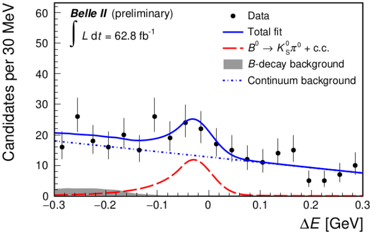


Status at Belle II

- Measurement of $\mathscr{B}$ and A_{CP} from a time-integrated analysis using $62.8\ fb^{-1}$
- Suppress continuum background with a tight (optimised) cut on a BDT classifier
- Simultaneous fit to ΔE and M_{bc}. Use flavour tagging (7 bins of q.r)
- Consistent with results from Belle/Babar.





$\mathscr{B}(B^0 \to K^0\pi^0) = [8.5^{+1.7}_{-1.6}(stat) \pm 1.2(syst)] \times 10^{-6}$ $\quad A_{K^0\pi^0} = -0.40^{+0.46}_{-0.44}(stat) \pm 0.04(syst)$

Enhancing the analysis

- Extend to a time-dependent analysis: access to both direct and mixing-induced CP asymmetries A_{CP} and S_{CP}
- Challenge: vertexing only with K_S^0, poor decay-time resolution.
- Cut away candidates with large decay-time uncertainty. Retrieve signal efficiency by relaxing the continuum-suppression cut, and fit the BDT output
  2D$(\Delta E, M_{bc})\rightarrow$ **4D**$(\Delta E, M_{bc}, \Delta t, C'_{out})$ fit
- Validate the analysis on data with $B^0 \rightarrow J/\psi K_S^0$, vertexing only with the K_S^0.
- Use the full 200 fb^{-1} data sample available. Target Lepton-Photon

Selection criteria

$B^0 \to K_s^0 \pi^0$ selection

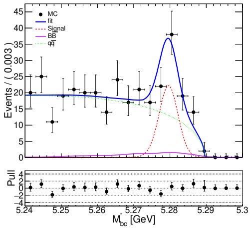
- Standard selection on π^0 and K_S^0 (see backup)
- Trained a FastBDT to suppress continuum background, reproduced selection performance of previous iteration. For the BDT used for Moriond 2021:
  $\rightarrow$ signal efficiency 16.37 %
  $\rightarrow$ surviving continuum 11 %
  $\rightarrow$ peaking background from listed decays (see backup)

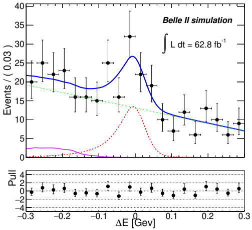
$B^0 \to J/\psi K_S^0$ selection

- Criterias are taken from `BELLE2-NOTE-PH-202`.

Starting from solid ground

- Results on 62.8 fb^{-1} approved for Moriond 2021 obtained by NTU group
- Before developing the new part of the analysis, check to start from the same basis
- Repeated the analysis on same MC sample with the new framework and checked consistency





Parameter	62.8 fb^{-1} MC cocktail
B.F.($\times 10^{-6}$)	$9.08^{+1.57}_{-1.50}$

Parameter	62.8 fb^{-1} MC cocktail
$\mathcal{B}$ [$\times 10^{-6}$]	$9.13^{+1.73}_{-1.59}$

BELLE2-NOTE-PH-2020-046

4D $(\Delta E, M_{bc}, \Delta t, C'_{out})$

Going time dependent

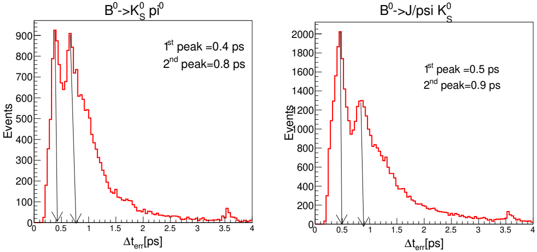
- Keep the same description in ΔE and M_{bc} of the time integrated analysis.
- Adding Δt variable in the fit:

$P_{sig}(\Delta t, q) = \frac{\exp^{-|\Delta t|/\tau_{B^0}}}{4\tau_{B^0}}([1 - q\Delta w + q\mu_i(1-2w)] + [q(1-2w) + \mu_i(1 - q\Delta w)](A_{CP}\cos(\Delta m_d \Delta t) - S_{CP}\cos(\Delta m_d \Delta t)))$

- Fix B lifetime and Δm_d to world-average value, keep only Yield, A_{CP} and S_{CP} free
- Flavour tagging parameters fixed from "*B*-flavor tagging at Belle II" (https://arxiv.org/pdf/2110.00790.pdf), using 7 bins of q.r
- Background modelled with empirical PDF determined from MC.

Decay-time uncertainty and time resolution

- Double peak observed in deltat err distribution, according to the first hits of K_S^0 tracks in VXD.
- Feature reproduced in the control channel



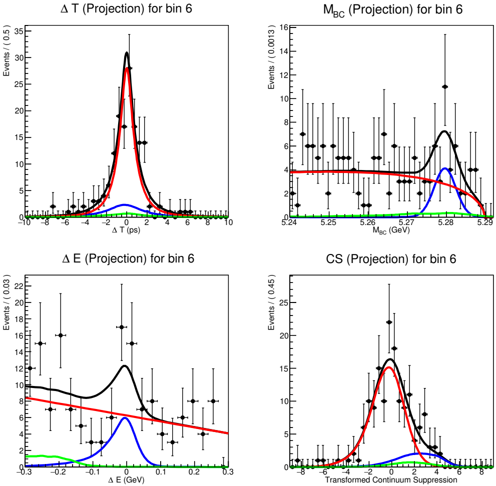
- Decay time resolution: average model with a double gaussian from deltat residual in MC.
- Cut away candidates with $\Delta t_{err} < 2.5$ which dilute A_{CP} and S_{CP} sensitivity. Loose 6.25 % single efficiency.
- Retrieve signal efficiency by loosening CS cut, from 9.99% to 16.37 %. Include CS output in the fit to gain signal-to-background separation

Analysis validation with toys and MC

- Check consistency with
  $\rightarrow$ toys: generate data from the PDF and fit back.
  $\rightarrow$ resampling signal dataset from corresponding MC (GSIM).
- Simulate different signal-to-background ratio in the q.r bins, but simplified approach first:
  $\rightarrow$ assume background PDF do not change in the 7 q.r bins (will check later)
- fix all shape parameters, only free parameters are: Yield, A_{CP}, and S_{CP}

Example of fit projections from MC test

Shown fit projection for candidates with q.r in [0.875,1.0]



- Rest of the bin fit projection shown in backup slide

Validation results

- Signal efficiency=16.37 % (all selection + loose cont. supp. cut $+\sigma_{\Delta t}$)

Pure toy

Parameter	Pull mean	Pull width	Average uncertainty
Signal Yield	-0.026	1.04	18
Continuum Yield	-0.011	0.99	69
A_{CP}	-0.017	1.03	0.297
S_{CP}	-0.063	1.04	0.431

GSIM toy

Parameter	Pull mean	Pull width	Average uncertainty
Signal Yield	-0.045	1.07	18
Continuum Yield	-0.046	1.08	71
A_{CP}	-0.11	1.01	0.277
S_{CP}	0.07	0.99	0.392

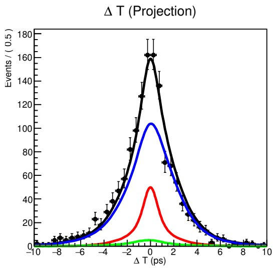
Control Sample study $B^0 \to J/\psi K_S^0$
B Lifetime
A_{CP} & S_{CP}

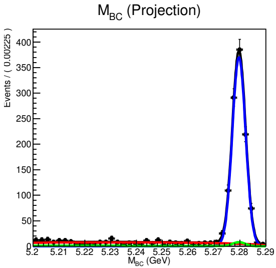
Control channel modelling

- Want to perform the full analysis on the $B^0 \to J/\psi K_S^0$ decay as a validation. Compare with known values, a measurement of
  $\to$ B^0 lifetime
  $\to$ A_{CP} and S_{CP}
- First, develop the analysis on simulation, as done for the rare decay
- Simplified fit: since $B^0 \to J/\psi K_S^0$ is much cleaner, don't need CS. Fit M_{bc} and Δt only.
- Same approach for flavour-tagging and time-dependent PDF:
  $\to$ 7 bins of q.r with parameters fixed fromm "*B*-flavor tagging at Belle II" (https://arxiv.org/pdf/2110.00790.pdf)
  $\to$ cut a $\Delta t_{err} < 2.5$ ps, and average resolution function (2 Gaussian)

B Lifetime fit

- $200\ fb^{-1}$ cocktail of signal, background are generated from PDFs.
- All shape parameters are fixed





Parameter	Fitted value	Expected value
Signal Yield	1045 ± 33	1044
Background Yield	275 ± 18	275
Lifetime (ps)	1.523 ± 0.136	1.52

Validation of TDCPV fitter

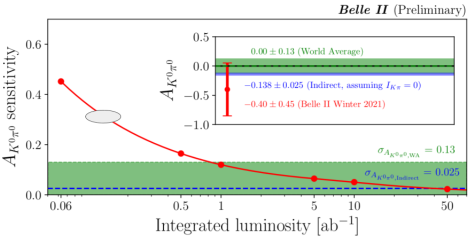
- Test with toys and full MC successful, found unbiased results for lifetime, A_{CP} and S_{CP}.
- Expected sensitivity on A_{CP} and S_{CP} from control sample on 200 fb^{-1}: 0.09 and 0.15 (details in backup).
- Validate the analysis with 3 times better precision than that on the $B^0 \to K_s^0 \pi^0$ decay.
- Next: run the analysis on data for $B^0 \to J/\psi K_S^0$

Summary

- Extend to a time-dependent analysis: access to both direct and mixing-induced CP asymmetries A_{CP} and S_{CP} from $B^0 \to K_S^0\pi^0$
- Perform the full analysis on the $B^0 \to J/\psi K_S^0$ decay as a validation.
- Validate the analysis with toys and MC
- Two groups are working on $B^0 \to K_s^0\pi^0$ time-dependent analysis
- Expect to have preliminary result in next winter conference.

Plans

- Calculate the systematics uncertainty
  $\rightarrow$ Vertex Reconstruction, Resolution function, Flavor Tagging, Physics parameters...
- Before going to unblind full dataset, first unbind Moriond 2021 dataset, measure $\mathscr{B}$ to check compatibility and move to the rest
- B lifetime, A_{CP} & S_{CP} measurement in control sample for full dataset
- Full analysis report will be ready by this month.



Thank You

Selection criteria

$B^0 \to K_s^0 \pi^0$ selection

- $120 < m_{\pi^0} < 145$ MeV and $|\cos\theta_H| < 0.98$
- Barrel $E_\gamma > 30$, Backward $E_\gamma > 60$ and Forward $E_\gamma > 80$ MeV
- $482 < m_{K_S^0} < 513$ MeV
- $5.24 < M_{bc} < 5.3$ GeV and $-0.3 < \Delta E < 0.3$ GeV

$B^0 \to J/\psi K_S^0$ selection

- Criterias are taken from `BELLE2-NOTE-PH-202`.
- $dr < 0.5$ cm, $|dz| < 3$ cm, for muon tracks.
- muonID(μ^+) or muonID(μ^+) > 0.2
- $2.80 < M_{J/\psi} < 3.40$ GeV and $482 < M_{K_S^0} < 513$ MeV
- $5.2 < M_{bc} < 5.3$ GeV and $|\Delta E| < 0.05$ GeV

- For CP-side: IP constraint and only K_S^0 vertexing
- For tag-side : IP constraint
- $\sigma_{\Delta t} < 2.5$ ps

Rare components investigation

2D (M_{bc}, ΔE) Extended Fit (Cont'd)

- Rare background contributing to the analysis region:

expected @ 62.8 fb^{-1}

$N = \int \mathscr{L}dt \cdot \sigma \cdot f^{+-} \cdot 2 \cdot \mathscr{B} \cdot \epsilon$

$N = \int \mathscr{L}dt \cdot \sigma \cdot f^{00} \cdot 2 \cdot \mathscr{B} \cdot \epsilon$

	Mode	$\mathcal{B}[10^{-6}]$ (PDG2020 Avg. [3])	ϵ[%]	Yield
B^+	$\rho^+ K^0$	$7.3^{+1.0}_{-1.2}$	1.05	5.5 ± 0.8
	$K^*(892)^+ \pi^0$	6.8 ± 0.9	0.85	4.1 ± 0.5
	$X_{s,u}\gamma$	349 ± 19	<0.01	0.7 ± 0.0
	$a_1(1260)^+ K^0$	35 ± 7	<0.01	0.1 ± 0.0
B^0	$f_2(1270) K^0$	$2.7^{+1.3}_{-1.2}$	0.52	1.0 ± 0.4
	$f_0(980) K^0$	4.1 ± 0.4	0.19	0.5 ± 0.1
	$X_{s,d}\gamma$	349 ± 19	<0.01	0.5 ± 0.0
	$K^0_S K^0_S$	0.61 ± 0.08	0.50	0.2 ± 0.0
	$K^0 \eta'$	66 ± 4	<0.01	0.1 ± 0.0
Sum				12.7 ± 1.1

dominant processes
$B \to K^0 \pi^+ \pi^0$
(PDG; PRD)

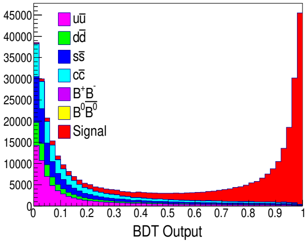
- Finally assign a Gauss(μ=12.7, σ=1.1) constraint on the normalization of rare background

Continuum suppression validation

- FatBDT as the multivariate classifier.
- Same number of signal and background events.
- 800 fb^{-1} for training and 400 fb^{-1} for testing.
- Use only continuum (u, d, s, c) background instead of generic$(u, d, s, c, B\bar{B})$ background.
- Same classifier input used(BELLE2-NOTE-PH-2020-046).

Classifier Output

Our study



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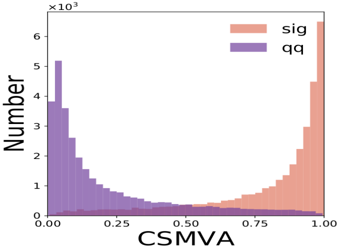


Background rejection comparison

Using our CS weight file

1) generic BKG to train CS

Cut	BKG rej.	$\#u\bar{u}$	$\#d\bar{d}$	$\#s\bar{s}$	$\#c\bar{c}$	$\#B^0\bar{B}^0$	$\#B^+B^-$	# signal
0.0		5434	2287	4180	4280	109	22	98
0.9	98.33 %	80	46	52	90	58	11	53

2) Continuum BKG to train CS

Cut	BKG rej.	$\#u\bar{u}$	$\#d\bar{d}$	$\#s\bar{s}$	$\#c\bar{c}$	$\#B^0\bar{B}^0$	$\#B^+B^-$	# signal
0.0		5434	2287	4180	4280	109	22	98
0.9	98.25 %	90	49	58	84	54	9	48

Using BELLE2-NOTE-PH-2020-046 CS weight file

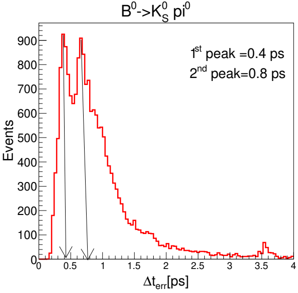
https://stash.desy.de/projects/B2B2C/repos/btohadronscripts/browse/BToCharmless_WithCorr_CSFBDT.root

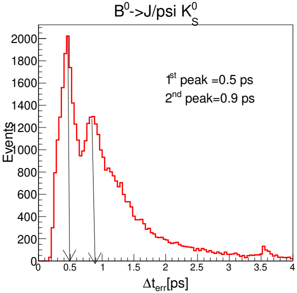
Cut	BKG rej.	$\#u\bar{u}$	$\#d\bar{d}$	$\#s\bar{s}$	$\#c\bar{c}$	$\#B^0\bar{B}^0$	$\#B^+B^-$	# signal
0.0		5434	2287	4180	4280	109	22	98
0.9	98.39 %	74	45	52	88	54	11	48

- Now we use the common BToCharmless weight file for CS

Δt_{err} distribution

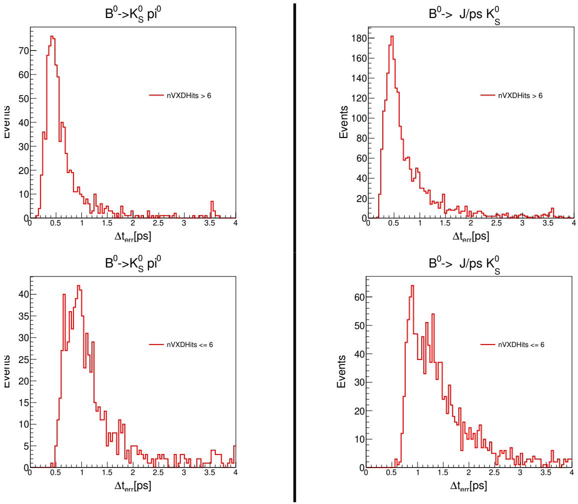
only K_S^0 vertexing





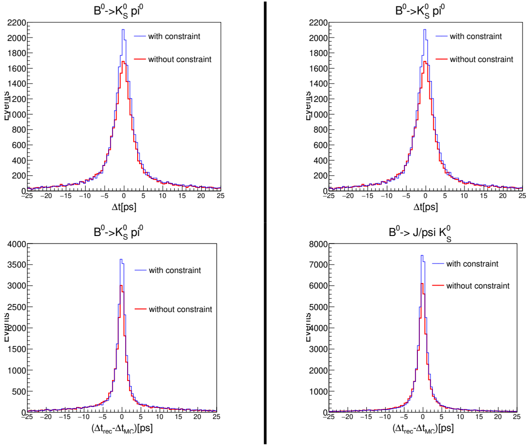
- After including only K_S^0 in the vertexing we get double peak in both cases.

Δt_{err} double peak



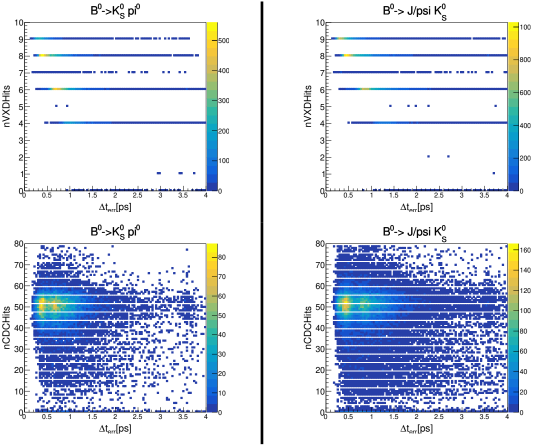
- We observe the decond peak due to fewer hits in VXD.

Effect of IP constraint



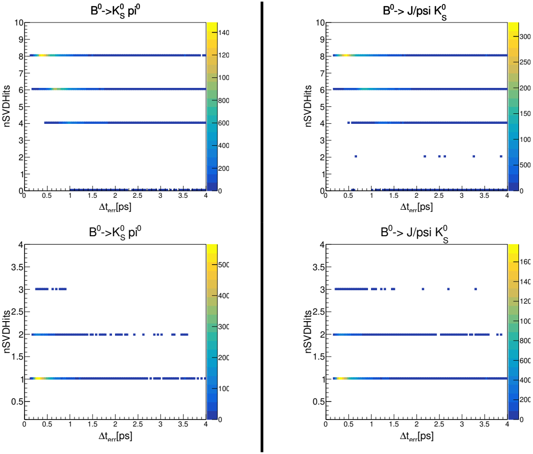
- After applying IP constraint in tag side Δt resolution improves.
- Similar trend is seen in the control channel .

Δt_{err} vs. Hits



- We plots number of hits in VXD and CDC to find out the double peak structure in the Δt_{err} distribution.

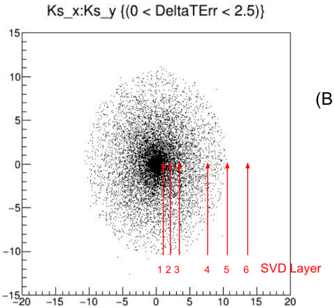
Δt_{err} vs. Hits



- We plots number of hits in VXD and CDC to find out the double peak structure in the Δt_{err} distribution.

DeltaTErr and Ks Vertex Position

- Location of Ks vertex on x-y plane
- Cut of 2.5 on DeltaTErr corresponds to the 5th layer of the SVD
- This means the cut requires two hits in the SVD



(B -> J/Psi Ks)

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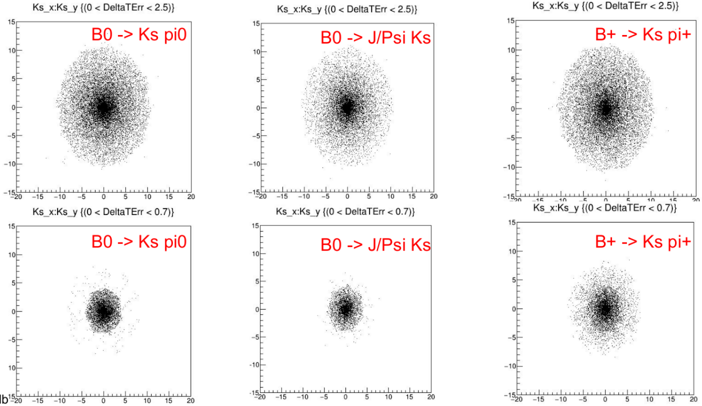
Tim Green, University of Melbourne

DeltaTErr and Ks Vertex Position

0 < DeltaTErr < 2.5

0 < DeltaTErr < 0.7
(First peak)



Tim Green, University of Melb

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Signal mode

Signal yield calculation

The expected signal yield is calculated as

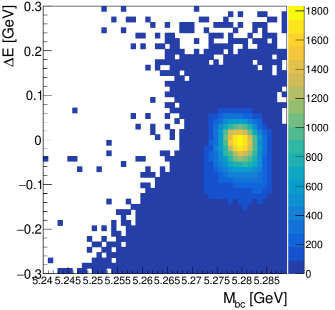
$$N_{sig}^{expected} = \mathscr{B} \cdot \epsilon \cdot \mathscr{B}_s \cdot 2 \cdot N_{B^0\bar{B}^0} \tag{1}$$

- $N_{B^0\bar{B}^0} = \int \mathcal{L} \cdot \sigma \cdot f^{00}$, where $\sigma = 1.110$ and $f^{00} = 0.487$
- $\mathscr{B}_s = 0.5$, probability of $K^0 \to K^0_S / K^0_L$
- $\mathscr{B}(B^0 \to K^0\pi^0) = 9.93 \times 10^{-6}$ (PDG value 2020)
- Signal efficiency=16.37 % (all selection + loose cont. supp. cut $+\sigma_{\Delta t}$)
- N_{sig}^{expt}=165 (200 fb^{-1})

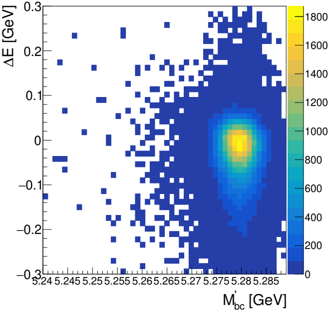
Modified M_{bc}

- $M_{bc} = \sqrt{E^{*2}_{beam} - \vec{p}^{*2}_{B}}$
- $\vec{p}^{*}_{B} = \vec{p}^{*}_{K^0_S} + \vec{p}^{*}_{\pi^0}$
- $\vec{p}^{*}_{B} = \vec{p}^{*}_{K^0_S} + \frac{\vec{p}_{\pi^0}{}^{*}}{|\vec{p}_{\pi^0}{}^{*}|}(\sqrt{(E^{*}_{beam} - E^{*}_{K^0_S})^2 - m^2_{\pi^0}}$

cor=0.143(signal)



cor=-0.03(signal)



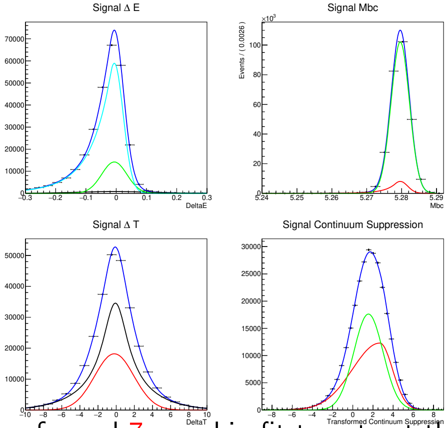
Signal Modeling

- Δt : RooBCPGenDecay PDF PDF convolved with double Gaussian:

  $$P_{sig}(\Delta t, q) = \frac{\exp^{-|\Delta t|/\tau_{B^0}}}{4\tau_{B^0}}([1 - q\Delta w + q\mu_i(1-2w)] + [q(1-2w) + \mu_i(1 - q\Delta w)](A_{CP}\cos(\Delta m_d \Delta t) - S_{CP}\cos(\Delta m_d \Delta t)))$$

  Core and tail Gaussian, $\tau_{B^0} = 1.520$ ps and $\Delta m_d = 0.507/\text{ps}$
- ΔE : Crystal Ball + double Gaussian with common mean
- M_{bc} : Crystal Ball + Gaussian, C'_{out} : Bifurcated + Gaussian

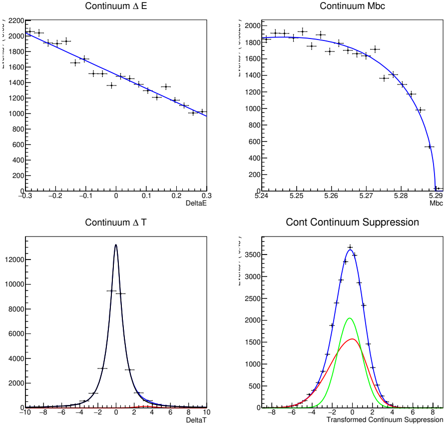
Example plot of integrated $q \cdot r$ bin



- In same way performed $7\ q \cdot r$ bin fit to extract the PDFs parameters

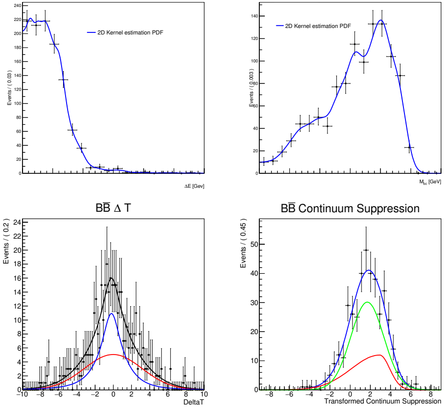
Continuum bkg modeling

- Δt : RooDecay PDF convolved with double Gaussian : $e^{-|t|/\tau}$ Core and tail Gaussian
- ΔE : Linear function
- M_{bc} : ARGUS function, C'_{out} : Bifurcated + Gaussian



$B\bar{B}$ bkg Modeling

- Δt : RooDecay PDF convolved with double Gaussian : $e^{-|t|/\tau}$
  Core and tail Gaussian
- 2D Kernel estimation PDF used for $\Delta E - M_{bc}$ modeling
- C'_{out} : Bifurcated + Gaussian

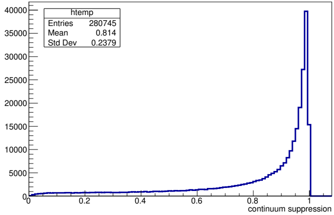


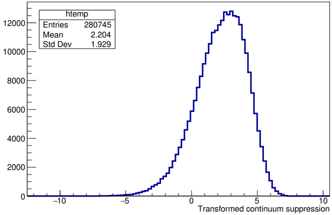
Adding extra dimension to the fitter

- We transform the BDT classifier output (C_{out}) to (C'_{out}) in order to parametrize using a simple PDF
- Transform continumm suppression variable is defined as

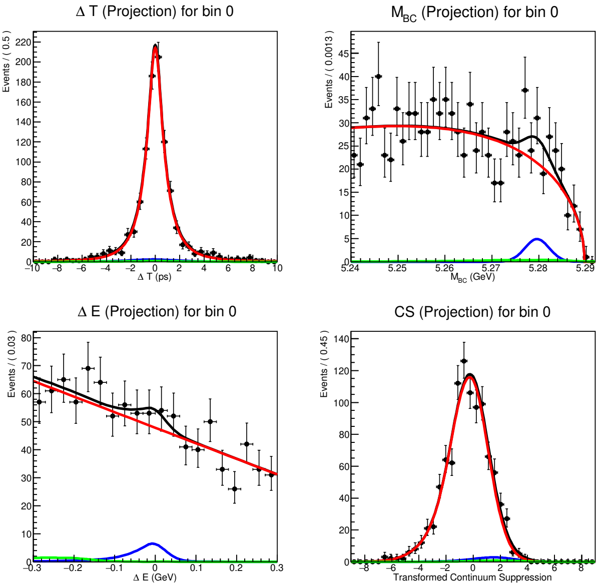
$$C'_{out} = log(\frac{C_{out} - C_{out_{min}}}{C_{out_{max}} - C_{out}}) \tag{2}$$

where $C_{out_{max}}$=0.999339 and $C_{out_{min}}$=0.6

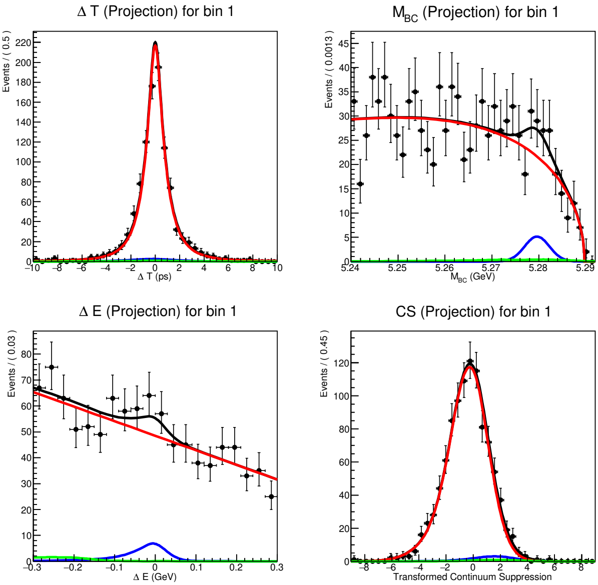




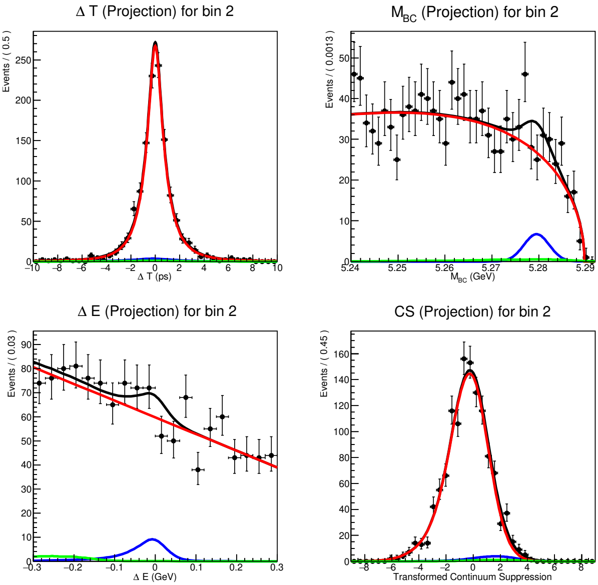
7-bin fit projection



7-bin fit projection

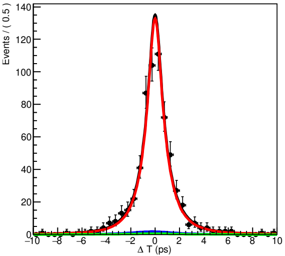


7-bin fit projection

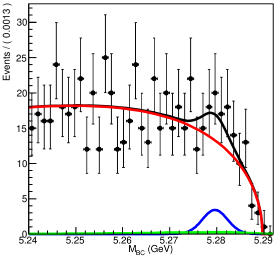


7-bin fit projection

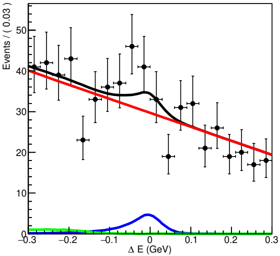
Δ T (Projection) for bin 3



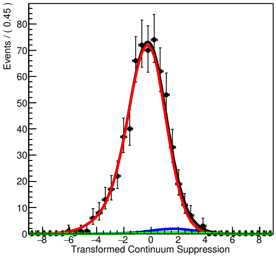
M_{BC} (Projection) for bin 3



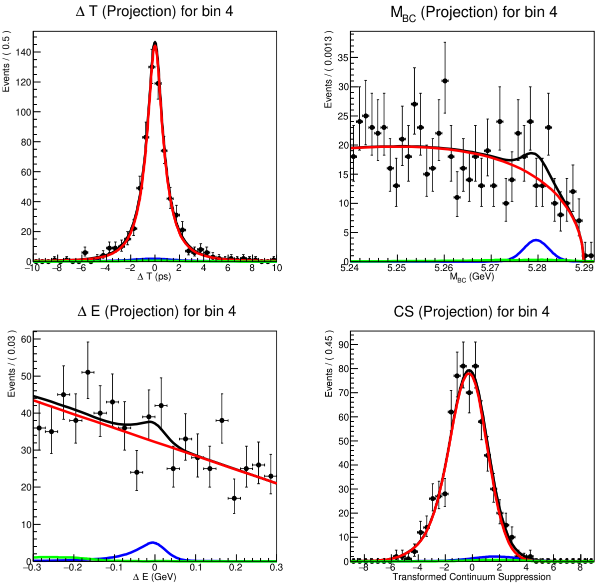
Δ E (Projection) for bin 3



CS (Projection) for bin 3

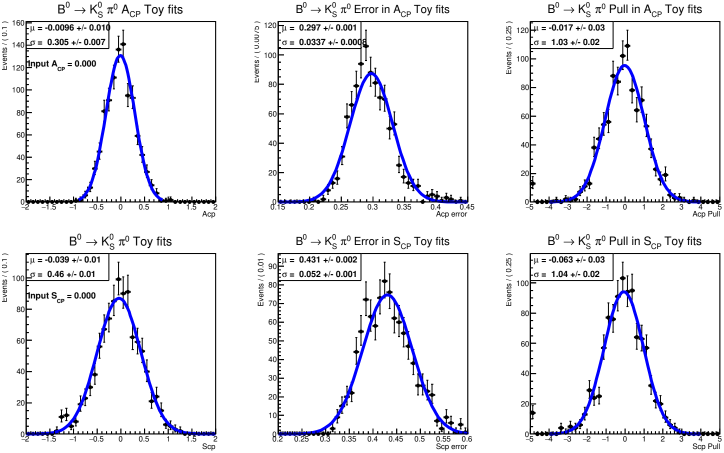


7-bin fit projection



Pure toy test

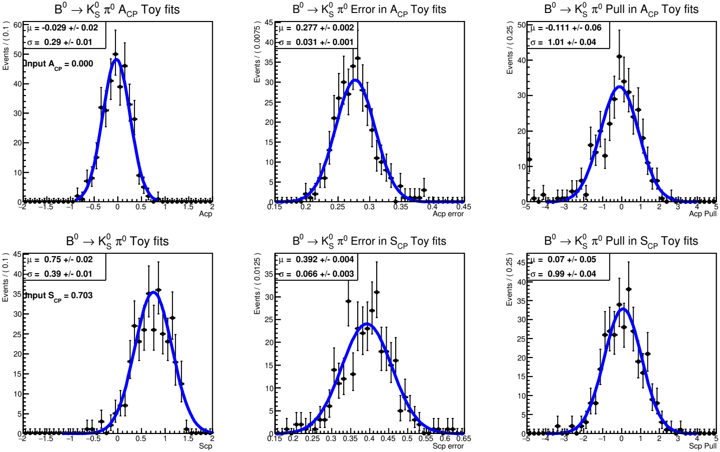
- To validate the fitter, 1000 toy experiments performed
- Signal, continuum and $B\bar{B}$ dataset are generated
- Expected A_{CP}: 0.0 and S_{CP}: 0.0



- There is no significant bias !

GSIM Toy test

- Signal dataset are used from the corresponding MC sample
- Continuum and $B\bar{B}$ dataset are generated using the PDF shape
- $\sin(2\beta) = sin(2\phi_1) = S_{CP} = 0.7032$, where $\beta = 0.39$ rad
- Expected A_{CP}: 0.0 and S_{CP}: 0.7032



- There is no significant bias !

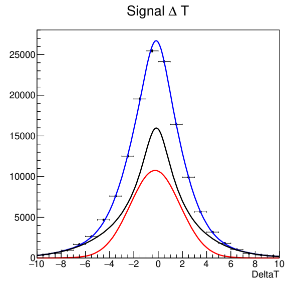
Control mode

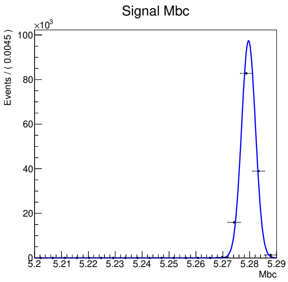
Signal Modeling

- Δt : RooBCPGenDecay PDF convolved with double Gaussian:

  $P_{sig}(\Delta t, q) = \frac{\exp^{-|\Delta t|/\tau_{B^0}}}{4\tau_{B^0}}([1 - q\Delta w + q\mu_i(1-2w)] + [q(1-2w) + \mu_i(1 - q\Delta w)](A_{CP}\cos(\Delta m_d \Delta t) - S_{CP}\cos(\Delta m_d \Delta t)))$

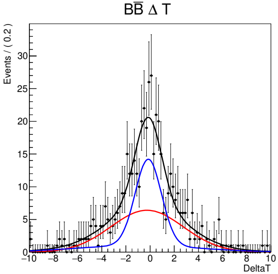
  Core and tail Gaussian
- M_{bc} : Crystal Ball function

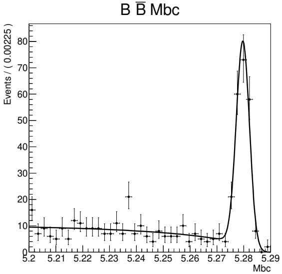




$B\bar{B}$ modeling

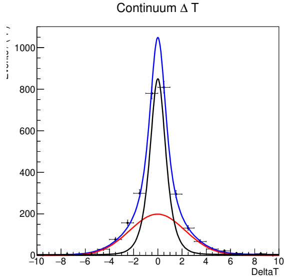
- Peaking compoment peaking at the true B mass ($2-3\%$ of signal events)
- Δt : RooDecay PDF convolved with double Gaussian : $e^{-|t|/\tau}$ Core and tail Gaussian
- M_{bc} : ARGUS + Gaussian function

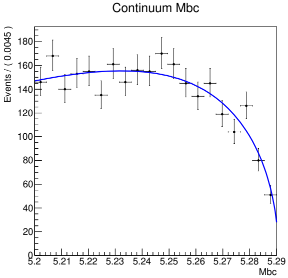




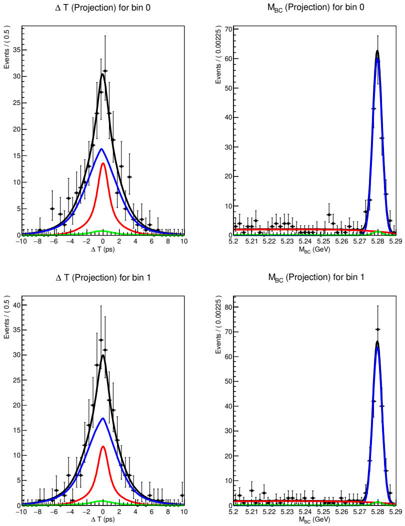
$q\bar{q}$ modeling

- Δt : RooDecay PDF convolved with double Gaussian : $e^{-|t|/\tau}$
  Core and tail Gaussian
- M_{bc} : ARGUS function

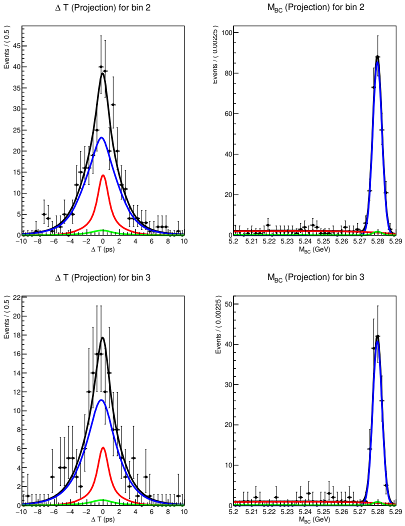




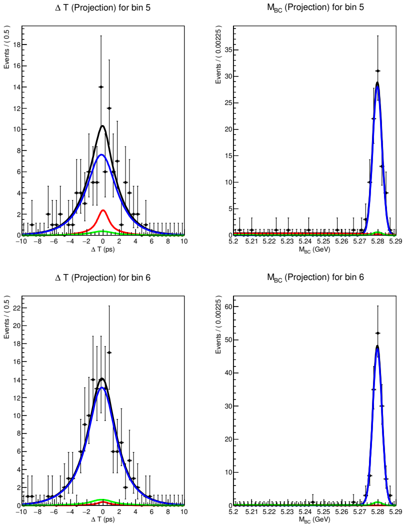
7-bin fit projection



7-bin fit projection



7-bin fit projection



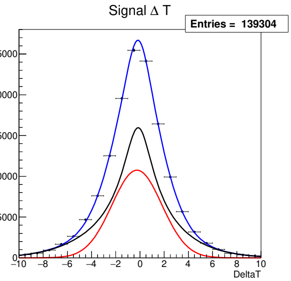
Lifetime fit on Signal MC

- Δt : RooBCPGenDecay PDF convolved with double Gaussian:

$$P_{sig}(\Delta t, q) = \frac{\exp^{-|\Delta t|/\tau_{B^0}}}{4\tau_{B^0}}([1 - q\Delta w + q\mu_i(1-2w)] + [q(1-2w) + \mu_i(1 - q\Delta w)](A_{CP}\cos(\Delta m_d \Delta t) - S_{CP}\cos(\Delta m_d \Delta t)))$$

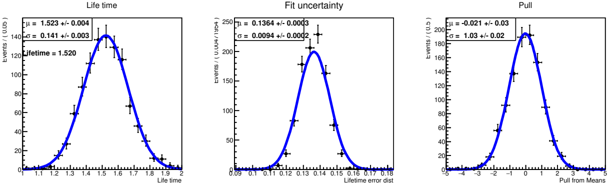
Core and tail Gaussian

$B^0 \to J/\psi K_S^0$



Lifetime (ps)	1.521 ± 0.011	1.52

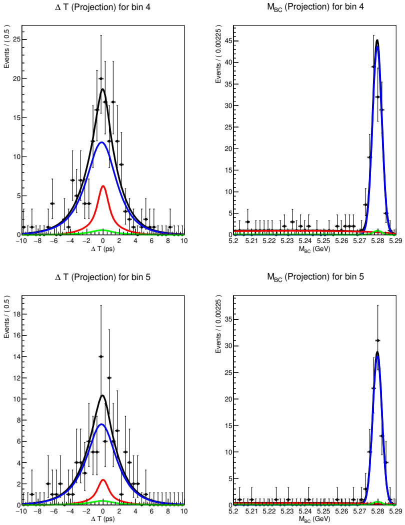
GSIM toy



Parameter	Fitted value	Expected value
Signal Yield	1045 ± 33	1044
Background Yield	275 ± 18	275
Lifetime (ps)	1.523 ± 0.136	1.52

7-bin fit projection

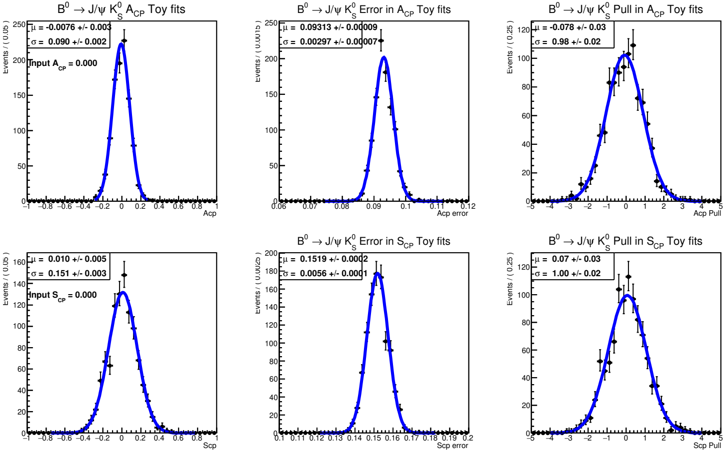
Example plot of few bin



- Rest of the bin fit projection shown in backup slide

Pure toy test

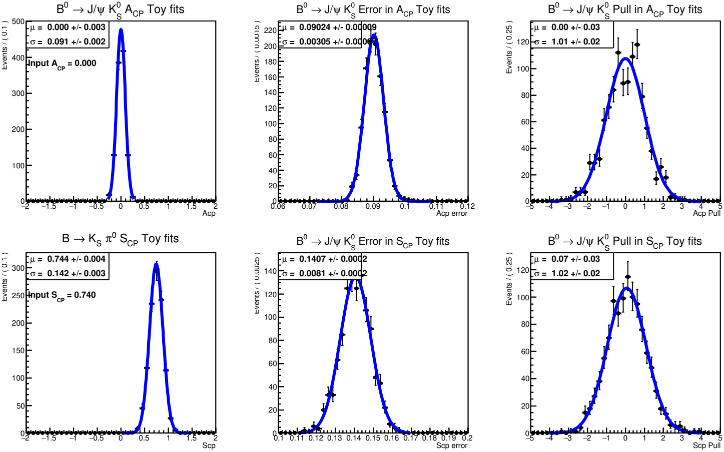
- To validate the fitter, 1000 toy experiments performed
- Signal, continuum and $B\bar{B}$ dataset are generated using the shape
- Expected A_{CP}: 0.0 and S_{CP}: 0.0



- There is no significant bias !

GSIM Toy test

- Signal dataset are used from the corresponding MC sample
- Continuum and $B\bar{B}$ dataset are generated using the PDF shape
- Expected A_{CP}: 0.0 and S_{CP}: 0.74



- There is no significant bias !

Toy results

- Expected signal yield= 1044 (200 fb^{-1})

Pure toy

Parameter	Fitted value	Expected value
Signal Yield	1043 ± 33	1044
Continuum Yield	275 ± 18	275
A_{CP}	-0.007 ± 0.093	0.0
S_{CP}	0.010 ± 0.151	0.0

GSIM toy

Parameter	Fitted value	Expected value
Signal Yield	1044 ± 33	1044
Continuum Yield	274 ± 18	275
A_{CP}	0.0 ± 0.09	0.0
S_{CP}	0.744 ± 0.140	0.74

- $S_{CP} = 0.749 \pm 0.055$ ($500 fb^{-1}$) BELLE2-NOTE-PH-202.

cosHelicityAngleMomentum selection

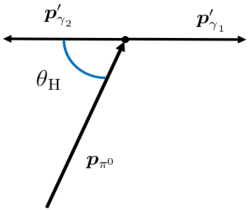
cosHelicityAngleMomentum

If the given particle has two daughters: cosine of the angle between the line defined by the momentum difference of the two daughters in the frame of the given particle (mother) and the momentum of the given particle in the lab frame.

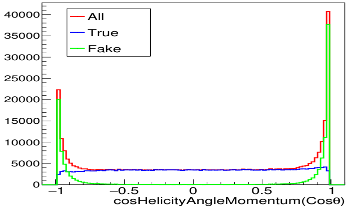
If the given particle has three daughters: cosine of the angle between the normal vector of the plane defined by the momenta of the three daughters in the frame of the given particle (mother) and the momentum of the given particle in the lab frame.

Otherwise, it returns 0.

(a) Defination



(b) Helicity Angle (θ_H)



(c) cosHelicityAngleMomentum